ABSTRACT

The aim of this paper is to present proper finite element (FE) models which can predict the dynamic characteristics and behaviour of a railway bridge under dynamic train loading for the purposes of fatigue design and assessment. As a case study, an existing six-span continuous welded plate girder railway bridge in Stockholm is investigated. A number of detailed three-dimensional FE models of the bridge are developed in terms of increasing complexity, starting from a beam-only model and refining it to an FE model consisting of shell elements and the combination of the two. Additional factors that are being investigated are the effects of different boundary conditions, the number of modelled spans and the effect of secondary elements such as bracings. Eigenvalue analysis of the bridge is first carried out in order to determine its dynamic characteristics such as dominant frequencies and mode shapes. These are compared between the different models and existing analytical solutions from the literature. Furthermore, available field measurements at different members on the bridge are compared with the results obtained from dynamic time-history analysis under the passage of a typical locomotive at various speeds. Based on the FE analyses on the bridge models and their validation with the field measurements, a number of conclusions and suggestions are made for the advanced dynamic modelling of bridges.

Keywords: Steel railway bridge, dynamic analysis, eigenvalues, field measurements, stresses.

1. INTRODUCTION

FE analysis becomes necessary before undergoing any retrofit or major repair. Moreover, it is necessary to study the structural behaviour of the existing bridge due to increasing train traffic. One of the most effective ways of investigating the dynamic behaviour of bridge under complicated loading and secondary load effects, such as out-of-plane bending of main girders, is through proper FE simulation. Secondary load effects on the existing bridge can arise from poor connections between the bridge members and their which, over time, may result in fatigue damage on bridges [1]. The accuracy of the FE analysis of any bridge model depends on the calibration with the existing bridge material parameters and the member behaviour under realistic loading. Bridge material behaviour depends on the structural ageing and environmental conditions, whereas the member behaviour depends on the structural details and the train load distribution.

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The aim of this paper is to investigate the effects of different modelling assumptions on the dynamic behaviour of steel bridges and to present guidelines on carrying out dynamic analyses on such bridges. As a case study, a welded steel girder bridge, in which fatigue cracks have been detected on the connection between the stiffeners and main girders, is analysed. A number of finite element models of the bridge with different degrees of complexity, combining beam and shell elements, are developed. Initially, a number of eigenvalue analyses are performed in order to determine the dominant frequencies of the bridge. This is followed by linear, time-history analyses of the bridge under the passage of selected trains. The results are compared with analytical solutions as well as field measurements which were carried out under the passage of a test locomotive over the bridge.

1.1 Case study bridge

In this paper, an existing six-span continuous welded plate girder railway bridge located in Stockholm has been taken to carry out the FE analyses. Figure 1 shows the plan and elevation of the bridge. This bridge was built over the stream Söderström in the mid-1950s to connect the northern and southern Sweden with two separate train tracks laid over wooden sleepers resting on the stringer beams. Almost 520 trains pass over this bridge every day, out of which commuter trains are the most frequent whereas the freight trains are comparatively much less frequent.

1.2 Field measurements

The division of structural design and bridges at the Royal Institute of Technology, Sweden (KTH) conducted field tests by passing a Rc6 locomotive over one track (west side track) of the bridge. The locomotive had a total weight of 78 tons and 4 axles with distances 2.7m+5.0m+2.7m. Strains and accelerations were measured at different locations of span 7–8 (see Figure 1), underneath the loaded track of the bridge, using 56 strain gauges and 5 accelerometers. Details of the strain gauge locations in span 7–8 are shown in Figure 2a.
Measurements were obtained at stringer and cross-girder mid-spans (points C, D, I), on the main girder (point A) and at the connection between the stringer and the cross-girder (point E). Figure 2b shows a typical cross section of a stringer mid-span location with the measurements points. As can be seen, measurements at four points (two on the top flange and two on the bottom flange) have been obtained. This was the case of all members i.e. stringers, cross-girders, and main girders. The field measurements for the Rc6 locomotive were conducted at different speeds i.e. 1, 10 and 52 km/h, the first speed (1 km/h) representing effectively the case of static loading. The MGC plus system with amplifiers of the ML801 type produced by Hottinger Baldwin Messtechnik was used for the data acquisition. Measurements were recorded at 400Hz.

1.3. Finite element model

The dynamic behaviour of the case study bridge was investigated under different degrees of complexity, using shell and/or beam elements. FE models of the bridge were developed using the commercial FE package ABAQUS [3]. Eight-noded, reduced integration shell elements (S8R) and three-noded, quadratic beam elements (B32) were used in the FE models. Single-span, three-span and six-span (full) bridge FE models were developed and analysed. Both simply supported (SS) and fixed support conditions were assumed in the single-span and three-span models at the two ends of the bridge in order to investigate the effect of boundary conditions. All members were tied to each other which are equivalent to assuming rigid connections between them.

As a first step, only span (7–8) of the full bridge was modelled first with shell elements and then with beam elements. The effect of bracings was also investigated via the single-span FE model by developing a shell model with and without bracings. Figure 3a shows the shell-element model of the single span with the bracings included. In all the shell-element models, the stiffeners in the main girders were also modelled. The effect of adjacent spans in the overall behaviour of the bridge was studied by developing three-span and six-span bridge models. In the case of the three-span bridge models, one model was developed using shell elements for all spans, as shown in Figure 3b, whereas the other model was developed by a combination of shell and beam elements. In the latter model, see Figure 3c, it can be seen that shell elements were employed for span 7–8, whereas beam elements were used to model the adjacent two spans. For the full bridge model, which is shown in Figure 3d, shell elements were used only for span 7–8, the remaining spans being modelled with beam elements. In both the three-span and six-span FE models, the intermediate supports were modelled as simply supported. Due to the high computational effort required, a full-shell model of the entire bridge was not attempted.

2. FE ANALYSES OF CASE STUDY BRIDGE

Eigenvalue analyses were carried out on all FE models of the bridge and the results, in the form of bridge periods (frequencies), were obtained and compared in order to assess the capability of different FE modelling detail levels on capturing fundamental dynamic properties of the bridge. Following the eigenvalue analysis, which was employed to obtain the fundamental dynamic properties of the bridge,
static and dynamic FE analyses under the passage of the Rc6 test locomotive were carried out to investigate its overall dynamic behaviour.

2.1. Eigenvalue analysis of bridge

To avoid the bridge structural resonance, the fundamental frequencies of the bridge structure should not match with the excitation frequency ($\nu/\ell$). This depends on the speed ($\nu$) of the passing train and the bridge span ($\ell$). Eigenvalues of an undamped mechanical system can be calculated based on the equation of motion as given in Frýba [4]. The bridge frequencies obtained from the FE analysis were compared with available empirical formulae suggested by Frýba [4] and the International Union of Railways [5]. These empirical expressions were developed through statistical evaluation and regression analysis of a large number of field measurements of bridge frequencies carried out in the past on different type of ballasted and unballasted bridges such as steel truss, plate girder, and concrete [4].

![Figure 3. FE models of the bridge](image)

2.2. Dynamic analysis of bridge

Two different types of dynamic analyses i.e. modal dynamic and implicit dynamic were undertaken to investigate the suitability of each to capture the dynamic behaviour of the bridge. Explicit dynamic analysis is computationally much more demanding than implicit analyses. Due to the large nature of the FE model, this type of analysis was excluded from this investigation. The difference between implicit and explicit dynamic analysis lies in the solution procedure of the dynamic equations of motion [3].
The fundamental equation of motion of the bridge system can be expressed in general form as given in Frýba [4].

The analyses were carried out on the full-span, beam-shell element FE model of the bridge (Figure 3d). A range of different train velocities were employed in the analyses and the results were compared with the available field measurements. The bridge was loaded with the test locomotive and the axle loads of the train (195kN) were applied directly to the top flange of the stringers ignoring any load distribution due to the effect of rails and sleepers. Loading was initiated from the start of span 5–6 since the field measurements showed that the investigated span 7–8 experienced the effect of the locomotive from that point onwards. It was then traversed in 1m steps until the middle of span 6–7 from which point onwards a smaller step size of 0.5m was used up to the middle of span 8–9 where the step was once again changed to 1m until the locomotive exited the bridge. The loads were applied as triangular distribution of forces as suggested in [6] and [7]. An artificial damping of 5%, a typical value for this type of bridges, was included by default in the implicit analysis. The modal dynamic analysis was carried out using 30 modes and a material damping ratio of $\zeta = 2.6\%$ which was obtained based on the first modal frequency of the full bridge [4]. A static analysis of the bridge was also carried out to compare the results with the field measurements obtained under a train velocity of 1 km/h which can effectively be considered as static loading.

3. RESULTS AND DISCUSSIONS

[Figures 4, showing mode shapes]

3.1 Eigenvalue analysis of bridge

Single-span bridge

As a first step, only the span of the bridge where the field measurements were obtained (span 7–8, see Fig. 1) was modelled as a single span using shell elements. Figure 4 shows the first mode shape behaviour of the bridge model without bracings, for two different boundary conditions i.e. simply supported (SS) and fixed, whereas Figure 5 shows the first mode shape behaviour of the bridge with the bracings included in the model. It can be seen from Figure 4 that the first mode shape obtained from the model without bracings is lateral bending with periods of 0.510 and 0.460 seconds for the SS and fixed boundary conditions, respectively. The addition of bracings in the model reduces the bridge period by 64% and 73% for the SS and fixed boundary conditions, respectively. This is an indication of the additional stiffness that is provided by the bracings and suggests that these should be modelled for the purposes of dynamic analyses. As a result, all subsequent analyses were carried out on models which included the bracing elements.

The inclusion of bracings in the bridge FE model with SS boundary conditions resulted in a vertical bending mode shape (see Figure 5). On the other hand, the mode shape in the fixed boundary condition model was found to be a combination of lateral bending and torsion (Figure 5b). This shows that the fixed boundary condition model is able to capture the out-of-plane behaviour of the main
members which was responsible for the observed fatigue cracking in the main girder web to stiffener connections. Comparing SS and fixed boundary conditions for the model with bracings, the latter results in a period which is 32% lower than the former.

Figure 5. First mode shape of the single-span shell element FE model with bracings

For comparison purpose with the shell element model, a beam element model of the span under consideration was also developed and analysed. Figure 6 shows the first mode shape obtained from the eigenvalue analysis which consists of vertical bending for both SS and fixed boundary conditions. The eigenvalue analysis for this model showed that, although they are computationally much cheaper, beam elements fail to capture the out-of-plane and torsional deformations of the main bridge girders.

Figure 6. First mode shape of the single-span beam element FE model with bracings

Three-span bridge model

The full-shell, three-span bridge model shown in Figure 3b was developed as an attempt to increase the accuracy of the obtained results. Figure 7 shows the first mode shape obtained from the eigenvalue analyses for two different boundary conditions at the two ends of the FE bridge model (SS and fixed). It can be seen that, irrespective of the boundary conditions, the first mode shape captures vertical bending of the bridge (Figures 8a and 8b). The fixed boundary condition results in a 20% reduction in the first period as compared to the SS case (0.181 vs. 0.151 seconds). The second mode obtained from the eigenvalue analysis captures the out-of-plane flexural and torsional behaviour of the main girders for both SS and fixed boundary condition assumptions.
In the case of SS boundary conditions, the first period of bridge was found to be very similar to the single-span model. On the other hand, in the case of fixed boundary conditions, modelling the bridge with three-spans resulted in an almost 20% increase in the bridge period as compared to the single-span model (0.123 vs. 0.151 seconds).

As an attempt to reduce the time required for the analysis, the adjacent spans to the investigated span were modelled using beam elements, which are computationally cheaper than shell elements. Figure 8 shows the first mode shape for the beam and shell element three-span bridge model for both boundary conditions. It can be seen that, similar to the full-shell element model described above, the first mode shape obtained for this model is vertical bending for both types of boundary conditions (see Figure 8a and 8b). The effect of fixed boundary conditions is to reduce the first time period by 20%.

Comparing the beam-shell bridge FE model with the full-shell FE model in terms of the obtained periods, it was found that the former results in a slight decrease in the period by a maximum 7%. This demonstrates the fact that, for time economy considerations, modelling the remaining spans of the bridge using beam elements does not decrease the accuracy of the results considerably.

Full bridge model

Figure 9 shows the first four mode shapes obtained from the eigenvalue analysis for the full-bridge FE model which was developed using shell elements for span 7–8 and beam elements for the remaining bridge. As it can be seen, the out-of-plane mode is captured here through mode 3, whereas the remaining modes all include vertical bending effects. This full-bridge model results in a fundamental period of 0.208 seconds which is considerably higher than the previous models. For the subsequent modes, small increases in the periods, as compared to the single- and three-span models, are also evident. This demonstrates the fact that for the purposes of dynamic analyses, modelling the entire bridge in the way shown in Figure 3d can be expected to provide a good prediction of the dynamic behaviour of the bridge.
Overall comparisons

Table 1 presents an overview of the results obtained from the eigenvalue analysis using the different finite element models. The periods of the first three modes of each model as well as the time required for each analysis and the number of elements in each FE model are shown in the table. The table clearly shows the time saving achieved by using beam elements in the model. For example, comparing the analyses time for the three-span shell element model with its beam-shell counterpart, it can be seen that the analysis time required for the latter is almost one-fifth of the time required for the former. In the case of the full-bridge model, which is a combination of shell and beam elements, the time required for the analysis in almost half of that required for the three-span shell element model and is, most probably, significantly lower than the time that would be required for a full-shell entire bridge model considering the non-linear relationship between the size of the FE model (number of elements) and analysis time.

Table 1. Comparison of the periods for the different FE models

<table>
<thead>
<tr>
<th>No. of Spans</th>
<th>Single-span</th>
<th>Three-span</th>
<th>Full bridge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>Shell Element</td>
<td>Beam Element</td>
<td>Shell</td>
</tr>
<tr>
<td>No. of elements</td>
<td>12139</td>
<td>1318</td>
<td>35174</td>
</tr>
<tr>
<td>Analysis time (s)</td>
<td>292</td>
<td>10</td>
<td>1446</td>
</tr>
<tr>
<td>Boundary conditions</td>
<td>SS Fixed</td>
<td>SS Fixed</td>
<td>SS Fixed</td>
</tr>
<tr>
<td>$T_1$ (s)</td>
<td>0.182</td>
<td>0.176</td>
<td>0.114</td>
</tr>
<tr>
<td>$T_2$ (s)</td>
<td>0.156</td>
<td>0.148</td>
<td>0.114</td>
</tr>
<tr>
<td>$T_3$ (s)</td>
<td>0.110</td>
<td>0.115</td>
<td>0.114</td>
</tr>
</tbody>
</table>

Table 2 shows the fundamental period obtained for the particular bridge being investigated through the available empirical formulae. Comparison of the empirical values with the results obtained from the FE eigenvalue analyses presented in Table 1 reveals a good agreement between the two sets of results.
The first natural frequency obtained from the full-bridge model lies well within the upper and lower limits suggested by the UIC [5] & BS EN 1991–2 [8].

Table 2. Comparison of the first period $T_1$ of the single-span FE models with available empirical formulae

<table>
<thead>
<tr>
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<tbody>
<tr>
<td></td>
<td>Shell Elements</td>
<td>Beam Elements</td>
<td>(1)</td>
</tr>
<tr>
<td>SS Fixed</td>
<td>0.182</td>
<td>0.123</td>
<td>0.176</td>
</tr>
<tr>
<td>$T_1$ (s)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(1) for railway bridges of all types, materials and structural systems,
(2) for steel plate girder bridges without ballast,
(3) unloaded railway bridges of all types and materials, lower limit (for span 20 ≤ $\ell$ ≤ 100m) and
(4) unloaded railway bridges of all types and materials, upper limit (for span 4 ≤ $\ell$ ≤ 100m)

3.2 Dynamic analysis of bridge

The full-span bridge FE model was used to carry out a number of initial parametric dynamic analyses under the passage of the test locomotive and to compare the results with the available field measurements in order to determine the most appropriate type of dynamic analyses. Both implicit and modal dynamic analyses using 30 modes were carried out using ABAQUS. The results obtained from the modal analysis did not show good agreement with the field measurements and hence they are excluded from further discussion. Since the available field measurements included the case of a “static” passage of the test locomotive over the bridge by a velocity of 1 km/h, a static FE analysis of the bridge was also carried out.

Figure 10 shows the comparison of the strain histories obtained from the static analysis with the field measurements at points S1 and S2 of location C (see Figure 3), which is located at the mid-length of the stringer, under a velocity of 1 km/h. It can be seen that good overall agreement between the results is obtained with the static FE analysis predicting slightly higher maximum strains at the peak points.

Figure 11 shows the comparison of strain histories obtained from the implicit dynamic analysis under velocities of 10 km/h with the field measurements under the same speeds. Similar comparison was made for train velocity of 52 km/h. Good agreement can be seen between the results in the case of the dynamic analyses with very good prediction of the maximum strains. Although the implicit dynamic analysis captures the overall trend of the strain history well, it produces some additional smaller strain cycles which is, however, expected to be insignificant in terms of fatigue assessment and fatigue damage calculations.
Mean stress ranges

The mean stress range $E[S_r]$ at a detail can be viewed as an important parameter in terms of fatigue assessment. The strain histories obtained from the passage of the test locomotive over the bridge were converted in stress histories and these were then analysed using the rainflow counting procedure to obtain the stress range histogram and, therefore, $E[S_r]$. Mean stress ranges obtained at the mid-span of the stringer and cross-girder, connections of the stringer with cross-girder and the main girder near to the loaded track of the bridge were compared. Locomotive speed of 1km/h (static case) shows that the stress ranges obtained from the FE static analysis are generally more conservative. The highest deviations were observed at the location of stringer mid-span.

There is a better agreement between the results obtained from the dynamic FE analysis and the field measurements for velocities of 10 and 52 km/h. The mean stress ranges obtained from the stringers of the bridge were higher than the cross-girder and main-girder. Overall, a good agreement of the mean stress range between field measurements and dynamic FE analysis under higher velocities is an indication that the fatigue life may be estimated with reasonable accuracy through results obtained from the FE model of a bridge in the form of Figure 3d.

4. CONCLUSIONS

For the case-study bridge, first the effect of modelling assumptions on dynamic behaviour were investigated on a number of different FE models by the eigenvalue analyses. It was found that secondary elements such as bracings may have a significant effect on the frequency of the bridge and it is suggested that they are modelled during an FE analysis. It was also found that in order to capture the out-of-plane and torsional behaviour of the main girders, which lead to the development of fatigue cracks on the case-study bridge, shell elements should be used in the FE models, as beams are unable to capture such type of behaviour. Modelling the investigated span of a bridge using shell elements and the remaining spans using beam elements is suggested as an economical and practical way of obtaining reasonably accurate results. Overall, good agreement between the dominant bridge frequencies obtained from the eigenvalue analysis and empirical results was observed. Later the full-bridge FE model was analysed dynamically under the passage of the test locomotive with different velocities. The comparison of the results obtained from dynamic FE analyses with available field measurements showed that implicit dynamic analysis is a reasonable and computationally efficient method of capturing the dynamic behaviour of a bridge and obtaining dynamic stress histories.

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REFERENCES


