

# Consistency of the Blind Source Separation Computed With Five Common Algorithms for Magnetoencephalogram Background Activity

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## Abstract

Blind source separation (BSS) is widely used to analyse brain recordings like the magnetoencephalogram (MEG). However, few studies have compared different BSS decompositions of real brain data. Those comparisons were usually limited to specific applications. Therefore, we aimed at studying the consistency (i.e., similarity) of the decompositions estimated for real MEGs from 26 subjects using five widely used BSS algorithms (AMUSE, SOBI, JADE, extended-Infomax and FastICA) for five epoch lengths (10 s, 20 s, 40 s, 60 s and 90 s). A statistical criterion based on Factor Analysis was applied to calculate the number of components into which each epoch would be decomposed. Then, the BSS techniques were applied. The results indicate that the pair of algorithms ‘AMUSE–SOBI’, followed by ‘JADE–FastICA’, provided the most similar separations. On the other hand, the most dissimi-

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lar outcomes were computed with ‘AMUSE–JADE’ and ‘SOBI–JADE’. The BSS decompositions were more similar for longer epochs. Furthermore, additional analyses of synthetic signals supported the results of the real MEGs. Thus, when selecting BSS algorithms to explore brain signals, the techniques offering the most different decompositions, such as AMUSE and JADE, may be preferred to obtain complementary, or at least different, perspectives of the underlying components.

*Keywords:* Algorithm comparison, Blind Source Separation (BSS), Consistency, Independent Component Analysis (ICA), Magnetoencephalogram (MEG)

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## 1 Introduction

The electroencephalogram (EEG) and the magnetoencephalogram (MEG) are the only techniques that measure the synchronous oscillations of the cortex directly and non-invasively. Whereas the former records the electrical brain activity, the latter reflects the corresponding magnetic fields [1]. These signals have slightly different characteristics. For instance, MEG is only affected by current flows oriented parallel to the scalp and it is less distorted than the EEG by extra-cerebral tissues [1]. Despite these subtle differences, similar problems are faced when analysing both recordings. Firstly, the signals acquired at a particular sensor are a weighted linear mixture of the underlying brain activity [2]. Therefore, the isolation and analysis of the electromagnetic activity generated by a specific source of interest is a complex task [2]. Moreover, the brain activity is usually recorded together with undesired signals (i.e., artefacts) of physiological or environmental origin [2, 3].

15 Blind Source Separation (BSS) is useful to overcome some difficulties  
16 encountered in EEG and MEG analysis [2, 3]. The BSS estimates the con-  
17 stituent sources (or components) of the observations assuming a linear mix-  
18 ture model [3]. Although the components and the mixing system are un-  
19 known, they can be estimated thanks to a minimal set of assumptions that  
20 includes the statistical independence of the sources [2–5].

21 BSS has been widely applied to EEG and MEG data [2, 3, 5]. For in-  
22 stance, diverse methodologies have been used to detect and remove the arte-  
23 facts [5–9]. BSS is also helpful to isolate brain activity related to specific  
24 brain functions [3, 4, 10] or to improve the discrimination of demented pa-  
25 tients against controls [11–13].

26 There is a wide variety of BSS techniques available and not all algorithms  
27 are based on the same principles. For a review see, for example, [3–5]. Theo-  
28 retical relationships exist among some of the metrics used in the algorithms.  
29 However, it may be difficult to select a priori the most appropriate algorithm  
30 for a particular application [6, 8]. These methods are data-driven and, by  
31 their own nature, exploratory [5].

32 In order to try to clarify the relationships between BSS techniques, a  
33 few studies have compared some algorithms (see [10] and references therein).  
34 However, most of these analyses were based on synthetic (i.e., artificial) sig-  
35 nals. For instance, three Higher-Order Statistics (HOS) algorithms were  
36 compared in [14]. However, basic hypotheses in HOS-BSS were violated in  
37 the experimental design: some synthetic sources were sub-Gaussian and, in  
38 some cases, moving sources were simulated [14]. This can limit the relia-  
39 bility of the results. Moreover, the analysis focused on acoustic signals and

40 the extension of those results to brain recordings is not straightforward [14].  
41 Computational and statistical comparisons among HOS methods were also  
42 performed with super-Gaussian synthetic signals [4]. The main conclusions  
43 supported the robustness of HOS techniques under slight violations of the  
44 assumptions. Additional analysis suggested that different techniques may  
45 reveal different components when applied to real signals [4].

46 Diverse studies have compared BSS algorithms in artefact removal from  
47 EEGs [8, 15–17]. The independence of the extracted components was checked  
48 in the removal of ocular artefacts [18, 19]. However, the most commonly  
49 used algorithms were left out of this analysis and the evaluation was done in  
50 terms of mutual information [18, 19]. This might bias the analysis in favour  
51 of those algorithms directly based on this metric. Moreover, the significance  
52 of the differences among algorithms was not tested [19]. Other analyses have  
53 evaluated the performance of BSS algorithms regarding the quality of their  
54 artefact removal [7]. Recently, an extensive study focused on EEG data has  
55 been published [10]. Nevertheless, it was entirely based on synthetic data  
56 [10]. The outputs of three common BSS algorithms have also been compared  
57 against a new BSS approach based on the short-time Fourier transform [20].  
58 This study suggests that, in the case of spontaneous activity, HOS methods  
59 tend to focus on the extraction of artefacts whereas a Second-Order Statistics  
60 (SOS) approach failed since it tended to extract components with very similar  
61 spectra [20]. However, this analysis was mainly carried out in the specific  
62 framework of the study of the phase differences between components with  
63 data from only one subject [20].

64 To sum up, most comparisons among BSS algorithms were carried out

65 with simulated signals only or in a very particular context, such as artefact  
66 removal [6, 8, 16, 17]. This may limit the application of the results to other  
67 settings. Moreover, a detailed study on the similarity of the decompositions  
68 for real brain recordings computed with different algorithms is lacking [2].

69 Thus, it is important to study the consistency (i.e., similarity) of the  
70 separations estimated from real electromagnetic recordings. This could lead  
71 to further understanding of the relationships among BSS techniques and  
72 to more informed decisions about which algorithms could offer complemen-  
73 tary perspectives in one particular study. By offering information about  
74 which BSS methods provide more similar results, the search for appropriate  
75 techniques for the problem at hand would be facilitated. To achieve this  
76 goal, real MEG background activity will be decomposed using five widely  
77 used BSS algorithms in the analysis of EEGs and MEGs: algorithm for  
78 multiple unknown signals extraction (AMUSE), second-order blind identifi-  
79 cation (SOBI), joint approximate diagonalisation of eigenmatrices (JADE),  
80 Lee-Sejnowski’s extended-Infomax algorithm and Hyvärinen-Oja’s FastICA  
81 algorithm. The results obtained from the real MEG activity will be com-  
82 plemented by measuring the quality of the BSS in a dataset of synthetic  
83 signals.

## 84 **2. Subjects and MEG Recording**

85 Twenty-six healthy elderly subjects without past or present mental disor-  
86 ders participated in this study (9 men and 17 women). Their mean age was  
87  $71.77 \pm 6.38$  years (mean  $\pm$  standard deviation, SD). These subjects are part  
88 of a larger database acquired to study the effects of Alzheimer’s disease in

89 the MEG (see, for instance, [12, 13]). We limited the analyses to the control  
90 subjects to avoid any bias in the results due to that dementia. All subjects  
91 gave their informed consent to participate in the current research, which was  
92 approved by the local ethics committee.

93 The MEG recording process was carried out in a magnetically shielded  
94 room with a 148-channel whole-head magnetometer (MAGNES 2500 WH,  
95 4D Neuroimaging) located in the MEG Centre Dr. Pérez-Modrego at the  
96 Complutense University of Madrid (Spain). During this procedure, the sub-  
97 jects lay on a patient bed with eyes closed in a relaxed state. They were  
98 asked to stay awake and not to move eyes and head. For each subject, five  
99 minutes of MEG recording were acquired at a sampling rate of 678.19 Hz.  
100 Then, the data were decimated to a sampling frequency of  $f_s = 169.55$  Hz.  
101 Afterwards, the recordings were processed with a band-pass FIR filter with  
102 cut-off frequencies at 0.5 Hz and 60 Hz. Finally, the MEGs were divided into  
103 epochs of 10 s, 20 s, 40 s, 60 s and 90 s (1695, 3390, 6780, 10170 and 15255  
104 samples, respectively).

### 105 **3. Blind Source Separation (BSS)**

#### 106 *3.1. Linear Mixing Model for BSS*

107 BSS techniques attempt to represent a set of  $m$  measured time-varying  
108 signals,  $\mathbf{x}(t) = [x_1(t), x_2(t), \dots, x_m(t)]^T$ , where  $^T$  denotes transposition,  
109 as a linear mixture of  $l$  latent underlying components (or sources),  $\mathbf{s}(t) =$   
110  $[s_1(t), s_2(t), \dots, s_l(t)]^T$ , given by a full-rank  $m \times l$  mixing matrix,  $\mathbf{A}$  [3–5].  
111 A vector  $\mathbf{n}(t) = [n_1(t), n_2(t), \dots, n_m(t)]^T$  can also be included in the model  
112 to account for measurement noise [3, 9, 21, 22]. Hence, the BSS model can

113 be represented as:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t). \quad (1)$$

114 In EEG and MEG analysis,  $\mathbf{x}(t)$  denotes the recordings, whereas  $\mathbf{s}(t)$  repre-  
115 sents either neural activity or interference signals of diverse origins [3].

116 Since only the observations  $\mathbf{x}(t)$  are available, several assumptions are  
117 needed to estimate  $\mathbf{A}$  and  $\mathbf{s}(t)$  [3, 4]. In addition to linearity, it is hypothe-  
118 sized that  $m \geq l$  and that the mixture is stationary. Moreover, the compo-  
119 nents are assumed to be mutually independent or, alternatively, decorrelated  
120 at any time delay [3, 5]. All these hypotheses have been validated for brain  
121 signals [2–5].

### 122 3.2. BSS Algorithms

123 Five BSS algorithms commonly used in the analysis of EEGs and MEGs  
124 were compared: AMUSE, SOBI, JADE, extended-Infomax and FastICA [2–  
125 4, 6–8, 12, 13].

126 AMUSE [23] and SOBI [24] are time-structure based methods, also known  
127 as SOS-BSS. They assume that the sources have no spatial-temporal correla-  
128 tions [3]. Thus, these techniques try to diagonalize a set of cross-covariance  
129 matrices computed from  $\mathbf{x}(t)$ . AMUSE only considers two time delays –  
130 usually  $\tau = 0$  and  $\tau = 1$  sample, which corresponds to  $\tau = 0.0059$  s at  
131  $f_s = 169.55$  Hz [23]. As a result, it orders the components by decreasing  
132 linear predictability, a criterion closely related to the signal spectral content  
133 [12, 13]. On the other hand, SOBI uses iterative procedures to simultane-  
134 ously diagonalise multiple temporal lags [24]. Similarly to [15], SOBI was  
135 applied with 50 consecutive time lags from  $\tau = 1$  sample to  $\tau = 50$  samples

136 ( $\tau = 0.2949$  s at  $f_s$ ). This choice was supported by the fact that this set of  
137 delays covered a wide time interval without extending beyond the support of  
138 the average autocorrelation function of the MEG recordings.

139 On the other hand, JADE [25], extended-Infomax [26] and FastICA [4]  
140 rely on HOS, that is, statistical parameters like negentropy or kurtosis. They  
141 look for non-Gaussian sources assuming that  $\mathbf{x}(t)$  are observations of random  
142 variables where the temporal order is irrelevant [3, 4]. In this study, FastICA  
143 was applied with the non-linearity  $\tanh(\cdot)$  and the deflationary approach  
144 [4]. This function was selected for being a good general-purpose function [4].  
145 The extended version of Infomax was used in order to estimate both sub- and  
146 super-Gaussian sources [26]. This version of the algorithm has been widely  
147 applied to EEG and MEG [8, 15, 16]. The number of each type of components  
148 was automatically determined [26]. JADE has no input parameters [4, 6, 25].

149 All these BSS algorithms are contained in the EEGLAB [27], FastICA  
150 [28] and ICALAB toolboxes [29].

### 151 *3.3. Preprocessing and Model Order Selection*

152 The implementation of most BSS algorithms assumes a noiseless mixture  
153 where  $m = l$  [3, 4]. However, EEG and MEG are affected by measurement  
154 noise whose power may not be negligible [9, 21, 22, 30, 31]. Furthermore, the  
155 number of channels in current EEG and MEG systems can be much larger  
156 than that of meaningful BSS components (i.e.,  $m > l$ ) [21, 30]. Hence, a  
157 suitable preprocessing is important to reduce the importance of the measure-  
158 ment noise and the dimensionality of the input signals of the BSS algorithms  
159 [3, 9, 21].

160 The preprocessing applied before a BSS algorithm is usually based on



161 Principal Component Analysis (PCA) [3]. Nevertheless, this approach has  
 162 some drawbacks as it implies a certain degree of arbitrariness in the estima-  
 163 tion of  $l$ . Moreover, it is not clear that the external noise is weak enough at  
 164 all sensors [3, 9, 21]. In contrast to PCA, we apply a preprocessing based on  
 165 factor analysis (FA) that can deal with different noise power at each sensor.  
 166 Moreover, the model order ( $l$ ) has been estimated with a statistical criterion:  
 167 the Minimum Description Length (MDL) [30]. The preprocessing variables  
 168 are computed for the range of possible  $l$  values and, for each of them, the  
 169 value of the statistical criterion MDL is computed. Then, the optimum  $l$  is  
 170 selected as the one providing the minimum MDL. A detailed description of  
 171 FA and the MDL can be found in [30] or [9]. This preprocessing was evalu-  
 172 ated in [9] using synthetic data. The results suggested that it provided more  
 173 accurate estimations of  $l$  than other commonly used PCA-based approaches.  
 174 Furthermore, other studies have found that FA is more parsimonious when  
 175 estimating the value of  $l$  in real EEGs and MEGs than classical PCA schemes  
 176 [21, 22].

### 177 3.4. Comparison of BSS Algorithms

178 A completely accurate quantification of the performance provided by a  
 179 BSS algorithm  $q$  can only be achieved if either the original mixing matrix,  $\mathbf{A}$ ,  
 180 or set of sources,  $\mathbf{s}(t)$ , is known [11, 32, 33]. This is the case when analysing  
 181 synthetic signals. For real EEG and MEG recordings, these data are not  
 182 available. However, the consistency of various BSS algorithms can still be  
 183 precisely computed [29]. In order to do so, two different BSS algorithms  
 184 (algorithm  $q$  and algorithm  $r$ ) must be applied to the same input data in  
 185 order to estimate the corresponding mixing matrices:  $\mathbf{A}^q$  and  $\mathbf{A}^r$  [29]. Then,

186 the columns of these matrices are normalized to unit length vectors and a  
 187 matrix  $\mathbf{P}^{qr}$ , whose size is  $l \times l$ , is computed as:

$$\mathbf{P}^{qr} = (\mathbf{A}^q)^{-1} \mathbf{A}^r. \quad (2)$$

188 If the two algorithms  $q$  and  $r$  provide exactly the same separation,  $\mathbf{P}^{qr}$   
 189 will be a generalized permutation matrix [4]. Similarly, the closer  $\mathbf{P}^{qr}$  is to  
 190 a permutation matrix, the more consistent the separations of the algorithms  
 191  $q$  and  $r$  are [29].

192 In order to measure the degree to which  $\mathbf{P}^{qr}$  is close to a permutation  
 193 matrix, we define the metric  $F$  as  $F = (F_1 + F_2) / 2$ , with  $F_1$  and  $F_2$  computed  
 194 as in [11]:

$$F_1 = \frac{1}{l} \sum_{i=1}^l \left[ \frac{1}{l-1} \left( \sum_{j=1}^l \frac{|p_{ij}|}{\max_k |p_{ik}|} - 1 \right) \right], \quad (3)$$

195 and

$$F_2 = \frac{1}{l} \sum_{j=1}^l \left[ \frac{1}{l-1} \left( \sum_{i=1}^l \frac{|p_{ij}|}{\max_k |p_{kj}|} - 1 \right) \right], \quad (4)$$

196 where  $p_{ij}$  denotes an element of  $\mathbf{P}^{qr}$  and  $l$  is the number of components.

197  $F_1$  measures the average coupling of other sources into one particular  
 198 component, whereas  $F_2$  accounts for the fact that two or more estimated  
 199 components represent exactly the same original source [11]. It is worth not-  
 200 ing that  $F_1$  and  $F_2$  are normalized so that their values do not depend on  
 201 the dimensions of  $\mathbf{P}^{qr}$ . Since  $F_1$  and  $F_2$  are bounded between 0 (for a per-  
 202 fect generalized permutation matrix) and 1,  $F$  also ranges between 0 and 1.  
 203 Hence, the lower the value of  $F$  for a pair of algorithms, the more consistent  
 204 they are (i.e., the outcomes of both algorithms are more similar).

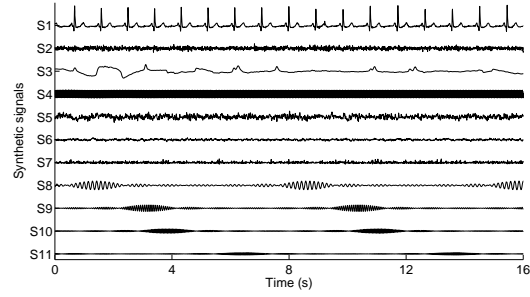
205 *3.5. Synthetic signals*

206 For the sake of completeness, synthetic signals are generated to evalu-  
207 ate the quality of the separations computed with AMUSE, SOBI, JADE,  
208 extended-Infomax and FastICA. However, it should be borne in mind that  
209 the reliability of any results computed from simulated data is limited.

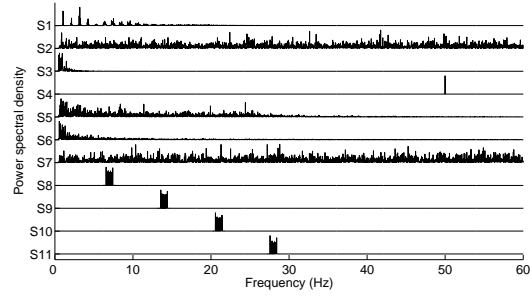
210 The synthetic signals used in this study were developed in [9]. They  
211 are composed by 11 inner components. These signals have the same sample  
212 frequency and were processed with the same filter as the real MEG recordings.  
213 Fig. 1 depicts one example of each synthetic source including their time plot,  
214 power spectral density and histogram. Additional details can be found in [9]:

- 215 1. S1 corresponds to a real electrocardiogram representing the cardiac  
216 artefact.
- 217 2. S2 is an inner white Gaussian noise source.
- 218 3. S3 is a real electrooculogram illustrating ocular activity.
- 219 4. S4 is a sine wave at 50 Hz.
- 220 5. S5 is a real MEG channel selected to have minimal artefactual activity.
- 221 6. S6 is a  $1/f$  noise source.
- 222 7. S7 is a white exponential noise source.
- 223 8. S8 to S11 represented rhythmic activity with main frequencies are 7 Hz,  
224 14 Hz, 21 Hz and 28 Hz.

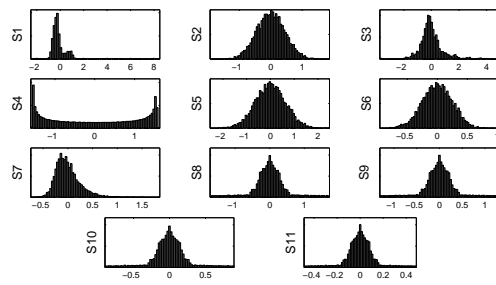
225 The synthetic signals allow to evaluate how close the mixing matrix com-  
226 puted with the BSS is to the actual one. In order to do so, the metric  $F$  is  
227 calculated from a matrix  $\mathbf{P}^{qr}$  where  $\mathbf{A}^q$  refers to the known synthetic mixing  
228 matrix and  $\mathbf{A}^r$  is estimated with a BSS technique.



(a) Time plot.



(b) Power spectral density.



(c) Histogram.

Figure 1: Example of synthetic sources. The time plot (a), power spectral density (b) and histogram (c) are shown.

229 To avoid any influence of the preprocessing, the number of mixtures is set  
230 to the number of components ( $m = l = 11$ ). The synthetic mixing matrix  
231 is created with a Gaussian process with zero mean and SD equal to one [9].  
232 In order to study the influence of the synthetic data length, epochs of 2 s,  
233 4 s, 8 s, 16 s, 32 s and 64 s are considered. For each length, 100 different  
234 instances of the synthetic signals are created with random delays or phases.

### 235 *3.6. Statistical Analysis*

236 Boxplots are used to visually summarise distributions of data. This dia-  
237 gram is composed of a box with three horizontal lines at the lower quartile,  
238 median and upper quartile values. The confidence interval of the median is  
239 indicated with a couple of notches. The boxplot also has two whiskers to  
240 show the extent of the rest of the data, which is estimated as 1.5 times the  
241 interquartile range. Values beyond the end of the whiskers are considered  
242 outliers and are marked with a ‘+’.

243 For the real MEG signals, a one-way ANalysis Of VAriance (ANOVA) is  
244 used to test whether the means of several groups are all equal. This procedure  
245 offers the possibility of partitioning the observed covariance in the data into  
246 components due to diverse explanatory variables (e.g., ‘Pair of algorithms’).  
247 Additionally, a quantitative predictor (i.e., covariate) can be removed from  
248 the samples by a regression in order to account for some variability and  
249 increase statistical power. In this case, the number of components ( $l$ ) can be  
250 taken as a covariate. The Scheffé’s correction will be applied in the post-hoc  
251 multiple comparison procedure. In the case of the synthetic data, it is not  
252 necessary to consider  $l$  as a covariate in the ANOVA since it has a constant  
253 value.

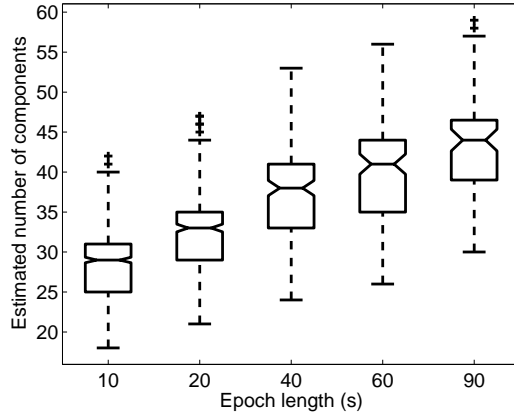


Figure 2: Boxplots showing the number of components ( $l$ ) estimated for the real MEG epoch lengths.

#### 254 4. Results

255 Our main objective was to study the consistency of real MEG data decom-  
 256 positions estimated with five common BSS algorithms for five epoch lengths:  
 257 10 s, 20 s, 40 s, 60 s and 90 s. Firstly, the value of  $l$  was estimated for  
 258 each case with the MDL [9, 30]. Fig. 2 shows the boxplots representing the  
 259 distributions of  $l$  for every epoch length. As it can be expected,  $l$  tended to  
 260 increase with the epoch length of the MEG signal.

261 Secondly, the MEG recordings were preprocessed with the optimal  $l$  value  
 262 estimated for each epoch. These preprocessed signals were decomposed with  
 263 the five BSS methods. Then, the matrices  $\mathbf{P}^{qr}$  were computed for each epoch  
 264 and pair of algorithms and characterized with the metric  $F$ . In order to  
 265 reduce the amount of data to be analysed, we studied only one matrix,  $\mathbf{P}^{qr}$ ,  
 266 instead of both  $\mathbf{P}^{qr}$  and  $\mathbf{P}^{rq}$ . This decision was supported by the fact that  
 267 the average absolute differences for the  $F$  metric between  $\mathbf{P}^{qr}$  and  $\mathbf{P}^{rq}$  were

268 always lower than 1.2%.

269 For each length, the  $F$  values obtained for every pair of algorithms were  
270 averaged. These results are depicted in Fig. 3, where all subplots are repre-  
271 sented with the colour scale used to represent the data [34]. Lower  $F$  values  
272 are related to more consistent (more similar) pairs of algorithms. For all  
273 epoch lengths, Fig. 3 suggests that the most consistent pair of algorithms  
274 was ‘AMUSE–SOBI’ (SOS-based methods), followed by the pair ‘JADE–  
275 FastICA’, which involve HOS. Moreover, Fig. 3 shows that the general level  
276 of consistency improved as the length of the real MEG epochs increased.  
277 This suggests that the separations provided by different algorithms tended  
278 to converge as larger signals were decomposed.

279 For each epoch length, a one-way ANOVA with the Scheffé’s multiple  
280 comparison procedure, ‘Pair of algorithms’ as the grouping factor and ‘Num-  
281 ber of estimated components’ ( $l$ ) as a covariate was used to statistically  
282 evaluate the differences in the  $F$  values.

283 For epochs of 10 s, there were significant differences in the  $F$  values as  
284 a consequence of the factor ‘Pair of Algorithms’, the covariate ‘Number of  
285 estimated components’ and their interaction ( $p \ll 0.0001$  in all cases). The  
286 slopes of the regression of  $F$  against  $l$  were significantly different from 0  
287 ( $p < 0.05$ ) for the pairs of algorithms ‘AMUSE–SOBI’, ‘AMUSE–extended-  
288 Infomax’ and ‘SOBI–extended-Infomax’. In the first case,  $F$  slightly in-  
289 creased with  $n$  (a larger number of components made the decompositions  
290 more different). For the other two pairs, more components produced lower  
291  $F$ . Finally, the post-hoc multiple comparison procedure confirmed that the  
292 level of consistency of the ‘AMUSE–SOBI’ pair was significantly lower from

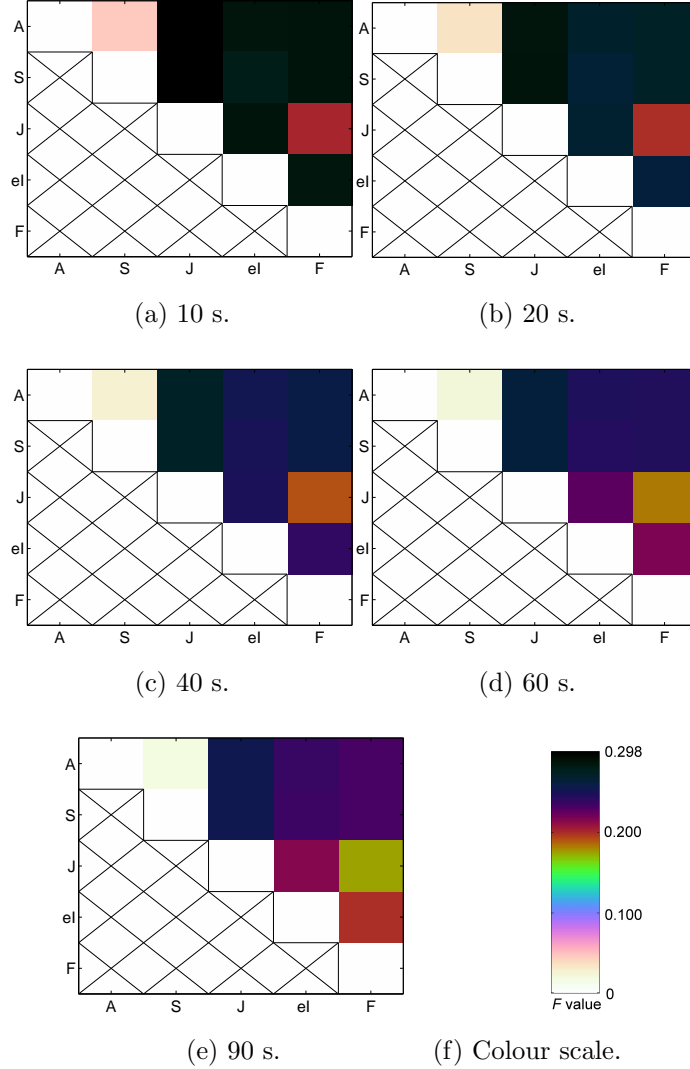


Figure 3: Average  $F$  values for each pair of BSS algorithms (A: AMUSE, S: SOBI, J: JADE, el: extended-Infomax, F: FastICA) and length of the real MEG epochs: (a) 10 s, (b) 20 s, (c) 40 s, (d) 60 s and (e) 90 s. For the sake of clarification, the zero-level of the  $F$  metric for redundant pairs has been included in the diagonal of each subplot. The colour scale [34] used to represent the  $F$  values appears in (f).



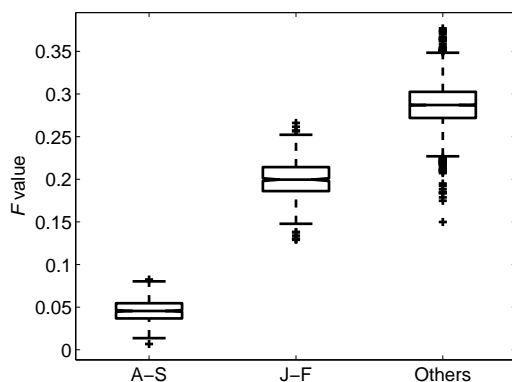


Figure 4: Boxplots showing the  $F$  values for the pairs of algorithms ‘AMUSE–SOBI’ (A-S) and ‘JADE–FastICA’ (J-F) and jointly for the other 8 pairs of algorithms (Others) for real MEG epochs of 10 s.

293 that of ‘JADE–FastICA’, and that the  $F$  values for these two pairs also dif-  
 294 fered significantly from the other eight pairs. Of note is that ‘AMUSE–JADE’  
 295 and ‘SOBI–JADE’ offered the most different separations. To illustrate this  
 296 statistical differences among pairs of algorithms, Fig. 4 shows the boxplots  
 297 of the  $F$  values for pairs ‘AMUSE–SOBI’, ‘JADE–FastICA’ and the rest of  
 298 pairs. It can be observed that ‘AMUSE–SOBI’ has the lowest  $F$  values,  
 299 followed by ‘JADE–FastICA’.

300 The results obtained for epochs of 20 s were very similar to those previ-  
 301 ously reported for 10 s, with the same level of significant differences in the  
 302 grouping factor and covariate. In this case, the regression of  $F$  against  $l$  was  
 303 significantly positive for ‘JADE–FastICA’ as well as for the cases reported  
 304 for the 10 s case.

305 The case of epoch length equal to 40 s presents slight deviations from the  
 306 previous results. The  $F$  values varied significantly with ‘Pair of Algorithms’

307 ( $p \ll 0.0001$ ), ‘Number of estimated components’ ( $p = 0.0100$ ) and their in-  
308 teraction ( $p \ll 0.0001$ ). The slopes of the regression for ‘AMUSE–extended-  
309 Infomax’, ‘AMUSE–FastICA’, ‘SOBI–extended-Infomax’ and ‘SOBI–FastICA’  
310 decreased with  $l$ , whereas the pair ‘JADE–FastICA’ offered less similar sep-  
311 arations for larger  $l$ . Likewise the previous cases, the post-hoc multiple com-  
312 parison procedure indicated that ‘AMUSE–SOBI’ and ‘JADE–FastICA’ of-  
313 fered the most similar decompositions between algorithms. The analysis also  
314 suggested that the outcomes of ‘AMUSE–JADE’ and ‘SOBI–JADE’ were the  
315 most dissimilar.

316 When epochs of 60 s were studied, the  $F$  values only presented significant  
317 differences for ‘Pair of Algorithms’ and its interaction with  $l$  ( $p \ll 0.0001$  in  
318 both cases). The slopes of  $F$  against  $l$  that are significantly different from zero  
319 are identical to those indicated in the analysis made for epochs of 40 s. The  
320 multiple comparison procedure suggested that ‘AMUSE–SOBI’ and ‘JADE–  
321 FastICA’, in that order, were the most consistent pairs of algorithms. There  
322 was also a tendency for ‘AMUSE–JADE’ and ‘SOBI–JADE’ to compute the  
323 least similar decompositions.

324 The decompositions of 90 s were also analysed. Only the ‘Pair of Algo-  
325 rithms’ and its interaction with  $l$  had significant  $p$  values ( $p \ll 0.0001$ ). The  
326 pairs ‘JADE–FastICA’ and ‘JADE–extended-Infomax’ offered more differ-  
327 ent separations when  $l$  increased ( $p < 0.05$ ). ‘AMUSE–extended-Infomax’,  
328 ‘AMUSE–FastICA’ and ‘SOBI–extended-Infomax’ had regression slopes sig-  
329 nificantly ( $p < 0.05$ ) lower than zero. For this epoch length, ‘AMUSE–JADE’  
330 and ‘SOBI–JADE’ computed the least consistent separations. On the other  
331 hand, four pairs of algorithms had significantly different population marginal

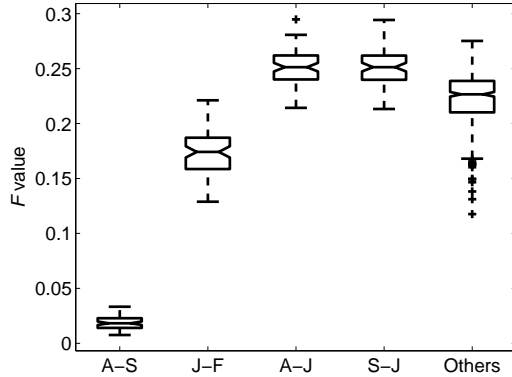


Figure 5: Boxplots showing the  $F$  values for the pairs of algorithms ‘AMUSE–SOBI’ (A-S), ‘JADE–FastICA’ (J-F), ‘AMUSE–JADE’ (A-J) and ‘SOBI–JADE’ (S-J) and jointly for the other 6 pairs of algorithms (Others) for real MEG epochs of 90 s.

332 means for the  $F$  values from the rest of pairs: ‘AMUSE–SOBI’, ‘JADE–  
 333 FastICA’, ‘extended-Infomax–FastICA’ and ‘JADE–extended-Infomax’ (from  
 334 more consistent to more dissimilar). The results for the most consistent and  
 335 dissimilar pairs of algorithms are illustrated with boxplots in Fig. 5.

336 Finally, synthetic data were used to complement the previous analyses.  
 337 AMUSE, SOBI, JADE, extended-Infomax and FastICA were used to esti-  
 338 mate the mixing matrix. The metric  $F$  was computed from a matrix  $\mathbf{P}^{qr}$   
 339 where  $q$  represented the known synthetic mixing matrix and  $r$  referred to  
 340 one of the BSS methods. Hence,  $F$  indicated the quality of the decomposi-  
 341 tion, with lower  $F$  values accounting for more accurate estimations. For each  
 342 synthetic epoch length, the  $F$  values of the 100 instances of the simulated  
 343 data were averaged. These results are illustrated in Fig. 6. Similarly to the  
 344 real recordings, Fig. 6 indicates that the accuracy of the separation increased

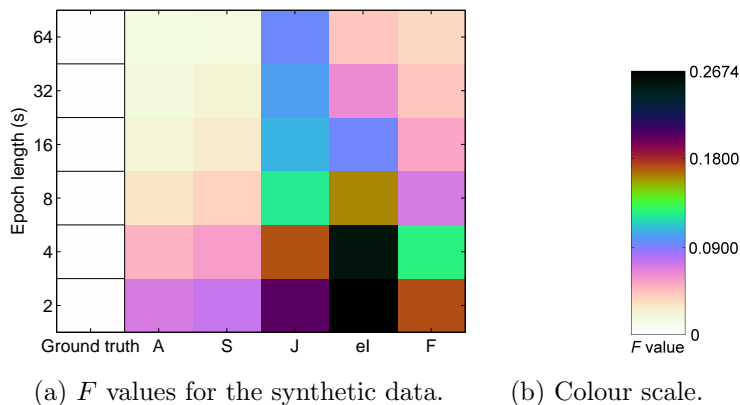


Figure 6: Average  $F$  values for the decompositions of the synthetic data computed with AMUSE (A), SOBI (S), JADE (J), extended-Infomax (el) and FastICA (F) for epochs of 64 s, 32 s, 16 s, 8 s, 4 s and 2 s. The zero-level of the  $F$  metric for a perfect decomposition is also plotted (Ground truth). The colour scale [34] used to represent the  $F$  values appears in (b).

345 with the epoch length and that AMUSE and SOBI offered similar levels of  
 346 accuracy in the BSS.

347 A Scheffé-corrected one-way ANOVA with ‘Algorithm’ as factor was ap-  
 348 plied to assess the differences in the  $F$  values of the synthetic signals. For  
 349 all epoch lengths, the differences in the separation accuracy were signifi-  
 350 cant ( $p \ll 0.0001$ ) and showed the same pattern in the multiple comparison  
 351 analysis. AMUSE and SOBI provided the best estimations of the synthetic  
 352 mixing matrix. Their level accuracy was significantly different from that of  
 353 HOS-BSS techniques. As for this type of methods, FastICA calculated more  
 354 accurate separations than JADE and extended-Infomax and the quality of  
 355 JADE did not improve with the signal length as much as in the other cases.  
 356 The differences among the three HOS-BSS approaches were significant for all

357 signal lengths.

## 358 5. Discussion and Conclusions

359 We compared five BSS algorithms in terms of the similarity between their  
360 decompositions for the real signals recorded from 26 subjects. We only eval-  
361 uated one matrix,  $\mathbf{P}^{qr}$ , for each pair of algorithms instead of both  $\mathbf{P}^{qr}$  and  
362  $\mathbf{P}^{rq}$  as the average differences for the  $F$  metric between  $\mathbf{P}^{qr}$  and  $\mathbf{P}^{rq}$  were  
363 always lower than 1.20%. By taking this decision, we tried to reduce the  
364 surplus complexity and redundancy of the problem. The results indicated  
365 that ‘AMUSE–SOBI’ and ‘JADE–FastICA’, in that order, are the pairs of  
366 algorithms that provide more similar BSS decompositions, while ‘AMUSE–  
367 JADE’ and ‘SOBI–JADE’ provided the most different outcomes. The sep-  
368 arations tended to be more similar for longer epochs. These results were  
369 supported by a complementary analysis of synthetic signals.

370 The preprocessing does not constitute a BSS algorithm itself. It relies on  
371 the classical projection technique of FA [9, 30, 31]. However, this preprocess-  
372 ing is important for several reasons [4, 5, 9, 21, 30]:

- 373 1. The number of inner meaningful components in real EEGs and MEGs  
374 may be less than the number of available channels.
- 375 2. A dimensionality reduction may sometimes be necessary to avoid “over-  
376 fitting”.
- 377 3. A dimensionality reduction may help to reduce the importance of the  
378 external noise.

379 The preprocessing included the estimation of the optimum number of com-  
380 ponents ( $l$ ).

381 The dependence of the preprocessing on the signal length was studied for  
382 real epochs of 10 s, 20 s, 40 s, 60 s and 90 s. As it was expected,  $l$  increased  
383 with the epoch length. This means that longer signals tend to be composed  
384 of more inner sources or, at least, need to consider more components to  
385 obtain an optimum decomposition. These results are supported by other  
386 contributions about the model order selection in EEG and MEG [21, 22].  
387 These studies investigated the performance of diverse approaches based on  
388 PCA and FA to estimate the number of BSS components in real EEG and  
389 MEG. Those results indicated that probabilistic PCA and FA models yield  
390 estimations of the dimensionality that are more reliable and independent of  
391 the signal power than commonly used PCA approaches [22]. The estimated  
392 values of  $l$  were about one third of the measurement space dimension [22].  
393 In our case, the number of components was usually lower than one third  
394 of channels, specially for the shorter epochs. This suggested that, in the  
395 case of MEG equipment, more channels do not necessarily reflect more brain  
396 signals [22]. What is more, the data dimension reduction is supported by the  
397 statistical properties of the signal and the FA models may offer an appropriate  
398 description of the brain recordings [21, 22].

399 The visual representation of the results provided by Fig. 3 clearly indi-  
400 cated that the pair ‘AMUSE–SOBI’ provided the most similar decomposition  
401 of the analysed MEGs. This result was confirmed by the fact that these tech-  
402 niques computed the most accurate decompositions of the synthetic data.  
403 The principle beneath these two techniques is the simultaneous diagonalisa-  
404 tion of several time-delayed cross-covariance matrices [3, 4]. We also found  
405 that the decompositions of JADE and FastICA are characterized by a high

406 degree of similarity for the real signals. This might be explained by the fact  
407 that the theoretical principles of both algorithms could be related [4].

408 Additionally, the algorithms tended to estimate more similar decomposi-  
409 tions of the real MEGs as longer epochs were considered. This may be due  
410 to the fact that, although  $l$  increased with the epoch length, the number of  
411 data samples considered increased more rapidly than the number of elements  
412 to be estimated in  $\mathbf{A}$  [2]. This fact is illustrated in Table 1, which depicts the  
413 number of available data samples and the median of the number of elements  
414 to be estimated in  $\mathbf{A}$  for each epoch length. It is clear that, the longer the  
415 epoch, the larger the number of samples available to estimate each element  
416 of  $\mathbf{A}$ . Hence, the decompositions may be considered more reliable for longer  
417 epochs [2], which could explain the ‘relative similarity’ of the BSS outcomes  
418 for long real MEG epochs. This result was supported by the analysis of syn-  
419 thetic data. Table 1 also depicts the number of synthetic samples available to  
420 compute the BSS. As the number of synthetic components was fixed, longer  
421 signals offered more accurate decompositions. Yet, it should be noticed that,  
422 for the synthetic data, the quality of the JADE separation did not improve  
423 as much as with the other techniques.

424 The statistical analysis carried out for every epoch length pointed out the  
425 statistical significance of the similarity between the decomposition computed  
426 by ‘AMUSE–SOBI’ and ‘JADE–FastICA’. On the other hand, the pairs  
427 ‘AMUSE–JADE’ and ‘SOBI–JADE’ used to provide the most dissimilar sepa-  
428 rations of the MEG signals. A relatively consistent pattern was that a larger  
429 number of components made the outcomes of the separations calculated by  
430 ‘AMUSE–SOBI’ and ‘JADE–FastICA’ slightly more different. Surprisingly,

Table 1: Ratios of the number of data samples for each epoch length divided by the median value of the number of elements in  $\mathbf{A}$  for the real MEG recordings and for the synthetic signals.

Real MEG recordings				
Epoch length	Data samples	Median of $l$	Elements in $\mathbf{A}$ ( $l^2$ )	Ratio
10 s	1695	29	841	2.02
20 s	3390	33	1089	3.11
40 s	6780	38	1444	4.67
60 s	10170	41	1681	6.05
90 s	15255	44	1936	7.88
Synthetic signals				
Epoch length	Data samples	Value of $l$	Elements in $\mathbf{A}$ ( $l^2$ )	Ratio
2 s	339	11	121	2.80
4 s	678	11	121	5.60
8 s	1356	11	121	11.21
16 s	2713	11	121	22.42
32 s	5425	11	121	44.83
64 s	10851	11	121	89.68



431 the pairs ‘AMUSE-extended-Infomax’ and ‘SOBI-extended-Infomax’ com-  
432 puted BSS decompositions that were slightly more similar for larger values  
433 of  $l$ .

434 A few studies have compared the performance of several BSS techniques  
435 from different perspectives. For instance, synthetic signals have been used  
436 to evaluate whether the BSS improved the automatic detection of artefacts  
437 in the EEG [7] or to assess the quality of the BSS decomposition [10]. Our  
438 results from simulated data agree with those of [10] in the sense that SOS-  
439 BSS techniques seemed to calculate more accurate decompositions. SOS-  
440 BSS methods also performed better than HOS-BSS techniques in a detailed  
441 analysis of the ocular artefact rejection for EEG [16, 17]. However, in the  
442 artefact detection problem, Infomax performed better than FastICA and  
443 SOBI in [7]. Artificially mixed EEG signals have also been analysed in [8]  
444 to compare the relative performance of a few BSS algorithms to isolate the  
445 artefacts. The results varied depending on which type of contamination was  
446 considered [8]. Real MEG signals were decomposed in [6] to evaluate their  
447 ability to extract artefacts by comparing the contaminated components of  
448 different algorithms with reference signals. Some of these previous studies  
449 used small datasets or small numbers of channels and the evaluation of the  
450 algorithms was frequently based on subjective criteria [10]. Moreover, it  
451 must be noted that different sets of synthetic data could produce different  
452 results. In contrast, we analysed the decompositions of real MEG recordings  
453 from 26 subjects globally to gain insight into the similarities between some of  
454 the most commonly used BSS algorithms. Instead of comparing a manually  
455 selected subset of components [6], the entire decomposition was assessed since

456 the metric  $F$  [11] was computed from the mixing matrices  $\mathbf{A}^g$  and  $\mathbf{A}^r$ .

457 It is important to note that the real sources are unknown, hence the term  
458 blind [5]. Therefore, assessing the performance of the BSS analysis is not  
459 straightforward at all since the separation cannot be absolutely validated  
460 for real data [3, 5]. Thus, the analyses of the real signals were exploratory  
461 and only aimed at measuring the similarity between the results of the BSS  
462 algorithms and not at evaluating the actual quality of the separation. This  
463 can only be achieved with some kind of synthetic signals [32, 33]. Yet, our  
464 complementary analysis of the simulated data supported the results derived  
465 from the real MEG recordings. This suggests that the results for each pair  
466 of algorithms are indeed due to the methodology of the BSS techniques and  
467 not to this particular application. Our study is also limited by the fact that  
468 only real signals of MEG background activity were studied. Additionally,  
469 only recordings from elderly people were analysed. Thus, the results might  
470 be difficult to generalise to younger subjects.

471 To sum up, this study evaluated the degree to which diverse BSS tech-  
472 niques provide similar decompositions for real MEG background activity.  
473 The most similar separations were computed with ‘AMUSE–SOBI’, followed  
474 by ‘JADE–FastICA’. The pairs ‘AMUSE–JADE’ and ‘SOBI–JADE’ used to  
475 provide the most dissimilar outcomes. Finally, the overall level of similarity  
476 increased as longer signals were decomposed. These results were supported  
477 by a study based on synthetic signals. Since diverse BSS methods may of-  
478 fer relatively different perspectives when applied to real signals [4], these  
479 results should be taken into account when deciding which BSS algorithms  
480 are to be applied to brain signals. For instance, if only two BSS are to be

481 selected for an exploratory analysis, the algorithms AMUSE and JADE will  
482 provide relatively different perspectives of the data and minimise the amount  
483 of redundant information.

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