Trade-off between Energy Efficiency and Spectral Efficiency in the Uplink of a Linear Cellular System with Uniformly Distributed User Terminals

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Abstract—In this paper, we propose a tight closed-form approximation of the Energy Efficiency vs. Spectral Efficiency (EE-SE) trade-off for the uplink of a linear cellular communication system with base station cooperation and uniformly distributed user terminals. We utilize the doubly-regular property of the channel to obtain a closed form approximation using the Marchenko Pasture law. We demonstrate the accuracy of our expression by comparing it with Monte-Carlo simulation and the EE-SE trade-off expression based on low-power approximation. Results show the great tightness of our expression with Monte-Carlo simulation. We utilize our closed form expression for assessing the EE performance of cooperation for both theoretical and realistic power models. The theoretical power model includes only the transmit power, whereas the realistic power model incorporates the backhaul and signal processing power in addition to the transmit power. Results indicate that for both power models, increasing the number of antennas leads to an improvement in EE performance, whereas, increasing the number of cooperating BSs results in a loss in EE when considering the realistic power model.

I. INTRODUCTION

In the past, communication network evolution has mainly been driven by spectral efficiency (SE) improvement. In recent years, the reduction in network energy consumption has become of great importance for network operators. So has the importance of the energy efficiency (EE) as a metric for network performance evaluation.

The SE is the traditional metric for measuring the efficiency of communication systems. It measures how efficiently a limited frequency resource (spectrum) is utilized, however, it fails to give any insight on how efficiently energy is utilized. A new metric that provides this insight, i.e. the bits-per-Joule (bits/J), was introduced in [1]. Then, the bits/J capacity of an energy limited wireless network was defined in [2] as the maximum amount of bits that can be delivered by the network per Joule it consumed to do so.

Research work on EE was initially motivated by limited power applications [1] such as underwater acoustic telemetry, sensor networks and home networks. Since most of these systems are operated on batteries, EE is a paramount factor for designing such networks. The global trend towards energy consumption reduction has led to the extension of the EE concept to unlimited power applications, e.g. devices with constant power supply such as base station (BS) and fixed relay terminal in cellular networks. Moreover, the available spectrum resource needs to be efficiently used for the transmission of information bits and, consequently, the SE also needs to be taken into account in the design of communication networks. However, the two objectives of minimizing the energy consumed in the network and maximizing the bandwidth efficiency, i.e. SE, are not achievable simultaneously and, hence, this creates the need for a trade-off.

The Shannon’s capacity theorem illustrates that there exists a trade-off between bandwidth, transmit power and the coding strategy implemented to achieve a certain rate $R$, in other words, the trade-off between EE and SE. The low-power approximation technique introduced in [3] has been used to investigate the EE-SE trade-off for single user, multi user [4], single relay networks [5], multiple relay networks [6] and BS cooperation [7]. As far as the power consumption model for the uplink of cellular system is concerned, three main power components can be distinguished: the users transmit power, BSs signal processing power and backhauling power. For instance, a theoretical power model that only takes into account the users transmit power has been utilized in the low-power approximation technique of [3]. Meanwhile in [8], [9], the authors considered the circuit power (signal processing power) in addition to the transmit power in their model for improving the EE of sensor networks, however, they did not consider the spectrum efficiency. Moreover, in [6], the authors considered the EE-SE trade-off of relay networks based on both the circuit and transmit powers but without including the backhauling power.

In this paper, we derive a closed-form approximation (CFA) of the EE-SE trade-off for the uplink of a linear cellular system with uniformly distributed user terminals (UTs) by considering a realistic power model and full BS cooperation. We revisit our previous work in [10], in which we derived a CFA of the EE-SE trade-off based on the Wyner cellular model, and extend it to a more realistic and generic model with uniformly distributed UTs, as it is shown in Fig. 1. Our CFA is based on random matrix theory for limiting eigenvalue distribution of large random matrices and exploits the doubly-regular property of the channel via the Marchenko Pasture law. Our approach has a considerable advantage over the approximation method in [3] and Monte-Carlo simulation. It is more accurate than
the former and over a wider range of SE values. It requires far less computational complexity than the latter and can be used for getting insight about the behavior of the EE-SE trade-off at low or high SE.

In Section II, we introduce the uplink cellular model with uniformly distributed UTs. In Section III, we first derive our CFA of the EE-SE trade-off for the uplink of cellular system with BS cooperation and joint decoding at the central processor by considering that UTs are uniformly distributed within cells, a multi-input multi-output (MIMO) Rayleigh fading channel between each user and the BS, as well as a theoretical power model. We numerically show its high accuracy by comparing it with Monte-Carlo simulation and the approximation method of [3]. We then derived the low-SE approximation of our CFA and numerically compared it with the minimum energy-per-bit-to-noise spectral density ratio of [3]. Next, Section IV presents the realistic power model of [11] for the uplink of cellular system and utilize our CFA for analyzing the impact of the power model, number of cooperating BS and the number of antennas on the EE. We show that for both the theoretical and realistic power models, the EE-SE trade-off for BS cooperation improves as the number of antennas at the nodes increases. In addition, our results indicate that the EE performance is highly dependent on the number of cooperating BSs when the realistic power model is considered. Finally, Section V concludes the paper.

In this paper, we use boldface letters to denote matrices and vectors and refer to the set of complex numbers as \( \mathbb{C} \). Let \( \mathbf{Z} \) be a matrix, then \( \text{tr}(\mathbf{Z}) \) denotes its trace, \( \mathbf{Z}^\dagger \) denotes its complex conjugate, \( \mathbf{Z}^T \) denotes its transpose, \( \mathbf{Z}^\star \) denotes its complex conjugate transpose, \( \text{det}(\mathbf{Z}) \) denotes its determinant, \( \| \mathbf{Z} \| \) denotes its Frobenius norm. In addition, \( \log(\cdot) \) denotes the logarithm to base 2, \( \mathbb{E}[\cdot] \) denotes the expectation, \( \odot \) denotes the Kronecker product, \( \odot \) denotes the Hadamard product and, \( \mathbf{I}_M \) is an identity matrix with size \( M \).

II. SYSTEM MODEL

We consider the uplink of a linear cellular system with \( K \) UTs uniformly distributed in each cell and \( M \) BSs (Fig. 1). Each UT and each BS is equipped with \( t \) and \( r \) antennas respectively. The received signal at the \( n^{th} \) BS is given by

\[
y^n[i] = \sum_{m=1}^{M} \sum_{k=1}^{K} \alpha_k^{nm} \mathbf{H}_{km}^n[i] x_k^n[i] + w^n[i],
\]

where \( i \) is the time index, \( x_k^n[i] \) is the transmitted vector of the \( k^{th} \) UT in the \( m^{th} \) cell, \( \mathbf{H}_{km}^n \) is the MIMO channel matrix between the \( k^{th} \) UT in cell \( m \) and the \( n^{th} \) BS, \( w^n[i] \) is the additive white Gaussian noise at the \( n^{th} \) BS with zero mean and \( \sigma^2 \) variance. In addition, the signal transmitted by the \( k^{th} \) UT must satisfy the following power constraint : \( \text{tr}(\mathbb{E}[x_k^n x_k^n^\dagger]) \leq P_k \). The interference scaling factors \( \alpha_k^{nm} \) for the transmission path between the \( k^{th} \) UT of the \( m^{th} \) cell and the \( n^{th} \) BS are obtained from the modified power-law path loss model given in [12] as

\[
\alpha_k^{nm} = \sqrt{L_0 \left( 1 + \frac{d_{km}^{nm}}{d_0} \right)^\eta},
\]

where \( d_{km}^{nm} \) is the distance between the \( k^{th} \) UT of the \( m^{th} \) cell and the \( n^{th} \) BS, \( \eta \) is the path loss exponent, \( L_0 \) is the power loss at a reference distance \( d_0 \). To simplify notation, we assume that all UTs transmit with equal power, i.e. \( P_k = P \forall \{k = 1, \ldots, K\} \), and that the UTs transmit power is normalized by the noise power such that \( \gamma = P/\sigma^2 \). Thus, the per-cell transmit power normalized by the receiver noise power is represented by \( \tilde{\gamma} \) where \( \tilde{\gamma} = K\gamma \). Omitting the time index \( i \), when the BSs cooperate to receive data from UTs, the overall system model can be illustrated by

\[
y = \tilde{\mathbf{H}} \mathbf{x} + \mathbf{w},
\]

where \( \mathbf{y} = [y^{(1)} \ldots y^{(M)}]^T \) is the joint received signal vector, \( \mathbf{x} = [x^{(1)}_1 \ldots x^{(K)}_1, \ldots, x^{(1)}_M \ldots x^{(K)}_M]^T \) is the transmitted signal vector and \( \mathbf{w} = [w^{(1)}_1 \ldots w^{(M)}_M]^T \) is the joint received noise vector. The aggregate channel matrix can be expressed as:

\[
\tilde{\mathbf{H}} = \mathbf{O}_V \odot \mathbf{H}_V,
\]

where \( \mathbf{H}_V \) is a \( Mr \times KMt \) matrix with all its elements equal to one and \( \mathbf{O}_V \) is a \( M \times KM \) deterministic matrix. As a result of the collocation of the multiple antennas at the UT and BS, \( \mathbf{O}_V = \mathbf{O} \odot \mathbf{J} \), where \( \mathbf{J} \) is a \( r \times t \) matrix with all its elements equal to one and \( \mathbf{O} \) is a \( M \times KM \) deterministic matrix given by

\[
\mathbf{O} = \begin{bmatrix}
\alpha_0 & \alpha_1 & \alpha_2 & \cdots & \alpha_{M-1} \\
\alpha_{-1} & \alpha_0 & \alpha_1 & \cdots & \vdots \\
\alpha_{-2} & \alpha_{-1} & \alpha_0 & \cdots & \alpha_2 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
\alpha_{-M+1} & \alpha_{-M} & \cdots & \alpha_{-2} & \alpha_{-1} & \alpha_0
\end{bmatrix},
\]

with \( \alpha_m = [\alpha_0^m \cdots \alpha_K^m] \) being a \( 1 \times K \) vector, containing the interference scaling coefficients between all users in the \( m^{th} \) cell and a reference BS. Assuming equal power allocation for all users, the optimal per-cell sum capacity when the number of cells tends to infinity is given by [13]

\[
C_{\text{opt}} = \lim_{M \to \infty} \frac{1}{M} \mathbb{E}\left[ \mathcal{J}(\mathbf{x}; \mathbf{y} \tilde{\mathbf{H}}) \right] = r\nu\frac{1}{\gamma/Kt} \mathbb{E}\left[ \ln(\tilde{\gamma}/Kt) \right],
\]
where \( \nu_{\mathbf{Z}}(y) \) is the Shannon transform of the random square Hermitian matrix \( \mathbf{Z} \) such that
\[
\nu_{\mathbf{Z}}(y) \triangleq \mathbb{E}[\log(1 + y\mathbf{Z})] = \int_0^\infty \log(1 + y\lambda)dF_{\mathbf{Z}}(\lambda),
\]
with \( F_{\mathbf{Z}}(\lambda) \) being the cumulative function of the asymptotic eigenvalue distribution of \( \mathbf{Z} \).

III. CFA of the EE-SE Trade-off for Uniformly Distributed UT

In this section, we utilize the doubly-regular property of our channel model to obtain the EE-SE trade-off. According to [13] a \( M \times KM \) matrix \( \mathbf{Z} \), which is both asymptotically row regular and asymptotically column regular, is referred to as asymptotically doubly-regular. Such a matrix satisfies that
\[
\lim_{M \to \infty} \frac{1}{M} \sum_{i=1}^{M} \mathbf{Z}_{i,j} = \frac{1}{KM} \sum_{j=1}^{KM} \mathbf{Z}_{i,j},
\]
i.e., the column average and row average are the same. For instance, uplink cellular models that meet the doubly-regular condition include: the Wyner model (linear-circular and the planar) [14], Letzepis model [15] and the scenario with uniformly distributed UTs given in (5) when the linear cellular architecture is modelled as circular.

For the Gaussian channel \( \mathbf{H} \) with \( CN(0, \mathbf{I}) \), the asymptotic eigenvalue distribution of \( \frac{1}{M} \mathbf{H}^{\dagger} \mathbf{H} \) converges to the non-random asymptotic eigenvalue distribution of the Marcenko-Pastur law [16] with Shannon transform given by:
\[
\nu_{\frac{1}{M} \mathbf{H}^{\dagger} \mathbf{H}}(y) \underset{M \to \infty}{\rightarrow} \nu_{MP}(y, \beta),
\]
where
\[
\nu_{MP}(y, \beta) = \log \left( \frac{1 + y - \Gamma(y, \beta)}{1 + y + \Gamma(y, \beta)} \right) + \frac{1}{\beta} \log \left( \frac{1 + y + \Gamma(y, \beta)}{1 + y - \Gamma(y, \beta)} \right) - \frac{1}{\beta y} \Gamma(y, \beta),
\]
\[
\Gamma(y, \beta) = \frac{\sqrt{1 + y(1 + \beta^2) - \sqrt{1 + y(1 - \beta^2)^2}}}{\beta},
\]
with \( \beta = \frac{Kt}{r} \) representing the ratio of the horizontal dimension to the vertical dimension of the matrix \( \mathbf{H} \). From [17], if \( \frac{1}{M} \mathbf{H}^{\dagger} \mathbf{H} \) is doubly-regular, the Marcenko-Pastur law and the Shannon transform of the limiting eigenvalue distribution of \( \frac{1}{M} \mathbf{H}^{\dagger} \mathbf{H} \) is given in [12] as
\[
\nu_{\frac{1}{M} \mathbf{H}^{\dagger} \mathbf{H}} \left( \frac{\beta}{Kt} \right) \simeq \nu_{MP} \left( q(\Omega) \frac{\gamma}{\beta} \right),
\]
where \( q(\Omega) = \| \Omega \|^2 / (KM^2) \). The per-cell sum-rate can thus be expressed as
\[
C_P = Kt \log \left( 1 + y - \Gamma(y, \beta) \right) + r \log \left( 1 + y + \Gamma(y, \beta) \right) - \frac{r}{y} \Gamma(y, \beta),
\]
where \( y = q(\Omega) \frac{\gamma}{\beta} \). We have proved in [10] that the per-cell sum-rate \( C_P \) in bit/s can be re-expressed as
\[
C_P = \frac{W}{2 \ln 2} \left( C_r + C_t \right),
\]
where \( C_r \) and \( C_t \) are given in [10], and \( W \) is the bandwidth.

By following our same reasoning as in [10] and replacing \( \gamma_0 \) in [10] with \( y = q(\Omega) \frac{\gamma}{\beta} \), where the cell signal to noise ratio is given by \( \tilde{\gamma} = \frac{R\bar{E}_b}{N_0^r W} \), we can approximate the EE-SE trade-off as follows
\[
\frac{E_b}{N_0} \approx \frac{\beta W \left( 1 + \frac{1}{\bar{W}_0(r, C_t, C_r)} \right) \left( 1 + \frac{1}{\bar{W}_0(f, r, C, \bar{C}_r)} \right) - 1}{2M Rq(\Omega)(1 + \beta)},
\]
where \( R \leq C_P \) is the achievable rate, \( C_t \) and \( C_r \) are functions of \( R \), \( E_b \) is the energy-per-bit and \( N_0 \) is the noise spectral density. The energy efficiency \( C_j \) which is equal to \( \frac{1}{\bar{W}_0} \) can equivalently be expressed as
\[
C_j \approx \frac{2M Rq(\Omega)(1 + \beta)}{\beta N_0 W \left( 1 + \frac{1}{\bar{W}_0(f, r, C, \bar{C}_r)} \right) \left( 1 + \frac{1}{\bar{W}_0(f, r, C, \bar{C}_r)} \right) - 1}.
\]

A. Low-SE Approximation of EE-SE Trade-off

In order to analyze the EE of BS cooperation in the uplink of cellular system at the low-SE, we derive a simplified version of (14) and compare it with the result from the approximation of [3]. In the case that \( \beta = 1 \), it can easily proved that (14) simplifies as
\[
\frac{E_b}{N_0} \approx \frac{W \left( 1 + \left( 1 + \left( W_0 \left( 2 - 2\left( \frac{2}{r} \right)^{\frac{1}{2}} e^{-\frac{x}{2}} \right) \right) \right) - 1 \right)^2}{4M Rq(\Omega)}. \tag{16}
\]
Then, by assuming that the spectral efficiency, \( C = \frac{R}{W} \approx 0 \) in (16) and using a similar approach as in [11], we obtain \( \frac{E_b}{N_0 \min, c} \) as
\[
\frac{E_b}{N_0 \min, c} \geq \frac{\ln(2)}{q(\Omega) M r}, \tag{17}
\]
which is the minimum achievable energy per bit. Based on the low-power regime approximation [3], \( \frac{E_b}{N_0 \min} \) is given by
\[
\frac{E_b}{N_0 \min} \geq \frac{KM t \ln(2)}{\mathbb{E} \left( \text{tr} \left( \tilde{\mathbf{H}}^{\dagger} \tilde{\mathbf{H}} \right) \right)}. \tag{18}
\]

B. Numerical Results

We fix the number of cooperating BSs to \( M = 10 \), number of UTs per cell to \( K = 50 \) and \( q^2 = N_0 W \) unless otherwise stated. The variance profile is obtained from (2) while other parameters used in our evaluation are listed in Table I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_{sp} )</td>
<td>58W</td>
</tr>
<tr>
<td>( e_c )</td>
<td>0.29</td>
</tr>
<tr>
<td>( c_{ba} )</td>
<td>1.4</td>
</tr>
<tr>
<td>( C_{ba} )</td>
<td>100Mbit/s</td>
</tr>
<tr>
<td>( p_0 )</td>
<td>50W</td>
</tr>
<tr>
<td>( W )</td>
<td>5MHz</td>
</tr>
<tr>
<td>( N_0 )</td>
<td>-169dBm/Hz</td>
</tr>
<tr>
<td>( L_0 )</td>
<td>34.5dB</td>
</tr>
<tr>
<td>( \eta )</td>
<td>3.5</td>
</tr>
<tr>
<td>( \delta_0 )</td>
<td>1</td>
</tr>
</tbody>
</table>
Figure 2 depicts the trade-off between EE and SE for various antenna combinations when using the theoretical power model. It could be observed that our CFA results closely match those obtained through Monte-Carlo simulation, whereas the low-power approximation approach of [3] is only accurate in the low-SE regime. Increasing the number of antennas at the UT or BS node results in an increase in the EE and SE of the system since the slope of the trade-off curve becomes steeper in this case. In Fig. 3, we show the great tightness between our low-SE approximation of the EE-SE trade-off in (17) with $E_b/N_0_{min}$ of [3] given in (18) for various number of antennas at the BS. Results in Figs. 2 and 3 emphasize that our CFA in (14) is as accurate as the previous existing approximation of [3] in the low-SE regime and far more accurate for other SE regimes.

In Fig. 4, we utilized our CFA to obtain the EE of BS cooperation based on the theoretical power model. Increasing the number of antennas at the nodes results in an increase in the EE as a result of the diversity gain. However, increasing the number of cooperating BS beyond three does not produce any significant improvement in terms of the EE. This is due to the fact that the first tier interference signal dominates other tiers, hence little gain is achieved in terms EE when $N_c > 3$.

IV. EE-SE TRADE-OFF FOR THE LINEAR CELLULAR ARCHITECTURE WITH A REALISTIC POWER MODEL

As an application for our CFAs in (14) and (15), we analyze the EE of BS cooperation for the uplink of cellular system when a realistic power model is considered. We have derived our EE-SE trade-off expression in Section III by considering only the transmit power. Whereas in this Section, we incorporate in our closed-form the realistic power model of [18] for BS cooperation, which defines the average consumed power $P_M$ as

$$P_M = aP_{sp} + P_{bh} + KP_{ms},$$

(19)

instead of $P_M = KP_{ms}$ in the theoretical model. This realistic model incorporates the per-cell signal processing power $P_{sp}$, the backhaul power $P_{bh}$ in addition to the total power consumed by the UT $P_{ms}$, which is an underestimation of $P_{ms}$. The effects of cooling and battery backup are also taken into account via the factors $c_c$ and $c_{bu}$ respectively, such that $a = r(1 + c_c)(1 + c_{bu})$. The backhaul power $P_{bh}$ is given as $P_{bh} = C_{bh}^c P_b$ Watts, where $C_{bh}^c$ is the capacity of the backhaul link with dissipation power $p_b$. Note that $P_{sp}$ is given as

$$P_{sp} = p_{sp}(0.8 + 0.1N_c + 0.1N_c^2),$$

(20)

where $p_{sp}$ is the base line signal processing power per-BS and $N_c$ is the number of cooperating BS. The effective energy efficiency $C_{jef}$ of the uplink cellular system with BS cooperation and joint user detection is given by

$$C_{jef} = \frac{C_j KP}{P_M},$$

(21)

where $C_j$ is expressed in (15).

Figure 5 shows the effective EE-SE trade-off which has been obtained via our CFA in (15) when the realistic power model of [18] is considered. In contrast with the result in Fig. 4 for the theoretical model, it can be observed in Fig. 5 that increasing $N_c$ leads to a reduction in EE, which is due to an increase in backhaul and signal processing consumed powers without any significant increase in the per-cell sum-rate. However, increasing the number of antennas at the nodes results in an increase of EE as in Fig. 4. Thus, the EE can be improved via antenna diversity but not via macro diversity when a realistic power model is considered.
Fig. 4. EE for various number of cooperating BS and antenna configuration \((r \times t)\) in the linear cellular system based on the theoretical power model with transmit power of 27dBm.

Fig. 5. EE for various number of cooperating BS and antenna configuration \((r \times t)\) in the linear cellular system based on the realistic power model with transmit power of 27dBm.

V. CONCLUSION

In this paper, we have derived a tight closed-form approximation of the EE-SE trade-off for the uplink of cellular system with BS cooperation by considering a linear cellular architecture with uniformly distributed UTs, MIMO Rayleigh fading channel and a theoretical power model. We then presented the realistic power model of [18] for the uplink of cellular system and incorporated it into our EE-SE trade-off expression.

The findings of this paper are summarised as follows: Our closed-form approximation for the EE-SE trade-off tightly match with the Monte-Carlo simulation and, thus, is very accurate. When only the transmit power is considered, i.e. theoretical power model, increasing the number of antennas at the UT or BS nodes results in an increase in both the EE and SE. When the signal processing and the backhaul powers are introduced, it was revealed that increasing the number of cooperating BS results in a reduction in EE performance.

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