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A FISCAL STIMULUS AND JOBLESS RECOVERY

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A Fiscal Stimulus and Jobless Recovery*

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Abstract
We analyse the effects of a government spending expansion in a dynamic stochastic general equilibrium (DSGE) model with Mortensen-Pissarides labour market frictions, deep habits and a constant-elasticity-of-substitution (CES) production function. The combination of deep habits and CES technology is crucial. The presence of deep habits enables the model to deliver output and unemployment multipliers in the high range of recent empirical estimates, while an elasticity of substitution between capital and labour in the range of available estimates allows it to produce a scenario compatible with the observed jobless recovery. An accommodative monetary policy with respect to the output gap alongside sticky prices plays an important role for the stabilisation properties of the fiscal stimulus.

Keywords: Fiscal policy; deep habits; labour market search-match frictions; unemployment; CES production function.

JEL Codes: E24; E62.

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1 Introduction

In the recent financial crisis an important dimension along which many governments have taken action has been fiscal policy. The economics profession and much of the academic discussion placed emphasis on the issue of whether and to what extent a fiscal stimulus delivers the dual outcome of (i) moderating the output collapse and (ii) boosting job creation. This assumes great importance also in the light of the jobless recovery that the US are experiencing in the aftermath of the Great Recession.

As shown in Figure 1, the cyclical component of hours worked per employee closely co-moves with the cyclical fluctuations of real output, and the cyclical component of unemployment is negatively correlated with that of output (see Table 1). However, while hours worked per employee and output have been on a recovery path from 2009 Q2 (the trough of the great recession), the unemployment rate has persistently remained well above average. In the recovery period, while the correlation between output and hours worked per employee is 0.99, the correlation between the unemployment rate and output is -0.18. Figure 1 also shows the well known fact in the business cycle literature that the unemployment rate is around ten times more volatile than output. Hours worked per employee are less volatile than output, but the volatility has the same order of magnitude.

<table>
<thead>
<tr>
<th>Correlation coefficients</th>
<th>Hours/employee - Output</th>
<th>Unemployment rate - Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995 Q1 - 2011 Q1</td>
<td>0.92</td>
<td>-0.63</td>
</tr>
<tr>
<td>2009 Q2 - 2011 Q1</td>
<td>0.99</td>
<td>-0.18</td>
</tr>
</tbody>
</table>

Table 1: Correlation coefficients between (i) hours worked per employee and real output; and (ii) the unemployment rate and real output. (Cyclical components: Percentage deviations from HP-trend for GDP and hours per employee, percentage deviations from the sample mean for the unemployment rate. Source: ALFRED, Federal Reserve Bank of St. Louis and authors’ computations).

In this paper we analyse the effects of a government spending expansion in a dynamic stochastic general equilibrium (DSGE) model with Mortensen-Pissarides labour market frictions, deep habits in private and public consumption and a constant-elasticity-of-substitution (CES) production function. The main results are that: (i) we obtain output and unemployment multipliers inside the range of empirical estimates without the need to introduce nominal rigidities and the imposition of the zero lower bound (ZLB) on the nominal interest rate; (ii) we can reproduce a fiscal expansion with low job creation; and (iii) we can simulate a fiscal stimulus that mitigates the output collapse in a recession but contains the rise in unemployment only marginally. This scenario is in line with what we observe in the data in the aftermath of the great recession.

On the size of fiscal multipliers the literature has provided a variety of results. Auerbach et al. (2010) describe the range of mainstream estimate for multiplier effects as “almost embarrassingly large”. Recent VAR estimates of the output multiplier are generally greater than those predicted by DSGE models with no zero-lower-bound constraints but still present values varying from 0.7 to 2.5.\(^1\)

\(^1\)Pessimistic estimates of the output multiplier (around 0.7) can be found in (Barro and Redlick, 2011; Ramey, 2009); some contributions find values around one (see Hall, 2009, among others); while other authors (see Blanchard and Perotti, 2002; Monacelli et al., 2010; Blinder and Zandi, 2010; Acciona et al., 2011; Fragetta and Melina, 2011, among others) report values above one. Auerbach and Gorodnichenko (2011) study asymmetries in the propagation of fiscal shocks in booms and downturns and report an output multiplier of up to 2.5 during recessions.
The main reason why government spending multipliers are small in models with rational expectation is to be found in the negative wealth effect triggered by the increase in government purchases. This crowds out private consumption and investment and makes output respond in a less than proportional way. The results in New-Keynesian (NK) models have also been shown to be dependent on the reaction of monetary policy: the more accommodative the monetary policy, the higher the fiscal multiplier. On the last point Canova and Pappa (2011) also provide empirical support using VARs. Moreover, substantially larger-than-one multipliers can be obtained in standard NK models if the ZLB binds.

To examine the issue of unemployment there is an increasing practice in macroeconomics to introduce Mortensen-Pissarides search-matching (MPMF) frictions into otherwise standard NK models (Campolmi et al., 2010; Faia et al., 2010; Monacelli et al., 2010). These models allow to obtain unemployment equilibria, investigate the traditional unemployment-inflation trade-off, and evaluate the policy effects on the extensive margin of employment.

A typical problem that arises in RBC and NK models featuring MPMF frictions is their difficulty in matching the unemployment volatility observed in the data (the so-called “unemployment volatility puzzle”). In the literature, this has mainly been addressed via the introduction of staggered nominal wages (Gertler and Trigari, 2006; Sala et al., 2008) although this practice has been criticised by Pissarides (2009). Di Pace and Faccini (2011) tackle the unemployment volatility puzzle via the introduction of

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2Woodford (2011) shows that the government spending multiplier is (i) necessarily below one in a neoclassical Real Business Cycle (RBC) model and exactly the same both in an RBC with monopolistic competition and in a sticky-price New-Keynesian (NK) model with strict inflation targeting; (ii) exactly one in an NK model with fixed real interest rate; (iii) somewhere between the two values in a model featuring a Taylor rule.

3Christiano et al. (2009) find that the spending multiplier may also reach 10 at the ZLB if the fiscal stimulus lasts for exactly the quarters when the ZLB is binding.

4Pissarides (2009) criticises their introduction as a device to solve the unemployment volatility puzzle on the grounds that while time-series estimates provide evidence for (average) sticky wages, panel-data estimates support the claim that wages in the new matches are pro-cyclical.
“deep habits” in consumption as in Ravn et al. (2006). The introduction of deep habits in a DSGE model imply also that a government spending expansion, even in the presence of flexible prices, reduces the mark-up, fosters the real wage, and crowds in private consumption.

These are desirable features as there is empirical evidence that (i) private consumption is typically crowded in by a government spending expansion as opposed to being crowded out as a canonical DSGE model predicts (Blanchard and Perotti, 2002; Gali et al., 2007; Pappa, 2009; Monacelli et al., 2010; Fragetta and Melina, 2011); (ii) the real wage increases after a government spending expansion (Pappa, 2005; Gali et al., 2007; Caldara and Kamps, 2008; Pappa, 2009; Fragetta and Melina, 2011) as opposed to falling as in the canonical model; (iii) the mark-up is typically countercyclical and, more specifically, Monacelli and Perotti (2008) and Canova and Pappa (2011) show that a government spending expansion is accompanied by a fall in the mark-up in the data. In the empirical literature there is also evidence that the elasticity of substitution between capital and labour is not one (Klump et al., 2007; Chirinko, 2008; Cantore et al., 2010a, 2011; León-Ledesma et al., 2010) and that factor shares are time-varying (Blanchard, 1997; Jones, 2003, 2005; McAdam and Willman, 2008; Ríos-Rull and Santaéulalia-Llopis, 2010). However, the standard use of Cobb-Douglas production functions prevents any model from replicating this regularity, as the Cobb-Douglas implies constant factor shares.

The model built in this paper is able to match all the mentioned empirical regularities by merging a standard RBC model with Mortensen-Pissarides labour market frictions, featuring both the intensive and the extensive margin of employment, deep habits in private and public consumption and a constant-elasticity-of-substitution (CES) production function. The combination of deep habits and CES technology is crucial for the jobless outcome of a fiscal stimulus. If the elasticity of substitution between capital and labour approaches one, i.e. the production function approximates a Cobb-Douglas, the presence of deep habits in consumption enables the model to deliver output and unemployment multipliers in the range of recent empirical estimates. As the elasticity of substitution is allowed to drop to values in the range of available estimates – i.e. the degree of complementarity between capital and labour increases – while the output multiplier falls only marginally, the unemployment multiplier experiences a sizeable contraction. The unequal effects on the output and unemployment multipliers depend on the fact that lowering the elasticity of substitution in the CES production function is equivalent to assuming that the technology is closer to the Leontief case, i.e. capital and labour are more complements than substitutes. Given that capital is unable to change instantaneously in response to the fiscal expansion, firms have smaller incentives to create new jobs through vacancy posting, being this a costly process. However, both the negative wealth effect (coming from the absorption of resources by the government) and the substitution of leisure with consumption (coming from the decline in the mark-up due to the presence of deep habits) still act in the same direction of causing a substantial increase in the supply of hours of work per employee.

In such a case, the expansion in output is driven relatively more by an increase in the hours of work of

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5This implies that households form habits on the consumption of varieties as opposed to overall consumption, which leads to counter-cyclical mark-ups also in a model with flexible prices. Under deep habits, monopolistically-competitive firms become more competitive because the elasticity of demand becomes procyclical. In addition, expected higher future profits, in a model with deep habits, induce firms, ceteris paribus, to post more vacancies and this leads to a greater amplification for the labour market tightness and, hence, equilibrium employment.

6In fact, the reduction in the mark-up determines a strong shift in labour demand which prevails on the shift in supply (determined by the negative wealth effect) and wages rise. Consumption rises because the negative wealth effect is offset by a strong substitution effect away from leisure and into consumption induced by the increase in wages.

7See Cantore et al. (2010b) for further details.
current employees rather than new job creation. Thus, the CES technology with an empirically supported elasticity of substitution proves to be a useful tool to simulate a fiscal stimulus that mitigates the output collapse in a recession but contains the rise in unemployment only marginally, being this in line with the observed jobless recovery.

We do not go as far as claiming that this is the only explanation for the jobless recovery and indeed this is still a very controversial issue in the literature. Possible explanations of the delay in the response of unemployment in recovery periods (observed from the 1991 recession onwards) have been associated with structural change stories such as the availability of a more flexible labour force (temporary workers and offshoring), the increase in health benefits and a rise in the speed of sectoral reallocations (see Groshen and Potter (2003), Andolfatto and MacDonald (2004) and Schreft et al. (2005) amongst others). However, Aaronson et al. (2004a) and Aaronson et al. (2004b) find little support for the structural change hypothesis. Some other authors have also given a cyclical explanation for the jobless recovery. Examples are Aaronson et al. (2004a), relating it to a negative labour supply shock; Bernanke (2003), focusing on a sluggish aggregate demand; and Bachmann (2011), calibrating a DSGE model with adjustment costs to the extensive margin. In this paper we link the jobless outcome of a fiscal stimulus to factor complementarity.

Finally, for the sake of comparability with the rest of the fiscal stimulus literature, we also explore the implications of price stickiness and monetary policy. In this case an accommodative monetary policy with respect to the output gap plays an important role for the size of fiscal multipliers and, more generally, for the expansionary effects of the fiscal stimulus.

The remainder of the paper is structured as follows. Section 2 briefly describes the flexible-price model with deep habits in consumption and labour market frictions. Section 3 illustrates the calibration. Section 4 presents the results in the flexible-price model and isolates the effects of several features of the model on the size of the output and unemployment multipliers. Section 5 provides a sticky-price extension. Finally, Section 6 concludes and sets the agenda for future research. The appendix provides the full model derivation, the symmetric equilibrium, the steady state and some sensitivity exercises.

2 The Model

The model structure is represented in Figure 2. Households have a fraction of their members employed and a fraction unemployed. They offer labour services and capital to firms, who hire members of the households via vacancy posting and operate in an imperfectly competitive market. Households and firms bargain over a wage. Households and the government exhibit deep habit formation in the consumption of differentiated goods produced by firms. The government buys a fraction of those goods, pays unemployment benefits to the unemployed members of households and finances its expenditures by taxing households (and issuing government bonds in one of the exercises presented below). In the NK extension of the model we add price stickiness and a central bank who takes the interest rate decision. As most of the building blocks of the model are standard in the literature, in what follows we present some less known features and we refer the reader to the appendix for the full model derivation.
2.1 Deep habits

Following Ravn et al. (2006), we assume that households exhibit external deep habit formation in consumption, i.e. habits are formed on the average consumption level of each variety of good.\(^8\) Under deep habits household \(j\)'s utility function is increasing in \((X^c_i)^j\), a habit-adjusted composite of differentiated consumption goods:

\[
(X^c_i)^j = \left[ \int_0^1 (C^j_{it} - \theta^c S^c_{it-1})^{1-\frac{1}{\eta}} di \right]^{1-\frac{1}{\eta}},
\]

where parameter \(\eta\) is the intratemporal elasticity of substitution across varieties, \(\theta^c \in (0, 1)\) is the degree of deep habit formation on each variety, and \(S^c_{it-1}\) denotes the stock of external habit in the consumption of good \(i\). The stock of external habit \(S^c_{it}\) evolves over time according to the following law of motion:

\[
S^c_{it} = \varrho^c S^c_{it-1} + (1 - \varrho^c) C^j_{it},
\]

where \(\varrho^c \in (0, 1)\) measures the speed of adjustment of the stock of external habit in the consumption of variety \(i\) to changes in the average level of consumption of the same variety \(C_{it}\).

Each household \(j\) solves a two-stage problem. Letting \(P_{it}\) be the price of variety \(i\), they first minimise total expenditure \(\int_0^1 P_{it} C^j_{it} di\) over \(C^j_{it}\) subject to (1). This leads to the optimal level of demand for each variety \(i\) for a given composite:

\[
C^j_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\eta} (X^c_i)^j + \theta^c S^c_{it-1},
\]

where \(P_t = \left[ \int_0^1 P_{it}^{-\eta} di \right]^{-\frac{1}{\eta}}\) is the nominal price index. Clearly, the level of demand for each variety \(i\) is characterised by a price-elastic component and a price-inelastic component. The second stage of the problem faced by household \(j\) at time \(t\) is the standard intertemporal utility maximisation.

Deep habits are present also in government consumption. From a technical point of view this is entirely

\(^8\)This consumption externality is also known as “catching up with the Joneses good by good”.
analogous to how deep habits are introduced in private consumption. From an intuitive point of view, this can be justified by assuming that households also derive habits from the consumption of government-provided goods. Alternatively, as in Leith et al. (2009), one can also argue that public goods are local in nature and households care about the provision of individual public goods in their constituency relative to other constituencies.\footnote{Controversies over “post-code lotteries” in health care and other local services (Cummins et al., 2007) and comparisons of regional per capita government spending levels (MacKay, 2001) suggest that households care about their local government spending levels relative to those in other constituencies.}

The differences in the transmission mechanism of a fiscal shock in a model with deep habits in consumption work through the fact that the mark-up is counter-cyclical under deep habits even if the model features fully flexible prices. Under deep habits the mark-up is counter-cyclical due to the co-existence of two effects: an \textit{intra-temporal effect} (or price-elasticity effect) and an \textit{inter-temporal effect}. The intra-temporal effect can easily be understood by looking at the demand faced by an individual firm $i$:

$$
ADt = C_{it} + G_{it} + I_{it} = \left( \frac{P_{it}}{f_t} \right)^{-\eta} (X_{it}^c + X_{it}^q + I_t) + \theta^c (S_{it-1}^c + S_{it-1}^q),
$$

where $G_{it}$ is the public consumption of variety $i$, $I_{it}$ is the component entering the investment aggregator $I_t$ (which is not subject to deep habits) and $X_{it}^q$ and $S_{it}^q$ are the public counterparts of the habit-adjusted consumption composite and the stock of habit for variety $i$. The right-hand side of the demand curve is given by the sum of a price-elastic term and a price-inelastic term. When the habit-adjusted aggregate demand $(X_{it}^c + X_{it}^q + I_t)$ rises, the “weight” of the price-elastic component of demand grows and the effective price elasticity of demand $\tilde{\eta}_{it} \equiv \frac{\partial ADt}{\partial P_{it}} \frac{P_{it}}{ADt} = \eta - \theta^c (\frac{S_{it-1}^c + S_{it-1}^q}{ADt})$ increases, as opposed to remaining constant and equal to $\eta$ as in the standard case ($\theta^c = 0$). The fact that the elasticity of demand is procyclical is one determinant for the price mark-up being counter-cyclical. The other determinant comes from the inter-temporal effect: the awareness of higher future sales coupled with the notion that consumers form habit at the variety level, makes firms inclined to give up some of the current profits – by temporarily lowering their mark-up – in order to lock-in new consumers into customer/firm relationships and charge them higher mark-ups in the future.

### 2.2 CES production function and “re-parametrisation”

We specialise the production function $F((ZK)_t, (ZN)_t n_t h_t)$ as a constant-elasticity-of-substitution (CES) production function:

$$
F((ZK)_t, (ZN)_t n_t h_t) = \left[ \alpha_K ((ZK)_t K_t) \frac{\sigma}{\sigma-1} + \alpha_N ((ZN)_t n_t h_t) \frac{1}{\sigma-1} \right]^{\frac{\sigma-1}{\sigma}}
$$

where $K_t$ is capital, $n_t$ is the number of employees, $h_t$ is hours worked per employee, $(ZK)_t$ and $(ZN)_t$ are capital and labour-augmenting technology shocks, $\sigma$ is the elasticity of substitution between capital and labour, and $\alpha_K$ and $\alpha_N$ are the so-called distribution parameters. Note that, unlike in the Cobb-Douglas case, the distribution parameters do not represent factor shares of income and are not dimensionless. In other words, these have dimensions that depend on the measurement units of capital and labour as discussed in Cantore and Levine (2011). As such, the distribution parameters are meaningless and cannot be calibrated. In this subsection, we show that once the capital share of income has been calibrated, $\alpha_K$
and $\alpha_N$ can be “re-parameterized”, i.e. expressed as functions of this share and of endogenous variables of the model, which in turn depend on the deep parameters. This procedure is conducted in the spirit of Cantore and Levine (2011).

As $\sigma \to 1$, the CES production function collapses to a Cobb-Douglas (CD) if and only if $\alpha_K + \alpha_N = 1$. While $\sigma \to 0$ leads to the Leontief case.

In the CES case, marginal products of capital and labour take the following forms:

$$F_{K,t} = \alpha_K (ZK)_t \left( \frac{Y_t}{K_t} \right)^{\frac{\sigma-1}{\sigma}} \frac{1}{\sigma},$$

$$F_{N,t} = \alpha_N (ZN)_t \left( \frac{Y_t}{n_t h_t} \right)^{\frac{1}{\sigma}} \frac{1}{\sigma}.$$  \hspace{1cm} (6)

Let variables without time subscript denote steady-state values and $S^K \equiv \frac{F_{K,K}}{Y} \in (0,1)$ be the calibrated capital share of income. Combining equation (5) with the definition of capital share and rearranging yields $\alpha_K$ as a function of the capital share and endogenous variables:

$$\alpha_K = S^K \left( \frac{Y}{(ZK)K} \right)^{\frac{\sigma-1}{\sigma}}.$$  \hspace{1cm} (7)

As $\sigma \to 1$, i.e. the production function tends to a CD, $\alpha_K \to S^K$. As the total products of capital and labour have to add up to total output, the following holds:

$$\frac{F_{N\text{h}}}{Y} = 1 - \frac{F_{K,K}}{Y} = 1 - S^K.$$  \hspace{1cm} (8)

Combining equations (6) and (8) allows us to recover $\alpha_N$:

$$\alpha_N = (1 - S^K) \left( \frac{Y}{(ZN)\text{n}} \right)^{\frac{\sigma-1}{\sigma}}.$$  \hspace{1cm} (9)

As $\sigma \to 1$, $\alpha_N \to (1 - S^K)$. Note that if the labour market is not Walrasian, i.e. it is characterised by wage bargaining and hiring costs, $(1 - S^K)$ does not represent the labour share, $S^N$, but it also includes the share of income that is wasted in the search-matching and bargaining process, $S^{SM} \equiv \frac{g(z)n}{Y}$, where $g(z)n$ represents total hiring costs, which are a function of the vacancy rate $z$. In equilibrium, $S^K + S^N + S^{SM} = 1$.

3 Parameter choice and calibration

To calibrate the model we assign numerical values to parameters in order to match a number stylised facts for the US economy in the post-WWII era. The time period in our model corresponds to one quarter in the data. Table 1 summarises the calibration exercise.

A set of parameters are simply set to values that are widely used in the literature. Namely we set the subjective discount factor, $\beta$, equal to 0.99, which implies a quarterly real interest rate of about 1%. The capital depreciation rate, $\delta$, and the coefficient of relative risk aversion, $\sigma_c$, are set equal to 0.025 and 2,
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$ 0.99</td>
</tr>
<tr>
<td>Capital share of income</td>
<td>$S^K$ 1/3</td>
</tr>
<tr>
<td>Capital depreciation rate</td>
<td>$\delta$ 0.025</td>
</tr>
<tr>
<td>Relative risk aversion</td>
<td>$\sigma_c$ 2</td>
</tr>
<tr>
<td>Elasticity of substitution in production function</td>
<td>$\sigma$ 0.4</td>
</tr>
<tr>
<td>Elasticity of substitution across varieties</td>
<td>$\eta$ 6</td>
</tr>
<tr>
<td>Investment adjustment cost parameter</td>
<td>$\gamma$ 3.24</td>
</tr>
<tr>
<td>Degree of deep habit formation</td>
<td>$\theta^c$ 0.86</td>
</tr>
<tr>
<td>Habit persistence</td>
<td>$\varphi^c$ 0.85</td>
</tr>
<tr>
<td>Job separation rate</td>
<td>$\lambda$ 0.103</td>
</tr>
<tr>
<td>Elasticity of matching to unemployment</td>
<td>$\omega$ 0.5</td>
</tr>
<tr>
<td>Firms’ bargaining power</td>
<td>$\epsilon$ 0.5</td>
</tr>
<tr>
<td>Share of government spending in output</td>
<td>$\bar{g}/\bar{y}$ 0.2</td>
</tr>
<tr>
<td>Persistence of government spending shock</td>
<td>$\rho_g$ 0.90</td>
</tr>
<tr>
<td>Persistence of tax shocks</td>
<td>$\rho_X$ 0.90</td>
</tr>
<tr>
<td>Convexity in hiring cost</td>
<td>$\psi$ 1</td>
</tr>
<tr>
<td>Elasticity of subst leisure/consumption</td>
<td>$\varrho$ set to target $\tilde{h} = 0.40$</td>
</tr>
<tr>
<td>Scaling factor in hiring cost function</td>
<td>$\chi$ set to target $\tilde{p} = 0.83$</td>
</tr>
<tr>
<td>Scaling factor in matching function</td>
<td>$\kappa$ set to target $\tilde{q} = 0.70$</td>
</tr>
<tr>
<td>Unemployment benefit</td>
<td>$w_u$ set to target $\tilde{\Theta} = 0.70$</td>
</tr>
</tbody>
</table>

Table 2: Baseline calibration

respectively, while the capital share of income, $S^K$, takes the conventional value of 1/3. The elasticity of substitution across varieties, is set to a rather standard value of 6, which implies a steady state markup of around 20% in the absence of deep habits.

When the production function takes the general CES form, we set the elasticity of substitution, $\sigma$, equal to 0.40, a value close to the empirical estimates in León-Ledesma et al. (2010). We obtain the Cobb-Douglas as a limiting case, by setting $\sigma \rightarrow 1$. The investment adjustment cost parameter is set equal to 3.24, the value estimated by Christiano et al. (2005). The degree of deep habit formation, $\theta^c$, and the habit persistence, $\varphi^c$, are set equal to 0.86 and 0.85, respectively. These are the same estimated values used in Ravn et al. (2006). We then set the convexity parameter in the hiring cost function to 1, which makes it quadratic as in Gertler and Trigari (2006) and Thomas (2008). The firms’ bargaining power, $\epsilon$, and the elasticity of matching to unemployment, $\omega$, are both set equal to 0.5. This choice satisfies the Hosios condition for the efficiency of the equilibrium. There is no reason to believe that this condition holds in practice, however this parameter choice is shared by most of the existing literature and hence allows comparability of the results. The value for the job separation rate, $\lambda$, is set equal to 0.103 to imply that jobs last on average 2 years and a half. This is in line with the calculations made by Shimer (2005). The persistence of fiscal shocks is set equal to 0.90, which is approximately the value observed in the data (see Monacelli et al., 2010, among others).

Finally, we set (i) the elasticity of substitution between leisure and consumption, $\varrho$; (ii) the scaling factor in the hiring cost function, $\chi$; (iii) the scaling factor in the matching function, $\kappa$, and (iv) the unemployment benefit, $w_u$, in order to match: (a) a steady-state share of hours worked over total hours,
\( \tilde{h} \), of 40%; (b) a steady-state job finding probability, \( \tilde{p} \), equal to 83%, as estimated by Shimer (2005);\(^{10}\) (c) a value for the vacancy filling probability, \( \tilde{q} \), equal to 70%, as in Trigari (2009); and (d) a ratio for the value of non-work to work activities (replacement ratio), \( \tilde{\Theta} = \frac{u_n - c_n}{\bar{F}_n} \) (i.e. the sum of unemployment benefits and the disutility of work over the marginal product of employment), equal to 70%, a value very close to the point estimate of 72% by Sala et al. (2008). As the value for the replacement ratio is debated in the literature and is an important determinant of the unemployment multiplier, we show sensitivity of our results to different magnitudes for this parameter in the appendix.

In addition to the explicitly-targeted steady-state values, this calibration implies reasonable “great ratios”; namely a consumption/output ratio of 61%, an investment/output ratio of 18% and a hiring costs/output ratio of 1%. The choice of the job separation rate, coupled with the job finding probability implies, through the Beveridge curve, a steady-state unemployment rate of approximately 11%.

4 Results

We present the results starting from a standard neoclassical (RBC) model with search and matching frictions in the labour market and adding deep habits and the CES technology one at a time. Subsection 4.1 presents the well known results that in the baseline RBC model output and unemployment multipliers are well below the range of available empirical estimates. It also shows some features at odds with the data, namely constant price mark-up and factor shares, a negative response of the real wage and a negative response of consumption following a government spending shock. Subsection 4.2 shows how, even in the absence of price stickiness and the imposition of a ZLB, the introduction of deep habits magnifies both output and unemployment multipliers. At the same time, in line with Ravn et al. (2006), now the mark-up falls, real wages rise and consumption is crowded in after an expenditure expansion. By introducing the CES production function in Subsection 4.3 we show that, as capital and labour became more complementary, the growth of output fostered by a government spending expansion is sustained relatively more by an increase in the intensive margin (current employees work longer hours) than an increase in the extensive margin (new job creation). Factor shares now present cyclical fluctuations. Subsection 4.4 shows how the magnitude of the responses of output and unemployment is altered by distortionary taxation and government debt. Finally, in Subsection 4.5, we explore the effects of a fiscal stimulus at a recession time, which fosters a jobless recovery.

4.1 Neoclassical benchmark with search-match frictions

In Figure 3 we plot the impulse responses of a number of fundamental macroeconomic variables to a government spending expansion of size 1% of output. Normalising the size of the fiscal shock as such allows us to interpret the output responses as fiscal multipliers. For unemployment, we report the absolute changes in percentage points that the increase in spending by 1% of output triggers.

\(^{10}\)Shimer (2005) estimated a monthly job finding probability of 0.45, which corresponds to a quarterly value of approximately 0.83.
Figure 3: A government spending expansion (1% of output, lump-sum taxes, balanced budget) in an RBC model augmented with Mortensen-Pissarides Matching Frictions: the effects of deep habits in consumption.

Note: Line marked by squares: RBC model with Mortensen-Pissarides Matching Friction (MPMF), Cobb-Douglas (CD) production function \((\sigma \to 1)\) and no habits in consumption \((\theta^c = \phi^c = 0)\). Line marked by circles: MPMF, CD production function, and deep habits in consumption \((\theta^c = 0.86\) and \(\phi^c = 0.85)\). Line marked by stars: MPMF, CES production function \((\sigma = 0.40)\) and deep habits in consumption. Responses of all variables but the unemployment rate are in percentage deviations from steady state. For the unemployment rate, absolute changes in percentage points are reported.

This can be regarded as a measure of the unemployment multiplier. This exercise is conducted under the assumption that the fiscal measure is fully financed by lump-sum taxes.\(^{11}\)

As a benchmark, we consider the effects of a government spending expansion in the neoclassical flexible-price benchmark with MPMF under the assumption that the production function is Cobb-Douglas, and that no deep habits in private and government consumption are formed. The results are in line with most

\(^{11}\)When lump-sum taxes are in place the timing of tax collection does not matter as the Ricardian equivalence holds. In other words, debt-financed fiscal expansions would yield the same effects.
of the recent theoretical fiscal stimulus literature: a fiscal expansion triggers a negative wealth effect, via an increase in tax obligations, that curbs consumption and boosts labour supply. In the context of MPMF, this has a negative effect on households’ reservation wage and a smaller positive effect on firms’ reservation wage. As a result, the share of the firms in the surplus from wage bargaining increases, which translates into more vacancies being posted, a tighter labour market, a reduction in equilibrium unemployment, and a fall in the real wage. The absorption of resources by the government is such that also private investment is crowded out and the real interest rate rises. As standard in flexible-price neoclassical models with imperfect competition, the price mark-up over the marginal cost remains constant. Another standard result – coming instead from the use of the Cobb-Douglas production function – is that capital and labour shares of income are also constant.

From a quantitative point of view, results are similar to existing contributions such as Campolmi et al. (2010) and Monacelli et al. (2010): government spending expansions yield output multipliers well below one (around 0.5 for our calibration) and almost negligible negative effects on unemployment.

The results in the flexible-price benchmark model contrast with much of the recent empirical literature, both from a quantitative and from a qualitative point of view. On the quantitative side, recent empirical estimates of the output multiplier are generally greater than those predicted by DSGE models with no zero-lower-bound constraints. On the size of the unemployment multiplier Monacelli et al. (2010) provide an estimate at peak of -0.6 percentage points after ten quarters, which per se may be regarded to be small, but it is an order of magnitude bigger than the multiplier predicted by the flexible-price benchmark with MPMF.\footnote{Brückner and Pappa (2010) report evidence according to which unemployment may also rise in response to a government spending shock and match this finding by including the labour-force participation rate into a New-Keynesian model through an insider/outside mechanism.} On the qualitative side, there is also empirical evidence that government spending expansions crowd in private consumption and boost \textit{both} hours worked \textit{and} the real wage (see Pappa, 2005; Gali et al., 2007; Pappa, 2009; Fragetta and Melina, 2011, among others). In addition, Monacelli and Perotti (2008) and Canova and Pappa (2011) find evidence for a fall in the price mark-up following a fiscal expansion.

\subsection*{4.2 Deep habits}

The introduction of deep habits in consumption yields a substantial improvement on the performance of the DSGE model in matching these empirical findings, even in the absence of price and/or wage rigidities and the zero-lower bound.\footnote{In the seminal work by Ravn et al. (2006), they already illustrate that a government spending expansion yields a crowding-in of private consumption as opposed to a crowding-out, when deep habits in private and public consumption are introduced into an otherwise standard flexible-price model with imperfect competition. In addition, Di Pace and Faccini (2011) find that deep habits in consumption have the property of considerably magnifying unemployment volatility also in a model with flexible wages, proposing a solution to Pissarides (2009)’s unemployment puzzle.}

In Figure 3 we show that by introducing deep habits in our model, not only are we able to match a number of empirical facts from a qualitative point of view, but we are also able to obtain output and unemployment multipliers closer to those computed in the SVAR literature, under a plausible calibration.

A government spending expansion, also under deep habits, causes a negative wealth effect. However, the drop in the mark-up, which in turn implies higher future sales, translates into more vacancy posting through the job creation condition. The higher labour market tightness implies a greater fall in the unemployment rate. This coexists not only with an increase in the intensive margin (hours worked) but
also with an increase in the real wage. The increase in the real wage is made possible by the greater increase in the firm’s reservation wage, which induces a rise in the bargained wage. The increase in equilibrium wage makes leisure relatively more expensive and causes a substitution effect towards consumption that more than compensates the negative wealth effect. As a result, consumption rises.

With a Cobb-Douglas production function and our baseline calibration the resulting output multiplier is around 1.7, a number in the high range of empirical estimates. The peak unemployment multiplier is -0.27 percentage points, which is of the same order of magnitude of the estimates reported by Monacelli et al. (2010), as opposed to the model without deep habits.

In sum, deep habits in private and public consumption are a useful addition to the DSGE model because through them – even in the absence of any sources of nominal stickiness and without the imposition of the ZLB – (i) the output multiplier of government spending can be considerably magnified up to values in the range of empirical estimates; (ii) the unemployment multiplier can be brought from near-zero to values of the same order of magnitude found in the data; (iii) private consumption is crowded in by government spending; (iv) the price mark-up drops; and (v) the real wage rises together with hours worked.

In the NK literature the fall in the mark-up and the increase in the real wage are matched to a certain extent by including price and/or wage stickiness. However, NK models manage to get only an initial positive response in the real wage – while the empirical literature finds a persistent positive increase – and the fall in the mark-up is not generally enough to push aggregate supply upward to such an extent that the fiscal multiplier is dramatically magnified. Consumption is still crowded out unless either (i) a non-additively separable utility function is adopted and the intertemporal elasticity of substitution of consumption is set to be low (i.e. $\sigma_c$, its inverse, is high) entailing strong intratemporal substitution effects between consumption and leisure (see for example Linnemann, 2006; Monacelli et al., 2010) or (ii) it has to be assumed that an implausibly high share of consumers exhibit a "rule-of-thumb" non-optimising behaviour (Gali et al., 2007).

### 4.3 CES production function

The empirical literature has not reached a consensus on the macroeconomic effects of fiscal policy. Nonetheless, if one wants to operate a synthesis of available empirical estimates on output and unemployment expenditures multipliers, it seems fair to conclude that, when the government purchases more goods and services from the private sector, this may yield a sizeable increase in real output, while the effect on new job creation is likely to be small (Brückner and Pappa (2010) claim that the effect may even be negative).

In this subsection we show that if the elasticity of substitution between capital and labour, $\sigma$, is allowed to drop from 1 (CD case) to values in the range of estimated values, this empirical regularity can be explained within a DSGE model with MPMF and deep habits in private and government consumption. Estimates of $\sigma$ are between 0.3 and 0.6 (Klump et al., 2007; Chirinko, 2008; Cantore et al., 2011, 2010a; León-Ledesma et al., 2010).

In Figure 3 we show that the introduction of the CES production function – obtained by setting $\sigma = 0.4$ – marginally diminishes the output multiplier to almost 1.4 (which is about 83% of the value obtained in the CD case), while the unemployment multiplier drops to -0.18 percentage points (about 67% of the value obtained in the CD case). In addition, factor shares react to the government spending expansion.\(^{14}\)

\(^{14}\)In the exercises we perform in this subsection and the rest of the paper, different responses of unemployment in absolute
Figure 4: Sensitivity of the results to different values of the elasticity of substitution between capital and labour, $\sigma$.

Note: Fiscal policy: government spending expansion (1% of output, lump-sum taxes, balanced budget). Model: RBC with Mortensen-Pissarides Matching Friction (MPMF), deep habits in consumption ($\theta_c = 0.86$ and $\varphi_c = 0.85$).

The unequal effects on the output and unemployment multipliers depend on the fact that lowering the elasticity of substitution in the CES production function is equivalent to assuming that the technology is closer to the Leontief case, i.e. capital and labour are more complements than substitutes. In Figure 4 we show that, as $\sigma$ is allowed to assume values lower than one, given that capital is unable to change instantaneously in response to the fiscal expansion, firms have smaller incentives to create new jobs through vacancy posting. However, both the negative wealth effect (coming from the absorption of resources by the government) and the substitution of leisure with consumption (coming from the decline in the mark-up due to the presence of deep habits) still act in the same direction of causing a substantial increase in the supply of hours of work.

Similarly to the comparison made above for the fiscal multipliers, we can quantitatively compare the impact responses of equilibrium hours worked, wage and vacancies obtained in the CD case ($\sigma \rightarrow 1$) with the CES case, i.e. the case in which $\sigma$ takes a value in the range of empirical estimates ($\sigma = 0.4$). With a CES the response of hours worked is around 80% of the response obtained with a CD, while the responses of the real wage and vacancies with a CES are around 62% and 67% of the responses delivered by a CD, respectively.

In Figure 5 we plot the peak elasticity of the unemployment rate to output in response to a government deviations, obtained by changing some parameter values, are comparable as we ensure that steady-state unemployment is the same across calibrations. This is allowed by the calibration strategy itself, which entails targeting a specific job finding probability $\bar{p}$ and setting the job separation rate $\lambda$. In fact, steady-state unemployment, through the Beveridge curve, is a function of only $\bar{p}$ and $\lambda$, i.e. $\bar{n} = \bar{p}/(\lambda + \bar{p})$. 

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spending expansion at different levels of the elasticity of substitution between capital and labour. When \( \sigma \) drops from 1 (CD case) to the lower bound of the range of empirical estimates \( \sigma = 0.3 \), the peak elasticity of the unemployment rate to output drops by around 20%.

In sum, if the technology operating in the economy is represented by a CES production function, as \( \sigma \) falls, the growth of output fostered by a government spending expansion is sustained relatively more by an increase in the intensive margin (current employees work longer hours) than an increase in the extensive margin (new job creation).\(^{15}\)

### 4.4 Debt-financed fiscal policy and distortionary taxation

In order to introduce government debt and distortionary taxes we set the steady-state tax rates on consumption, the labour income and the return on capital to the values reported by Christiano et al. (2010), i.e. \( \tau_c = 0.05 \), \( \tau^w = 0.24 \), and \( \tau^k = 0.32 \). In addition, we let the government accumulate public debt, while tax rates react to government debt according to a feedback rule. We set the response coefficient of the tax rates to government debt \( \rho_{XB} = 0.02 \), the value used by Monacelli et al. (2010). Figure 6 shows that the introduction of distortionary taxes alters the magnitude of the responses of output and unemployment, but unemployment is affected more. Such reductions in the multipliers are (i) due to the distortion on equilibrium employment triggered by the increase in the tax rates following the fiscal expansion and (ii) to the dynamics of the fiscal instruments implied by the feedback rule. In fact, as consumption and the sources of income are taxed more, the tax-adjusted value of non-work activity increases. This reduces the total surplus of employment. In addition, as the imposed feedback rule implies a gradual return of the tax instruments to their steady state value, this implies also postponement of work activities.

\(^{15}\)In the appendix we also show that, as the technology tends to Leontief, the calibration of the bargaining parameter becomes increasingly less important for the equilibrium outcome. Rowthorn (1999) also emphasises the role of CES technology with an elasticity \( \sigma \) below unity for explaining European unemployment persistence despite moves towards greater labour market flexibility as captured by an increase in the firm's bargaining power in our model.
4.5 Jobless recovery

In this subsection we investigate the low-job-creation feature of the fiscal stimulus in a case in which the latter takes place at a recession time. In accordance with the findings of subsection 4.3, we show that a lower-than-one factor elasticity of substitution delivers a more jobless outcome as the output contraction is mitigated more by an increase in the hours of work than by vacancy posting and job creation.

For illustrative purposes, we simulate a recession by means of a negative technology shock. Figure 7 shows the responses of output and unemployment in the cases in which the production function is a CD and a CES with $\sigma = 0.4$ (bold lines in the first and second row of Figure 7). The size of the shock is chosen in order to make output contract by around 7.5% from steady state at peak when the production function is a CES. This is approximately the size of the deviation of the US output from potential in the second quarter of 2009 (the trough of the great recession according to the National Bureau of Economic Research), using the series available in ALFRED (Federal Reserve Bank of St. Louis). The same shock makes output contract less (6%) when the production function is CD. In addition the model predicts that unemployment increases at peak by more than 4 percentage points in the CES case and by 2.5 percentage points in the CD case. In the same charts, we show the mitigatory effects of a fiscal stimulus (dashed lines). In particular, we proxy the fiscal stimulus with a government spending expansion of 5% of output, approximately the expenditure expansion foreseen by the ARRA.\footnote{Blinder and Zandi (2010, table 10) report that the total more-than $1$-trillion 2009 stimulus package in the US was split into a total of $682$ billion for spending increases and $383$ billion for tax cuts. Given that the 2009 US GDP at current prices was $14$ trillion, the spending increases were 4.9$\%$ of GDP.
Figure 7: A fiscal stimulus in a recession.

Note: Model: RBC with Mortensen-Pissarides Matching Friction (MPMF) and deep habits in consumption ($\theta^c = 0.86$ and $\varphi^c = 0.85$). Recession driven by a negative technology shock that leads to a peak output contraction of around 7.5% from steady state with a CES production function. Fiscal stimulus: government spending expansion of 5% of output; lump-sum taxes; balanced budget. First row (CD): simulated output and unemployment responses in the absence and with the fiscal stimulus under a Cobb-Douglas technology ($\sigma \rightarrow 1$). Second row (CES): responses under a CES technology ($\sigma = 0.40$). Third row (CD and CES): ratios of impulse responses with and without the fiscal stimulus under CE and CES technologies.

It is evident that while the fiscal stimulus has similar effects in terms of output stabilisation, unemployment stabilisation is considerably less pronounced under the CES production function. The third row of Figure 7 plots the ratios of the impulse responses with the fiscal stimulus activated with respect to the impulse responses with no fiscal stimulus, in the two alternative cases of CD and CES. In the experiment proposed here, the output contraction in the presence of the fiscal stimulus is around 40% of the contraction in the no-fiscal policy scenario under CD and around 25% under CES. The rise in unemployment in the presence of the fiscal stimulus is instead 50% less pronounced under CD and around 20% under CES. In other words, at a recession time, the model with a CES production function predicts that a government spending expansion fosters a considerably more jobless recovery. As explained in Subsection 4.3, the intuition is to be found in a higher degree of factor complementarity, which is in line with empirical estimates, coupled
with the inability of capital to react quickly to shocks and the costly nature of job creation. Given that capital is unable to change instantaneously, if capital and labour are complementary enough, in response to the fiscal expansion firms have smaller incentives to create new jobs through vacancy posting, which is a costly process. However, both the negative wealth effect (coming from the absorption of resources by the government) and the substitution of leisure with consumption (coming from the decline in the mark-up due to the presence of deep habits) still act in the same direction of causing an increase in the supply of hours of work per employee, which is sufficient to foster a tangible increase in output. Therefore, the mitigatory effects of the fiscal stimulus on the output contraction are driven relatively more by the fact that current employees work longer hours rather than because of the creation of more jobs.

5 The fiscal stimulus in a NK extension of the model

This section offers a new-Keynesian (NK) extension of the model that includes sticky prices and monetary policy. Price stickiness is introduced as in Rotemberg (1982), i.e. by assuming that changing prices costs resources\(^\text{17}\) while monetary policy is set by imposing a Taylor rule

\[
\log \left( \frac{R^n_t}{R^n} \right) = \rho_r \log \left( \frac{R^n_{t-1}}{R^n} \right) + (1 - \rho_r) \left[ \rho_\pi \log \left( \frac{\Pi_t}{\Pi} \right) + \rho_y \log \left( \frac{Y_t}{Y} \right) \right], \tag{10}
\]

where \(R^n_t\) is the nominal interest rate, \(\Pi_t\) is the gross inflation rate, and \(\rho_r\), \(\rho_\pi\) and \(\rho_y\) are parameters. A Fisher equation links the ex-post real interest rate \(R_{t+1}\) to the nominal interest rate:

\[
R_{t+1} = E_t \left[ \frac{R^n_t}{\Pi_{t+1}} \right]. \tag{11}
\]

5.1 Results

Woodford (2011) shows that adding sticky prices into an otherwise standard DSGE model enhances the effects of a government spending expansion. Jacob (2011) argues that if price stickiness is added into a model with deep habit formation the countercyclical movement that the government spending shock induces in the mark-up is milder, that private consumption may still be crowded out as in traditional RBC and NK models and, consequently, the output multiplier becomes small. We show that, with deep habit formation, the addition of price stickiness may indeed soften the effects of a government spending expansion. However, we also find that (i) for an empirically plausible degree of deep habit formation and price stickiness the effects of a fiscal stimulus in terms of consumption and investment crowding-ins, the decline in the mark-up, the increase in the real wage, and the sizes of the output and unemployment multipliers are quite robust to the introduction of price stickiness; and (ii) Jacob’s result (as evident also in the robustness exercises of his paper) is dependent on the assumption that the Taylor rule has a strong monetary response to the output gap that makes the nominal interest rate counteract the output expansion to an extent that the effects of the fiscal expansion are offset.

\(^{17}\)The use of price-adjustment costs as in Rotemberg (1982) is shared by virtually all papers featuring deep habits in consumption as it is a rather straightforward addition from a technical point of view. By contrast using Calvo-type contracts introduces firm-specific habit effects which are more difficult to handle.
NKPC in quarterly terms. Namely, the log-linearised NKPC as response of prices to the marginal cost and hence it is impossible to compare the deep habits New-Keynesian Phillips Curve (NKPC) slope to the Calvo analogue. Hence, following Jacob (2011), we interpret the slope of the standard forward-looking NKPC in quarterly terms. Namely, the log-linearised NKPC assumes the following form: 
\[ \Pi_t = \beta E_t \Pi_{t+1} + \kappa MC_t, \]
where \( \kappa = \frac{\eta - 1}{\xi} \) under Rotemberg pricing and \( \kappa = \frac{(1-\beta)(1-\xi)}{\xi} \) under Calvo contracts, where \( \xi^* \) is the Calvo parameter that

As a result, it is not price stickiness per se that subverts the effects of a government spending expansion, but an aggressive monetary response that goes exactly in the opposite direction of output growth, which is the primary goal of the fiscal stimulus itself.

Figure 8 shows the effects of an expansion of government expenditures at different degrees of price stickiness in two alternative scenarios. First, in the top panel of Figure 8, we assume that the nominal interest rate exhibits persistence in line with the data (\( \rho_y = 0.8 \)) and that the monetary authority reacts only to inflation (\( \rho_{\Pi} = 2 \)) and not to the output gap (\( \rho_y = 0 \)). We explore increasing degrees of price stickiness, \( \xi \). A \( \xi = 29.41 \) corresponds to a Calvo contract average duration of around 3 quarters for our calibration.\(^\text{18}\) When we introduce price stickiness, the effects of the fiscal expansion become softened by

\(^{\text{18}}\)Both parameter values are the posterior estimates found by Smets and Wouters (2007).

\(^{\text{19}}\)Jacob (2011) shows that for a given value of Rotemberg adjustment costs, the introduction of deep habits reduces the response of prices to the marginal cost and hence it is impossible to compare the deep habits New-Keynesian Phillips Curve (NKPC) slope to the Calvo analogue. Hence, following Jacob (2011), we interpret the slope of the standard forward-looking NKPC in quarterly terms. Namely, the log-linearised NKPC assumes the following form: 
\[ \Pi_t = \beta E_t \Pi_{t+1} + \kappa MC_t, \]
where \( \kappa = \frac{\eta - 1}{\xi} \) under Rotemberg pricing and \( \kappa = \frac{(1-\beta)(1-\xi)}{\xi} \) under Calvo contracts, where \( \xi^* \) is the Calvo parameter that
the decrease in the rate of inflation. This occurs if the shift in the aggregate supply, due to the presence of a high level of deep habits, is relatively strong given the shift in the aggregate demand due to the government spending expansion. However, the effects of a government spending expansion are similar to those obtained in the flexible-price. In the lower panel of Figure 8, we set a Taylor rule featuring a strong response to the output gap ($\rho_y = 0.5$). In this case if prices are sticky, the nominal interest rate reacts positively to the rise in output despite the fall in inflation, the real interest rate reaction becomes positive and offsets the effects of the fiscal expansion.

Figure 9 shows the impact responses (peak responses for unemployment) (i) at different levels of monetary policy response to the output gap and (ii) at different degrees of deep habit formation. Surfaces show that, even at high degrees of deep habit formation, a substantial monetary policy response to the output gap may offset the expansionary effects of a government spending expansion. In particular, unemployment may also rise and consumption crowded out if $\rho_y$ is above 0.4, while the output multiplier falls below one with a $\rho_y$ above 0.6. Is the observed response parameter $\rho_y$ so high and should it be so from an optimal policy perspective? In the empirical DSGE model literature, estimates of the value of $\rho_y$ are typically low. For example in Smets and Wouters (2007) in a standard NK model with superficial habit, no unemployment and Cobb-Douglas production, estimated using US data by Bayesian methods over 1984:1-2004:1, a posterior mean corresponding to $\rho_y = 0.08$ is obtained. These findings are typical of this literature. In the optimal policy literature optimised interest rate rules using a welfare criteria also find a weak long-run response of the interest rate to the output gap; for example, Schmitt-Grohe and Uribe (2007) find $\rho_y = 0.1$ and Levin et al. (2006), Levine et al. (2008) and Levine et al. (2011) all find its welfare-improving contribution to be so small as to be ignored in their optimised rules.  

6 Concluding remarks

We have analysed the effects of a government spending expansion in a DSGE model with Mortensen-Pissarides labour market frictions, deep habits in private and public consumption and a constant-elasticity-of-substitution (CES) production function.

The combination of deep habits and CES technology is crucial. The presence of deep habits enables the model to deliver output and unemployment multipliers in the high range of recent empirical estimates, while an elasticity of substitution in the range of available estimates allows it to produce a scenario compatible with the observed jobless recovery. In other words, factor complementarity proves to be a determinant of the jobless outcome of a fiscal stimulus.

Further, if sticky prices are added into the model, an accommodative monetary policy with respect to the output gap, plays an important role for the size of fiscal multipliers and, more generally, for the expansionary effects of the fiscal stimulus.

determines the average quarterly duration of contracts $\frac{1}{1-\xi}$. Given a certain $\xi$, it is straightforward to induce the implied analogous contract duration in the Calvo world.

20 These results abstract from active fiscal stabilisation policy. Our findings suggest that when fiscal rules are added their efficacy would require an even weaker response of interest rates to the output gap. This is confirmed in Schmitt-Grohe and Uribe (2007). Indeed they devote a whole subsection to “the importance of (monetary policy) not responding to output”. For a study of optimal monetary and fiscal policy in a new Keynesian model with deep habit see Leith et al. (2009). In both these models, there is no unemployment and Cobb-Douglas production is assumed, so important features of our set-up are missing. Nonetheless their optimised interest rate rules in conjunction with fiscal stabilisation of debt also feature a weak long-run response to the output gap.
The results presented in this paper are an important starting point for future research: (i) it would be interesting to empirically evaluate the building blocks of the model via the comparison of the marginal likelihood in a Bayesian estimation setting; (ii) given the binding ZLB for the monetary policy rate in the latest recession, it would be worth investigating to what extent this features affect our results; (iii) the model is well-suited for the design of optimal fiscal and monetary rules. In particular, given the sensitivity of the results to the monetary response, examining optimised Taylor rules would be a useful exercise.

These are basically two ways of introducing a nominal interest rate ZLB. In the deterministic setting of this paper the Taylor rule is allowed to remain in force as long as the ZLB is not reached. When this does happen a residual adjustment is added to the rule that avoids a negative interest rate (see, for example, Christiano et al. (2010)). In a stochastic setting a Monte-Carlo approach is needed that uses the previous technique for each stochastic draw (see Coenen et al. (2004)). When it comes to optimal policy a desirable property of a monetary rule is that the ZLB is only reached very infrequently. This outcome is achieved by raising the steady-state inflation rate and penalising the interest rate variability in an optimal fashion – see Levine et al. (2008).
References


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Appendix

A Full model derivation

A.1 Search-match technology

The labour market is characterised by standard Mortensen-Pissarides search-match frictions in which firms fill jobs by posting vacancies. Let \( n_t \) be the number of employed workers and total population be normalised to one. Conventionally, we assume that the number of new hires or “matches”, \( M_t \), is a Cobb-Douglas function of unemployed workers, \( u_t \equiv 1 - n_t \), and vacancies, \( v_t \), \( M_t = \kappa u_t^\omega v_t^{1-\omega} \), where \( \kappa \) represents the efficiency of the matching process and \( \omega \in (0, 1) \) is the elasticity of the number of matches to unemployment. Thus, the current probability that a worker finds a match is \( p_t = \frac{M_t}{u_t} = k \left( \frac{v_t}{u_t} \right)^{\omega-1} = k \theta_t^{1-\omega} \), where \( \theta_t \equiv \frac{v_t}{u_t} \) is commonly labelled as the labour market “tightness”. The more vacancies are posted, given a certain level of unemployment, the tighter the labour market is said to be. Analogously, the current probability that a firm fills a vacancy is given by \( q_t = \frac{M_t}{v_t} = k \theta_t^\omega \). Both firms and workers take \( p_t \) and \( q_t \) as given. The two probabilities are linked by \( p(\theta_t) = \theta_t q(\theta_t) \) and \( q'(\theta_t) < 0, p'(\theta_t) > 0 \).

The law of motion of aggregate employment can be written as:

\[
n_{t+1} = M_t + (1 - \lambda)n_t,
\]

where \( \lambda \) is an exogenous job destruction rate.

A.2 Households

The economy is populated by a continuum of identical households indexed by \( j \in [0, 1] \) who have preferences over a continuum of differentiated consumption varieties indexed by \( i \in [0, 1] \). Household members can be either employed or unemployed. The employed at firm \( i \in [0, 1] \) earn a real wage \( w_{it} \) and suffer disutility from working, while the unemployed receive an unemployment benefit \( w_{ut} \). Following Ravn et al. (2006), we assume that households exhibit external deep habit formation in consumption, i.e. habits are formed on the average consumption level of each variety of good. Let \( n_{jt} \) be the number of employed household members, and \( h_{jt} \) be the hours that each employed individual devotes to work activities. Then, the total hours of labour supplied by household \( j \) is \( N_{jt} \equiv n_{jt} h_{jt} \). Let the total number of household members be normalised to one, so that \( n_{jt} \) can be interpreted as an employment rate. Let also the total time available to individuals be normalised to one. Then, the leisure time for the employed members of household \( j \) is \( l_{jt} \equiv 1 - h_{jt} \), while the unemployed “enjoy” leisure \( l_{jt} = 1 \).

Then, the representative household’s instantaneous utility function is given by:

\[
U((X_{jt}^i)^{l_{jt}}, n_{jt}, 1 - h_{jt}) = n_{jt}U((X_{jt}^i)^{l_{jt}}, 1 - h_{jt}) + (1 - n_{jt})U((X_{jt}^i)^{l_{jt}}, 1),
\]

where \((X_{jt}^i)^{l_{jt}}\) is a habit-adjusted composite of differentiated consumption goods:

\footnote{We also assume that workers can perfectly insure themselves against idiosyncratic shocks, i.e. that income is pooled between the employed and the unemployed.}
\[
(X_t^i)^j = \left[ \int_0^1 (C_{it}^j - \theta^c S_{it-1}^c)^{1-\frac{1}{\theta}} \, dt \right]^{\frac{1}{1-\theta}},
\]  
(A.3)

parameter \( \eta \) is the intratemporal elasticity of substitution across varieties, \( \theta^c \in (0,1) \) is the degree of deep habit formation on each variety, and \( S_{it-1}^c \) denotes the stock of external habit in the consumption of good \( i \). The stock of external habit \( S_{it}^c \) evolves over time according to the following law of motion:

\[
S_{it}^c = \varrho^c S_{it-1}^c + (1 - \varrho^c)C_{it},
\]  
(A.4)

where \( \varrho^c \in (0,1) \) measures the speed of adjustment of the stock of external habit in the consumption of variety \( i \) to changes in the average level of consumption of the same variety.

For household \( j \), the Beveridge curve is given by:

\[
n_t^{j+1} = (1 - \lambda)n_t^j + p(\theta_t)(1 - n_t^j).
\]  
(A.5)

Let us also assume that household \( j \) has \( K_t^j \) capital holdings, which evolve according to the following law of motion:

\[
K_{t+1}^j = (1 - \delta)K_t^j + I_t^j \left[ 1 - S \left( \frac{I_t^j}{P_{t-1}} \right) \right],
\]  
(A.6)

where \( \delta \) is the capital depreciation rate, \( I_t^j \) is investment taking place at time \( t \), and \( S(\cdot) \) represents an investment adjustment cost satisfying \( S(1) = S'(1) = 0 \) and \( S''(1) > 0 \). We assume that investment is also a composite of differentiated goods; however it does not exhibit deep habit formation, i.e. \( I_t^j = \left[ \int_0^1 (I_t^j)^{1-\frac{1}{\eta}} \, dt \right]^{\frac{1}{1-\eta}} \). Expenditure minimisation leads to the optimal level of demand of investment goods for each variety \( i \):

\[
I^*_t = \left( \frac{P_t}{P_t} \right)^{-\eta} I_t^j,
\]  
(A.7)

where \( P_t = \left[ \int_0^1 P_{it}^{1-\eta} \, dt \right]^{\frac{1}{1-\eta}} \) is the nominal price index.

Each household \( j \) solves a two-stage problem. Letting \( P_it \) be the price of variety \( i \), they first minimise total expenditure \( \int_0^1 P_it C_{it}^j \, di \) over \( C_{it}^j \), subject to (A.3). This leads to the optimal level of demand for each variety \( i \) for a given composite:

\[
C_{it}^j = \left( \frac{P_t}{P_t} \right)^{-\eta} (X_t^i)^j + \theta S_{it-1}^c,
\]  
(A.8)

which is characterised by a price-elastic component and a price-inelastic component.

By multiplying both sides of equation (A.8) by \( P_t \), integrating across varieties, and using the definition of nominal price index, we obtain the nominal value of the habit-adjusted consumption composite \( P_t(X_t^j)^j = \int_0^1 P_t C_{it}^j - \theta S_{it-1}^c \, di \), which can be rearranged to write the household’s real consumption expenditure \( C_t^j \) as a function of the consumption composite and the stock of habit: \( C_t^j = (X_t^i)^j + \Omega_t \), where \( \Omega_t = \theta^c \int_0^1 P_{it}^{1-\eta} S_{it-1}^c \, di \).

The second stage of the problem faced by household \( j \) at time \( t \) is choosing paths for the habit-adjusted
consumption composite \((X_t^c)^j\), capital \(K_{t+1}^j\), investment \(I_t^j\), and government real bond holdings \(B_t^j\), which pay the gross real interest rate \(R_{t+1}\) one period ahead, to maximise lifetime utility:

\[
H_t^j \left( n_t^j, K_t^j, B_t^j \right) = \max_{(X_t^c)^j, K_{t+1}^j, I_t^j} \left\{ U \left( (X_t^c)^j, n_t^j, 1 - h_t^j \right) + \beta E_t H_t^j \left( n_{t+1}^j, K_{t+1}^j, B_{t+1}^j \right) \right\}, \tag{A.9}
\]

where \(\beta \in (0, 1)\) is the discount factor, subject to the law of motion of capital (A.6) and the following budget constraint:

\[
(1 + \tau_t^C) \left( (X_t^c)^j + \Omega_t \right) + I_t^j + \tau_t + B_t^j = (1 - \tau_t^W) n_t^j h_t^j w_t + (1 - n_t^j) w_u + (1 - \tau_t^K) R_t^K K_t^j + R_t B_{t-1}^j + \int_0^1 J_{it} \, di, \tag{A.10}
\]

where \(\tau_t^C\), \(\tau_t^W\) and \(\tau_t^K\) are tax rates on consumption, labour income and the return on capital, respectively; \(\tau_t\) is a lump-sum tax; \(R_t^K\) is the rental rate of capital; and \(\int_0^1 J_{it} \, di\) represents firms’ profits.

The first-order condition with respect to the consumption composite \((X_t^c)^j\) implies that the Lagrange multiplier on the household’s budget constraint (A.10) is equal to \(\Lambda_t^j = \frac{U_{t,t+1}^j}{1 + \tau_t^C}\), where \(U_{t,t+1}^j\) is the marginal utility of the consumption composite. Let \(\Lambda_t^j Q_t^j\) be the multiplier on the capital accumulation equation (A.6), and \(Q_t^j\) represent Tobin's Q. Then, the first-order condition with respect to capital, \(K_t^j\), yields the following Euler equation:

\[
Q_t^j = E_t \left\{ D^j_{t,t+1} \left[ \left( 1 - \tau_t^k \right) R_t^K + (1 - \delta) Q_{t+1}^j \right] \right\}, \tag{A.11}
\]

where \(D^j_{t,t+1} = \beta \frac{U_{t+1,t}}{U_{t,t+1}^j} \frac{1 + \tau_t^C}{1 + \tau_t^k}\) is the stochastic discount factor. The first order condition with respect to investment \(I_t^j\) yields the following:

\[
\left\{ Q_t^j \left( 1 - S \left( \frac{I_t^j}{I_{t-1}^j} \right) \right) - S' \left( \frac{I_t^j}{I_{t-1}^j} \right) \left( \frac{I_t^j}{I_{t-1}^j} \right)^2 \right\} + E_t \left( D^j_{t,t+1} Q_{t+1}^j S' \left( \frac{1 + \lambda}{I_t^j} \right) \left( \frac{1 + \lambda}{I_t^j} \right)^2 \right) = 1; \tag{A.12}
\]

while the first order condition with respect to real government bonds implies:

\[
1 = E_t \left[ D^j_{t,t+1} R_{t+1} \right]. \tag{A.13}
\]

Employment \(n_t^j\) is determined as a result of a Nash wage bargaining, as described below. The surplus of the household in the bargaining, \(S_t^{w,j}\), can be computed as the value of having an additional household member employed. By using the envelope condition for employment, we obtain:

\[
(S_t^{w})^j = H_{nt}^j \left( n_t^j, K_t^j, B_t^j \right) = (1 - \tau_t^W) w_k h_t^j - \left[ w_u - \frac{U_{n,t}^j}{U_{x,t}^j} \right] + (1 - \lambda - p(\theta_t)) E_t \left[ D_{t,t+1} (S_{t+1}^{w})^j \right], \tag{A.14}
\]
which implies that the surplus from employment for the household is increasing in the net labour income plus the expected value from being employed the next period and decreasing in the opportunity costs.

Finally, hours of work $h^j_t$ are chosen in a way that makes the bargain efficient, as again shown below.

### A.3 Government

In each period $t$, the government allocates spending $P_tG_t$ over differentiated goods sold by retailers in a monopolistic market to maximise the quantity of a habit-adjusted composite good:

$$X^g_t = \left[ \int_0^1 (G_{it} - \theta^c S^g_{it-1})^{1-\eta} di \right]^{\frac{1}{1-\eta}},\quad (A.15)$$

subject to the budget constraint $\int_0^1 P_d G_d \leq P_t G_t$, where $\eta$ is the elasticity of substitution across varieties, $S^g_{it-1}$ denotes the stock of habits for government expenditures, which evolves as:

$$S^g_{it} = \varphi t S^g_{it-1} + (1 - \varphi t) G_{it}.\quad (A.16)$$

At the optimum:

$$G_{it} = \left( \frac{P_d}{P^t} \right)^{-\eta} X^g_t + \theta^c S^g_{it-1}.\quad (A.17)$$

Aggregate real government consumption $G_t$ is set as an exogenous process:

$$\log \left( \frac{G_t}{\bar{G}} \right) = \rho_G \log \left( \frac{G_{t-1}}{\bar{G}} \right) + \epsilon_t^g,\quad (A.18)$$

where $\bar{G}$ is the steady-state level of government spending, $\rho_G$ is an autoregressive parameter and $\epsilon_t^g$ is a mean zero, i.i.d. random shock with standard deviation $\sigma^G$.

The government budget constraint will then read as follows:

$$B_t = R_t B_{t-1} + G_t + (1 - n_t) w_u - \tau_t - \tau^C_t C_t - \tau^W_t w_t n_t h_t - \tau^K_t R^K_t K_t,\quad (A.19)$$

while taxes are set according to the following feedback rule:

$$\log \left( \frac{X_t}{\bar{X}} \right) = \rho_X \log \left( \frac{X_{t-1}}{\bar{X}} \right) + \rho_{XB} \frac{B_{t-1}}{Y_{t-1}} + \epsilon_t^X,\quad X_t = (\tau, \tau^C, \tau^W, \tau^K),\quad (A.20)$$

where $\rho_X$ are autoregressive coefficients; $\bar{X}$ are steady state values; $\epsilon_t^X$ are serially uncorrelated, normally distributed shocks with zero mean and standard deviations $\sigma^X$, and $\rho_{XB}$ is the responsiveness of tax $X$ to the debt-to-GDP ratio.

We set steady-state government debt equal to zero in steady state, implying also that the government runs a balanced budget in steady state. To explore the benchmark scenario of lump-sum taxes and fully financed lump-sum taxation, it suffices to set the tax rates and government debts constantly equal to zero,

$$B_t = \tau^C_t = \tau^K_t = 0, \text{ and } \tau_t = G_t + (1 - n_t) w_u.$$
**A.4 Firms**

A continuum of monopolistically competitive firms indexed by \( i \in [0,1] \) uses capital, \( K_{it} \), and labour, \( N_{it} \equiv n_{it}h_{it} \) to produce differentiated goods \( Y_{it} \), which are sold at price \( p_{it} \equiv P_u/P_t \). The technology used in the production process is represented by \( F((ZK)_tK_{it}, (ZN)_tn_{it}h_{it}) \), where \((ZK)_t\) and \((ZN)_t\) are a capital-augmenting technology shock and a labour-augmenting technology shock, respectively.

Employment at firm \( i \) evolves over time according to the following law of motion:

\[
    n_{it+1} = (1 - \lambda)n_{it} + q(\theta_t)v_{it}, \tag{A.21}
\]

where \( \theta_t \) is treated as exogenous by the firm.

In addition, the firm faces hiring costs, \( HC_{it} \), of posting \( v_{it} \) vacancies and employing \( n_{it} \) workers given by:

\[
    HC_{it} = g(z_{it})n_{it}; \quad g', g'' \geq 0, \tag{A.22}
\]

where \( z_{it} \equiv \left( \frac{n_{it}}{v_{it}} \right) \) is the vacancy ratio.\(^2\)

The firm rents capital services from households at a rental rate \( R^K_t \), takes employment \( n_{it} \) as given at time \( t \), and maximises the following flow of discounted profits:

\[
    J_t(n_{it}) = E_t \left\{ \sum_{s=0}^{\infty} D_{t,t+s} \left[ p_{it}(C_{it+s} + G_{it+s} + I_{it+s}) - HC_{it+s} - w_{it+s}n_{it+s}h_{it+s} - R^K_{it+s} K_{it+s} \right] \right\}, \tag{A.23}
\]

with respect to \( K_{it+s}, n_{it+s}, v_{it+s}, C_{it+s}, G_{it+s}, S^c_{it+s}, \theta_{it+s}, \lambda_{it+s} \) and \( p_{it+s} \equiv P_{it+s}/P_{it+s} \) subject to (A.21), (A.22), the demand for good \( i \) in the form of private consumption \( C_{it} \), (A.8), government consumption \( G_{it} \), (A.17), and investment, (A.7), the laws of motion of the stocks of habit for households, (A.4), and the government, (A.16), and the firm’s resource constraint:

\[
    C_{it+s} + G_{it+s} + p^{-1}_{it+s}I_{it+s} = F((ZK)_tK_{it}, (ZN)_tn_{it}h_{it}) = Y_{it}. \tag{A.24}
\]

The corresponding first-order conditions for this problem are:

\[
    R^K_t = MC_t F_{K,it}, \tag{A.25}
\]

\[
    \mu_{it} = (MC_t F_{N,it} - w_{it})h_{it} + g'(z_{it})z_{it} - g(z_{it}) + (1 - \lambda)E_t[D_{t,t+1}\mu_{it+1}], \tag{A.26}
\]

\[
    g'(z_{it}) = q(\theta_t)E_t[D_{t,t+1}\mu_{it+1}], \tag{A.27}
\]

\[
    \nu_t^c = p_{it} - MC_t + (1 - g^c)\lambda_t^c, \tag{A.28}
\]

\[
    \lambda_t^c = E_tD_{t,t+1}(\theta^c\nu_{t+1}^c + g^c\lambda_{t+1}^c), \tag{A.29}
\]

\(^2\)Note in the original Pissarides model \( g(z_{it}) = cz_{it} \) so that hiring costs per vacancy posted are constant.
\[ \nu_t^q = p_{it} - MC_t + (1 - \varrho)\lambda_t^q, \quad (A.30) \]

\[ \lambda_t^q = E_t D_{t,t+1}(\theta^g \nu_{t+1}^q + \varrho^g \lambda_{t+1}^q), \quad (A.31) \]

\[ C_{it} + G_{it} + (1 - \eta)\nu_{it}^{-\eta}I_t + \eta MC_{it} p_{it}^{-\eta}I_t - \eta \nu_{it}^c p_{it}^{-\eta}X_t^c - \eta \nu_{it}^t p_{it}^{-\eta}X_t^g = 0. \quad (A.32) \]

Variables \( MC_t \), \( \mu_{it} \), \( \nu_{it}^c \), \( \lambda_t^c \), \( \nu_{it}^q \), \( \lambda_t^q \) are the Lagrange multipliers associated to constraints (A.24), (A.21), (A.8), (A.4), (A.17), (A.16), respectively. In particular, \( MC_t \) is the shadow value of output and represents the firm’s real marginal cost.

If we denote the nominal marginal cost with \( MC_t^n \), the gross mark-up charged by final good firm \( i \) can be defined as \( M_{it} \equiv P_{it}/MC_t^n = P_{it}/MC_t = p_{it}/MC_t \). In the symmetric equilibrium all final good firms charge the same price, \( p_{it} = P_t \), hence the relative price is unity, \( p_{it} = 1 \). It follows that, in the symmetric equilibrium, the mark-up is simply the inverse of the marginal cost.

By combining equations (A.28), (A.30) and (A.32), substituting for the demands for \( C_{it} \) and \( G_{it} \), (A.8) and (A.17), and rearranging, the optimal pricing decision in the symmetric equilibrium can be written as follows:

\[
\begin{align*}
\left\{ \begin{array}{l}
(X_t^c + X_t^g + I_t) \left[ 1 - \frac{\eta}{\varrho^c} MC_t \right] \\
+ \frac{\eta}{\varrho^c} (1 - \varrho^g) [\lambda_t^c X_t^c + \lambda_t^g X_t^g] - \frac{\varrho^c}{\varrho^g} (S_{t-1}^c + S_{t-1}^g) = 0.
\end{array} \right.
\end{align*}
\] (A.33)

The surplus of the firm from employment at the margin is represented by \( \mu_{it} \):

\[ S_{it}^j = \mu_{it}, \quad (A.34) \]

while \( F_{K, it} \) represents the marginal product of capital, and \( F_{N, it} \) represents the marginal product of labour. Note that (A.26) uses the fact that the product of an employee is given by \( F_{n, it} = F_{N, it} h_{it} \) at the margin.

Iterating (A.26) one period forward and combining it with (A.27) yields the following vacancy equation or job creation condition:

\[
\frac{g'}{(z_{it+1})} = E_t \left[ D_{t,t+1} \mu_{it+1} \right] = E_t \left\{ D_{t,t+1} \left[ (MC_t F_{N, it+1} - w_{it+1}) h_{it+1} + g'(z_{it+1}) z_{it+1} \right] \right. \left. - g(z_{it+1}) + (1 - \lambda) \frac{g'(z_{it+1})}{(\theta_{it+1})} \right\}. \quad (A.35)
\]

Clearly, in the absence of hiring costs, \( g(z_{it+1}) = g'(z_{it+1}) = 0 \), (A.35) becomes \( MC_t F_{N, it} = w_{it} \), the competitive labour market outcome.

### A.5 Wage bargaining and hours worked

Let \( \epsilon \in [0,1] \) denote the firm’s bargaining power and \( S_{it}^w \) be the surplus of a household negotiating with firm \( i \). Then, Nash bargaining implies that the real wage maximise the weighted product of the worker’s and the firm’s surpluses from employment:
\[
\max_{w_{it}} (S_{w_{it}}^{w})^{1-\epsilon} (S_{s_{it}}^{s})^{\epsilon}
\]

The solution to problem (A.36) yields the following surplus-splitting rule:

\[
S_{w_{it}}^{w} = \frac{1-\epsilon}{\epsilon} (1 - \tau_{t}^{w}) S_{s_{it}}^{s}.
\]  

(A.37)

The introduction of the distortionary labour tax makes the workers actual bargaining power fluctuate along the business cycle and reduces the share of the workers in the bargaining itself. Substituting for (A.14), (A.34), and (A.26) into (A.37) and rearranging yields the following wage equation:

\[
w_{it} h_{it} = (1-\epsilon) \left[ MC_{t} F_{N, it} h_{it} - g(z_{it}) + g'(z_{it}) z_{it} + \theta_{t} g'(z_{it}) \right] + \epsilon \left[ w_{u} - \frac{U_{n, t}}{U_{x, t}} \right].
\]  

(A.38)

Condition (A.38) implies that the wage paid to the employee is a weighted average of the marginal product of the employee plus the savings from job continuation, net of the cost of posting vacancies, and the opportunity cost of working, which is increasing in the unemployment benefits, the disutility of working activities and the labour income tax.

Finally, hours are chosen to achieve an efficient bargain. This means that in equilibrium the marginal product of labour must be equal to the marginal rate of substitution between leisure and the consumption composite:

\[
F_{N, it} = -\frac{U_{n, t}}{U_{x, t}}.
\]  

(A.39)

A.6 Equilibrium

In equilibrium all markets clear. The resource constraint completes the model:

\[
Y_{t} = C_{t} + I_{t} + G_{t} + g(z_{t}) n_{t}.
\]  

(A.40)

The system of equations describing the full equilibrium is summarised in Appendix B.

A.7 Introducing sticky prices

The introduction of sticky prices changes the problem of final good firms \( i \in (0, 1) \) presented in Section A.4 in that they now choose the price level, \( P_{it} \), instead of the relative price, \( p_{it} \), and they face quadratic price adjustment costs \( \frac{\xi}{2} \left( \frac{P_{it+1}}{P_{it}} - 1 \right)^{2} \), where parameter \( \xi \) measures the degree of price stickiness. Thus, the profit function now reads as follows:

\[
J_{t}(n_{it}) = E_{t} \left\{ \sum_{s=0}^{\infty} D_{t+s} \left[ \frac{P_{it+s}}{P_{it+s-1}} (C_{it+s} + G_{it+s} + I_{it+s}) - HC_{it+s} - w_{it+s} n_{it+s} h_{it+s} - R_{t+s} K_{it+s} + \frac{\xi}{2} \left( \frac{P_{it+s}}{P_{it+s-1}} - 1 \right)^{2} \right] \right\},
\]  

(A.41)

The first-order conditions with respect to \( K_{it+s}, n_{it+s}, v_{kt+s}, C_{it+s}, S_{it+s}, G_{it+s}, S_{it+s}^{g} \) remain unaltered relative to the flexible-price case, while taking the first-order condition with respect to the price level \( P_{it+s} \) leads to the following:
\[
\begin{align*}
\left\{ \frac{P_{t+1}}{P_{t}} (C_{it} + G_{it}) - \xi \left( \frac{P_{t+1}}{P_{t+2}} - 1 \right) \frac{P_{t}}{P_{t-1}} + (1 - \eta) \left( \frac{P_{t+1}}{P_{t}} \right)^{1-\eta} I_{t} \right. \\
+ \eta M C_{t} \left( \frac{P_{t+1}}{P_{t}} \right)^{-\eta} I_{t} - \eta \nu_{t}^{c} \left( \frac{P_{t+1}}{P_{t}} \right)^{-\eta} X_{t}^{c} - \eta \nu_{t}^{g} \left( \frac{P_{t+1}}{P_{t}} \right)^{-\eta} X_{t}^{g} \\
+ \xi A_{l,t+1} \left[ \left( \frac{P_{t+1}}{P_{t}} - 1 \right) \frac{P_{t+1}}{P_{t}} \right] \right\} = 0. \quad (A.42)
\end{align*}
\]

Similar algebraic manipulations to those described in Section 2.4 lead to the following optimal pricing decision in the symmetric equilibrium:\textsuperscript{3}

\[
\begin{align*}
\left\{ \left( X_{t}^{c} + X_{t}^{g} + I_{t} \right) \left[ 1 - \frac{\eta}{\eta-1} M C_{t} \right] \\
+ \frac{\nu}{\eta-1} (1 - \varphi) \left[ \lambda_{t}^{c} X_{t}^{c} + \lambda_{t}^{g} X_{t}^{g} \right] - \frac{\varphi}{\eta-1} (S_{t}^{c} + S_{t}^{g}) \\
+ \xi E_{t} A_{l,t+1} [\Pi_{t+1} (\Pi_{t+1} - 1) - \Pi_{t} (\Pi_{t} - 1)] \right\} = 0, \quad (A.43)
\end{align*}
\]

where \( \Pi_{t} \equiv \frac{P_{t}}{P_{t-1}} \) is the gross inflation rate. Note that the pricing equation (A.43) collapses to the analogous flexible-price equation (A.32) when \( \xi = 0 \). Furthermore, when \( \xi > 0 \), real cost \( \frac{\xi}{2} \left( \frac{P_{t+1}}{P_{t}} - 1 \right)^{2} \) enters the economy’s resource constraint.

### A.8 Functional forms

Equation (A.2) specialises as a non-additively-separable utility function:

\[
U(X_{t}^{c}, n_{t}, 1 - h_{t}) = n_{t} \frac{X_{t}^{c(1-\varphi)} (1 - h_{t})^{\varphi}}{1 - \sigma_{c}} - 1 + (1 - n_{t}) \frac{X_{t}^{c(1-\varphi)(1-\sigma_{c})}}{1 - \sigma_{c}} - 1,
\]

where \( \sigma_{c} > 0 \) is the coefficient of relative risk aversion, and \( \varphi \) is the elasticity of substitution between leisure and consumption. When \( \sigma_{c} \to 1 \), preferences are represented by an additively separable utility function; while in the case of full employment, i.e. \( n_{t} \to 1 \), the equation reads as a standard utility function in consumption and leisure compatible with balanced growth.

Investment adjustment costs take the form of a quadratic function:

\[
S \left( \frac{I_{t}}{I_{t-1}} \right) = \frac{\mu}{2} \left( \frac{I_{t}}{I_{t-1}} - 1 \right)^{2}, \quad \mu > 0,
\]

while we allow for a convex hiring cost function, i.e. \( g(z_{t}) \) specialises as follows:

\[
g(z_{t}) = \frac{\chi}{1 + \psi} z_{t}^{1+\psi}, \quad \psi > 0.
\]

### B Symmetric equilibrium

Production function and marginal products:

\[
F((ZK)_{t}K_{t}, (ZN)_{t}n_{t}h_{t}) = \left[ \alpha_{K} ((ZK)_{t}K_{t})^{\sigma_{K}} + \alpha_{N} ((ZN)_{t}n_{t}h_{t})^{\sigma_{N}} \right]^{\frac{1}{\sigma_{K} + \sigma_{N}}}
\]

\textsuperscript{3}Equation (A.43) is obtained by combining equations (A.28), (A.30) and (A.42), substituting for the demands for \( C_{it} \) and \( G_{it} \), (A.8) and (A.17), and rearranging.
\[ F_{K,t} = \alpha_K(ZK)_t^{\frac{\sigma-1}{\sigma}} \left( \frac{Y_t}{K_t} \right)^{\frac{1}{\sigma}} \] (B.2)

\[ F_{N,t} = \alpha_N(ZN)_t^{\frac{\sigma-1}{\sigma}} \left( \frac{Y_t}{n_t h_t} \right)^{\frac{1}{\sigma}} \] (B.3)

Utility function, marginal utilities and deep habits in consumption:

\[ U(X^c_t, n_t, 1 - h_t) = n_t \left[ (X^c_t)^{(1-\varphi)(1-h_t)} \right]^{1-\sigma_c} - 1 + (1 - n_t) \frac{(X^c_t)^{(1-\varphi)(1-h_t)} - 1}{1-\sigma_c} \] (B.4)

\[ U_{x,t} = (1 - \varphi) \left( X^c_t \right)^{(1-\varphi)(1-\sigma)} \left[ 1 + n_t \left( (1 - h_t) \varphi(1-\sigma) - 1 \right) \right] \] (B.5)

\[ U_{n,t} = \frac{(X^c_t)^{(1-\varphi)(1-\sigma)}}{1-\sigma} \left[ (1 - h_t) \varphi(1-\sigma) - 1 \right] \] (B.6)

\[ U_{hn,t} = -\varphi \left( X^c_t \right)^{(1-\varphi)(1-\sigma)} (1 - h_t) \varphi(1-\sigma) \] (B.7)

\[ S^c_t = \varphi S^c_{t-1} + (1 - \varphi^c) C_t \] (B.8)

\[ C_t = X^c_t + \theta^c S^c_{t-1} \] (B.9)

Intertemporal investment/consumption decisions:

\[ K_{t+1} = (1 - \delta)K_t + I_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right] \] (B.10)

\[ S \left( \frac{I_t}{I_{t-1}} \right) = \frac{\gamma}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \] (B.11)

\[ Q_t = E_t \left\{ D_{t,t+1} \left[ \left( 1 - \tau^k_{t+1} \right) R^K_{t+1} + (1 - \delta)Q_{t+1} \right] \right\} \] (B.12)

\[ 1 = Q_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) - S' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] + E_t \left\{ D_{t,t+1}Q_{t+1}S' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right\} \] (B.13)

\[ D_{t,t+1} = \beta \frac{U_{x,t+1}}{U_{x,t}} \frac{1 + \tau^c_t}{1 + \tau^c_{t+1}} \] (B.14)

\[ 1 = E_t \left[ D_{t,t+1}^t R_{t+1} \right] \] (B.15)
\[ MC_t F_{K,t} = B_t^K \]  

\textbf{Hiring decisions and wage bargaining:}

\[ g_t = \frac{\chi}{1 + \psi} z_t^{1+\psi} \]  

(B.17)

\[ g_{z,t} = \chi z_t^\psi \]  

(B.18)

\[ n_{t+1} = (1 - \lambda)n_t + q(\theta_t)v_t \]  

(B.19)

\[ g'(z_t) = E_t \left\{ D_{t,t+1} \left[ (MC_t F_{N,t+1} - w_{t+1})h_{t+1} + g'(z_{t+1})z_{t+1} 
- g(z_{t+1}) + (1 - \lambda) \frac{g'(z_{t+1})}{q(\theta_{t+1})} \right] \right\} \]  

(B.20)

\[ w_t h_t = (1 - \epsilon) \left[ MC_t F_{N,t} h_t - g(z_t) + g'(z_t)z_t + \theta_t g'(z_t) \right] + \epsilon \left[ \frac{w_u - U_{u,t}}{U_{u,t}} \right] \]  

(B.21)

\[ F_{N,t} = -\frac{U_{nb,t}}{U_{C,t}} \]  

(B.22)

\[ z_t = \frac{v_t}{n_t} \]  

(B.23)

\[ \theta_t = \frac{v_t}{u_t} \]  

(B.24)

\[ u_t = 1 - n_t \]  

(B.25)

\[ q_t = k \theta_t^{-\omega} \]  

(B.26)

\[ p_t = \theta_t q_t \]  

(B.27)

\textbf{Further firms' decisions:}

\[ 1 - MC_t + (1 - g^c)\lambda_t^c = \nu_t^c \]  

(B.28)

\[ E_t D_{t,t+1}(\theta^c\nu_{t+1}^c + \phi^c\lambda_{t+1}^c) = \lambda_t^c \]  

(B.29)
1 - MC_t + (1 - \phi^c) \lambda_t^g = \nu^q_t 

(B.30)

E_t D_{t,t+1}(\theta^c \nu^g_{t+1} + \phi^c \lambda^g_{t+1}) = \lambda^g_t

(B.31)

\left\{ \begin{array}{l}
(X^c_t + X^g_t + I_t) \left[ 1 - \frac{\phi}{\eta - 1} MC_t \right] \\
+ \frac{\phi}{\eta - 1} (1 - \phi^c) [\lambda_c^c X^c_t + \lambda^g X^g_t] - \frac{\phi^c}{\eta - 1} (S^c_{t-1} + S^g_{t-1}) \\
+ \xi E_t A_{t,t+1} [\Pi_{t+1} (\Pi_{t+1} - 1) - \Pi_t (\Pi_t - 1)]
\end{array} \right\} = 0

(B.32)

Government budget constraint and fiscal rules:

\begin{align*}
B_t &= R_t B_{t-1} + G_t + (1 - n_t) w_t \tau_t - \tau^C_t C_t - \tau_t^W t w_n h_t - \tau_t^K R_t^k K_t \\
S^g_t &= \phi^c S^q_{t-1} + (1 - \phi^c) G_t \\
G_t &= X^g_t + \theta^c S^q_{t-1}
\end{align*}

(B.33)

(B.34)

(B.35)

\log \left( \frac{G_t}{G} \right) = \rho_G \log \left( \frac{G_{t-1}}{G} \right) + \epsilon_t^G

\log \left( \frac{X_t}{X} \right) = \rho_X \log \left( \frac{X_{t-1}}{X} \right) + \rho_{XY} \frac{B_{t-1}}{y_{t-1}} + \epsilon_X, \quad X_t = (\tau, \tau^c, \tau^w, \tau^k)

\log \left( \frac{R^n_t}{R^n} \right) = \rho_n \log \left( \frac{\Pi_t}{\Pi} \right) + \rho_y \log \left( \frac{Y_t}{Y} \right)

\frac{R_t}{\Pi_{t+1}} = E_t \left[ \frac{R^n_t}{\Pi_{t+1}} \right]

(B.36)

(B.37)

(B.38)

(B.39)

(B.40)

C Steady state

Steady-state values of the employment rate, n, hours worked, h, and the marginal cost, MC, solve simultaneously the wage equation, (B.21), the economy’s resource constraint, (B.38), and the pricing equation, (B.39).
(B.32), while the value of the remaining unknowns in the system of equations reported in Appendix A can be found recursively by using the following relationships:

\[
\begin{align*}
ZN &= (ZN)_0 \\
ZK &= (ZK)_0 \\
\bar{Y} &= Y_0 \\
\bar{D} &= \beta \\
\bar{Q} &= 1 \\
\bar{\Pi} &= 1 \\
\bar{R}^K &= \frac{R + \delta}{1 - \gamma^K} \\
\bar{K} &= \left(\frac{K}{Y}\right)Y \\
\bar{T} &= \delta\bar{K} \\
\bar{G} &= \left(\frac{G}{Y}\right)Y \\
\bar{S}\bar{\theta} &= \bar{G} \\
\bar{X}^{\bar{y}} &= (1 - \theta^c)\bar{G} \\
\bar{\pi} &= 1 - \bar{\pi} \\
\bar{p} &= \frac{\lambda\pi}{1 - \bar{\pi}} \\
\bar{\theta} &= \left(\frac{\bar{p}}{\kappa}\right)^{1-\omega} \\
\bar{\eta} &= \kappa\bar{\theta}^{-\omega} \\
\bar{\sigma} &= \bar{\pi}\bar{\theta}
\end{align*}
\]
\[ z = \frac{\pi}{\pi} \]  
(C.19)

\[ \mathcal{g} = \frac{\chi}{1 + \psi} e^{1 + \psi} \]  
(C.20)

\[ \overline{g_z} = \chi \overline{z} \psi \]  
(C.21)

\[ F_N = \alpha_N MC \left( ZN \right)^{\frac{\sigma - 1}{\sigma}} \left( \frac{Y}{Nh} \right)^{\frac{1}{\sigma}} \]  
(C.22)

\[ F_n = \overline{F}_N \overline{h} \]  
(C.23)

\[ x^* = \frac{1 - \vartheta}{\vartheta} \overline{F}_N \frac{1 + \overline{n}}{(1 - h)^{\varphi(1 - \sigma_c) - 1}} \]  
(C.24)

\[ c^* = \frac{x^*}{1 - \vartheta} \]  
(C.25)

\[ s^* = c^* \]  
(C.26)

\[ u_n = \frac{(x^*)^{(1 - \vartheta)(1 - \sigma_c)}}{1 - \sigma_c} \left( (1 - \overline{h})^{\varphi(1 - \sigma_c) - 1} \right) \]  
(C.27)

\[ u_x = (1 - \vartheta) \left( x^* \right)^{(1 - \vartheta)(1 - \sigma_c) - 1} \left( 1 + \overline{n} \left( (1 - \overline{h})^{\varphi(1 - \sigma_c) - 1} \right) \right) \]  
(C.28)

\[ w = \frac{1}{Dh} \left[ -\frac{\pi}{\vartheta} + D \left( \overline{F}_n - \mathcal{g} + \overline{g}_z + (1 - \lambda) \frac{\overline{g}_z}{\vartheta} \right) \right] \]  
(C.29)

## D Sensitivity exercises

### D.1 Bargaining power

In Figure D.1 we show how the combination of different elasticities of substitutions (\( \sigma \)) and different levels of firms’ bargaining power (\( \epsilon \)) affect the response of output, unemployment and the real wage to a government spending shock. In the left column, the technology is almost Leontief (\( \sigma = 0.10 \)), in the right column it approximates a Cobb-Douglas (\( \sigma \to 1 \)), while the central column features an intermediate elasticity of substitution in the range of empirical estimates (\( \sigma = 0.40 \)). Impulse responses are drawn with \( \epsilon = \{0.10, 0.50, 0.90\} \).

If \( \sigma = 0.10 \), despite the use of deep habits in consumption, by which the mark-up responds negatively to a government spending shock, the real wage declines as the strong complementarity between capital and labour induces firms to post relatively less vacancies and to a lower reservation wage for them. In other words labour demand (through vacancy posting) “shifts” less than labour supply. This scenario predicts an output multiplier less than unity and a small or even positive response of unemployment (if firms get 90% of total surplus in the wage bargaining).
Figure D.1: Sensitivity of output and unemployment multipliers to changes in the firms’ bargaining power.

Note: Fiscal policy: government spending expansion (1% of output, lump-sum taxes, balanced budget). Model: RBC with Mortensen-Pissarides Matching Friction (MPMF) and deep habits in consumption ($\theta^c = 0.86$ and $\varphi^c = 0.85$). Responses of output and the real wage are in percentage deviations from steady state. For the unemployment rate, absolute changes in percentage points are reported. $\epsilon = \text{firms’ bargaining power}; \sigma = \text{elasticity of substitution between labour and capital.}$

If the technology is sufficiently away from Leontief, the greater firms’ share in the wage bargaining, the smaller the increase in the real wage and the reduction in unemployment, given the smaller incentive for households to sign labour contracts, keeping how they value non-work activities relative to work activities (replacement ratio) constant. While output is not greatly affected by the calibration of the factor elasticity of substitution and the bargaining parameter, the unemployment response is considerably affected by both choices. In addition, as the technology tends to Leontief, the calibration of the bargaining parameter becomes increasingly less important for the equilibrium outcome.

D.2 Hagedorn and Manovskii effect

A common result in the MPMF literature is that unemployment volatility importantly depends on the calibration of the replacement ratio, $\bar{\Theta}$, i.e. the value of non-work to work activities. The higher is the steady-state value of non-work to work activities, the higher is the volatility of unemployment. In the literature $\bar{\Theta}$ ranges between Shimer (2005)’s 0.40 and Hagedorn and Manovskii (2008)’s 0.95. In Figure D.2 we show the sensitivity of the output and unemployment multipliers to the replacement ratio in the model with deep habits in consumption and the CES production function. Increasing $\bar{\Theta}$ increases the magnitudes of both multipliers, however the output multiplier changes only marginally. Even using the CES production function (with $\sigma = 0.4$) – and hence incorporating a mechanism that moderates the
D.3 Quantitative implications of the choice of the replacement ratio and the bargaining power

In Table D.1 we report the impact output multipliers and the unemployment peak multipliers obtained with different parameterisations: (i) our baseline value of the replacement ratio (\(\Theta = 0.7\)), which is close to the estimate of 0.72 of Sala et al. (2008) versus the value used in the baseline calibration of Monacelli et al. (2010) (\(\Theta = 0.9\)), which is in the high range of empirical estimates; (ii) our baseline value for the firms’ bargaining power (\(\epsilon = 0.5\)) versus two extreme cases in which either the workers or the firms get almost the whole surplus (\(\epsilon = 0.1\) or \(\epsilon = 0.9\), respectively); (iii) the CD production function (\(\sigma \rightarrow 1\)) versus a CES with \(\sigma = 0.4\) (our baseline value).

As noted above, while the unemployment multiplier is very sensitive to the choice of the replacement ratio, the output multiplier barely changes. Keeping \(\sigma\) constant, as firms gain a bigger share of the surplus from employment, while the output multiplier slightly increases, the unemployment multiplier significantly drops.
In relative terms (last column), almost irrespective of how the surplus is split between workers ($\epsilon$) and firms and how workers value non-work activities with respect to work activities ($\Theta$), when $\sigma$ drops from 1 (CD case) to 0.4, while the output multiplier is around 4/5 of the value obtained in the CD case; the unemployment multiplier is around or even below 2/3 of the value delivered by the CD case. In sum, the increasingly jobless stimulus obtainable as $\sigma$ drops is robust to the calibration of the replacement ratio and the bargaining power parameter.

Table D.1: The impact of the fiscal stimulus in different scenarios

<table>
<thead>
<tr>
<th>$\Theta$</th>
<th>$\epsilon$</th>
<th>(A) $\Delta Y/$G</th>
<th>(B) $\Delta Y/$G</th>
<th>$\Delta u/$G</th>
<th>(B)/(A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>0.1</td>
<td>$\sigma \to 1$</td>
<td>1.69</td>
<td>1.40</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Delta Y/$G</td>
<td>-0.31</td>
<td>-0.21</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Delta u/$G</td>
<td>-0.27</td>
<td>-0.18</td>
<td>0.67</td>
</tr>
<tr>
<td>0.9</td>
<td>0.5</td>
<td>$\epsilon = 0.5$</td>
<td>1.79</td>
<td>1.46</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Delta Y/$G</td>
<td>-0.14</td>
<td>-0.08</td>
<td>0.57</td>
</tr>
<tr>
<td>0.9</td>
<td>0.1</td>
<td>$\epsilon = 0.1$</td>
<td>1.68</td>
<td>1.39</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Delta Y/$G</td>
<td>-0.82</td>
<td>-0.53</td>
<td>0.65</td>
</tr>
<tr>
<td>0.9</td>
<td>0.5</td>
<td>$\epsilon = 0.5$</td>
<td>1.71</td>
<td>1.41</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Delta Y/$G</td>
<td>-0.70</td>
<td>-0.44</td>
<td>0.63</td>
</tr>
<tr>
<td>0.9</td>
<td>0.9</td>
<td>$\epsilon = 0.9$</td>
<td>1.78</td>
<td>1.45</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Delta Y/$G</td>
<td>-0.30</td>
<td>-0.19</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Note: Government spending expansion (1% of output, lump-sum taxes, balanced budget) in a model augmented with Mortensen-Pissarides Matching Frictions and deep habit formation. Impact output multipliers and peak unemployment multipliers are reported.)