Codebook-Based Precoding and Power Allocation for Nonregenerative Dual Hop Relay Systems

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***Abstract*—In this paper, we consider the precoding problem in the nonregenerative dual hop multi-input multi-output (MIMO) relay system. We propose a joint beamforming and power allocation method for a limited feedback system where a codebook-based beamformer is considered for each hop. The destination node selects the optimal beamforming codeword by relying on full channel knowledge of both source-relay and relay-destination channels for each hop from the code book and also compute the optimal power allocation coefficients for each substream. The index of the codewords and a quantized version of the power allocation coefficients are conveyed back to the source and relay nodes. The source and relay use the selected beamforming matrices and power allocation values to precode the data stream. In order to demonstrate the effectiveness of the proposed method, the performance of our joint beamforming and power allocation method is compared with previous codebook-based algorithms.**

I. INTRODUCTION

Relay networks were first introduced in the seventies [1]–[3]. Recently, cooperative relay communication has attracted considerable attention [4]-[12] and is supported by recent standardization activities like the ones in IEEE802.16 j and m Task Groups [4]. This technique is essential for providing reliable transmission, high throughput and broad coverage in cellular systems as well as ad hoc networks [5]. Deployment of multi-array antennas at the source, relay and destination nodes and application of multi-input multi-output (MIMO) techniques such as beamforming and space time codes have significantly boosted the performance of cooperative communication [6]. In [1], the capacity of a full duplex single antenna relay system has been studied and a cut set bound is introduced as an upper bound on the capacity of the full duplex relay channel. Then, the upper and lower bounds of the MIMO relay channel capacity of the full duplex system have been derived in [7]. In [8] and [9], tighter bounds on the capacity have been obtained and the gap between the achievable rate and the capacity has been quantified in [10] for the MIMO case. As far as precoding and power allocation for relay networks are concerned, numerous schemes have been proposed in the literature [11]-[14]. For instance, optimal power allocation methods under total power constraint at the relay have been proposed in [11] and [12]. Whereas, joint source and relay precoder designs with different optimization criteria have been proposed in [13] and [14] by assuming that channel state information (CSI) of both channels is available at the source node. The destination node could obtain CSI by using pilot symbols which are sent on the channel. Moreover CSI is usually conveyed back from the destination to the relay or/and source node via a feedback link. However, feedbacking CSI may impose a huge overhead on the system and may prove to be very difficult to implement in a practical system because of the fast changes in the quality of mobile channels, especially in fast fading conditions. In order to mitigate this problem, limited feedback precoding techniques have been introduced in point to point MIMO communication for reducing the feedbacking overhead by deploying partial or imperfect channel knowledge precoding techniques. Deployment of limited feedback techniques has been extended to relay networks in [15]-[19]. A novel limited feedback precoding method is codebook-based precoding, which was first introduced in [20] and has recently been utilized in relay networks, [18] and [19]. However, the power is not optimally allocated in these works and the available power at the source and relay nodes is uniformly divided between sub channels of different strengths, which consequently result in a waste of power. In this paper, we propose a limited feedback precoding and power allocation for dual hop relay networks. Unlike the previous works, codebook-based beamforming and power allocation are jointly integrated in our method. The destination node selects the optimal beamforming matrix from the codebook and compute the optimal power allocation for each hop by relying on the CSI of both source-relay and relay-destination channels. Then, the index of the selected beamforming matrices and a quantized version of the computed power allocation coefficients are conveyed back to the relay node and the source node (via the relay) through a limited feedback link. Thus, on the one hand, the overhead of the feedback link is dramatically reduced by using this precoding technique. On the other hand, the performance of the system in terms of spectral efficiency is considerably improved in comparison with previous codebook-based methods that use uniform power allocation. This spectral efficiency gain is obtained with a tolerable increase in the feedback link rate, which is needed for transmitting the quantized power allocation values over the two feedback links.

This paper is organized as follows. In, Section II, the system model of the nonregenerative half duplex relay system is introduced and its aggregate mutual information is formulated. In Section III, our novel codebook-based precoding method is presented, a selection criterion for codebook-based beamforming is obtained and an optimal power allocation method is derived. Numerical results of Section IV indicate that our novel precoding technique outperforms the codebook-based precoding method with uniform power allocation and the system with no precoding at the source node. Conclusions are drawn in section V.

Notation

We use uppercase boldface to denote matrices, lowercase boldface to denote vectors, ^{h} to denote the conjugate transposition, $^{-1}$ to denote the matrix inversion, $\mathbf{I}_M$ to denote the $M \times M$ identity matrix, $\|.\|_F$ to denote the matrix Frobenius norm, $\mathrm{tr}(.)$ to denote the trace of a matrix, $\det(.)$ to denote the determinant of a matrix, $\mathbb{C}^p$ to denote p-dimensional complex vector space, $\mathcal{CN}(\boldsymbol{\mu}, \mathbf{R})$ to denote the complex normal distribution with independent real and imaginary parts with mean vector equal to $\boldsymbol{\mu}$ and covariance matrix equal to $\mathbf{R}$.

II. SYSTEM MODEL

We consider a relay system composed of three nodes, as depicted in Fig. 1, where a source node, a relay node and a destination node are equipped with p, q and l antennas, respectively. The typical half duplex transmission scheme is considered, where in the first phase the source node broadcasts its data to the relay node. The baseband discrete time equivalent received signal at the relay node can be written as

$$\mathbf{y}_\mathrm{r} = \mathbf{H}_\mathrm{sr}\mathbf{F}_\mathrm{s}\mathbf{s} + \mathbf{n}_\mathrm{r}, \tag{1}$$

where $\mathbf{s}$ is the transmit data vector which consists of N independent data symbols. Moreover, each of them is selected from a constellation $\mathcal{W}$ and has unit power, such that the covariance of the transmit vector is an identity matrix, $\mathbf{R}_\mathrm{s} = \mathbf{I}_N$. At the source, the transmit vector is multiplied by the precoder matrix $\mathbf{F}_\mathrm{s}$, which is the product of a power allocation matrix with a beamforming matrix, i.e. $\mathbf{F}_\mathrm{s} = \mathbf{B}_\mathrm{s}\boldsymbol{\Phi}_\mathrm{s}$, where $\boldsymbol{\Phi}_\mathrm{s}$ is a N by N diagonal matrix that contains the power allocation coefficients for each sub stream and $\mathbf{B}_\mathrm{s}$ is a p by N unitary beamforming matrix which multiplexes the substreams into the desired configuration. A slow time varying memory-less MIMO Rayleigh fading channel is assumed between the source and the relay nodes such that $\mathbf{H}_\mathrm{sr} \in \mathbb{C}^{q \times p}$ has independent identically distributed (i.i.d.) elements, which are distributed according to $\mathcal{CN}(0, 1)$. In addition, $\mathbf{n}_\mathrm{r}$ is the additive Gaussian noise at the relay node with distribution $\mathcal{CN}(0, \mathbf{R}_{\mathbf{n}_\mathrm{r}})$. The average transmit power at the source node is limited by P_1 such that the power constraint for the source could be written as $\mathrm{P_s} : \mathrm{tr}(\mathrm{E}[\mathbf{F}_\mathrm{s}\mathbf{s}\mathbf{s}^h\mathbf{F}_\mathrm{s}^h]) \leq P_1$. The relay node precodes the received symbol $\mathbf{y}_\mathrm{r}$ by multiplying it by the relay precoder matrix $\mathbf{F}_\mathrm{r}$. Hence, the transmit vector at the relay is $\mathbf{x}_\mathrm{r} = \mathbf{F}_\mathrm{r}\mathbf{y}_\mathrm{r}$, where $\mathbf{F}_\mathrm{r}$ is the product of the relay power allocation matrix $\boldsymbol{\Phi}_\mathrm{r}$ with the relay beamformer $\mathbf{B}_\mathrm{r}$ and a pre-rotating matrix. In the second phase, the relay node transmit the precoded streams to the destination node. Hence, the received signal at the destination can be given by

$$\mathbf{y}_\mathrm{d} = \mathbf{H}_\mathrm{rd}\mathbf{x}_\mathrm{r} + \mathbf{n}_\mathrm{d} = \underbrace{\mathbf{H}_\mathrm{rd}\mathbf{F}_\mathrm{r}\mathbf{H}_\mathrm{sr}\mathbf{F}_\mathrm{s}}_{\mathbf{H}}\mathbf{s} + \underbrace{\mathbf{H}_\mathrm{rd}\mathbf{F}_\mathrm{r}\mathbf{n}_\mathrm{r} + \mathbf{n}_\mathrm{d}}_{\mathbf{n}}, \tag{2}$$

where $\mathbf{H}_\mathrm{rd} \in \mathbb{C}^{l \times q}$ is the relay-destination channel matrix with i.i.d. elements that are distributed according to $\mathcal{CN}(0, 1)$ and $\mathbf{n}_\mathrm{d}$ is the additive Gaussian noise vector at the destination node with distribution $\mathcal{CN}(0, \mathbf{R}_{\mathbf{n}_\mathrm{d}})$. The power constraint for relay node limits the transmit power of the relay as $\mathrm{P_r} : \mathrm{tr}(\mathrm{E}[\mathbf{F}_\mathrm{r}\mathbf{y}_\mathrm{r}\mathbf{y}_\mathrm{r}^h\mathbf{F}_\mathrm{r}^h]) \leq P_2$. Note that the number of substreams that are multiplexed together should be equal or less to the rank of each subchannels such that $N \leq R = \min(p, q, l)$. The system model of the relay channel can be summarized as $\mathbf{y}_\mathrm{d} = \mathbf{H}\mathbf{s} + \mathbf{n}$, where $\mathbf{H} = \mathbf{H}_\mathrm{rd}\mathbf{F}_\mathrm{r}\mathbf{H}_\mathrm{sr}\mathbf{F}_\mathrm{s}$ is the equivalent channel matrix and $\mathbf{n}$ is a non-white Gaussian noise vector equal to $\mathbf{H}_\mathrm{rd}\mathbf{F}_\mathrm{r}\mathbf{n}_\mathrm{r} + \mathbf{n}_\mathrm{d}$. Thus, the aggregate mutual information of the half duplex relay system is given by [4]

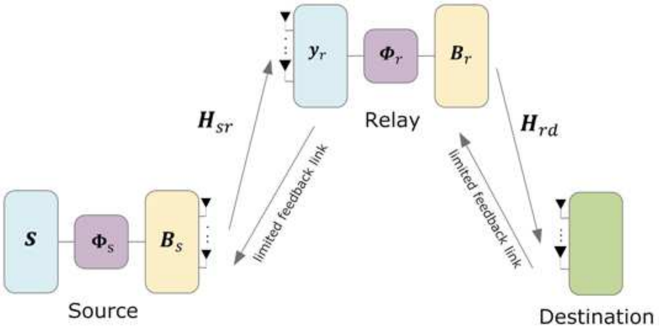

Fig. 1. Block diagram of a dual hop relay system with limited feedback links

$$I(\mathbf{y}_\mathrm{d}, \mathbf{s}) = (1/2)\log(\det(\mathbf{I}_l + \mathbf{H}\mathbf{R}_\mathrm{s}\mathbf{H}^h\mathbf{R}_\mathbf{n}^{-1})), \tag{3}$$

where $\mathbf{R}_\mathrm{s}$ is the transmit covariance matrix, which is assumed to be an identity matrix, and $\mathbf{R}_\mathbf{n}$ is the noise covariance matrix, which can be expressed as follows

$$\begin{aligned} \mathbf{R}_\mathbf{n} &= \mathrm{E}[\mathbf{n}\mathbf{n}^h] = \mathrm{E}[(\mathbf{H}_\mathrm{rd}\mathbf{F}_\mathrm{r}\mathbf{n}_\mathrm{r} + \mathbf{n}_\mathrm{d})(\mathbf{H}_\mathrm{rd}\mathbf{F}_\mathrm{r}\mathbf{n}_\mathrm{r} + \mathbf{n}_\mathrm{d})^h] \\ &= \mathbf{R}_{\mathbf{n}_\mathrm{d}} + \mathbf{H}_\mathrm{rd}\mathbf{F}_\mathrm{r}\mathbf{R}_{\mathbf{n}_\mathrm{r}}\mathbf{F}_\mathrm{r}^h\mathbf{H}_\mathrm{rd}^h \end{aligned}. \tag{4}$$

Note that the factor (1/2) accounts for the two phase transmission in (3). By assuming white additive noise at both relay and destination node and by substituting $\mathbf{R}_\mathbf{n}$ in (4), the mutual information between transmit symbol $\mathbf{s}$ and receive symbol $\mathbf{y}_\mathrm{d}$ would be

$$\begin{aligned} I(\mathbf{y}_\mathrm{d}, \mathbf{s}) &= (1/2)\log(\det(\mathbf{I}_l + \mathbf{H}_\mathrm{rd}\mathbf{F}_\mathrm{r}\mathbf{H}_\mathrm{sr}\mathbf{F}_\mathrm{s}\mathbf{F}_\mathrm{s}^h\mathbf{H}_\mathrm{sr}^h\mathbf{F}_\mathrm{r}^h\mathbf{H}_\mathrm{rd}^h \\ &\quad \times(\sigma_{\mathbf{n}_\mathrm{d}}^2\mathbf{I}_l + \sigma_{\mathbf{n}_\mathrm{r}}^2\mathbf{H}_\mathrm{rd}\mathbf{F}_\mathrm{r}\mathbf{F}_\mathrm{r}^h\mathbf{H}_\mathrm{rd}^h)^{-1})) \end{aligned}. \tag{5}$$

The performance of the system could be improved by optimally designing the beamforming and power allocation matrices of the source and relay nodes. We consider the aggregate mutual information as the performance metric and aim at maximizing it under two power constraints $\mathrm{P_s}$ and $\mathrm{P_r}$ at the source and relay nodes, respectively.

III. CODEBOOK-BASED PRECODING AND POWER ALLOCATION

In this section we propose a limited feedback precoder design which optimizes the beamforming and power allocation matrices at both the source and relay nodes. We assume that the relay node knows $\mathbf{H}_\mathrm{sr}$ and that the destination node knows both $\mathbf{H}_\mathrm{sr}$ and $\mathbf{H}_\mathrm{rd}$. First, we address the beamforming selection criterion in parts A and B and discuss how to optimally allocate the power later in part C.

A. Optimal design for beamformer selection criterion

The channel matrices can be rewritten as $\mathbf{H}_{\text{sr}} = \mathbf{U}_{\text{sr}}\mathbf{\Sigma}_{\text{sr}}\mathbf{V}_{\text{sr}}^{h}$, $\mathbf{H}_{\text{rd}} = \mathbf{U}_{\text{rd}}\mathbf{\Sigma}_{\text{rd}}\mathbf{V}_{\text{rd}}^{h}$ by using singular value decomposition, where $\mathbf{V}_{\text{sr}} \in \mathbb{C}^{p\times p}$, $\mathbf{V}_{\text{rd}} \in \mathbb{C}^{q\times q}$, $\mathbf{U}_{\text{sr}} \in \mathbb{C}^{q\times q}$ and $\mathbf{U}_{\text{rd}} \in \mathbb{C}^{l\times l}$ are unitary right and left singular matrices of source-relay and relay-destination channels. In addition, $\mathbf{\Sigma}_{\text{sr}} \in \mathbb{C}^{q\times p}$ and $\mathbf{\Sigma}_{\text{rd}} \in \mathbb{C}^{l\times q}$ are diagonal matrices with diagonal elements $\{\sigma_{\text{sr},i}\}_{i=1}^{R_1}$ and $\{\sigma_{\text{rd},i}\}_{i=1}^{R_2}$, which are ordered singular values of the channel matrices. We design two independent codebooks for the source and relay nodes. The codebooks, which are a finite set of codewords, are designed offline and are known by each node

$$\begin{aligned} \mathcal{B}_{\text{s}} &= \{\mathbf{B}_{\text{s},1}, \mathbf{B}_{\text{s},2}, , \mathbf{B}_{\text{s},M}\} \\ \mathcal{B}_{\text{r}} &= \{\mathbf{B}_{\text{r},1}, \mathbf{B}_{\text{r},2}, , \mathbf{B}_{\text{r},M}\} \end{aligned}. \tag{6}$$

The destination node selects the optimal beamforming matrix from the codebooks for each source-relay and relay-destination channels and transmit back the index of the optimal codeword to the source and relay nodes. In fact, the codeword that best-represents the optimal unquantized beamforming matrix is selected according to an appropriate selection criterion in the codebook-based beamforming. Thus, two problems should be addressed; first, the design of the codebook and, second, the derivation of the selection criterion. The codebook should be designed in a way that the distortion caused by the quantization process is minimized. Design of the optimal codebooks for MIMO systems have been studied in [18] and [20]. It is known that Grassmannian quantizer is the most appropriate technique to design the codebooks [20] for Rayleigh fading channel, since the optimal unquantized beamformer is uniformly distributed on the unit sphere. Here, we address the derivation of the selection criterion for the optimal codeword, i.e. quantized beamformer. When full channel knowledge is available, the optimal precoding matrices are [6]

$$\begin{aligned} \mathbf{F}_{\text{s}} &= \overline{\mathbf{V}}_{\text{sr}}\mathbf{\Phi}_{\text{s}} \\ \mathbf{F}_{\text{r}} &= \overline{\mathbf{V}}_{\text{rd}}\mathbf{\Phi}_{\text{r}}\overline{\mathbf{U}}_{\text{sr}}^{h}, \end{aligned} \tag{7}$$

where $\overline{\mathbf{V}}_{\text{sr}}$ and $\overline{\mathbf{V}}_{\text{rd}}$ are the matrices of the first N columns of the right singular value matrices of the channels and $\overline{\mathbf{U}}_{\text{sr}}$ is the matrix of the first N columns of the left singular value matrix of $\mathbf{H}_{\text{sr}}$. Consequently, the optimal unquantized beamformer for source and relay is $\mathbf{B}_{\text{s}} = \overline{\mathbf{V}}_{\text{sr}}$ and $\mathbf{B}_{\text{r}} = \overline{\mathbf{V}}_{\text{rd}}$. The Receiver selects the optimal quantized beamformer codewords from the codebooks $\mathbf{B}_{\text{s}}$ and $\mathbf{B}_{\text{r}}$ by using the knowledge about the optimal unquantized beamformer. The selection criterion for maximizing the mutual information is given by

$$\{\mathbf{B}_{\text{s}}, \mathbf{B}_{\text{r}}\} = \underset{\mathbf{B}_{\text{s},i}\in\mathcal{B}_{\text{s}}, \mathbf{B}_{\text{r},i}\in\mathcal{B}_{\text{r}}}{\arg\max} I(\mathbf{y}_{\text{d}}, \mathbf{s}). \tag{8}$$

An exhaustive search among all the possible codeword combinations of the two codebooks can be performed for solving this optimization problem. However, the computational complexity of such an algorithm is huge. Consequently, we propose another selection criterion that dramatically reduces the computation by splitting the problem of joint selection of codewords into two independent search processes and maximizing a lower bound on the mutual information.

B. Suboptimal codeword selection criterion

The mutual information could be lower bounded as follows

$$\begin{aligned} I(\mathbf{y}_{\text{d}}, \mathbf{s}) \geq I_1(\mathbf{y}_{\text{d}}, \mathbf{s}) &= (1/2)\log(\det(\mathbf{I} + (\Sigma_{\text{rd}}\mathbf{V}_{\text{rd}}^{h}\mathbf{B}_{\text{r}}\mathbf{\Phi}_{\text{r}}\Sigma_{\text{sr}}\mathbf{V}_{\text{sr}}^{h} \\ &\times \mathbf{B}_{\text{s}}{\mathbf{\Phi}_{\text{s}}}^{2}\mathbf{B}_{\text{s}}\mathbf{V}_{\text{sr}}\Sigma_{\text{sr}}\mathbf{\Phi}_{\text{r}}\mathbf{B}_{\text{r}}^{h}\mathbf{V}_{\text{rd}}\Sigma_{\text{rd}})(\sigma_{\mathbf{n}_{\text{d}}}^{2}\mathbf{I} + \sigma_{\mathbf{n}_{\text{r}}}^{2}\Sigma_{\text{rd}}\mathbf{V}_{\text{rd}}^{h} \cdot \\ &\times \mathbf{B}_{\text{r}}{\mathbf{\Phi}_{\text{r}}}^{2}\mathbf{B}_{\text{r}}^{h}\mathbf{V}_{\text{rd}}\Sigma_{\text{rd}})^{-1})) \end{aligned} \tag{9}$$

By substituting $\mathbf{F}_{\text{s}} = \mathbf{B}_{\text{s}}\mathbf{\Phi}_{\text{s}}$ and $\mathbf{F}_{\text{r}} = \mathbf{B}_{\text{r}}\mathbf{\Phi}_{\text{r}}\overline{\mathbf{U}}_{\text{sr}}^{h}$ in (5) and using the property of unitary matrix $\det(\mathbf{U}\mathbf{A}\mathbf{U}^{h}) \geq \det(\mathbf{A})$ [21], where $\mathbf{U}$ is a unitary matrix. After straightforward simplifications, $I_1(\mathbf{y}_{\text{d}}, \mathbf{s})$ in (9) is further lower bounded by

$$\begin{aligned} I_1(\mathbf{y}_{\text{d}}, \mathbf{s}) \geq I_2(\mathbf{y}_{\text{d}}, \mathbf{s}) &= (1/2)\sum_{i=1}^{N}\log(1 + (\sigma_i^2(\Sigma_{\text{sr}}\mathbf{V}_{\text{sr}}^{h}\mathbf{B}_{\text{s}})\phi_{\text{s},i}^2 \\ &\times \sigma_i^2(\Sigma_{\text{rd}}\mathbf{V}_{\text{rd}}^{h}\mathbf{B}_{\text{r}})\phi_{\text{r},i}^2(\sigma_{\mathbf{n}_{\text{d}}}^2 + \sigma_{\mathbf{n}_{\text{r}}}^2\sigma_i^2(\Sigma_{\text{rd}}\mathbf{V}_{\text{rd}}^{h}\mathbf{B}_{\text{r}})\phi_{\text{r},i}^2)^{-1}) \end{aligned}. \tag{10}$$

Then, $I_2(\mathbf{y}_{\text{d}}, \mathbf{s})$ in (10) can be approximated as

$$\begin{aligned} I_2(\mathbf{y}_{\text{d}}, \mathbf{s}) &\approx (1/2)\sum_{i=1}^{N}\log(\sigma_i^2(\Sigma_{\text{sr}}\mathbf{V}_{\text{sr}}^{h}\mathbf{B}_{\text{s}})\phi_{\text{s},i}^2) \\ &\times \sum_{i=1}^{N}\log\left(\frac{\sigma_i^2(\Sigma_{\text{rd}}\mathbf{V}_{\text{rd}}^{h}\mathbf{B}_{\text{r}})\phi_{\text{r},i}^2}{\sigma_{\mathbf{n}_{\text{d}}}^2 + \sigma_{\mathbf{n}_{\text{r}}}^2\sigma_i^2(\Sigma_{\text{rd}}\mathbf{V}_{\text{rd}}^{h}\mathbf{B}_{\text{r}})\phi_{\text{r}}^2}\right) \end{aligned} \tag{11}$$

for large signa-to-noise ratio (SNR) values. We consider equation (11) as the lower bound for mutual information. This relation can be seen as the product of two terms, where the first term only depends on the source-relay channel and the second one is a function of the relay-destination channel. Hence, this lower bound can be maximized by maximizing each part independently. As both expressions are monotonic functions, the lower bound would be maximized if $\sum_{i=1}^{N}\sigma_i^2(\Sigma_{\text{sr}}\mathbf{V}_{\text{sr}}^{h}\mathbf{B}_{\text{s}})$ and $\sum_{i=1}^{N}\sigma_i^2(\Sigma_{\text{rd}}\mathbf{V}_{\text{rd}}^{h}\mathbf{B}_{\text{r}})$ are maximized. Thus, the selection criterion for each codebook is obtained separately as

$$\mathbf{B}_{\text{s}} = \underset{\mathbf{B}_{\text{s},i}\in\mathcal{B}_{\text{s}}}{\arg\max}\|\Sigma_{\text{sr}}\mathbf{V}_{\text{sr}}^{h}\mathbf{B}_{\text{s},i}\|_F^2 \tag{12}$$

$$\mathbf{B}_{\text{r}} = \underset{\mathbf{B}_{\text{r},i}\in\mathcal{B}_{\text{r}}}{\arg\max}\|\Sigma_{\text{rd}}\mathbf{V}_{\text{rd}}^{h}\mathbf{B}_{\text{r},i}\|_F^2 \tag{13}$$

by considering that $\sum_{i=1}^{N}\sigma_i^2(\mathbf{A}) = \|\mathbf{A}\|_F^2$.

C. Power allocation method

By subsisting the optimal unquantized precoders in the formula (5) and then, by simplifying this expression, the sum rate can be written as

$$\begin{aligned} I(\mathbf{y}_{\text{d}}, \mathbf{s}) &= (1/2)\log(\det(\mathbf{I} + (\Sigma_{\text{rd}}\mathbf{\Phi}_{\text{r}}\Sigma_{\text{sr}}{\mathbf{\Phi}_{\text{s}}}^{2}\Sigma_{\text{sr}}\mathbf{\Phi}_{\text{r}}\Sigma_{\text{rd}}) \\ &\times(\sigma_{\mathbf{n}_{\text{d}}}^2\mathbf{I} + \sigma_{\mathbf{n}_{\text{r}}}^2\Sigma_{\text{rd}}{\mathbf{\Phi}_{\text{r}}}^{2}\Sigma_{\text{rd}})^{-1})) \\ &= (1/2)\sum_{i=1}^{N}\log\left(1 + \frac{\sigma_{\text{sr},i}^2\phi_{\text{s},i}^2\sigma_{\text{rd},i}^2\phi_{\text{r},i}^2}{\sigma_{\mathbf{n}_{\text{d}}}^2 + \sigma_{\mathbf{n}_{\text{r}}}^2\sigma_{\text{rd},i}^2\phi_{\text{r},i}^2}\right) \end{aligned}. \tag{14}$$

In addition, the power constraints simplify as

$$\begin{aligned} \text{P}_{\text{s}} &: \sum_{i=1}^{N}\phi_{\text{s},i}^2 \leq P_1 \\ \text{P}_{\text{r}} &: \sum_{i=1}^{N}\phi_{\text{r},i}^2(\sigma_{\text{sr},i}^2\phi_{\text{s},i}^2 + \sigma_{\mathbf{n}_{\text{r}}}^2) \leq P_2 \end{aligned}. \tag{15}$$

The destination node computes the optimal power allocation matrices $\mathbf{\Phi}_\mathrm{s}$ and $\mathbf{\Phi}_\mathrm{r}$, then, conveys back their quantized versions along with the index of the optimal beamformer codeword. In order to find the optimal power allocation matrices, the following optimization problem should be solved

$$\begin{aligned} &\max_{\{\phi^2_{\mathrm{s},i}\}_{i=1}^N,\{\phi^2_{\mathrm{r},i}\}_{i=1}^N} \quad I(\mathbf{y}_\mathrm{d},\mathbf{s}) \\ &s.t. \;\; \mathrm{P}_\mathrm{s}, \mathrm{P}_\mathrm{r} \end{aligned} \quad , \tag{16}$$

which is an optimization of two sets of variables at the same time. We adopt the recursive algorithm introduced in [22] to obtain the optimal power allocations at both source and relay nodes. In this method the problem is split in two parts. First it is assumed that P_2 is uniformly allocated to the streams in the second hop, so that $\{\phi^2_{\mathrm{s},i}\}_{i=1}^N$ are optimized by classical water-filling method and, then, the values of $\{\phi^2_{\mathrm{r},i}\}_{i=1}^N$ can be derived by replacing the values of $\{\phi^2_{\mathrm{s},i}\}_{i=1}^N$ in (14) and solving a concave problem. This procedure is repeated until the algorithm converges. By applying this method, the diagonal elements of $\mathbf{\Phi}_\mathrm{s}$ and $\mathbf{\Phi}_\mathrm{r}$ can be expressed as

$$\begin{aligned} \phi^2_{\mathrm{s},i} &= \max\left(\xi_1 - \frac{\sigma^2_{\mathbf{n}_\mathrm{d}} + \sigma^2_{\mathbf{n}_\mathrm{r}}\sigma^2_{\mathrm{rd},i}\phi^2_{\mathrm{r},i}}{\sigma^2_{\mathrm{sr},i}\sigma^2_{\mathrm{rd},i}\phi^2_{\mathrm{r},i}}, 0\right) \\ \phi^2_{\mathrm{r},i} &= \left(-1 + \sqrt{1 + \frac{4\sigma^2_{\mathrm{rd},i}\xi_2}{\sigma^2_{\mathrm{sr},i}\phi^2_{\mathrm{s},i} + \sigma^2_{\mathbf{n}_\mathrm{r}}}}\right)/(2\sigma^2_{\mathrm{rd},i}) \end{aligned} \quad , \tag{17}$$

where ξ_1 and ξ_2 satisfy the two power constraints. Then, the power allocation values for each substream are quantized using a scalar uniform quantizer.

In a nutshell, the different steps of our joint codebook-based beamforming and power allocation method are summarized below. The destination node:

1) Selects the optimal quantized beamformer for the source node by applying the selection criteria in (12);
2) Selects the optimal quantized beamformer for the relay node by applying the selection criteria in (13);
3) Finds the optimal power allocation values for each substream by using the recursive method, which is introduced in part C;
4) Quantizes these power allocation values by using a uniform scalar quantizer;
5) Transmits back the index of the selected beamforming codewords and quantized power allocation values to the relay and source nodes.

IV. Numerical Results

Monte-Carlo simulations are performed in order to illustrate the performance of our proposed joint beamforming and power allocation method. Grassmann manifold packing method with Chordal distance is used to design the two independent codebooks of each hop. Subspace packing, which is available in [23], is used as the set of codewords. We have assumed memory-less Rayleigh fading channel with elements distributed according to $\mathcal{CN}(0,1)$ for each hop.

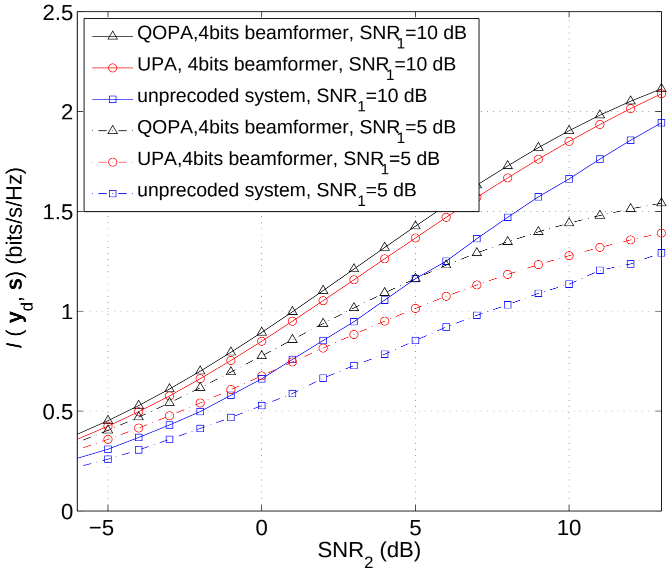

Fig. 2. Mutual information comparison of precoders with quantized optimal power allocation and uniform power allocation method, for a system with 2 substreams and 2 antennas at each node. SNR of the first link is fixed at 5 and 10 dB.

A. Limited feedback beamforming and power allocation

In this simulation, we consider a three node relay channel with 2 antennas at each node, 2 substreams are multiplexed together in order to be sent over the relay system. We compare the achievable rate of the system by using the proposed joint codebook-based beamforming and power allocation method with the codebook-based equal power allocation method and also a system with no feedback link. The SNR of the first hop is fixed at 5 and 10 dB and the mutual information is depicted as a function of the SNR of relay-destination channel (SNR$_2$). We have used a 4bits (16 codewords) codebook for each hop and quantized the power allocation coefficients by using a 4 level uniform quantizer for each substream. We denote our proposed method as quantized optimal power allocation (QOPA) and the previous method in [19] as uniform power allocation (UPA). As depicted in figure 2. The graphs confirm that our method outperforms the previous codebook-based method.

In fact, the joint beamforming and power allocation method prevents the waste of power and increases the power efficiency of the system by allocating power to each subchannel according to their strengths. It is clear that the improvement is made at the expense of extra feedback link overhead (2bits per subchannel in this simulation).

B. Effect of feedback link rate on the system performance

In this simulation, we study the effect of the feedback link rate on the precoder ability to adapt the transmission strategy according to the channel quality. We consider the same simulation parameters as in the previous subsection. We have fixed the power allocation rate to 2 bits per subchannel but with different codebook rates (sizes). We compare the

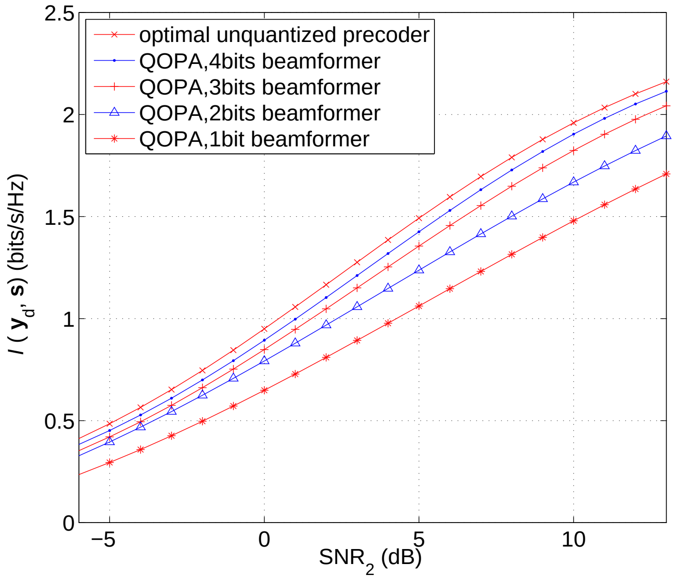

Fig. 3. Mutual information comparison for a codebook based beamformer and quantized optimal power allocation with different codebook size. A system with 2 substreams and 2 antennas at each node is assumed. SNR of the first link is fixed at 10 dB.

performance of our method for several codebook sizes of 2, 4, 8 and 16 and simulate the system with unquantized (optimal) precoder. As it can be seen in Fig. 3, the mutual information increases when the cardinality of the codebook increases and the beamformer with 4 bits codebook has a performance close to the unquantized one. This demonstrates the huge savings in terms of system resources (feedback) that can obtained when using codebook-based precoders.

V. CONCLUSION

In this paper, we have proposed a joint beamforming and power allocation method for nonregenerative relay system with limited feedback link. We have derived the beamforming selection criterion and also computed the optimal power allocation coefficients. The destination node selects the optimal beamformer from the codebooks $\mathcal{B}_s$ and $\mathcal{B}_r$ by using the CSI of both the source-relay and relay-destination channels. It also computes the optimal power allocation coefficients and then quantizes them. Finally, it informs the source and relay nodes about the chosen beamformer and power allocation values by conveying back the index of the optimal codeword and the quantized power allocation vectors. The source and relay nodes will pre-multiply the transmit signal by these precoding matrices to adapt their transmission strategy according to the channel quality. The simulation results show that our joint codebook-based beamforming and power allocation scheme increases the spectral efficiency of the system in comparison with existing codebook-based beamforming method. In addition, they also indicate that the proposed method can reach close to optimal unquantized performance but with a lower complexity. In our future works, we will consider the direct link between source and destination and aim at finding the beamforming matrix and power allocation coefficients for the source node based on both source-relay and source-destination channels.

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