Effect of fracture on the reliability of a moment resisting frame under earthquake loading

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ABSTRACT: The fractures observed following the Northridge earthquake in welded beam-to-column connections have been linked to a number of different material and manufacturing parameters such as fracture toughness, crack size and yield strength all of which are in general random. In a previous study of a simple sway frame, where randomness of the above variables was considered, fracture of the connections was found to result in the reduction of the frame's lateral stiffness with an accompanying reduction of the natural frequency of the frame. In this paper, the linear elastic behaviour of a multi degree-of-freedom frame under earthquake loading is examined probabilistically as an extension of the previous investigation. The results demonstrate that most of the connections fail early on in the seismic event and that fracture probabilities are heavily dependant on the storey on which they are located. Removal of the backing bar is found to reduce the failure probability of the connection without, however, being able to prevent fracture when used as a sole measure.

1 INTRODUCTION

The damage in the moment resisting frames due to the Northridge earthquake, which was in the form of brittle fractures in the connections, has led to intensive research being carried out to identify the causes of this type of damage. Factors, which may have lead to the fracture of the connections have been identified as the stress concentration due to the backing bar (Chen 1997, Joh and Chen 1997, Miller 1998, Barsom 2003, Kuntiyawichai and Burdekin 2003), the low fracture toughness of the weld metal (Chen 1997, Joh and Chen 1997, Kaufmann et al. 1997, Miller 1998, Ricles et al. 2002), high residual stresses (Zhang and Dong 2000, Matos and Dodds 2001, Righiniotis et al. 2002), increased triaxiality at the connection (Chen 1997, Schafer et al. 2000, Shama et al. 2003), restricted welding access (Kaufmann et al. 1997, Miller 1998) and high strain rates (Chen 1997, Joh and Chen 1997, Miller 1998, Matos and Dodds 2002).

Due to the large variability in the material properties and the resulting poor behaviour of the Northridge connections, probabilistic fracture mechanics analyses have been used in the past to study the connections' fracture resistance by taking into account several of the above mentioned factors (Joh and Chen 1999, Matos and Dodds 2001, Righiniotis and Imam 2004). Finite element (FE) analyses have also

been widely used to study the behaviour of the connections (Yang and Popov 1995, Ojdrovic and Zarghamee 1997, Popov et al. 1998, Jon and Chen 1999, Chi et al. 2000a, 2000b, Matos and Dodds 2001). By examining the stress intensity value at the critical location of the bottom flange weld and the backing bar, these FE analyses have confirmed the poor behaviour of the connections during the Northridge earthquake.

In an attempt to quantify the dynamic nature of the problem, dynamic analyses of moment resisting frames have also been carried out by taking into account the fracture behaviour of the connections (Luco and Cornell 2000, Wang and Wen 2000a, 2000b, Foutch and Yun 2002, Lee and Foutch 2002, Wen and Song 2003, Righiniotis 2004, Rodgers and Mahin 2004). These investigations have found that fracture generally resulted in the reduction of the stiffness of the building with an accompanying increase in the natural period.

This investigation extends the work of the first author on the study of the pre- and post-fracture behaviour of a single degree-of-freedom (SDOF) frame (Righiniotis 2004), to a multi degree-of-freedom (MDOF) frame. Statistical scatter in material properties (fracture toughness and yield strength) as well as in crack depth and connection residual strength are considered. The results of this investigation are presented in terms of the Cumulative Distribution Function (CDF) of the time to first fracture as well as in terms of fragility curves for a single earthquake record and for two connections.

2 JOINT STIFFNESS AND FRACTURE MOMENT

In the aftermath of the Northridge earthquake, it was found that the majority of the beam flanges were partially fused (Kaufmann et al. 1997). This lack of fusion, which was caused by restricted welding access, effectively amounted to the introduction of a sharp notch. Furthermore, the backing bar, which according to pre-Northridge construction practice was left in place, increased the deleterious effect of the notch. This led researchers to the assumption that the backing bar and pre-existing flaw could be modelled as a sharp crack of an effective length equal to the sum of the thickness of the backing bar and the flaw depth (Kaufmann et al. 1997, Ricles et al. 2002). Here, the stress concentration effect caused by the backing bar is accounted for through the appropriate stress magnification factor.

The presence of the crack at the interface of the beam and column flanges increases the flexibility of the joint. The stiffness of the cracked flange K_s may be expressed as (Righiniotis et al. 2002)

$$K_s = \frac{E\, I_b}{6\pi(1-\nu^2)}\left[(D-t)\, f_{bt} + t\, f_{bb}\right]^{-1} \tag{1}$$

where E is the Young's modulus, I_b is the beam's second moment of area, D and t are defined in Figure 1 and f_{bt}, f_{bb} are flexibility coefficients given in the Appendix.

By assuming that the bottom flange and crack act as springs in series, which are in parallel to the spring that represents the remaining section, the joint stiffness may be derived as (Righiniotis 2004)

$$K_{total} = K_i \left\{ R + \frac{1-R}{1+(1-R)(K_i / K_s)} \right\} = K_i\, F_i \qquad i = 1, 2 \tag{2}$$

where K_i is the rotational stiffness of the beam ($K_1 = 2EI_b/L_b$, $K_2 = 4EI_b/L_b$) and R is the ratio of the stiffness of a connection with a completely severed flange to the un-cracked connection stiffness K_i. Note that the approach presented here assumes that a cracked or indeed a fully severed bottom flange only affects the moment-rotation characteristics of the joint. Moreover, Eq. (2) indicates that when $K_s \rightarrow 0$ (severed flange), the joint stiffness equals the stiffness of the remaining section, whereas when $K_s \rightarrow \infty$ (un-cracked flange), the joint stiffness becomes equal to K_i.

The fact that a crack is present in the bottom flange not only affects the stiffness of the joint but also determines whether fracture will occur or not. Based on a Linear Elastic Fracture Mechanics fracture criterion, the bending moment corresponding to fracture will be given as (Righiniotis et al. 2002)

$$M_f = \left(2.24\sqrt{\frac{Q}{\pi\, a}}\, K_{Ic} - 1.12\,\sigma_y\, Y_t \right) \frac{I_b}{Y_1\left[(D-t)Y_t + t\, Y_b\right]} \tag{3}$$

where a is the flaw depth (see Fig. 1), K_{Ic} is the fracture toughness of the weld metal, σ_y is its yield stress, Y_1, Y_t and Y_b are stress magnification factors given in the Appendix and Q is a non-dimensional factor of the cracked geometry, also given in the Appendix. It has to be noted that in Eq. (3), the second term in brackets, which represents the welding residual stress contribution may be seen to reduce the connection's fracture moment.

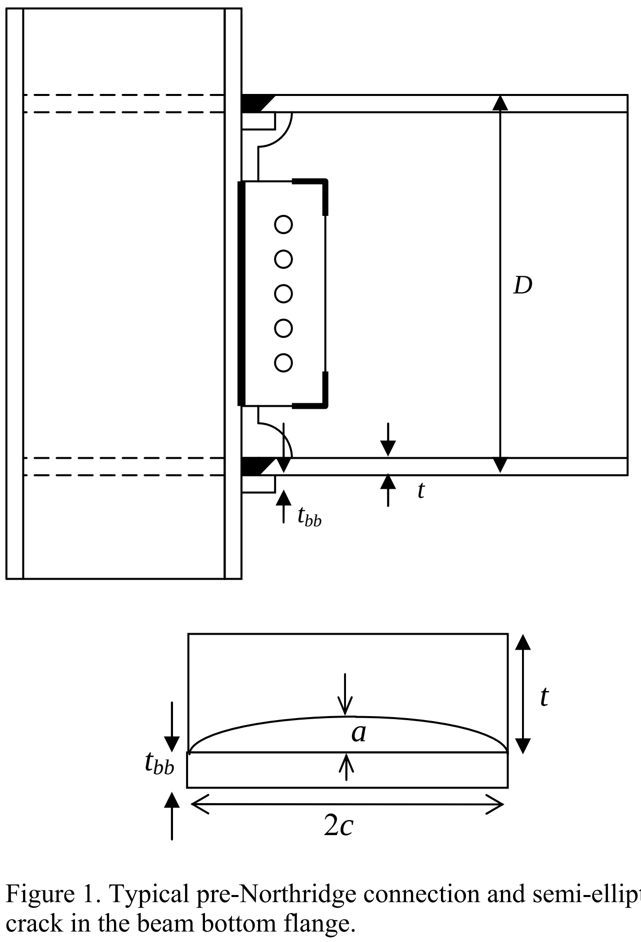

Figure 1. Typical pre-Northridge connection and semi-elliptical crack in the beam bottom flange.

3 FRAME DYNAMIC ANALYSIS

From the preceding discussion it becomes obvious that cracked connections (prior to the fracture event) and severed connections (following the fracture event) will alter the dynamic characteristics of a moment-resisting frame. This change comes about through the introduction in the stiffness matrix of the moment-rotation stiffness coefficients given by Eq. (2). In this paper, the stiffness matrix is condensed and the mass of the frame is assumed to be lumped at the floors. A 2% Rayleigh damping is assumed for the first two modes of vibration.

The frame considered here is a two-storey, three-bay frame (see Fig. 2). The beams and columns are assumed to have a length of 4m and 3m, respectively. The beams are assumed to consist of W24x76 sections and the columns of W14x193 sections, throughout the structure. For the purposes of calculating the fracture moment and cracked flexibility coefficients (see Section 2 and the Appendix), the thickness of the backing bar is taken equal to 9.5 mm, which is a typical thickness used in Northridge (Chi et al. 2000a, Matos and Dodds 2001). Assuming axial inextensibility of the beams, the frame consists of a total of 16 DOF (14 rotational at the joints and 2 translational at the floors).

Under the stated assumptions the equations of motion become (Chopra 2001)

$$\mathbf{M}\,\ddot{\mathbf{u}} + \mathbf{C}\,\dot{\mathbf{u}} + \mathbf{K_t}\,\mathbf{u} = -\mathbf{M}\,\mathbf{i}\,\ddot{u}_g \tag{4}$$

where $\mathbf{u}$ and its derivatives are column-vectors, which contain the translational degrees of freedom, $\mathbf{M}$ and $\mathbf{C}$ are the mass and damping matrices, $\mathbf{K_t}$ is the condensed stiffness matrix, $\mathbf{i}$ is a column vector containing 1's and $\ddot{u}_g$ is the horizontal ground acceleration. Assuming elastic behaviour of the frame, Eqs. (4) are here solved using modal analysis and Newmark's method under a linear acceleration scheme.

The crack opening bending moments, which are developed at the joints (left and right for internal joints), are compared with the fracture moment M_f (see Eq. 3). Although the fracture event will sever the flange leading to a new stiffness matrix, this sequential treatment is not pursued here. Instead, the stiffness matrix is assumed to retain its initial cracked form throughout the entire seismic event.

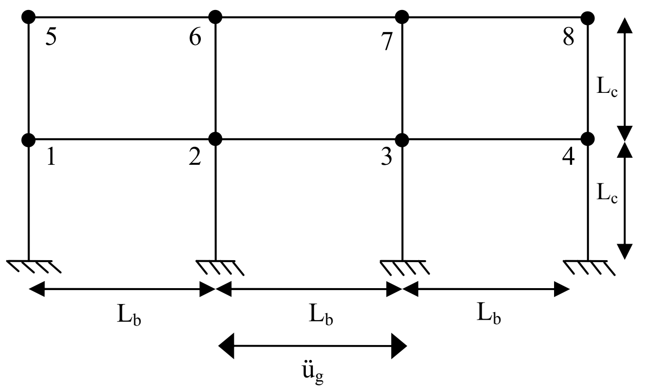

Figure 2. MDOF frame.

4 PROBABILISTIC TREATMENT

Randomness of the Northridge phenomenon is here addressed through the manufacturing and material parameters. Accordingly, the problem is randomised through the crack depth, fracture toughness and yield stress. The rationale behind these choices is given by Righiniotis and Imam (2004). Table 1 lists the assumed distributions for these three parameters. Also shown in Table 1 is the normalised residual connection strength, which is also assumed to be random. The choices for R are here taken arbitrarily.

The frame analyses presented in Section 3 are carried out for the Arleta 090 accelerogram using Monte Carlo simulation with 5000 samples. Failure probabilities are defined for different connections according to

$$P_f = P[M \geq M_f] \tag{5}$$

where M is the applied moment. The time to fracture t_f may be evaluated numerically through the Newmark algorithm. Within the Monte Carlo scheme it is a straightforward matter to evaluate the corresponding cumulative distribution function (CDF) $F(t_f)$ according to

$$F(t_f) = \int_0^{t_d} f(t_f)\,dt_f \tag{6}$$

where t_d is the duration of the seismic event.

Table 1. Random variables (Righiniotis and Imam 2004, Righiniotis 2004).

Variable	Distribution	Mean	CoV
a	LN	3.77 mm	0.73
K_{Ic}	Weibull	65 MPa $m^{1/2}$	0.13
σ_y	LN	292 MPa	0.09
R	LN	0.4	0.2

5 RESULTS AND DISCUSSION

Figure 3 depicts the variation of the change in the two natural periods of the frame with the non-dimensional residual strength R of the connections. The results are plotted assuming that all of the connections have severed flanges ($a = t$) and a deterministic R. The % change in period is reported in relation to the case of no cracks being present in the frame. The case of R = 0 corresponds to the case where the frame consists of hinges at the connections, whereas, at the other extreme, the case R = 1 corresponds to the unrealistic assumption that a severed bottom flange does not affect the connection's rotational stiffness.

As expected, an increase in the residual stiffness implies smaller changes to the vibration characteristics of the frame. Furthermore, Figure 3 demonstrates that in this case, the largest effect of R is manifest on the second mode. By comparison, the changes in the fundamental period of a simple sway frame reported by Righiniotis (2004), were found to be approximately 7% and 10% for R = 0.44 and R = 0.2, respectively. Here, the corresponding changes in T_1 are 3.4% and 6% for the same values of R. Figure 4 depicts the deterministically calculated change in the Rayleigh parameters for different residual connection stiffnesses.

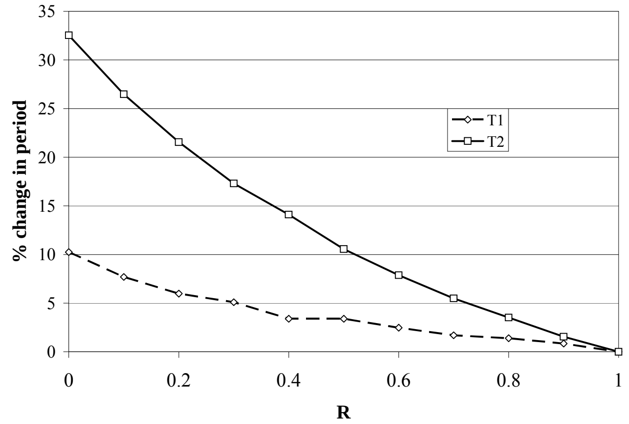

Figure 3. Variation of the change in the natural periods with R.

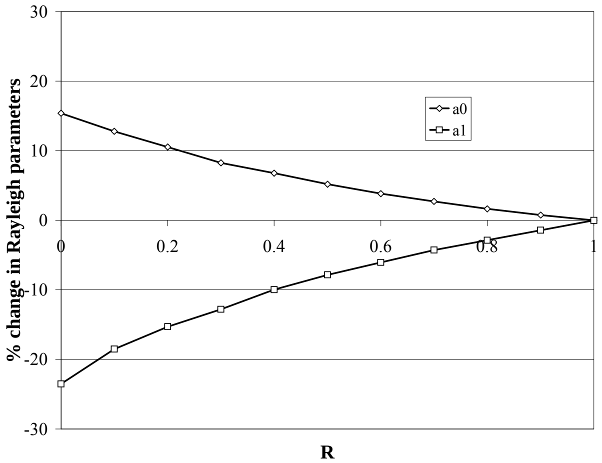

Figure 4. Variation of the change in the Rayleigh parameters with R.

As can be seen in Figure 4, when compared to the un-cracked structure, the fully cracked structure consistently corresponds to an increase in the first Rayleigh parameter (a_0, associated with $\mathbf{M}$) and a decrease in the second Rayleigh parameter (a_1, associated with $\mathbf{K}$), irrespective of R. The latter change is more pronounced than the former over the entire range of R values.

Figure 5 depicts the CDFs of t_f for Connection 2. The results for both faces of the connection (L-Left, R-Right) are shown in Figure 5. Although both faces tend to fracture within the third and sixth second of the motion, the left face appears to fracture within a narrower time frame, with the majority of the samples (80%) failing approximately 3.5 seconds in the event. For the right face, the majority of the fractures take place after 4 (55%) and 5 seconds of motion (30%).

Figure 6 shows the CDFs of t_f for Connection 6. Here, both faces fail at approximately the same time and within a very narrow time frame. Accordingly, fracture takes place after approximately 3.6 seconds of motion. The results presented in Figures 5 and 6 would of course change if fracture sequence effects were taken into account but nevertheless demonstrate that fracture takes place very shortly after the beginning of this particular seismic event.

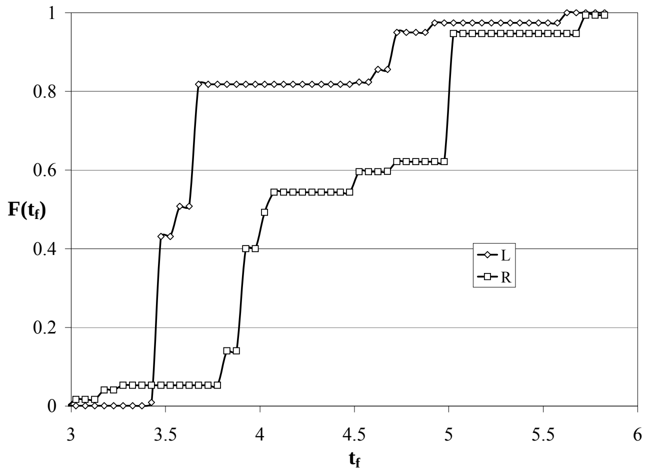

Figure 5. CDF of time to fracture for Connection 2.

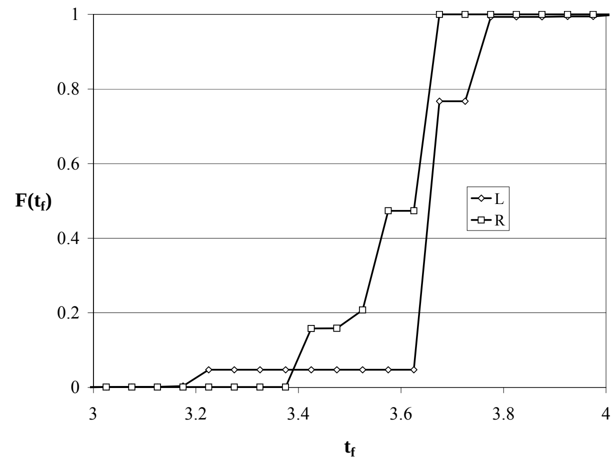

Figure 6. CDF of time to fracture for Connection 6.

Figures 7 and 8 depict the fragility curves for Connections 2 and 6 (see Fig. 2), respectively, of the frame being subjected to the Arleta record. Although different scaling schemes are currently under investigation by the authors, here a simple scaling scheme of a single record is employed. The record is scaled in terms of the actual peak ground acceleration A_g.

The figures report failure probabilities for both ends of the connection (L = left, R = right). The notation BB appearing in Figures 7 and 8 is used to indicate that these results are obtained assuming that the backing bar is present. For Connection 2, the actual record results in a failure probability of 39% for the left side of the connection and a failure probability of 25% for the right side. By contrast, Connection 6 is far more susceptible to fracture where both the left and right sides of the connection fracture with a probability of almost 100%. Figures 7 and 8 indicate that different sides of the connection can fracture with different probabilities. However, since in the present treatment sample realisations are identical for all of the connections, connections pertaining to the same floor will all fail with the same probability. Figures 7 and 8 also depict the results of the analyses for the case when backing bars are removed from all connections indicated in the legend as No BB. The results are obtained by specifying $Y_1 = 1$, in which case, the flexibility coefficients and the fracture moment become solely functions of Y_t and Y_b (see Appendix). Accordingly, backing bar removal is here seen to affect not only the resistance to fracture but also the stiffness of the frame. At high scaling values it can be seen that the failure probabilities drop considerably as a result of backing bar removal. For Connection 2, the results are almost identical for both faces of the connection irrespective of the scaling value (maximum difference less than 1%). In the case of Connection 6, larger differences may be observed between the two faces. For both connections it can be seen that at low earthquake intensities, backing bar removal results in slightly higher failure probabilities. This is because, although the fracture moment increases considerably, the vibration characteristics of the building change albeit by a small amount. This in turn affects the global seismic response. Since sample values for crack sizes etc are the same for both faces of a connection, the differences in the fragility curves L and R shown in Figures 7 and 8 should be attributed to the directionality of seismic input. This effect appears to be negligible for Connection 2 with the backing bar removed and, depending on the scaling factor, significant in all other cases. Overall, it can be said that although removing the backing bar results in substantial improvements in fracture reliability, backing bar removal alone cannot mitigate against fracture. The resulting failure probabilities are non-negligible especially at the levels of ground acceleration experienced at Northridge (3.5% minimum, 10% maximum).

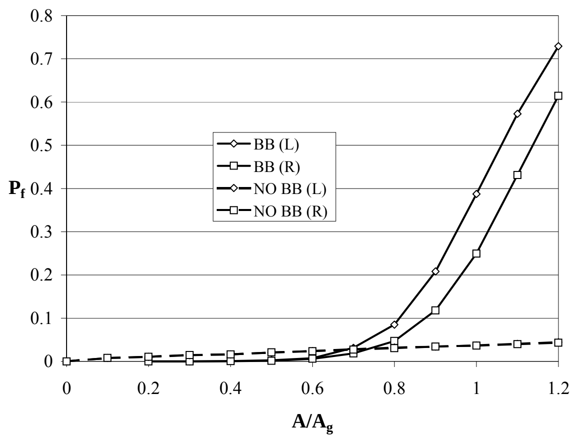

Figure 7. Fragility curves for the first fracture event of Connection 2.

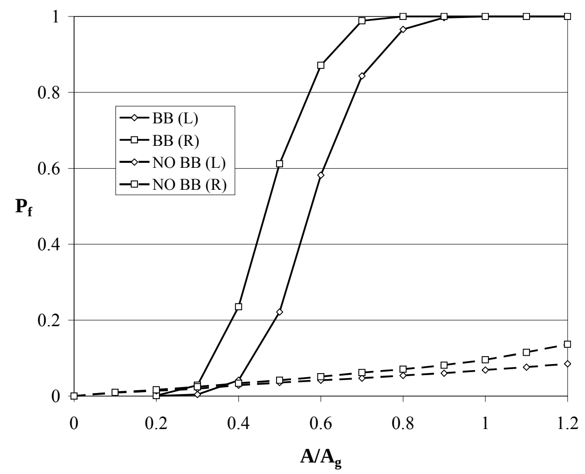

Figure 8. Fragility curves for the first fracture event of Connection 6.

6 CONCLUSIONS

In this paper, the behaviour of moment resisting welded steel connections with reference to fracture was investigated in a probabilistic way. Randomness was considered in terms of the fracture toughness, crack size, yield stress and the connections' residual stiffness. The methodology presented elsewhere for a simple sway frame (Righiniotis 2004) was extended to capture the behaviour in a two-storey, three-bay frame, assuming linear elastic behaviour. It was found that the frame's natural periods as well as the Rayleigh parameters were affected by the connections' residual strength. For two of the connections under a single ground record investigated here, fracture was found to take place very early on in the event. For one of the two connections (Connection 2), both faces were found to fracture over a larger time frame, while for the other (Connection 6), both faces failed within a narrow time band. Fragility curves for two of the frame's connections pertaining to the first fracture event under a single record were presented. It was found that for the selected earthquake record, the first storey connections would fracture with a probability of 39% (left face) and 25% (right face). By contrast, the second storey connections would fracture with a probability of almost 100% for both faces. Backing bar removal was found to improve the connections' reliability significantly, but as a sole measure, it was found that it could not mitigate against fracture. Future developments of the methodology presented here that are currently under way include:

- Updating of the stiffness matrix to capture fracture sequence effects.
- Investigation of spatial variability with reference to different connections.
- Investigation of the connections' non-linear, post fracture behaviour.
- Development of fragility curves incorporating all of the above, by using an appropriate scaling scheme.

APPENDIX

The flexibility coefficients are given as (Righiniotis et al. 2002)

$$f_{bt}(v) = \frac{1}{1.12^2} \int_0^v \frac{x}{Q} Y_t \, Y_b \, Y_1^2 \, dx \tag{A1}$$

$$f_{bb}(v) = \frac{1}{1.12^2} \int_0^v \frac{x}{Q} (Y_b \, Y_1)^2 \, dx \tag{A2}$$

where $v = a/t$ and the Y factors appearing in Eqs. (A1) and (A2) are given by (Righiniotis et al. 2002)

$$\begin{aligned} Y_t &= 3.776 - 64.493\left(\frac{a}{t}\right) + 627.2\left(\frac{a}{t}\right)^2 - \\ &2771\left(\frac{a}{t}\right)^3 + 6140\left(\frac{a}{t}\right)^4 - 6583\left(\frac{a}{t}\right)^5 + 2741\left(\frac{a}{t}\right)^6 \end{aligned} \tag{A3}$$

$$\begin{aligned} Y_b &= 2.168 - 27.1\left(\frac{a}{t}\right) + 237.7\left(\frac{a}{t}\right)^2 - 1003\left(\frac{a}{t}\right)^3 + \\ &2168\left(\frac{a}{t}\right)^4 - 2290\left(\frac{a}{t}\right)^5 + 944.8\left(\frac{a}{t}\right)^6 \end{aligned} \tag{A4}$$

$$Y_1 = 1.081\left(\frac{(a/t)}{(a/t) + (t_{bb}/t)}\right)^{-0.33} \tag{A5}$$

Finally, the factor Q is given as

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \tag{A6}$$

where, here c is taken to be equal to half the width of the beam flange (see Fig. 1). In general, c will be random but experimental evidence has demonstrated that the lack of fusion was more or less present throughout the width of the flange (Matos and Dodds 2001).

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