The g factor of the $2^+_1$ state of $^{170}$Hf was measured by perturbed $\gamma$-$\gamma$ angular correlation in a static external magnetic field. The result, $g(2^+_1) = 0.28(5)$, extends the systematics of g factors of even-even Hf isotopes to $N = 98$ and enables a better test of theoretical models. The $g(2^+_1)$ experimental values of these isotopes exhibit a remarkable constancy as a function of neutron number. This phenomenon, which was also observed for other isotopic chains in the Gd–W range, is explained in terms of a recently proposed empirical model.

DOI: 10.1103/PhysRevC.76.047308

PACS number(s): 21.10.Ky, 21.60.Cs, 21.60.Ev
35° and 145°. Therefore, the eight detectors were arranged so that 12 of the 28 possible pairs were at these angles. Of the remaining pairs, eight were at 70°, 110°; four were at 75°, 105°; and the remaining four were at 180°. The data acquisition system was set so that coincidence matrices of all possible pairs of detectors were recorded. In the data sorting, the events from detector pairs at the same angle were sorted in the same two-dimensional $\gamma$-$\gamma$ coincidence matrix. The signs of the angles (positive or negative) with respect to the external field were determined using the convention as given by Dorum and Selsmark [9]. In addition to the 779–101 keV cascade, we also determined using the convention as given by Dorum and Selsmark [9]. In addition to the 779–101 keV cascade, we also determined using the convention as given by Dorum and Selsmark [9]. In addition to the 779–101 keV cascade, we also determined using the convention as given by Dorum and Selsmark [9].

The results of the double ratio $R(\theta, B)$ for all four cascades mentioned above for the two experimental runs are given in Table I. To obtain the experimental value of the $g$ factor, we calculated the function $R(145°, B)$ as a function of the $g$ factor for the $0^+ \rightarrow 2^+_1 \rightarrow 0^+_1$ cascade. This was done for the two values of the magnetic field 4.40 and 5.85 T. The $g$ factor is deduced by comparing the experimental values in Table I with the calculated functions. The results are presented in Fig. 2. Two values of $g(2^+_1)$ were obtained for the two values of the magnetic field: 0.28(7) and 0.28(6) for 4.40 and 5.85 T, respectively. The average of the two values gives the final result of this experiment:

$$g(2^+_1)_{\text{exp}} = 0.28(5).$$

Note that the experimental double ratio $R(145°, B)$ for the $0^+_2 \rightarrow 2^+_1 \rightarrow 0^+_1$ cascade is higher for the higher value of the magnetic field, as expected. All the other experimental values in columns four and five of Table I were extracted from the data as a consistency check. In columns six and seven we present the calculated values of the double ratios, obtained using the value of the $g$ factor given by Eq. (2). At 180°, the expected value of the double ratio for all the cascades is 1.00. Comparison of columns four, five, six, and seven of Table I clearly shows that all the experimental values of the double ratio are, within the experimental error, in good agreement with

<table>
<thead>
<tr>
<th>Cascade (keV)</th>
<th>Spin sequence</th>
<th>Angle</th>
<th>$R_{\text{exp}}(\theta, 4.40 \text{ T})$</th>
<th>$R_{\text{exp}}(\theta, 5.85 \text{ T})$</th>
<th>$R_{\text{calc}}(\theta, 4.40 \text{ T})^a$</th>
<th>$R_{\text{calc}}(\theta, 5.85 \text{ T})^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>779–101</td>
<td>$0^+_2 \rightarrow 2^+_1 \rightarrow 0^+_1$</td>
<td>145°</td>
<td>1.51(12)</td>
<td>1.62(11)</td>
<td>1.48(8)</td>
<td>1.61(8)</td>
</tr>
<tr>
<td>860–101</td>
<td>$2^+_1 \rightarrow 2^+_1 \rightarrow 0^+_1$</td>
<td>145°</td>
<td>1.08(5)</td>
<td>1.10(4)</td>
<td>1.080(10)b</td>
<td>1.095(10)b</td>
</tr>
<tr>
<td>987–101</td>
<td>$3^+_1 \rightarrow 2^+_1 \rightarrow 0^+_1$</td>
<td>145°</td>
<td>0.80(6)</td>
<td>0.88(5)</td>
<td>0.920(10)b</td>
<td>0.900(10)b</td>
</tr>
<tr>
<td>221–101</td>
<td>$4^+_1 \rightarrow 2^+_1 \rightarrow 0^+_1$</td>
<td>145°</td>
<td>1.01(3)</td>
<td>1.02(2)</td>
<td>1.025(5)</td>
<td>1.033(5)</td>
</tr>
<tr>
<td>779–101</td>
<td>$0^+_2 \rightarrow 2^+_1 \rightarrow 0^+_1$</td>
<td>180°</td>
<td>1.01(7)</td>
<td>1.06(11)</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>860–101</td>
<td>$2^+_1 \rightarrow 2^+_1 \rightarrow 0^+_1$</td>
<td>180°</td>
<td>1.00(6)</td>
<td>1.09(8)</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>987–101</td>
<td>$3^+_1 \rightarrow 2^+_1 \rightarrow 0^+_1$</td>
<td>180°</td>
<td>0.97(9)</td>
<td>0.99(11)</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>221–101</td>
<td>$4^+_1 \rightarrow 2^+_1 \rightarrow 0^+_1$</td>
<td>180°</td>
<td>0.99(3)</td>
<td>0.99(3)</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

$^a$The values of $R_{\text{calc}}$ and its error bars, where given, were obtained using the value $g_{\text{exp}} = 0.28(5)$ (see text).

$^b$The double ratio was calculated assuming pure $E2$ character for the first transition of the cascade.
the calculated ones. This can be considered as experimental evidence that systematic errors are smaller than the statistical errors.

We now discuss this new experimental value within the systematics of $g$ factors of $2^+_1$ states in the even-even Hf isotopes. In Fig. 3 we present the existing data and the result of the present work as a function of the neutron number $N$. The value reported in this work extends the systematics to $N = 98$ and thus enables a better test of theoretical models. We first attempt to interpret the data in terms of two models: the hydrodynamical model [4] and the proton-neutron version of the Interacting Boson Model (IBA-2) (see, for example, Ref. [10] for treatment of $g$ factors in IBA-2). In the $F$-spin symmetry limit of the IBA-2, the values of $g(2^+_1)$ are given by the simple relation

$$g(2^+_1) = \frac{g_\pi N_\pi + g_\nu N_\nu}{N_\pi + N_\nu}, \quad (3)$$

where $N_\pi, N_\nu$ are the numbers of proton, neutron bosons and $g_\pi, g_\nu$ are the boson $g$ factors. For the latter parameters we use the values $g_\pi = 0.63$ and $g_\nu = 0.05$ [11]. Because IBA-2 is a valence model, only the valence particles contribute to the magnetic moment, and therefore the $N$-dependence prediction for the $g$ factors is quite strong, as shown in Fig. 3. We also note the change of slope of the IBA-2 calculated values at midshell ($N = 104$). The vibrational and rotational predictions of the hydrodynamical model, presented in Fig. 3, show a weak $N$ dependence that is due to the fact that in this model all nucleons are assumed to contribute to the magnetic moment. Although the error bars are relatively large, the data in Fig. 3 indicate an $N$ dependence even weaker than that predicted by the hydrodynamical model. This behavior of the $g(2^+_1)$ data vs $N$ has also been observed for the even-even Yb isotopes [2]. However, while for the Yb isotopes the experimental values were between the rotational and vibrational values, in the present case both these calculations overestimate the data. We conclude that both the hydrodynamical and the IBA-2 models do not provide a good description of the data. In a recent work, Zhang et al. [5] have proposed a simple phenomenological model to explain the constancy of $g$ factors in deformed nuclei. This model assumes that a reduction in the effective proton-neutron interaction across the midshell region can be simulated by introducing effective proton and neutron boson numbers. This is the same effect that causes the saturation of $B(E2)$ values in the same region. The reduction of the boson numbers is phenomenologically described in Ref. [5] by the equation

$$N_{\tau}^{eff} = N_{\tau}(1 - N_{\tau} f) \quad (\tau = \pi, \nu). \quad (4)$$

The parameter $f$ was found to be $f = 0.05$ from a fit to all existing experimental $g$-factor data for even-even isotopes from Gd to W and for the range of neutron numbers from $N = 88$ to $N = 112$. The predictions of the simple phenomenological model for the Hf isotopes can be calculated by inserting the effective boson numbers given above Eq. (4) into Eq. (3). The results of this calculation are also presented in Fig. 3. We see that the data are in good agreement with the phenomenological model. The new result reported in this Brief Report provides further experimental support for this model and for the concept of effective boson numbers. This phenomenological concept was used in the past [11] to explain anomalies of $g$-factor and $B(E2)$ data in the region of nuclei around $A = 150$. More theoretical work is needed to provide a microscopical basis for the phenomenological scheme mentioned above, which was found [5] to provide a consistent interpretation for both $g$-factor and $B(E2)$ data. Further experimental data will also contribute to a better understanding of the nuclear structure in this region. In particular, from Fig. 3 we see that $g$-factor values at $N = 94, 96, 100,$ and 102 are of special interest.
to substantiate the conclusions of the present work. These experiments are now being planned.

The authors are indebted to the staff of the Wright Nuclear Structure Laboratory for the skillful operation of the tandem accelerator and to Mr. Walter R. Garnett, Jr., for extensive technical support. A.W., Z.B., K.D., and N.P. acknowledge the hospitality of the WNSL during the performance of the experiment. P.H.R. is grateful for support from the EPSRC (UK), the Yale University Flint Fund, and the Science Development Fund. This work was supported by the U.S. DOE under Grants DE-FG02-91ER-40609, DE-FG02-88ER-40417, DE-FG52-06NA-26206, and DE-FG02-05ER-41379; by the U.S. NSF under Grant PHY-0245018; by the Yale Flint Fund; and by the YUUP under Project DPT2006K-120470.