R&D Subsidies, Spillovers and Privatization in Mixed Markets

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Abstract

We examine the use of subsidies to R&D in a mixed and a private duopoly market. We show that the socially optimal R&D subsidy is increasing in the degree of spillovers but it is lower in the private duopoly. The optimal R&D subsidy leads to an increase in total R&D and production, however, it does not lead to the equalisation of per firm output and therefore to an efficient distribution of production costs. We also find that privatization of the public firm reduces R&D activity and welfare in the duopoly market. This result stands even when optimal R&D subsidies are provided.

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Keywords: mixed duopoly, process innovation, R&D subsidies, privatization, spillovers.

1 Introduction

The social benefits and costs associated with the existence of public firms and, as a consequence, the advisability or not of privatization have been the focus of debate in both the academic and the political world. The academic literature on mixed oligopoly has shown that, in the absence of subsidies, privatizing a public firm improves social welfare under a number of different assumptions.[1] However, it has also been shown that if firms' outputs are subsidized privatization does not improve welfare (White, 1996; Pal and White, 1998; Poyago-Theotoky, 2001 and Fjell and Heywood, 2004) .

The above mentioned contributions focused on production related inefficiencies and the role of output subsidies in correcting them. In the absence of subsidies, output levels are suboptimal (as the private firm produces too little) and the distribution of costs across firms in is inefficient (as the public firm tends to produce more but at a higher marginal cost than a private firm). White (1996), Pal and White (1998) and Poyago-Theotoky (2001) showed that if the policy maker uses output subsidies to correct those market failures, social welfare is unaffected by privatization (this is the so-called "irrelevance result"). More recently, Fjell and Heywood (2004) showed that, in fact, social welfare may even decrease if the (ex-)public firm is a Stackelberg leader after privatization. All these results hold in the absence of R&D investment.

Interestingly, the issues of R&D competition and R&D subsidies in the context of mixed oligopolies have been explored relatively less extensively. This clearly contrasts with the key role of R&D subsidies and the role of public firms in facilitating innovation and the development of national innovation systems (Hart, 1998; Katz, 2001). For example, public firms are key players in sectors such as health care, bio-agriculture and energy sectors which are all highly R&D intensive (Aanestad et al., 2003, Oehmke, 2001 and Godø et al., 2003).[2] Moreover, public subsidies to R&D are routinely

[1] Namely, that the public firm is less efficient than the private firm and the marginal cost of production is linear, the market is contestable, the number of private firms is large enough and there are economies of scale or the public firm is incurring losses (De Fraja and Delbono 1989, 1990; Estrin and de Meza, 1995; Anderson, de Palma and Thisse, 1997; Matsumura and Kanda, 2005).

[2] There are even some examples of newly established public firms, such as Crown Fibre Holdings Ltd. a public company in New Zealand that will provide broadband access. See http://www.beehive.govt.nz/release/govt+releases+broadband+investment+proposal.

used by governments to encourage technological innovations. For example, the EU is running the Seventh Framework Programme (FP7) to foster innovation in the EU with an overall budget of €50500 million for the period 2007-2014.[3] Thus, the study of R&D activity and R&D subsidies in the context of mixed oligopolies does not only have a purely academic interest but also clear policy relevance.

Some contributions that have studied R&D activity in mixed markets (Delbono and Denicolò, 1993 and Poyago-Theotoky, 1998) show that a public firm is an effective policy tool in that it achieves efficient outcomes in terms of R&D investment. In the same context, Ishibashi and Matsumura (2006) have shown that if innovation size is endogenized, the presence of a public firm might not be enough to warrant a welfare maximizing outcome. All these contributions focused on patent races (where a new product or a new process are introduced) and did not incorporate R&D subsidies into the frame of analysis.[4]

In this paper, we emphasize this rather neglected aspect in the mixed oligopoly literature, by concentrating on the effect of R&D subsidies in the context of a non-tournament R&D competition model (as opposed to a patent race) where there are distinct appropriability issues as exemplified by the presence of spillovers. We *thus* turn our attention to a mixed oligopoly investing in efficiency-enhancing R&D, that is, R&D investment leading to a reduction in the marginal cost of production. This type of R&D has become particularly relevant in a number of sectors in recent years (for example, health care and energy).[5] It is well-known that the existence of spillovers discourages this type of R&D by private firms as firms cannot fully appropriate the returns to their R&D investments. Therefore, *our first objective* is to study to which extent the provision of subsidies to R&D can alleviate

[3] Another example comes from research in fuel cells and hydrogen technologies in Norway where public funding to R&D was approximately US€18 million in 2001 (Godϕ et al. 2003).

[4] See also Lin and Ogawa (2005), Matsumura and Matsushima (2004) and Nishimori and Ogawa (2002).

[5] For example, due to the steady rise in costs in recent years, firms in the health care sector are trying to innovate to boost their competitiveness. Moreover, policy makers are also calling for more efficiency in this sector (see, for example CBO Testimony by Peter Orszag "Growth in Health Care Costs"). The climate change is also posing significant challenges in terms of energy efficiency to a wide range of industries. A number of governments are subsidizing energy efficiency innovations, including the EU in its FP7 framework.

this problem. To the best of our knowledge, our paper is the first one to study the effect of R&D subsidies in a mixed oligopoly.

As mentioned before, regardless of the existence of R&D competition, mixed markets are affected by two production related inefficiencies (suboptimal output levels and inefficient distribution of production costs). One can argue that an R&D subsidy could be used to tackle not only the inefficiencies related to the R&D activity but also, at least partly, the ones related to production. The reason for this is that the efficiency-enhancing R&D activity will affect output by stimulating production (via the reduction in marginal cost) and by consequence the distribution of production costs (depending on the equilibrium levels of investment by the competing firms). Thus, a *second objective* in our study is to analyze to which extent a subsidy to R&D will have the same or similar effects to an output subsidy.

Our *third objective* is to offer some tentative policy guidelines regarding the use of subsidies before and after privatization and to analyze whether privatization would be welfare-enhancing both in the presence and in the absence of R&D subsidies. This should shed some light on the difference the presence of the subsidy makes to the change in welfare and R&D following privatization. In fact, in the absence of R&D subsidies, it has been shown that privatizing the public firm tends to reduce R&D activity and welfare (Tomaru, 2007, and Heywood and Ye, 2009).[6] Here we want to examine whether this result stands if the government subsidizes R&D activity.

We propose a model that uses a homogeneous good Cournot duopoly undertaking cost-reducing (process) innovation in the presence of spillovers. We introduce subsidies to R&D in the context of a mixed duopoly. We assume that both the public firm and the private firm are ex-ante equally efficient and keep the assumption of increasing marginal cost present in White (1996), Pal and White (1998), Poyago-Theotoky (2001), Fjell and Heywood (2004) among others others. These two assumptions jointly imply that any efficiency differential between the private and the public firm is not exogenously imposed but may endogenously arise from firms' output and R&D choices.

Our results show that the optimal R&D subsidy is increasing in the de-

[6] In particular, Tomaru (2007) shows that privatising the public firm reduces welfare in the context of an international duopoly. Heywood and Ye (2009) show that the optimal extent of (partial) privatization is lower with R&D competition than without it, other things being equal.

gree of spillovers and it is lower in the private relative to the mixed market. R&D subsidies boost R&D, output and welfare in both the private and the mixed markets. We argue that an R&D subsidy may partly serve the same purpose as an output subsidy (as it leads to an increase in total output and may correct to certain extent the inefficient distribution of production costs) although it does not guarantee the equalization of output and therefore marginal costs. Our findings suggest that total R&D investment will decrease following privatization even when R&D subsidies are provided. The privatization of the public firm tends to increase total profits but always reduces consumer and social welfare. Interestingly, the private firm may actually loose out with a privatization in the absence R&D subsidies if the degree of spillovers is high enough.

We structure the paper as follows: In section 2 we present the model. In sections 3 and 4 we solve respectively the mixed and private duopoly cases (with and without subsidies) and study the impact of R&D subsidies on R&D investments and output. In section 5, we compare the private duopoly and mixed duopoly with and without subsidies and discuss the implications for policy-making, with an emphasis on the issue of privatization. Section 6 presents our final remarks. Proofs to propositions and lemmata are relegated to the Appendix.

2 The model

We consider a simple market setting consisting of two firms competing in output. We compare two market structures: a mixed duopoly (where one of the two firms is public) and a private duopoly. A private firm is profit-maximizing while the public firm is assumed to maximize social welfare.[7] In the case of the mixed duopoly, we denote with subscript 0 the public firm and with subscript 1 the private firm. Demand is linear and given by $P(Q) = a - Q$, where $Q = q_i + q_j$, $i \neq j$, $i, j \in \{0, 1\}$ and $Q \leq a$.

Firms invest in R&D to lower their marginal cost of production (process innovation). Then the effective level of R&D, X_i, represents the aggregate

[7] This assumption is standard in the literature on mixed oligopoly (see e.g., Anderson et al. (1997), De Fraja and Delbono (1989), Fjell and Heywood (2004), Pal and White (1998), Poyago-Theotoky (2001) and White (1996, 2002)). A different assumption, such as a weighed objective function for the public firm combining profit and welfare maximization, is useful for addressing the issue of partial-privatization, e.g., Matsumura (1998), but lies outside the scope of the present paper.

reduction in firm i's marginal cost due to R&D, and has two components: the own R&D-output level, x_i, and the competitor's, x_j, influencing firm i via spillovers

$$X_i = x_i + \beta x_j,\ i \neq j,\ i, j \in \{0, 1\}, \tag{1}$$

where the extent of information leakage or degree of spillovers amongst firms is captured by the parameter β, which is exogenously given ($0 \leq \beta \leq 1$).

To avoid situations where the private firm is driven out of the market, we assume the existence of diminishing returns to scale by introducing a quadratic term related to production in the firms' cost function.[8] Thus, firm i's total cost function depends on its level of production, q_i, and the effective level of R&D, X_i,

$$C_i(q_i, X_i) = (c - X_i)q_i + q_i^2,\ \ a > c > 0,\ \ \ i \in \{0, 1\}. \tag{2}$$

This modelling of the cost function reflects the fact that the public firm is 'ex ante' equally efficient as the private firm; that is, in the absence of R&D and for given quantity the cost of production is the same for either firm. Given $C_i(q_i, X_i)$, the marginal cost of production is

$$mc_i = \frac{\partial C_i}{\partial q_i} = (c - X_i) + 2q_i. \tag{3}$$

This increasing marginal cost leads to a higher unit cost for the public firm, as *ceteris paribus* the public firm produces more than the private firm in equilibrium. The realization of this efficiency differential depends on the firm's R&D and production levels. In other words, we do not assume the existence of a cost differential between firms but rather, such a differential arises endogenously in equilibrium (see below). Notice that the effective level of R&D, X_i, affects only the intercept of the marginal cost (i.e., it shifts the marginal cost curve downwards) but not its slope.[9]

R&D is costly with its cost given by $\Gamma(x_i) = \gamma x_i^2$, $\gamma > 0$. This reflects diminishing returns to R&D investment (or effort/input) x_i. For tractability, we set $\gamma = 1$ which ensures non-negativity of all variables. By spending x_i^2 in R&D, a firm can lower its costs by x_i due to its own research effort and by

[8] With a constant marginal cost, the public firm could maximise welfare by pricing at marginal cost and serving the full market. The assumption of increasing marginal cost has been widely adopted in the literature on mixed oligopoly.

[9] This is the same effect that process R&D has on production costs in d'Aspremont and Jacquemin (1988) and followers, where production costs are linear.

an additional amount βx_j via unpaid appropriation of some part of the rival firm's effort. Further, the government subsidizes the R&D level of each firm. Each firm receives a subsidy s per unit of R&D output, $S(x_i) = sx_i$.[10,11] Thus, the private firm maximizes the following

$$\pi_i = P(Q)q_i - C_i(q_i, X_i) - \Gamma(x_i) + S(x_i), \tag{4}$$

whereas the public firm maximizes social welfare defined as the sum of consumer surplus (CS) and producer surplus (PS) net of R&D subsidies.

$$SW = \underbrace{\frac{Q^2}{2}}_{\text{CS}} + \underbrace{(\pi_i + \pi_j)}_{\text{PS}} - \underbrace{s(x_i + x_j)}_{\text{Subsidy}}, \tag{5}$$

which after aggregating yields

$$SW = \frac{Q^2}{2} + \sum_i [(a - q_i - q_j)q_i - C_i(q_i, X_i) - x_i^2]. \tag{6}$$

Note that the subsidy cancels out when aggregating. This implies that the subsidy has no *direct* effect on social welfare, and hence on the objective function of the public firm. However, the public firm's R&D (and output) will still be affected by the subsidy indirectly, through its impact on the private firm's R&D choice.

To study the effects of R&D subsidization on innovation along with the effects of privatization on innovation, firm profitability and welfare, we consider a multi-stage game with observable actions. The time structure of the game unfolds as follows: in stage one, the government chooses the level of the subsidy to R&D to maximize social welfare. In stage two, firms make simultaneously their R&D decisions and then play a standard Cournot game in the third stage. We solve the game for a mixed duopoly (where one of the firms maximizes social welfare while the other maximizes output) and for a private oligopoly (where both firms are profit maximizers). We denote with superscript m the mixed duopoly and with p the private duopoly. The game is solved by backward induction. For comparison purposes, we also

[10] Our results remain robust in the case that the government subsidizes R&D expenditure. The calculations in that case are available from the authors upon request.

[11] The government uses a uniform R&D subsidy. The case of differential or targeted R&D subsidies, although interesting, lies outside the scope of the present paper.

obtain the equilibrium results for the mixed and the private duopolies in the absence of R&D subsidies. [12]

3 Mixed duopoly

In this section we study the Subgame Perfect Nash Equilibrium (SPNE henceforth) for a mixed duopoly. In the last stage, each firm chooses the quantity that maximizes its objective function, taking the quantity of the other firm as given. Solving the system of first-order conditions (FOC henceforth) of the relevant maximization problems, yields the following Cournot-Nash equilibrium quantities[13]

$$q_0^m(x_0, x_1) = \frac{3(a-c) + (4-\beta)x_0 + (4\beta - 1)x_1}{11}, \tag{7}$$

$$q_1^m(x_0, x_1) = \frac{2(a-c) + (3\beta-1)x_0 + (3-\beta)x_1}{11}. \tag{8}$$

Note that $4-\beta \geq 4\beta - 1$ (and $3-\beta \geq 3\beta - 1$), implying that a firm's own R&D contributes more to its output than to its rival's output (except for $\beta = 1$). After substituting these equilibrium quantities into social welfare and into the private firm's profit function, we proceed to solve the R&D stage.

3.1 R&D stage

In the second stage, the public firm chooses R&D (cost reduction) to maximize welfare whereas the private firm decides on its R&D to maximize profit. Given q_0^m and q_1^m from above, the relevant FOCs give rise to the following R&D best-response functions

$$r_0(x_1) = \frac{(31 + 28\beta)(a-c) - [14 - \beta(87 - 14\beta)]x_1}{197 + 14\beta(2 - 3\beta)}, \tag{9}$$

[12] In these cases, the game is reduced to the last two stages and the subsidy rate is zero. To avoid unnecessary duplications, we will solve the more general cases (mixed and private duopolies with subsidies) and will obtain the equilibrium results for the cases without subsidies by setting $s = 0$ after having obtained the solutions to the second stage.

[13] We have checked that the second order conditions (and stability conditions where relevant) for the solutions to all the stages of both the mixed and the private duopoly games. They are fulfilled in all cases. Although not reported here, they are available from the authors upon request.

$$r_1(x_0) = \frac{8(3-\beta)(a-c) - 4(3-\beta)(1-3\beta)x_0 + 121s}{206 + 4\beta(6-\beta)}. \qquad (10)$$

We make a couple of observations regarding the best-response functions above. First, the best-response function of the public firm does not depend on the subsidy, s. This implies that the subsidy does not affect directly the level of R&D investment of the public firm. In fact the subsidy only affects indirectly the public firm through the level of R&D investment of its counterpart. This is because the subsidy has no effect on the objective function of the public firm (it is an inflow for the public firm but an outflow for the government, therefore, it cancels out in the social welfare function). However, the subsidy still affects the equilibrium level of R&D by the public firm through the effect it exerts on the private firm's R&D. Second, the slope of $r_0(x_1)$ and $r_1(x_0)$ is negative for low values of β and positive for higher values of β, implying that R&D is a strategic substitute/complement depending on the extent of spillovers. The following lemma elaborates.

Lemma 1 *In the mixed duopoly, R&D is*
i) a strategic substitute for both firms for $\beta < 0.17$ *,*
ii) a strategic substitute for the private firm but a strategic complement for the public firm for $0.17 < \beta < 0.33$ *and*
iii) a strategic complement for both firms for $\beta > 0.33$.

Lemma 1 shows that R&D is initially a strategic substitute and becomes a strategic complement, as spillovers intensify, for both the private and the public firm. The intuition for this is the following. An increase in the R&D investment by firm i leads in first instance to a decrease in the output by its competitor (which now is comparatively less efficient), thereby reducing its incentives to conduct R&D.[14] However, a second effect arises due to the existence of spillovers: An increase in the R&D investment by firm i leads to a decrease in firm j's marginal cost through the spillovers, which will have a positive effect on firm j's output and incentives to conduct R&D. If the degree of spillovers is high enough, the second effect will outweigh the first one and as a consequence, the R&D investment on both firms are positively related (strategic complements). Lemma 1 also states that the threshold value of the spillovers which turns R&D into a strategic complement (rather

[14] As R&D investment carries a fixed cost, the higher the output by firm i, the higher the profitability of conducting R&D for firm i.

than substitute) is lower for the public firm; this is due to the fact that the public firm places a higher value to the spillovers than the private firm, as it internalizes their social value.

Solving the system of (9) and (10) we obtain the R&D equilibrium outcomes as a function of the subsidy s

$$x_0^m(s) = \frac{2[25 + 2\beta(18 - \beta)](a - c) - s[14 - \beta(87 - 14\beta)]}{2[167 + 2\beta(25 - \beta)(1 - \beta)]} \tag{11}$$

$$x_1^m(s) = \frac{4(9 - \beta^2)(a - c) + s[197 + 14\beta(2 - 3\beta)]}{2[167 + 2\beta(25 - \beta)(1 - \beta)]}. \tag{12}$$

The equilibrium quantities can also be rewritten as

$$q_0^m(s) = \frac{2[53 + \beta(31 - 18\beta)](a - c) + s[-23 + \beta(102 + \beta - 14\beta^2)]}{2[167 + 2\beta(25 - \beta)(1 - \beta)]}, \tag{13}$$

$$q_1^m(s) = \frac{11\,[2(3 + \beta)(a - c) + s(5 - \beta(2 - \beta))]}{2[167 + 2\beta(25 - \beta)(1 - \beta)]}. \tag{14}$$

Setting $s = 0$ in (11), (12), (14) and (13), we obtain the **SPNE solutions for the mixed duopoly without subsidies**. Table 1 summarizes these results.

Table 1: Mixed Duopoly without Subsidies ($s = 0$)	
$q_0^m\|_{s=0} = \frac{[53+\beta(31-18\beta)](a-c)}{167+2\beta(25-\beta)(1-\beta)}$	$q_1^m\|_{s=0} = \frac{11(3+\beta)(a-c)}{167+2\beta(25-\beta)(1-\beta)}$
$x_0^m\|_{s=0} = \frac{[25+2\beta(18-\beta)](a-c)}{167+2\beta(25-\beta)(1-\beta)}$	$x_1^m\|_{s=0} = \frac{2(9-\beta^2)(a-c)}{167+2\beta(25-\beta)(1-\beta)}$
$\pi_0^m\|_{s=0} = \frac{(2184+1486\beta-2143\beta^2-972\beta^3+320\beta^4)(a-c)^2}{(167+2\beta(25-\beta)(1-\beta))^2}$	
$\pi_1^m\|_{s=0} = \frac{2(3+\beta)^2(103+12\beta-2\beta^2)(a-c)^2}{(167+2\beta(25-\beta)(1-\beta))^2}$	
$CS^m\|_{s=0} = \frac{2(43+21\beta-9\beta^2)^2(a-c)^2}{(167+2\beta(25-\beta)(1-\beta))^2}$	
$SW^m\|_{s=0} = \frac{(7736+6550\beta-2495\beta^3-1728\beta^3+478\beta^4)(a-c)^2}{(167+2\beta(25-\beta)(1-\beta))^2}$	

It is relatively straightforward to observe from the results in Table 1 that, in the absence of subsidies, the public firm invests more on R&D than the private firm, that is, $x_0^m|_{s=0} > x_1^m|_{s=0}$.[15] The derivatives of $x_0^m|_{s=0}$ and $x_1^m|_{s=0}$ with respect to β are:

[15]It is also interesting to note that the public firm's profits are unambiguosly positive for any β.

$$\left.\frac{\partial x_0^m}{\partial \beta}\right|_{s=0} = \frac{2(a-c)(2381+966\beta+811\beta^2-72\beta^3+2\beta^4)}{[167+2\beta(25-\beta)(1-\beta)]^2}, \quad (15)$$

$$\left.\frac{\partial x_1^m}{\partial \beta}\right|_{s=0} = \frac{2(a-c)(-455+602\beta-104\beta^2+2\beta^4)}{[167+2\beta(25-\beta)(1-\beta)]^2}. \quad (16)$$

From (15), it is clear that the public firm's R&D investment is increasing in β even without R&D subsidies, since investing in R&D is more socially desirable the higher the degree of spillovers is.[16] In contrast, the private firm's R&D investment is only increasing in β for very high degrees of spillover ($\beta > 0.88$) in the absence of subsidies. In other words, as the degree of spillovers increases, the private firm tends to free-ride more heavily on the public firm's investment, except when the degree of spillovers is almost perfect.

From Table 1, it is easy to see that the public firm produces more than the private firm in the equilibrium without subsidies, that is, $q_0^m|_{s=0} > q_1^m|_{s=0}$. However, the public firm also invests more on R&D than the private firm ($x_0^m|_{s=0} > x_1^m|_{s=0}$). As a consequence, it is yet unclear whether the marginal costs of production of the public firm will be lower than that of private firm in the equilibrium.[17] Substituting $x_0^m|_{s=0}$, $x_1^m|_{s=0}$, $q_0^m|_{s=0}$, $q_1^m|_{s=0}$ into (3) yields

$$mc_0^m|_{s=0} = c + \frac{(81+8\beta-34\beta^2+2\beta^3)(a-c)}{167+2\beta(25-\beta)(1-\beta)}, \quad (17)$$

$$mc_1^m|_{s=0} = c + \frac{(43-3\beta-34\beta^2+2\beta^3)(a-c)}{167+2\beta(25-\beta)(1-\beta)}. \quad (18)$$

Comparing $mc_0^m|_{s=0}$ and $mc_1^m|_{s=0}$, we now establish that the public firm's marginal cost of production is higher than that of the private firm in equilibrium, indicating that there are inefficiencies related to the distribution of the productions costs (it would best if the last units of output produced by the public firm had been produced by the private firm).

Next, we proceed to examine the effect that a positive R&D subsidy has in correcting the inefficiencies associated to innovation and output and

[16] The higher β is, the more effectively knowledge is transferred across firms and therefore, the higher the efficiency gains R&D investment

[17] Recall that the marginal cost of production is increasing in the output level.

marginal cost levels by means of comparative statics.

3.1.1 R&D subsidization and R&D output (cost reduction)

The following lemma shows the effect of changes in the subsidy rate on firms' R&D output.

Lemma 2
i) The private firm's R&D is increasing in the subsidy for all $\beta \in [0, 1]$.
ii) The public firm's R&D is increasing in the subsidy if $\beta \geq 0.17$ *and is decreasing in the subsidy if* $\beta < 0.17$.
iii) Total R&D output, $x_0^m + x_1^m$*, is increasing in the subsidy for all* $\beta \in [0, 1]$.

Part (i) of Lemma 2 follows from the best-response function of the private firm, which shifts out as s increases (see equation (10)). It follows that the private firm's R&D investment is always increasing in the subsidy, due to the fact that the subsidy is a net inflow for the private firm. However, the same does not apply to the public firm, as the subsidy also enters the social welfare function as a cost, cancelling out the positive effect of the subsidy. Thus, the subsidy has only an indirect effect on the public firm's behavior through the private firm's R&D, as explained above. The relationship between the two firms' R&D investments (subtitution or complementarity) will determine the effect of an increase in the subsidy on the public firm's R&D. We know from Lemma 1 that R&D investments are strategic substitutes (complements) from the point of view of the public firm if $\beta < (\geq) 0.17$. Thus, if $\beta \geq (<)\ 0.17$, an increase in s leads to an increase in the private firm's R&D investment in first instance and, as a consequence of this, to an increase (decrease) in the public firm's investment due to strategic complementarity (substitution), as stated in part (ii).

Finally, part (iii) shows that the decrease in the public firms' R&D level (if it takes place) will be outweighed by the increase in the private firm's, resulting in an increase in the total R&D. That is, the public firm will not reduce its R&D investment (if at all) as s increases to the extent of lowering aggregate R&D levels.

3.1.2 R&D subsidization and output levels (costs distribution)

In the next lemma, we analyze the effect of changes in the subsidy rate on the equilibrium output levels.

Lemma 3 *i) The output of the private firm is increasing in the subsidy for all* $\beta \in [0, 1]$.
ii) The output of the public firm is increasing in the subsidy if $\beta \geq 0.23$ *and decreasing if* ($\beta < 0.23$).
iii) Total output, $q_0^m + q_1^m$, *is increasing in the subsidy.*

Lemma 3 states that a threshold value for the spillover exists such that the net impact of the subsidy on the public firm's output can be positive or negative. Two effects are interacting and determining this result: (a) The R&D subsidy affects the public firm's output via the effect it exerts on its own R&D. In particular, we know from Lemma 2 that the public firm's R&D is decreasing (increasing) in s for $\beta < (>) 0.17$. (b) The R&D subsidy impacts the public firms' output via the output of the private firm. An increase in the subsidy leads to an increase in the private firm's R&D and output, which in turn leads to a decrease in the public firm's output. The indirect effect in (b) will only be compensated by the effect in (a) for $\beta > 0.23$. Regarding the private firm, the result is clear-cut: a higher subsidy will always lead to higher output, as the positive effect of the private R&D on the private firm's output dominates the negative effect through the public firm's output. As well, total output is increasing in s, which highlights the positive association between R&D and output decisions. Thus, an R&D subsidy may serve a similar purpose to an output subsidy, as it boosts total output levels.

White (1996) has shown that an output subsidy results in the redistribution of output from the (ex-post) higher marginal cost public firm to the lower-marginal cost private firm. Here we want to explore whether an R&D subsidy could serve a similar purpose. Interestingly, we can infer from Lemmata 2 and 3 that the effectiveness of an R&D subsidy in redistributing costs depends crucially on the extent of spillovers. In fact, both R&D investments and outputs by the two firms converge as s increases for low values of β. Therefore, at least for low values of β, the R&D subsidy could improve the distribution of production costs across firms. In the next section, we derive the optimal R&D subsidy and analyze whether it can lead to the equalization of output and consequently marginal costs across firms.

In addition, we investigate the role of the subsidy on social welfare.

3.2 Optimal R&D subsidy and its effects

The government chooses the value of the R&D subsidy that maximizes welfare. Substituting equilibrium R&D and quantities into the social welfare objective function and solving the FOC with respect to s, we find the equilibrium R&D subsidy.

$$s^m = \frac{2[3+\beta(32+17\beta-9\beta^2)](a-c)}{162+\beta[56-\beta(101-7\beta^2)]} \tag{19}$$

Using this we calculate the equilibrium solutions for the mixed duopoly with optimal subsidies, summarized in Table 2.

Table 2: Mixed Duopoly - Equilibrium Solutions ($s = s^m$)

$q_0^m\|_{s=s^m} = \frac{3[17+5\beta(2-\beta)](a-c)}{162+\beta[56-\beta(101-7\beta^2)]}$	$q_1^m\|_{s=s^m} = \frac{11(3-\beta)(1+\beta)(a-c)}{162+\beta[56-\beta(101-7\beta^2)]}$
$x_0^m\|_{s=s^m} = \frac{[24+7\beta(5+\beta(1-\beta))](a-c)}{162+\beta[56-\beta(101-7\beta^2)]}$	$x_1^m\|_{s=s^m} = \frac{[21+\beta(38+7\beta(1-\beta))](a-c)}{162+\beta[56-\beta(101-7\beta^2)]}$
$\pi_0^m\|_{s=s^m} = \frac{[2169+\beta(3126+907\beta+110\beta^2-174\beta^3-266\beta^4+77\beta^5)](a-c)^2}{[162+\beta(56-\beta(101-7\beta^2))]^2}$	
$\pi_1^m\|_{s=s^m} = \frac{[1863+\beta(2880+966\beta+114\beta^2-169\beta^3-266\beta^4+77\beta^5)](a-c)^2}{[162+\beta(56-\beta(101-7\beta^2))]^2}$	
$CS^m\|_{s=s^m} = \frac{2[42+13\beta(2-\beta)]^2(a-c)^2}{[162+\beta(56-\beta(101-7\beta^2))]^2}$	
$SW^m\|_{s=s^m} = \frac{[45+14\beta(2-\beta)](a-c)^2}{162+\beta[56-\beta(101-7\beta^2)]}$	

From table 2, it is straightforward to see that when R&D subsidies are provided the public firm produces more and invests more on R&D than the private firm. That is, $q_0^m > q_1^m$ and $x_0^m > x_1^m$.

The following Proposition characterizes the optimal R&D subsidy.

Proposition 1 *In the mixed duopoly, the optimal R&D subsidy is positive and increasing in the magnitude of the spillovers.*

According to Proposition 1, the R&D subsidy in the mixed duopoly is positive and increasing in the spillovers. The rationale behind this result stems from the role that the R&D subsidy plays in correcting a market failure associated with the imperfect appropriability of R&D due to the existence of spillovers. Next, we elaborate on this.

Recall that we have stated before that, in the absence of R&D subsidies, the existence of spillovers generates a free-riding problem, as the private firm tends to invest less on R&D as β increases, except for very high βs.

An optimal subsidy to R&D alleviates this problem by inducing the private firm to invest more as β increases. This can be seen from the derivative of x_1^m with respect to β:

$$\left.\frac{\partial x_1^m}{\partial \beta}\right|_{s=s^m} = \frac{(a-c)(4980+\beta\gamma_2)}{[162+\beta(56-\beta(101-7\beta^2))]^2}, \tag{20}$$

where $\gamma_1 = (6510+828\beta-1372\beta^2-91\beta^3-98\beta^4+49\beta^5) > 0\ \forall\beta\ ,\ \beta \in [0,1]$. Moreover, as in the case without subsidies, the public firm's R&D investment is increasing in β when optimal subsidies to R&D are provided:

$$\left.\frac{\partial x_0^m}{\partial \beta}\right|_{s=s^m} = \frac{(a-c)(4326+\beta\gamma_1)}{[162+\beta(56-\beta(101-7\beta^2))]^2}, \tag{21}$$

where where $\gamma_2 = (7116+525\beta-1456\beta^2-28\beta^3-98\beta^4+49\beta^5) > 0$. As before, the reason is that investing in R&D is more socially desirable the higher the degree of spillovers. Thus, when an optimal subsidy to R&D is used, the R&D levels of **both** the private and the public firm are increasing in the degree of spillovers. This emphasizes the role of the R&D subsidy in correcting the free-riding problem that exists in the absence of subsidization.[18]

We concluded the previous subsection leaving open the question whether an optimal R&D subsidy would lead to the equalization of firms' outputs and therefore costs. We have previously argued (see section 3.2) that a positive R&D subsidy will tackle the inefficiencies derived from R&D competition and could potentially address (at least partly) the inefficient distribution of production costs. q_0^m and q_1^m are reported in Table 2. It is relatively straightforward to see that $q_0^m > q_1^m$ for any $0 < \beta < 1$. However, we also know that $x_0^m > x_1^m$, which implies that the intercept of marginal cost function is lower for the public firm than for the private firm. Substituting x_0^m, x_1^m, q_0^m, q_1^m into (3), we find the marginal cost for the two firms in

[18] Interestingly, total R&D investment levels are increasing in β not only when optimal R&D subsidies are provided but also if subsidies are not provided (since $\left.\frac{\partial x_0^m}{\partial \beta}\right|_{s=0} > Abs\left[\left.\frac{\partial x_1^m}{\partial \beta}\right|_{s=0}\right]$). In other words, the mere presence of the private firm partially corrects the problem derived from the imperfect appropriability of the R&D results. However, as shown in Lemma 2, total R&D investment is higher in the presence of subsidies than in their absence. This highlights the additional benefits of employing R&D subsidisies even in cases where a public firm is being used by the government as a policy tool to tackle the inefficiencies derived from R&D activity.

equilibrium:

$$mc_0^m|_{s=s^m} = c + \frac{(78 + 4\beta - 75\beta^2 + 7\beta^4)(a-c)}{167 + 2\beta(25-\beta)(1-\beta)} \quad (22)$$

$$mc_1^m|_{s=s^m} = c + \frac{(45 - 18\beta - 64\beta^2 + 7\beta^4)(a-c)}{167 + 2\beta(25-\beta)(1-\beta)}. \quad (23)$$

Comparing the marginal cost in equilibrium of the public and the private firm under subsidization, we can state the following:

Lemma 4 $mc_0^m|_{s=s^m} > mc_1^m|_{s=s^m}$ *for all* $\beta \in [0, 1]$.

In other words, the public firm's marginal cost in equilibrium is higher than that of the private firm. An optimal R&D subsidy fails to bring about the equalization of output across firms and therefore it does not lead to an efficient distribution of the marginal costs. Our results show that R&D subsidies will not be enough to correct the inefficiencies related to distribution of production costs when there are spillovers either.[19] Thus, the use of an R&D subsidy only achieves a second best outcome. In fact, given the existence of both output and R&D related subsidies, achieving the social optimum (first best) would require the use of both R&D and output related instruments. In fact, in a model with R&D competition but without spillovers, Zikos (2007) has showed that a combination of taxes on R&D and a subsidy to output, could correct both R&D and production related inefficiencies. However, it can be argued that the public may not be likely to accept a policy involving taxation of R&D investment and therefore such a policy may not be implementable by the government. Hence, the relevance of studying the second best. Our findings point towards the trade-off facing policy makers when using R&D subsidies in mixed markets with spillovers: R&D subsidies encourage total R&D investment and counteract the private firms' incentive to free-ride on the public firm's investment but will do this at the expense of perpetuating the inefficient allocation of production costs.[20]

[19] Note that the R&D subsidy does not affect the equilibrium outcomes in the last stage (output).

[20] The reader may wonder whether the SPNE investments are higher than those that minimise cost. Interestingly, the magnitude of the subsidy does not affect the cost minimising investments and, in fact, both firm's cost minimising investments are higher than the SPNE R&D investments both without and with an (optimal) R&D subsidy. Details of these calculations are available from the authors upon request.

Before closing this section, we compare welfare levels in the mixed oligopoly with and without subsidies. The following result obtains

Lemma 5 *$SW^m|_{s=s^m} > SW^m|_{s=0}$ and $\frac{\partial(SW^m|_{s=s^m} - SW^m|_{s=0})}{\partial\beta} > 0$ for all $\beta \in [0,1]$.*

That is, social welfare will be higher in the mixed oligopoly when an (optimal) R&D subsidy is used than without a subsidy even without spillovers. Furthermore, this difference is strictly increasing in the degree of spillovers. It is worth noting that this result has been derived in a context where subsidies do not involve any deadweight loss. However, it implies that there is some scope to assume deadweight losses in order to provide subsidies, as they will boost welfare, particularly as the degree of spillovers intensifies.

The following proposition summarizes the discussion presented in this section:

Proposition 2 *In the mixed duopoly, the use of an optimal R&D subsidy leads to an increase in total R&D investment, output and welfare although is not sufficient to guarantee the efficient distribution of production costs.*

4 Private duopoly

In this case both firms are profit-maximizers. In the final stage of the game, both firms choose output to maximize profits. Solving the system of the associated FOCs, we obtain the stage-three equilibrium outputs, which is common for both regimes (with and without subsidies):

$$q_i^p = \frac{3(a-c) + (4-\beta)x_i + (4\beta - 1)x_j}{15}, \quad i \neq j, \quad i,j \in \{0,1\}. \tag{24}$$

Substituting these into the profit function of both firms and solving the system of FOCs, we obtain the following R&D best-response functions

$$r_i^p(x_j) = \frac{12(4-\beta)(a-c) - 4(4-\beta)(1-4\beta)x_j + 225s}{2[193 + 2\beta(8-\beta)]}, \ i \neq j, \ i,j \in \{0,1\}. \tag{25}$$

As in the mixed duopoly, R&D is initially a strategic substitute and becomes a strategic complement, as the degree of spillovers increases.[21] Solving the system of the R&D best-response functions, we find the equilibrium

[21] Here, the threshold value for the spillover degree is $\beta = 0.25$.

R&D outputs

$$x_i^p(s) = \frac{4(4-\beta)(a-c)+75s}{2[67-2\beta(3-\beta)]}, \quad i \in \{0,1\}. \tag{26}$$

Similarly to the effect of the subsidy on private R&D in the mixed duopoly, note that R&D is also positively related to the subsidy in the private duopoly. The equilibrium output as function of the subsidy is written as

$$q_i^p(s) = \frac{15\,[2(a-c)+s(1+\beta)]}{2[67-2\beta(3-\beta)]}, \quad i \in \{0,1\}. \tag{27}$$

In this case too, the quantities produced depend positively on the amount of the subsidy with this effect being the outcome of the positive R&D-output association. Substituting equilibrium R&D and equilibrium quantities into the expression for social welfare and performing the maximization with respect to s we obtain

$$s^p = \frac{2(1+11\beta)(a-c)}{3[22-3\beta(2+\beta)]}. \tag{28}$$

The result above is the equilibrium optimal R&D subsidy in the private duopoly. Analogously to s^m, one can easily establish that s^p is positive and increasing in β.

The R&D and output equilibrium solutions for the public and the private firm's R&D in the absence of subsidies are readily obtained from x_i^p and q_i^p by setting $s=0$ and $s=s^p$ respectively,summarized in the following tables.

Table 3: Private Duopoly – Equilibrium Solutions without Subsidies ($s=0$)
$q_i^p\|_{s=0} = \frac{15(a-c)}{67-6\beta+2\beta^2}$, $i \in \{0,1\}$
$x_i^p\|_{s=0} = \frac{2(4-\beta)(a-c)}{67-6\beta+2\beta^2}$, $i \in \{0,1\}$
$\pi_i^p\|_{s=0} = \frac{2(193+16\beta-2\beta^2)(a-c)^2}{(67-6\beta+2\beta^2)^2}$, $i \in \{0,1\}$
$CS^p\|_{s=0} = \frac{450(a-c)^2}{(67-6\beta+2\beta^2)^2}$
$SW^p\|_{s=0} = \frac{2(611+32\beta-4\beta^2)(a-c)^2}{(67-6\beta+2\beta^2)^2}$

Table 4: Private Duopoly Equilibrium Solutions with Optimal Subsidies ($s = s^p$)
$q_i^p\|_{s=s^p} = \frac{5(a-c)}{22-3\beta(2+\beta)}$, $i \in \{0,1\}$
$x_i^p\|_{s=s^p} = \frac{3(1+\beta)(a-c)}{22-3\beta(2+\beta)}$, $i \in \{0,1\}$
$\pi_i^p\|_{s=s^p} = \frac{[43+\beta(6+13\beta)](a-c)^2}{[22-3\beta(2+\beta)]^2}$, $i \in \{0,1\}$
$CS^p\|_{s=s^p} = \frac{50(a-c)^2}{[22-3\beta(2+\beta)]^2}$
$SW^p\|_{s=s^p} = \frac{6(a-c)^2}{22-3\beta(2+\beta)}$

For completeness, we also analyze the role of the subsidies in the private duopoly in correcting the market failures associated with the existence of spillovers. It is interesting to note that without subsidies, the derivative of x_i^p with respect to β is negative:

$$\left.\frac{\partial x_i^p}{\partial \beta}\right|_{s=0} = -\frac{2(43+16\beta+2\beta^2)(a-c)}{[67-2\beta(3-\beta)]^2} < 0. \tag{29}$$

That is, as β increases, firms tend to invest less on R&D. The reason for this is that firms tend to free-ride more on each other's R&D efforts as β increases. Again, this is socially inefficient because R&D becomes more socially desirable as the degree of spillovers increases. Analogously to the mixed oligopoly case, a R&D subsidy alleviates this problem.

In contrast, in the case with an optimal subsidy, the derivative of x_i^p with respect to β is positive:

$$\left.\frac{\partial x_i^p}{\partial \beta}\right|_{s=s^p} = \frac{3(28+6\beta+3\beta^2)(a-c)}{(67-6\beta+2\beta^2)^2} > 0. \tag{30}$$

Thus, in the private duopoly case, when subsidies are provided, R&D is increasing in the degree of spillovers. This implies that an R&D subsidy counteracts the incentives to reduce the level of R&D investment as the degree of spillovers increases. Furthermore, the total level of R&D will be higher when subsidies are used than when they are not. This can easily be seen from the fact that the equilibrium level of R&D (26) is increasing in s. Finally, comparing welfare levels with and without subsidies (see Tables 3 and 4), we can state that the use of an optimal subsidy boosts social welfare in the private oligopoly.[22] The next proposition summarizes the discussion

[22] Moreover, as in the case of the mixed oligopoly, the higher the degree of spillovers is, the higher the difference between the level of social that could be achieved with optimal

presented in this section:

Proposition 3 *In the private duopoly, the use of an optimal R&D subsidy leads to an increase in total R&D investment, output and welfare.*

5 Welfare comparisons

In this section we compare R&D output and quantity produced across the two market configurations and provide some tentative policy guidelines with respect to privatizing the public firm. We capture privatization in a very simple way: the private duopoly is equivalent to a setup where the public firm maximizes its own profit; in other words, the public firm is privatized. We pay particular attention to the effect of the presence of the R&D subsidy to the changes in R&D, output and welfare levels following the privatisation of the public firm. In order to do this, we compare the two settings (mixed and private market) in detail both with and without R&D subsidization.

5.1 No R&D subsidies ($s = 0$)

Without subsidies to R&D in either the mixed or the private duopoly, the following holds:[23]

Proposition 4 *When R&D subsidies are not provided in either the mixed or the private duopoly, the following hold:*

(i) $x_0^m|_{s=0} > x_0^p|_{s=0}$ *for all* $\beta \in [0, 1]$,
(ii) $x_1^m|_{s=0} < (\geqslant)\ x_1^p|_{s=0}$ *if* $\beta < (\geqslant) 0.98$,
(iii) $q_0^m|_{s=0} > q_0^p|_{s=0}$ *for all* $\beta \in [0, 1]$,
(iv) $q_1^m|_{s=0} > (\leqslant)\ q_1^p|_{s=0}$ *if* $\beta > (\leqslant) 0.76$,
(v) $\pi_0^m|_{s=0} < \pi_0^p|_{s=0}$ *for all* $\beta \in [0, 1]$, *and*
(vi) $\pi_1^m|_{s=0} < (\geqslant)\ \pi_1^p|_{s=0}$ *if* $\beta < (\geqslant) 0.75$.

The intuition behind the remarks about the public firm's behavior in the absence of R&D subsidies (that is, parts (i), (iii) and (v)) seems quite clear:

R&D subsidies and without subsidies.

[23] Our analysis in this subsection (comparison without subsidies, $s = 0$) is similar to the analysis in Heywood and Ye (2009). Our analysis differs from theirs in that we consider intermediate values of spillovers (which are critical for some of our results) but we do not contemplate the case of partial privatization. Therefore, our results in this section complement the results in Heywood and Ye (2009).

As the public firm does not maximize profits but social welfare, it will tend to invest and produce more and obtain lower profits than a private firm in a private duopoly.

Although the results regarding the private firm appear less intuitive, they can be reconciled when taking into account the existence of spillovers. Part (iv) states that, in the absence of subsidies, a private firm may produce more in a mixed market than in a private one for high enough degrees of spillovers. In principle, a private firm tends to produce more in a private market than in a mixed one.[24] However, a private firm will experience a higher reduction in its marginal cost through spillovers in a mixed market than in a private market, since its competitor in a mixed market invests more on R&D than a private firm would do in a private market, particularly as the degree of spillovers increases. This second effect (higher efficiency gains through spillovers in a mixed market) may lead the private firm to produce more and obtain higher profits in the mixed market for sufficiently high βs, as parts (ii) and (vi) state. Interestingly, this effect could even induce the private firm to invest more in R&D in a mixed market than in a private one, although this requires practically perfect spillovers as stated in part (ii).

The next proposition contains the comparison between the mixed and the private market in terms of total R&D, output and profits.

Proposition 5 *In the absence of R&D subsidies:*
(i) $x_0^m|_{s=0} + x_1^m|_{s=0} > 2\, x_i^p|_{s=0}$ *and* $q_0^m|_{s=0} + q_1^m|_{s=0} > 2\, q_i^p|_{s=0}$ *for all* $\beta \in [0,1]$*;*
(ii) $2\, \pi_i^p|_{s=0} > \pi_0^m|_{s=0} + \pi_1^m|_{s=0}$ *for all* $\beta \in [0,1]$.

Although in the absence of subsidies, the private firm may invest less on R&D and produce less output in a mixed oligopoly than in a private one, the higher R&D and production activity by the public firm will compensate for this, leading to higher aggregate R&D and output levels in the mixed than in the private market. Both firms's profits increase with privatization. Even when the public firm's profits will be reduced after the privatization ($\beta > 0.75$), the aggregate profits will still be higher after the privatization.

Finally, regarding social welfare, the following result obtains:

[24] As a reaction to its competitor which will tend to produce more if it is public than if it is private.

Proposition 6 *In the absence of subsidies to R&D,* $SW^m|_{s=0} > SW^p|_{s=0}$ *for all* $\beta \in [0, 1]$.

The above proposition states that, even without subsidization, social welfare is higher in a mixed oligopoly than in a private duopoly and therefore, privatization would result in a reduction in total surplus. The presence of the public firm in the market boosts aggregate R&D investment and output. This result is in line with the previous literature (Tomaru, 2007 and Heywood and Ye, 2009). Furthermore, our analysis also indicates the existence of a public firm in the market can even benefit its (private) competitor via spillovers. Thus, a policy involving privatization without using R&D subsidies may even damage the private firm's profits, particularly for high degrees of spillovers.

5.2 Optimal R&D subsidies (s^m,s^p)

With optimal subsidies to R&D in both the mixed and the private duopoly, the following holds:

Proposition 7 *When socially optimal subsidies to R&D are provided both in the mixed and the private duopoly, the following hold:*

(i) $x_0^m|_{s=s_m} > x_0^p|_{s=s_p}$ *for all* $\beta \in [0, 1]$,
(ii) $x_1^m|_{s=s_m} < (\geqslant)\ x_1^p|_{s=s_p}$, *if and only if* $\beta < (\geqslant) 0.44$,
(iii) $q_0^m|_{s=s_m} > q_0^p|_{s=s_p}$ *for all* $\beta \in [0, 1]$,
(iv) $q_1^m|_{s=s_m} > q_1^p|_{s=s_p}$ *for all* $\beta \in [0, 1]$,
(v) $\pi_0^m|_{s=s_m} < (\geqslant)\ \pi_0^p|_{s=s_p}$, *if and only if* $\beta < (\geqslant) 0.65$ *and*
(vi) $\pi_1^m|_{s=s_m 0} < \pi_1^p|_{s=s_p}$ *for all* $\beta \in [0, 1]$.

Parts (i) and (iii) provide the same qualitative results as in the case without R&D subsidies. That is, the public firm invests more and produces more than a firm in a private duopoly. Part (ii) states that the private firm invests more in R&D in a mixed duopoly than in a private duopoly for intermediate to high degrees of spillovers. As a public firm will invest more on R&D than a firm in a private market (see part (i)) and the R&D investments are strategic complements for high degrees of spillovers and substitutes for lower degrees of spillovers (as lemma (1) shows), it follows that the private firm will tend to invest more (less) in R&D in a mixed market than in a private one for high (low) degrees of spillovers.

Furthermore, a private firm produces more and obtains more profits in a private duopoly than in the mixed duopoly for any β, as parts (iv) and (vi) state. This differs from the result presented in the case without subsidies, where we argued that the private firm could benefit from the presence of a public firm via spillovers for sufficiently high β and this could compensate for the fiercer competitor in the mixed market. With R&D subsidies, there is not such a large difference between the efficiency gains from spillovers in the mixed and in the private market, as the R&D subsidy stimulates investment by the competing private firm. Thus, the negative effect from the more intense competition in the mixed market prevails. Part (v) suggests that a public firm could obtain higher profits than a firm in a private market if $\beta > 0.65$. In addition to facing less strong competition in the mixed than in the private market,[25] the public firm may even obtain higher efficiency gains via spillovers in a mixed market, as long as the spillover degree is high enough.[26] Thus, for high degrees of spillovers, the two effects go in the same direction and the public firm obtains higher profits in the mixed market than if it were privatized.

Proposition 8 *When optimal R&D subsidies are provided:*
(i) $x_0^m|_{s=s_m} + x_1^m|_{s=s_m} > 2\, x_0^p|_{s=s_p}$ *and* $q_0^m|_{s=s_m} + q_1^m|_{s=s_m} > 2\, q_1^p|_{s=s_p}$ *for all* $\beta \in [0,1]$*;*
(ii) $\pi_0^m|_{s=s_m} + \pi_1^m|_{s=s_m 0} < (\geqslant) 2\, \pi_0^p|_{s=s_p}$ *if* $\beta < (\geqslant) 0.94$.

The above proposition states that, as in the case without R&D subsidies, the presence of a public firm boosts total R&D investment and output when optimal R&D subsidies are provided. Total profit tends to be higher in the private market than in the mixed market when optimal R&D subsidies are provided. This does not hold for very high degrees of spillovers. When (optimal) R&D subsidies are provided, the private firm will always be worse-off in a mixed market than in a private market. However, the public firm may obtain higher profits in the mixed market than if privatised and even compensate the lower profits by the private firm for very high degrees of spillovers.

Bringing together the effect of a privatization on consumer surplus and on producer surplus, we can state the following:

[25] Because a private firm produces less in a mixed market than in a private one.

[26] For $\beta > 0.44$, a private firm invests more on R&D in a mixed market than in a private market if R&D subsidies are provided, as part (ii) states.

Proposition 9 *Under a policy of providing socially optimal subsidies to R&D,* $SW^m|_{s=s^m} > SW^p|_{s=s^p}$ *for all* $\beta \in [0, 1]$.

Although the privatization of the public firm would reduce aggregate output levels and thus lower consumer surplus, it would also lead to higher producer surplus not only for the private firm but also for the privatized (ex-public) firm as long as spillovers are not too high. It turns out that the former negative effect (lower consumer surplus) dominates the latter positive one (higher producer surplus unless spillovers are almost perfect), inducing a decline in social welfare following privatization.

Comparing the optimal subsidy to R&D in the mixed and in the private market, we establish the following:

Proposition 10 $s^m > s^p$ *for all* $\beta \in [0, 1]$.

In other words, the government should provide a larger subsidy in the mixed market than in the fully private market, *ceteris paribus*. The intuition for this results from the fact that output will be higher in a mixed market than in a private market, therefore rendering investing in R&D more socially profitable in a mixed than in a private market.

5.3 Discussion

The results obtained in the two previous subsections yield several interesting insights into a class of questions relevant to policy making. According to these results, privatization does reduce consumer surplus both with and without R&D subsidies, not only because the public firm produces more than a private firm but also because the presence of the public firm boosts total R&D spending which in turn favours output. When optimal R&D subsidies are provided, total profits will generally decrease after privatization. Without R&D subsidies, however, total profits are bound to increase with privatization although the private firm may actually loose out due to benefiting less from R&D spillovers in the private market. All in all, in both cases, the increase in total profits will not outweigh the decrease in consumer surplus, rendering lower levels of social welfare. Thus, we can state that privatising the public firm lowers social welfare and R&D even when (optimal) R&D subsidies are provided. Hence, our results offer some support to the view against the widespread adoption of privatization programmes.[27] In addition to these results, we have shown in section 3 that R&D subsidies boost total R&D investments, total output and social welfare in the mixed (and also in the private) duopoly. This highlights the additional benefits of using an R&D subsidy to tackle market failures even when a public firm is already present in the market.

The reader may wonder whether our results regarding the effects of privatization can be reconciled with the existing empirical evidence. Interestingly, Katz (2001) provides evidence that recently privatized Latin-American firms scaled down their R&D activities following their privatization . Likewise, Munari (2002) and Munari et al. (2002) presented similar evidence of restructuring and scaling down of R&D activity in several case-studies about firms from different countries.[28] Using a panel of recently privatized firms, Munari and Sobrero (2005) showed that firms tend to patent more after being privatized. This is not necessarily in contrast with our results.[29] In fact, the results of Munari and Sobrero (2005) indicate that the increase in patent activity is not the result of increases in R&D investment (in fact, they find that firms invest less in R&D after being privatized). Rather, it is the result of the change in firms' objectives following privatization (from maximizing social welfare to maximizing profits), which leads firms' to restructure their R&D activity and focus on innovations with higher commercial value (perhaps in detriment of those with higher social value).[30]

In this paper, we have assumed the existence of two firms. It is therefore interesting to check whether our result regarding the desirability of privatization extends to an oligopoly. Robustness checks reveal that privatization increases total surplus only if the number of private firms is sufficiently large, in line with the literature (e.g., De Fraja and Delbono, 1989).[31] The rea-

[27] Following White (1996), we have also compared the case where a subsidy is provided in the mixed oligopoly with a private oligopoly without subsidies. In such a case, social welfare also decreases with the privatisation of the public firm.

[28] In only one case privatization was not followed by a decrease in R&D activity. This is the case of the Nippon Telegraph and Telephone. Interestingly, the authors argue that in the Japanese case, law restrictions are in place so that R&D activity is not diminished in consideration of social welfare following a privatization.

[29] In fact, our model is not directly comparable in this case, because we focus on R&D activity when there are spillovers and where innovations cannot be patented.

[30] Similar observations are made in Ansal and Soyak (1999) in two case studies about Turkish firms.

[31] The calculations are available from the authors upon request.

son for this is that privatizing the public firm improves productive efficiency (firms in the private oligopoly produce the same amount of output and so operate at equal costs) but reduces the level of industry output. The gains in terms of productive efficiency outweigh the losses in terms of allocative efficiency only when the number of private firms is relatively large. In other words, in markets with a relatively small number of firms, privatization would lower social welfare, even if (optimal) R&D subsidies were provided by the government. In markets with a relatively large number of firms, we conjecture that a combination of R&D and output policy tools would be necessary to make the privatization of the public firm irrelevant from the point of view of social welfare. In other words, we expect that the so-called "irrelevance" result (that is, the privatization of the public firm not affecting social welfare) identified in White (1996), Pal and White (1998), Poyago-Theotoky (2001) and Zikos (2007) could be regained if a combination of R&D and output subsidies or taxes were implemented.

6 Concluding remarks

Although the literature on R&D has studied extensively the issue of R&D investment in the presence of spillovers, very little attention has been paid to the presence of public firms and the role of public policy in this context. However, there is strong empirical evidence pointing to the importance of the public sector in highly innovative industries. In this paper we extend and enrich the relevant literature by introducing a public firm in the context of a duopoly with spillovers and cost-reducing R&D in order to study the role of subsidies to R&D and the impact of privatization of the public firm on R&D and welfare both in the presence and in the absence of R&D subsidies.

Our findings suggest that the optimal R&D subsidy is positive and increasing in the degree of spillovers. We have shown that an R&D subsidy leads to an increase in total R&D (as it tackles the problems derived from the imperfect appropiability of R&D results) and, in a similar fashion to an output subsidy, it also leads to an increase in total output levels. However, an R&D subsidy does not lead to the equalization of output and therefore to an efficient distribution of production costs across firms. We have also shown that the policy maker should adjust downwards the subsidy rates following privatization of the public firm, as welfare maximization requires higher subsidization rates in the mixed than in the private duopoly.

According to our results, privatization reduces consumer surplus, irrespective of the use of R&D subsidies. This occurs not only because the public firm tends to produce more than a private firm but also because the presence of the public firm boosts total R&D spending which stimulates production. When optimal R&D subsidies are provided both before and after privatization, total profits will tend to increase after privatization. Without R&D subsidies, however, total profits necessarily increase with privatization although, perhaps surprisingly, the profits of the private firm may actually be lower after a privatization. The reason for this is that a private firm may benefiting more from knowledge spillovers when competing with a public firm than when competing with a private firm, since the public firm tends to invest more on R&D than a private firm. All in all, the increase in total profits will not outweigh the decrease in consumer surplus, rendering lower levels of social welfare. This result stands even if the policy maker provides optimal R&D subsidies. Thus, privatization cannot be recommended in our context, at least, in cases where number of private firms in the market is relatively small.

More generally, our analysis has indicated that policy makers face a trade-off between R&D spending and productive efficiency when designing optimal policies for market intervention. While the presence of a public firm may increase total spending in R&D this tends to come at the cost of introducing another type of distortion related to the composition of R&D and related cost asymmetry. An R&D subsidy may boost R&D investment and consequently output but does not eliminate completely the inefficient distribution of production costs. Hence, a public firm and/or R&D subsidies may be useful as policy instruments, although with certain limitations.

It should be stressed that these policy implications have been derived within a rather limited context and care should be taken with generalizing them. However, even within this limited context, it is clear that the conventional presumption about the desirability and efficiency of privatization can be overturned when specific features, such as R&D and appropriability issues, are considered. Further research is certainly welcome to better understand the circumstances under which the pursuit of privatization policies may be desirable from a social viewpoint. In particular, it would be worthwhile to allow more general demand and cost functions in the frame of analysis.

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7 Appendix

7.1 Proof of Lemma 1

Proof. By differentiating (9) we obtain $\partial r_0(x_1)/\partial x_1 \geq (\leq)0$ if and only if $\beta \geq (\leq)0.17$. Next, differentiating (10) yields $\partial r_1(x_0)/\partial x_0 \geq (\leq)0$ if and only if $\beta \geq (\leq)0.33$. Combining these two observations the result follows. ■

7.2 Proof of Lemma 2

Proof. Differentiating $x_0^m(s)$ we obtain $\frac{\partial x_0^m}{\partial s} = \frac{-14+\beta(87-14\beta)}{2H}$, where $H = 167 + 2\beta(25-\beta)(1-\beta) > 0\ \forall\beta\ ,\ \beta \in [0,1]$. Given that the denominator is positive, it follows that $\frac{\partial x_0^m}{\partial s} \geq (\leq)0$ if and only if $\beta \geq (\leq)0.17$. Differentiation of $x_1^m(s)$ yields $\frac{\partial x_1^m}{\partial s} = \frac{197+14\beta(2-3\beta)}{2H} > 0\ \forall\beta$. Finally, $\frac{\partial(x_0^m+x_1^m)}{\partial s} = \frac{183+115\beta-56\beta^2}{2H} > 0\ \forall\beta$. ■

7.3 Proof of Lemma 3

Proof. For part (i) note that $\frac{\partial q_0^m}{\partial s} = \frac{-23+\beta(102+\beta-14\beta^2)}{2H} \geq 0$ if and only if $\beta \geq 0.23$ as $H = 167 + 2\beta(25-\beta)(1-\beta) > 0\ \forall\beta\ ,\ \beta \in [0,1]$. For part (ii), $\frac{\partial q_1^m}{\partial s} = \frac{11[5-\beta(2-\beta)]}{2H} > 0\ \forall\beta$. Finally, $\frac{\partial(q_0^m+q_1^m)}{\partial s} = \frac{16+\beta(40+6\beta-7\beta^2)}{2H} > 0\ \forall\beta$. ■

7.4 Proof of Proposition 1

Proof. From (19), $\frac{ds^m}{d\beta} = \frac{K(a-c)}{B^2}$, where $(a-c) > 0$, $K = 2(5016 + 6114\beta - 190\beta^2 - 1092\beta^3 + 237\beta^4 - 238\beta^5 + 63\beta^6) > 0$, $B = 162 + \beta[56 - \beta(101 - 7\beta^2)] > 0\ \forall\beta\ ,\ \beta \in [0,1]$. It follows that $\frac{ds^m}{d\beta} > 0$. Next, note that $s^m\ |_{\beta=0} = 27(a-c) > 0$ and hence by continuity $s^m > 0\ \forall\beta$. ■

7.5 Proof of Lemma 4

Proof. From (22) and (23), we know that $mc_0^m|_{s=s^m} > mc_1^m|_{s=s^m}$ for any β, since $(78 + 4\beta - 75\beta^2 + 7\beta^4) > (45 - 18\beta - 64\beta^2 + 7\beta^4)\ \forall\beta$. ■

7.6 Proof of Lemma 5

Proof. From tables 3 and 4:

$$SW^m|_{s=s^m} - SW^m|_{s=0} = \frac{(a-c)^2(197+28\beta-42\beta^2)(3+32\beta+17\beta^2-9\beta^3)^2}{(167+2\beta(25-\beta)(1-\beta))^2(167+2\beta(25-\beta)(1-\beta))}$$

As $(167 + 2\beta(25 - \beta)(1 - \beta)) > 0$ and $(197 + 28\beta - 42\beta^2) > 0$ for any $\beta \epsilon [0,1]$, we know that $SW^m|_{s=s^m} - SW^m|_{s=0} > 0$ for any $\beta \epsilon [0,1]$. Moreover, $\frac{\partial SW^m|_{s=0}}{\partial\beta} = \frac{2\Theta_1(a-c)^2}{(167+2\beta(25-\beta)(1-\beta))^3}$ and $\frac{\partial SW^m|_{s=s^m}}{\partial\beta} = \frac{2(a-c)^2\Theta_2}{(162+56\beta-101\beta^2+7\beta^4)^3}$ are both positive since $\Theta_1 = (160125+224129\beta-31620\beta^2+46038\beta^3+11048\beta^4-5184\beta^5+956\beta^6) > 0$ and $\Theta_2 = (1008+2277\beta+1022\beta^2-630\beta^3-294\beta^4+98\beta^5) > 0$. It is tedious but straightforward to check that $\frac{\partial SW^m|_{s=s^m}}{\partial\beta} > \frac{\partial SW^m|_{s=0}}{\partial\beta}$ for $\forall\beta$. Thus, we know that $\frac{\partial SW^m - SW^m|_{s=0}}{\partial\beta} > 0\ \forall\beta$. ■

7.7 Proof of Proposition 2

Proof. From lemma (5), we know that welfare is higher with optimal subsidies than without subsidies in the mixed oligopoly. From lemma (4), we know that $mc_0^m|_{s=s^m} > mc_1^m|_{s=s^m}$. From lemmata (2) and (3), we know that total R&D and output levels are increasing in the subsidy rate and we know that the optimal subsidy is always positive from proposition (1). The rest of the proposition follows. ■

7.8 Proof of Proposition 3

Proof. It is straightforward to see from (26) and (27) that both $x_i^p(s)$ and $q_i^p(s)$ are increasing in s. Moreover, $s^p = \frac{2(1+11\beta)(a-c)}{3[22-3\beta(2+\beta)]} > 0$. Thus, we know that $\Sigma\, x_i^p|_{s=s^p} > \Sigma\, x_i^p|_{s=0}$ and $\Sigma\, q_i^p|_{s=s^p} > \Sigma\, q_i^p|_{s=0}$. From tables 3 and 4 $SW^p|_{s=s^p} - SW^p|_{s=0} = \frac{50(1+11\beta)^2(a-c)^2}{(67-6\beta+2\beta^2)^2(22-6\beta-3\beta^2)^2} > 0\ \forall\beta$. ■

7.9 Proof of Proposition 4

Proof. (i) $x_0^m|_{s=0} - x_i^p|_{s=0} = \frac{3(113+732\beta+72\beta^2-12\beta^3)(a-c)}{FC} > 0\,\forall\beta$; (ii) $x_1^m|_{s=0} - x_i^p|_{s=0} = \frac{(-65-87\beta+209\beta^2-54\beta^3)(a-c)}{FC} < 0$ if and only if $\beta < 0$ as $F = 67 - 6\beta + 2\beta^2 > 0, C = 167 + 50\beta - 52\beta^2 + 2\beta^3 > 0$ and $(a-c) > 0$; (iii) $q_0^m|_{s=0} - q_i^p|_{s=0} = \frac{(1046+1009\beta-506\beta^2+140\beta^3-36\beta^4)(a-c)}{FC} > 0\ \forall\beta$; (iv) $q_1^m|_{s=0} - q_i^p|_{s=0} = \frac{(-294-211\beta+780\beta^2-8\beta^3)(a-c)}{FC} < (>)0$ if $\beta < (>)0.76$; (v) $\pi_0^m|_{s=0} - \pi_i^p|_{s=0} = \frac{G(a-c)^2}{F^2C^2} < 0$ if and only if $\beta < 0.65$, since $G = -961178 - 2423930\beta - 4834531\beta^2 + 50920\beta^3 + 504176\beta^4 - 526160\beta^5 + 128764\beta^6 - 12528\beta^7 + 1296\beta^8 < 0\ \forall\beta$ and (vi) $\pi_1^m|_{s=0} - \pi_i^p|_{s=0} = \frac{L(a-c)^2}{F^2C^2} < (>)0$ for

$\beta < (>)0.75$, since $L = -1221274 - 1155618\beta + 3060979\beta^2 + 1218192\beta^3 - 492662\beta^4 - 14640\beta^5 + 8384\beta^6 - 432\beta^7 < (>)0$ for $\beta < (>)0.75$. ■

7.10 Proof of Proposition 5

Proof. In the mixed duopoly, total R&D, quantities and profits are given as follows:

$x_0^m|_{s=0} + x_1^m|_{s=0} = \frac{[43+\beta(36+4\beta)](a-c)}{C}$,
$q_0^m|_{s=0} + q_1^m|_{s=0} = \frac{2[43+21\beta-9\beta^2](a-c)}{C}$ and
$\pi_0^m|_{s=0} + \pi_1^m|_{s=0} = \frac{[4038+\beta(2938-1829\beta-972\beta^2+316\beta^3)](a-c)^2}{C^2}$.

In the private duopoly, since firms are symmetric, total R&D, quantities and profits are given as follows:

$2(x_i^p|_{s=0}) = \frac{4(4-\beta)(a-c)}{F}$,
$2(q_i^p|_{s=0}) = \frac{30(a-c)}{F}$ and
$2(\pi_i^p|_{s=0}) = \frac{4[4-\beta](a-c)^2}{F^2}$.

We then calculate $x_0^m|_{s=0} + x_1^m|_{s=0} - 2(x_i^p|_{s=0}) = \frac{\delta(a-c)}{FC}$ where $F = 67-6\beta+2\beta^2$ and $C = 167+50\beta-52\beta^2+2\beta^3$, as in the proof of proposition 4. Since $F > 0$, $C > 0\ \forall\beta$ and $\delta = 209 + 2022\beta + 634\beta^2 - 144\beta^3 > 0\ \forall\beta$, it follows that $x_0^m|_{s=0} + x_1^m|_{s=0} > 2(x_i^p|_{s=0})\ \forall\beta$. Next, $q_0^m|_{s=0} + q_1^m|_{s=0} - 2(q_i^p|_{s=0}) = \frac{\varpi(a-c)}{FC}$; the result then follows from the fact that $\varpi = 376 + 399\beta + 137\beta^2 + 66\beta^3 - 18\beta^4 > 0\ \forall\beta$, i.e. $q_0^m + q_1^m > 2q_i^p\ \forall\beta$. Next, we calculate $\pi_0^m|_{s=0}+\pi_1^m|_{s=0}-2(\pi_i^p|_{s=0}) = \frac{\varkappa(a-c)^2}{(FC)^2}$. This expression is negative as $\varkappa = -175678 + 16974\beta + 172593\beta^2 - 35234\beta^3 - 41246\beta^4 + 11104\beta^5 - 264\beta^6 + 16\beta^7 < 0\ \forall\beta$. For $\beta = 0$, $\nu = -305496 < 0$. ■

7.11 Proof of Proposition 6

Proof. From the equilibrium solutions for social welfare without subsidies (see Tables 1 and 2), $SW^m|_{s=0} - SW^p|_{s=0} = \frac{(a-c)^2}{F^2C^2}[\Gamma - \Psi]$ where $\Gamma = 646546 + 990910\beta + 3208497\beta^2 + 2677780\beta^3 + 208460\beta^6 + 1944\beta^8 > 0$ and Ψ=$727266\beta^4 + 791480\beta^5 + 20304\beta^7 > 0$. As in the proof of proposition 4, $F = 67 - 6\beta + 2\beta^2 > 0$ and $C = 167 + 50\beta - 52\beta^2 + 2\beta^3 > 0\ \forall\beta$. It is easy to see $\Gamma > \Psi\ \forall\beta$. Thus, $SW^m|_{s=0} - SW^p|_{s=0} > 0$. ■

7.12 Proof of Proposition 7

Proof. (i) $x_0^m|_{s=s^m} - x_i^p|_{s=s^p} = \frac{(42-28\beta+7\beta^2+2\beta^3)(a-c)}{EB} > 0\ \forall\beta$; (ii) $x_1^m|_{s=s^m} - x_i^p|_{s=s^p} = \frac{(-24+56\beta-2\beta^2-7\beta^3)(a-c)}{EB} < 0$ if and only if $\beta < 0.44$ as $E = 22 -$

$3\beta(2+\beta) > 0$, $B = 162 + \beta[56 - \beta(101 - 7\beta^2)] > 0$ and $(a-c) > 0$; (iii) $q_0^m|_{s=s^m} - q_i^p|_{s=s^p} = \frac{2(156+37\beta-79\beta^2+5\beta^4)(a-c)}{EB} > 0\ \forall\beta$; (iv) $q_1^m|_{s=s^m} - q_i^p|_{s=s^p} = \frac{2(-42+3\beta+16\beta^2-\beta^4)(a-c)}{EB} < 0\ \forall\beta$; (v) $\pi_0^m|_{s=s^m} - \pi_i^p|_{s=s^p} = \frac{F(a-c)^2}{E^2B^2} < 0$ if and only if $\beta < 0.65$, since $F = -78696 + 2712\beta + 227748\beta^2 + 19852\beta^3 - 151922\beta^4 + 5948\beta^5 + 30320\beta^6 - 1774\beta^7 - 2259\beta^8 + 84\beta^9 + 56\beta^{10} < 0$ if and only if $\beta < 0.65$ and (vi) $\pi_1^m|_{s=s^m} - \pi_i^p|_{s=s^p} = \frac{L(a-c)^2}{E^2B^2} < 0\ \forall\beta$, since $L = -226800 - 35568\beta + 350624\beta^2 + 18812\beta^3 - 167832\beta^4 + 4154\beta^5 + 30515\beta^6 - 1558\beta^7 - 2214\beta^8 + 84\beta^9 + 56\beta^{10} < 0\ \forall\beta$. ■

7.13 Proof of Proposition 8

Proof. In the mixed duopoly, total R&D, quantities and profits are given as follows:

$x_0^m|_{s=s^m} + x_1^m|_{s=s^m} = \frac{[45+\beta(73+14\beta(1-\beta)](a-c)}{B}$

$q_0^m|_{s=s^m} + q_1^m|_{s=s^m} = \frac{2[42+13\beta(2-\beta)](a-c)}{B}$ and

$\pi_0^m|_{s=s^m} + \pi_1^m|_{s=s^m} = \frac{[4032+\beta(6006+1873\beta+224\beta^2-343\beta^3-532\beta^4+154\beta^5)](a-c)^2}{B^2}$.

In the private duopoly, since firms are symmetric, R&D, quantities and profits are given as follows:

$2(x_i^p|_{s=s^p}) = \frac{6(1+\beta)(a-c)}{E}$

$2(q_i^p|_{s=s^p}) = \frac{10(a-c)}{E}$ and

$2(\pi_i^p|_{s=s^p}) = \frac{2[43+\beta(6+13\beta)](a-c)^2}{E^2}$.

We then calculate $x_0^m|_{s=s^m} + x_1^m|_{s=s^m} - 2(x_i^p|_{s=s^p}) = \frac{\xi(a-c)}{EB}$ where $\xi = 18+28\beta+5\beta^2-5\beta^3 > 0\ \forall\beta$ and B and E are defined as in proof of proposition 7 ($E = 22 - 3\beta(2+\beta)$ and . $B = 162 + \beta[56 - \beta(101 - 7\beta^2)]$).Recall that $B > 0$, $E > 0\ \forall\beta$. Thus, it follows that $x_0^m|_{s=s^m} + x_1^m|_{s=s^m} > 2(x_i^p|_{s=s^p})\ \forall\beta$. Next, $q_0^m|_{s=s^m} + q_1^m|_{s=s^m} - 2(q_i^p|_{s=s^p}) = \frac{2\rho(a-c)}{EB}$; the result then follows from the fact that $\rho = 114 + 40\beta - 63\beta^2 + 4\beta^4 > 0\ \forall\beta$, i.e. $q_0^m + q_1^m > 2q_i^p$. Next, we calculate $\pi_0^m|_{s=s^m} + \pi_1^m|_{s=s^m} - 2(\pi_i^p|_{s=s^p}) = \frac{\nu(a-c)^2}{(EB)^2}$. This expression is positive whenever $\nu > 0$, where $\nu = -305496 - 32856\beta + 578372\beta^2 + 38664\beta^3 - 319754\beta^4 + 10102\beta^5 + 60835\beta^6 - 3332\beta^7 - 4473\beta^8 + 168\beta^9 + 112\beta^{10}$. For $\beta = 0$, $\nu = -305496 < 0$, while for $\beta = 1$, $\nu = 22342 > 0$. Then by continuity, there exists a critical value of the spillover parameter $\bar{\beta}$, defined as $\bar{\beta} = \{\beta \mid \nu = 0\}$ with $\bar{\beta} \in (0.028, 1)$. Straightforward calculation yields $\bar{\beta} = 0.94$. Thus, if $\beta < 0.94$, $\pi_0^m|_{s=s^m} + \pi_1^m|_{s=s^m} < 2(\pi_i^p|_{s=s^p})$ and if $\beta \geq 0.94$, the reverse holds. ■

7.14 Proof of Proposition 9

Proof. From (19) and (28) it follows that $s^m - s^p = \frac{2(36+220\beta+4\beta^2-77\beta^3+2\beta^4+4\beta^5)(a-c)}{3EB}$ where $E = 22-3\beta(2+\beta) > 0$ and $B = 162 + \beta[56 - \beta(101 - 7\beta^2)] > 0\ \forall\beta$. Thus, it is immediate to see that $s^m - s^p > 0\ \forall\beta$. ■

7.15 Proof of Proposition 10

Proof. From the equilibrium solutions for social welfare (see Tables 2 and 4), $SW^m|_{s=s^m} - SW^p|_{s=s^p} = \frac{(18+10\beta-5\beta^2)(a-c)^2}{EB}$ where $(18+10\beta-5\beta^2) > 0$, $E = 22-3\beta(2+\beta) > 0$ and $B = 162 + \beta[56 - \beta(101 - 7\beta^2)] > 0\ \forall\beta$. Thus, it is immediate to see that $SW^m|_{s=s^m} - SW^p|_{s=s^p} > 0\ \forall\beta$. ■