A study of the Okubo-Zweig-Iizuka rule violations in $\eta_c \to VV$

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We discuss the Okubo-Zweig-Iizuka (OZI) rule violation effects in $\eta_c \to VV$ in light of the new data from BES Collaboration. In particular, a possible non-vanishing branching ratio for $\eta_c \to \omega\phi$ provides a hint of the degrees of OZI violations based on a recent factorization proposed for charmonium hadronic decays. The violation mechanism is studied via an intermediate meson exchange model. The results are consistent with the experimental observations.

The new data from BES Collaboration for $\eta_c \to VV$ [1], where V denotes vector meson, revive the question about the role played by non-perturbative QCD processes. In particular, an upper limit for $\eta_c \to \omega\phi$ branching ratio, which is the same order of magnitude as that for $\eta_c \to \phi\phi$, initiates renewed interests in possible OZI-rule [2] violations in charmonium hadronic decays. Since the flavor wavefunctions of ω and ϕ are nearly ideally mixing, i.e. ϕ meson is a pure $s\bar{s}$ and ω, pure $n\bar{n} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$, in the limit where the OZI-rule applies, the branching ratio for the doubly OZI disconnected (DOZI) process $\eta_c \to \omega\phi$ is expected to vanish, or at least be much smaller than the singly OZI disconnected (SOZI) processes such as $\eta_c \to K^*\bar{K}^*$, etc. Therefore, a possible non-negligible branching ratio for $\eta_c \to \omega\phi$ would be a direct evidence for the break-down of pQCD in the low-lying charmonium hadronic decays, for which the dynamical reason needs to be understood. Some theoretical attempts towards a dynamical understanding of $\eta_c \to VV$ can be found in the literature [3, 4, 5, 6, 7, 8].

A dynamical prescription for the OZI violations in the charmonium hadronic decays can be the intermediate meson exchange mechanism [9, 10]. As proposed in Ref. [11], the DOZI processes could be dual with the intermediate meson exchanges, which is in analogue with Geiger and Isgur's study [12] of OZI violation mechanisms. For $J/\psi \to V f_0$ [11], we found that the OZI violations via the intermediate meson exchanges accounted for the branching ratio patterns arising from $J/\psi \to V f_0^i$, where $i = 1, 2, 3$ denote $f_0(1710)$, $f_0(1500)$ and $f_0(1370)$. Similarly, this prescription can be applied to $\eta_c \to VV$, where the intermediate meson exchanges could be a natural explanation for a non-vanishing $\eta_c \to \omega\phi$ decay.

In this letter, we provide a coherent analysis for $\eta_c \to VV$ based on a recently proposed factorization scheme [13]. By determining the OZI-violation parameter defined in the factorization, we will be able to predict the range of branching ratio magnitude for $\eta_c \to \omega\phi$. We then investigate this DOZI process by applying the intermediate meson exchange mechanism. Conclusion and some discussions will be given in the end.

In the factorization scheme [13], the DOZI process is distinguished from the SOZI ones by the gluon counting rule. Namely, we assume that the transition amplitude of DOZI process has a strength of r times that of SOZI due to additional gluon exchange. We also introduce the SU(3) flavor breaking parameter R, of which its deviation from unity reflects the change of couplings due to the mass difference between u/d and s. The third parameter t is introduced to describe the coupling difference between gluon-$q\bar{q}$ and gluon-glueball. For $\eta_c \to VV$, the transitions will not involve parameters t since we treat η_c as a pure $c\bar{c}$ state. Finally, the gluon-$q\bar{q}$ coupling is defined by parameter g_0. As shown in Ref. [13], g_0 is a stable quantity insensitive to $\chi_{0,2} \to VV$ and PP. This property reflects that the gluon-$q\bar{q}$ coupling has a universal value at the same energy region.

Based on the above simple scheme, the transition amplitudes for $\eta_c \to VV$ can be expressed as

$$
\begin{aligned}
\langle\phi\phi|\hat{V}_{gg}|\eta_c\rangle &= g_0^2 R^2 (1 + r) \\
\langle\omega\omega|\hat{V}_{gg}|\eta_c\rangle &= g_0^2 (1 + 2r) \\
\langle\omega\phi|\hat{V}_{gg}|\eta_c\rangle &= g_0^2 r R \sqrt{2} \\
\langle K^{*+}K^{*-}|\hat{V}_{gg}|\eta_c\rangle &= g_0^2 R \\
\langle\rho^+\rho^-|\hat{V}_{gg}|\eta_c\rangle &= g_0^2 \; .
\end{aligned}
\tag{1}
$$

where $\hat{V}_{gg}$ is the $\eta_c \to gg \to (q\bar{q})(q\bar{q})$ potential. The amplitudes for other charge combinations of $K^*\bar{K}^*$ and $\rho\rho$ are the same.

A commonly used form factor is adopted in the calculation of the partial decay widths:

$$
\mathcal{F}^2(\mathbf{p}) = p^{2l} \exp(-\mathbf{p}^2/8\beta^2) \; ,
\tag{2}
$$

where $\mathbf{p}$ and l are the three momentum and relative angular momentum of the final-state mesons, respectively, in the η_c rest frame. We adopt $\beta = 0.5$ GeV, which is the same as in Refs. [13, 14, 15, 16]. Such a form factor will largely account for the size effects from the spatial wavefunctions of the initial and final state mesons.

In Ref. [1], branching ratios for $\eta_c \to \rho\rho$, $K^*\bar{K}^*$, $\phi\phi$ are measured with improved error bars compared with PDG [17], and upper limits for $\eta_c \to \omega\omega$ and $\omega\phi$ are estimated. In particular, the upper limit, $BR_{\eta_c \to \omega\phi} < 1.3 \times 10^{-3}$, is the same order of magnitude as $BR_{\eta_c \to \phi\phi} = (2.5 \pm 0.5 \pm 0.9) \times 10^{-3}$, which implies that large OZI violations might exist in $\eta_c \to VV$.

Adopting the data of Ref. [1], we make two fittings in the factorization scheme. The parameters are listed in Table I and the fitting results are presented in Table II. In Fit-I, the three parameters, r, R and g_0, are determined by fitting the branching ratios for $\eta_c \to \rho\rho$, $K^*\bar{K}^*$ and $\phi\phi$. Since there are three parameters to be fitted by three experimental data, the negligibly-small χ^2 reflects the fact that one can also determine the parameters by solving the three variable equations. Interestingly, large errors are found with the OZI violation parameter r. With these fitted parameters the predicted branching ratios for $\eta_c \to \omega\omega$ are nearly as large as two times the experimental upper limit. This reflects the lack of information about the parameter correlations among the three channels: $\rho\rho$, $K^*\bar{K}^*$ and $\phi\phi$. For instance, the decay of $\eta_c \to \rho\rho$ only involves g_0, thus, can be directly determined by the branching ratio without interferences with the $K^*\bar{K}^*$ and $\phi\phi$ channels. Due to this, the price to pay for a "precise" fit for $\rho\rho$, $K^*\bar{K}^*$ and $\phi\phi$ is that the predictions for $\omega\omega$ and $\omega\phi$ channels will bear much larger uncertainties though they can still be regarded as in agreement with the data (see Fit-I of Table II).

In order to put more constraints on the determination of the parameters and examine their sensitivity to the experimental upper limits for $\eta_c \to \omega\omega$ and $\omega\phi$, we set their branching ratios to be $BR_{\eta_c \to \omega\omega} = (3.1 \pm 3.1) \times 10^{-3}$ and $BR_{\eta_c \to \omega\phi} = (0.6 \pm 0.6) \times 10^{-3}$, i.e. assuming that the most probable values are at the half values of their upper limits. We then obtain Fit-II in Table I and II with a reasonably small χ^2, i.e. $\chi^2 = 0.5$. The fitted branching ratios for $\eta_c \to \phi\phi$ and $\rho\rho$ are found smaller than the experimental central values, while that for $K^*\bar{K}^*$ turns out to be larger though all these are well within the uncertainty of the data. Nevertheless, we find that the fitted values for $\omega\omega$ and $\omega\phi$ are now well below the experimental upper limits.

Compared with Fit-I, the uncertainties for r in Fit-II are improved; g_0 does not experience significant changes; and the SU(3) flavor symmetry seems to be better respected. This feature not only confirms that the new data from BES are more consistent with the expectations based on the SU(3) flavor symmetry [1], but also suggests that the OZI violations are phenomena different from the SU(3) flavor symmetry breaking. Moreover, the value, $g_0 \sim 0.36$ GeV$^{1/2}$, is stable and close to that found in $\chi_{c0,2} \to VV$ ($g_0 \simeq 0.25$ GeV$^{1/2}$) [13]. Since g_0^2 is proportional to the value of the $c\bar{c}$ wavefunction at its origin, these results indicate some dynamical similarities between $\eta_c \to VV$ and $\chi_{c0,2} \to VV$, and can be regarded as a consistent test of the factorization scheme.

To show the uncertainties bearing with the predictions for $\eta_c \to \omega\phi$ and $\omega\omega$, we also present the root mean square errors for these two channels in both fittings.

Note that η_c has large branching ratios to $\rho\rho$ and $K^*\bar{K}^*$ in its two-body decays. These transitions can occur via SOZI processes, while $\eta_c \to \omega\phi$ can only occur via DOZI ones. This is a natural explanation for the branching ratio suppression for $\eta_c \to \omega\phi$ in comparison with that for $\eta_c \to K^*\bar{K}^*$ and $\rho\rho$. However, note that both ω and ϕ have sizeable couplings to $K^*\bar{K} + c.c.$ and $\rho\pi$ [17]. It implies that the $K^*\bar{K}^*$ (and/or $\rho\rho$) rescattering into $\omega\phi$ via kaon (and/or pion) as shown by Fig. 1, could lead to sizeable contributions from the DOZI processes and be the major source for the OZI violations.

To estimate the meson exchange contributions to $\eta_c \to \omega\phi$, we evaluate the meson loop transitions and express the transition amplitude as follows:

$$M_{fi} = \int \frac{d^4 p_2}{(2\pi)^4} \delta^4(P_0 - P_\phi - P_\omega) \sum_{K^* pol.} \frac{T_a T_b T_c}{a_1 a_2 a_3} \mathcal{F}(p_2^2) \, , \tag{3}$$

where the vertex functions are

$$T_a \equiv \frac{ig_a}{M_0} \epsilon_{\alpha\beta\gamma\delta} p_1^\alpha \epsilon_1^\beta p_3^\gamma \epsilon_3^\delta,$$

$$T_b \equiv \frac{ig_b}{M_\phi} \epsilon_{\mu\nu\xi\tau} p_\phi^\mu \epsilon_\phi^\nu p_1^\xi \epsilon_1^\tau,$$

$$T_c \equiv \frac{ig_c}{M_\omega} \epsilon_{\lambda\iota\kappa\sigma} p_\omega^\lambda \epsilon_\omega^\iota p_3^\kappa \epsilon_3^\sigma, \tag{4}$$

where g_a, g_b and g_c are coupling constants at the meson interaction vertices (see Fig. 1); Note that the tensor part of the vector meson propagator will not contribute. The four-vectors, P_0, P_ϕ and P_ω, are momenta for the initial η_c and final state ϕ and ω mesons, while p_1, p_2 and p_3 are four momenta for the intermediate mesons, respectively. Quantities, $a_1 = p_1^2 - m_1^2$, $a_2 = p_2^2 - m_2^2$ and $a_3 = p_3^2 - m_3^2$, are the denominators of the propagators of intermediate mesons.

By applying the Cutkosky rule to the loop integration, we can reduce the transition amplitude to be

$$M_{fi} = \frac{-ig_a g_b g_c |\mathbf{p}_3|}{128\pi^2 M_\phi M_\omega M_0^2} \epsilon^{\lambda\iota\kappa\delta} P_{\omega\lambda}\epsilon_{\omega\iota} P_{\phi\kappa}\epsilon_{\phi\delta}\mathcal{I}, \tag{5}$$

with

$$\mathcal{I} \equiv \int \frac{p_2^2[2p_2^2 + 2(P_\phi^2 - p_1^2) - P_\phi \cdot P_\omega]\mathcal{F}(p_2^2)}{p_2^2 - m_2^2} d\Omega . \tag{6}$$

In the above the integration is over the azimuthal angles of the momentum $\mathbf{p}_1$ respect to the momentum of the final state ϕ meson, and $p_2^2 = (P_\phi - p_1)^2 = M_\phi^2 + m_1^2 - 2E_\phi E_1 + 2|\mathbf{P}_\phi||\mathbf{p}_1|\cos\theta$. The coupling constant g_a can be determined by the experimental data for $\eta_c \to K^* \bar{K}^*$, which is an SOZI process and one of the largest channels in the η_c decay:

$$g_a^2 = \frac{4\pi M_0^2}{|\mathbf{p}_1|^3}\Gamma_{\eta_c \to K^*\bar{K}^*} , \tag{7}$$

where $\Gamma_{\eta_c \to K^*\bar{K}^*} = 10.4 \times 10^{-3}\Gamma_{tot} = 0.18$ MeV.

Since the exchanged particles are correlated at those three vertices and the charge combination factor has been included in the derived coupling constant g_a, we determine the g_b and g_c by the SU(3) relation in comparison with $g_{\omega\rho^0\pi^0}$:

$$\begin{aligned} g_b &= g_{\phi K^{*0}\bar{K}^0} = g_{\phi K^{*+}K^-} = \sqrt{2}g_{\omega\rho^0\pi^0} = \sqrt{2}g_{\omega\rho^+\pi^-}, \\ g_c &= g_{\omega K^{*0}\bar{K}^0} = g_{\omega\bar{K}^{*0}K^0} = g_{\omega K^{*+}K^-} = g_{\omega K^{*-}K^+} = g_{\omega\rho^0\pi^0}, \end{aligned} \tag{8}$$

with $g_{\omega\rho^0\pi^0}^2 \simeq 84$ determined in vector meson dominance model in $\omega \to \pi^0 e^+ e^-$ [17]. However, the SU(3) flavor symmetry is generally broken. For example, the SU(3) flavor symmetry gives $g_{\phi\rho\pi} = 0$. But we know that the decay of $\phi \to \rho\pi$ has sizeable branching ratios [17]. To include the contributions from Fig. 1(b), we adopt $BR_{\phi\to\rho\pi+\pi^+\pi^-\pi^0} = 15.4\%$ as an upper limit to derive $g_{\phi\rho\pi}$.

We also note that the application of the Cutkosky rule to the meson loop integration for $\eta_c \to \omega\phi$ requires a sufficient consideration of the non-locality of the VVP couplings in the final state. As shown by Eq. (6), the integrand has a power of p_2^2, which suggests that the meson loop integral may not be necessarily suppressed compared with the tree diagram without form factor. Therefore, a form factor to take care of the non-locality of the two VVP vertices are needed. This issue is correlated to the off-shell effects from the exchanged light pseudoscalar meson. Following the argument of Ref. [18], we adopt a dipole form factor to account for the non-locality for the two VVP vertices and off-shell effects from the exchanged meson. This should be a natural treatment for the intermediate meson exchange model. As follows, we will list the integration results for three cases: i) with no form factor; ii) with a monopole form factor; and iii) with a dipole form factor. We then concentrate on the numerical results from (iii) due to the reason mentioned above.

i) With no form factor, i.e. $\mathcal{F}(p_2^2) = 1$, the integral becomes:

$$\mathcal{I} = \frac{8\pi|\mathbf{P}_\phi||\mathbf{p}_1|}{C}\left[2 - \frac{2(A-B)C}{B^2} - \frac{(A-B)(C-B)}{B^3}\ln\frac{1-B}{1+B}\right], \tag{9}$$

where the factors are

$$\begin{aligned} A &\equiv \frac{2|\mathbf{P}_\phi||\mathbf{p}_1|}{M_\phi^2 + m_1^2 - 2E_\phi E_1} \\ B &\equiv \frac{2|\mathbf{P}_\phi||\mathbf{p}_1|}{M_\phi^2 + m_1^2 - 2E_\phi E_1 - m_2^2} \\ C &\equiv \frac{4|\mathbf{P}_\phi||\mathbf{p}_1|}{4M_\phi^2 - 4E_\phi E_1 - P_\phi \cdot P_\omega}. \end{aligned} \tag{10}$$

ii) With a monopole form factor, i.e. $\mathcal{F}(p_2^2) = (\Lambda^2 - m_2^2)/(\Lambda^2 - p_2^2)$, where Λ is the cut-off energy, the integral becomes:

$$\mathcal{I} = 4\pi(\Lambda^2 - m_2^2)\left[2 + \frac{D(A-B)(C-B)}{ABC(B-D)}\ln\frac{1-B}{1+B} + \frac{B(A-D)(C-D)}{ACD(B-D)}\ln\frac{1-D}{1+D}\right], \tag{11}$$

where A, B, and C are the same as defined in (i), and

$$D \equiv \frac{2|\mathbf{P}_\phi||\mathbf{p}_1|}{M_\phi^2 + m_1^2 - 2E_\phi E_1 - \Lambda^2}. \tag{12}$$

iii) With a dipole form factor, i.e. $\mathcal{F}(p_2^2) = [(\Lambda^2 - m_2^2)/(\Lambda^2 - p_2^2)]^2$, the integral becomes

$$\begin{aligned}
\mathcal{I} = \; & 4\pi \frac{BD(\Lambda^2 - m_2^2)^2}{ACD_s} \left[\frac{2(C-D)(A-D)}{D(B-D)(1-D^2)} + \frac{(A-B)(C-B)}{B(B-D)^2} \ln \frac{1-B}{1+B} \right. \\
& \left. + \left(\frac{AC}{BD^2} + \frac{(A-B)(B-C)}{B(B-D)^2} \right) \ln \frac{1-D}{1+D} \right],
\end{aligned} \tag{13}$$

where A, B, C and D are the same as defined in (i) and (ii), and $D_s \equiv M_\phi^2 + m_1^2 - 2E_\phi E_1 - \Lambda^2$. Meanwhile, to test the sensitivity of the results to the cut-off energies, we adopt $\Lambda = 0.7$, 1.0 and 1.2 GeV as the lower and upper limit in the calculations.

With the above parameters, the intermediate meson exchanges of $K^*\bar{K}^*$ and $\rho\rho$ are calculated as leading OZI violation effects for $\eta_c \rightarrow \omega\phi$. We first discuss these two processes separately, and then consider their interferences based on the SU(3) relation.

In Table III, the branching ratios for $\eta_c \rightarrow \omega\phi$ are listed with the dipole form factor. It shows that the range of $\Lambda = 0.7 \sim 1.2$ GeV leads to about one order of magnitude changes to the predicted branching ratios. Although some cautions have to be taken due to the sensitivity of the results to the cut-off energies, we also note that the commonly adopted range of $\Lambda = 0.7 \sim 1.2$ GeV has been useful for providing the magnitude of contributions from the intermediate meson exchange processes. We find that the intermediate $K^*\bar{K}^*$ rescattering has dominant contributions to the $\eta_c \rightarrow \omega\phi$ transitions, while the $\rho\rho$ rescattering is more than an order of magnitude smaller. In particular, the predicted upper limit for the branching ratio $\eta_c \rightarrow K^*\bar{K}^* \rightarrow \omega\phi$ is in good agreement with the experimental upper limit [1].

As shown by the results with both $K^*\bar{K}^*$ and $\rho\rho$ contributing, the interference leads to about 2/3 enhancement to the branching ratio with the exclusive $K^*\bar{K}^*$ rescattering for $\Lambda = 1.2$ GeV. With smaller cut-off energies, the enhancement becomes more significant, but generally does not change the order of magnitudes. This is an indication that for the purpose of understanding the possible non-vanishing $\eta_c \rightarrow \omega\phi$ decay, the estimate of the $K^*\bar{K}^*$ contributions is nearly sufficient. Certainly, more rigorous study of the intermediate meson exchange contributions including more meson loops should be carried out with future improved experimental data.

In light of the improved branching ratios for $\eta_c \rightarrow \rho\rho$, $K^*\bar{K}^*$ and $\phi\phi$, and upper limits for $\omega\omega$ and $\omega\phi$ from BES collaboration, we investigate the implication of OZI violations in the decays of $\eta_c \rightarrow VV$ based on a factorization scheme recently developed [13, 16]. It shows that sizeable OZI violations can occur under the present experimental accuracy. The process of $\eta_c \rightarrow \omega\phi$ thus would be a unique test of such effects, and we find the branching ratio is at order of 10^{-4}.

We then apply an intermediate meson exchange model to investigate the OZI violation mechanism in $\eta_c \rightarrow \omega\phi$. Similar to what was found in $J/\psi \rightarrow Vf_0$ decays [11], we find that the intermediate $K^*\bar{K}^*$ rescattering can contribute to $\eta_c \rightarrow \omega\phi$ with non-negligible magnitudes of about 10^{-4}. In contrast, the contribution from the exclusive intermediate $\rho\rho$ rescattering is relatively small. But we find that its interference with the $K^*\bar{K}^*$ can produce sizeable effects. Although the outputs of the intermediate meson exchange model still bear uncertainties due to its model-dependent feature, we find that its agreement with the factorization scheme is very impressive. In another word, the intermediate meson exchange could be a dynamic mechanism which leads to the OZI-rule violations in those low-lying charmonium decays [11]. Nevertheless, the factorization scheme highlights interesting features arising from the η_c decays, such as the correlations between the OZI rule violations and SU(3) flavor symmetry breaking. Further experiments at BES concentrating on $\eta_c \rightarrow \omega\phi$ should be able to disentangle this long-standing question.

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Parameters	Fit-I	Fit-II
r	0.28 ± 0.73	-0.16 ± 0.15
R	0.83 ± 0.29	1.02 ± 0.23
g_0	0.36 ± 0.04	0.35 ± 0.04
χ^2	5.0×10^{-9}	0.5

TABLE I: The parameters determined in Fit-I and Fit-II.

Branching ratios ($\times 10^{-3}$)	BES data [1]	Fit-I	Fit-II
$\rho\rho$	$12.5 \pm 3.7 \pm 5.1$	12.5	10.7
$K^*\bar{K}^*$	$10.4 \pm 2.6 \pm 4.3$	10.4	13.6
$\phi\phi$	$2.5 \pm 0.5 \pm 0.9$	2.50	2.16
$\omega\omega$	< 6.3	10.1 (**10.5**)	1.67 (**1.06**)
$\omega\phi$	< 1.3	0.81 (**4.28**)	0.33 (**0.65**)

TABLE II: The branching ratios for $\eta_c \to VV$. The data are from BES [1]. Fit-I is obtained by fitting the BES data for $\eta_c \to \phi\phi$, $K^*\bar{K}^*$ and $\rho\rho$, while Fit-II are obtained by including the data for $\eta_c \to \omega\omega$ and $\omega\phi$ at the half values of their upper limits. The bold numbers in the brackets are the root mean square errors.

Λ (GeV)	0.7	1.0	1.2
$K^*\bar{K}^*$	1.550×10^{-5}	4.76×10^{-4}	1.59×10^{-3}
$\rho\rho$	7.90×10^{-6}	4.53×10^{-5}	1.02×10^{-4}
$K^*\bar{K}^* + \rho\rho$	4.55×10^{-5}	8.15×10^{-4}	2.50×10^{-3}

TABLE III: The branching ratios for $\eta_c \to \omega\phi$ with the intermediate $K^*\bar{K}^*$, $\rho\rho$ and the sum of both at different cut-off energies.
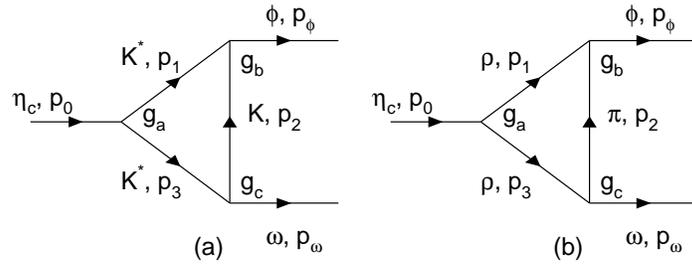
FIG. 1: Schematic pictures for the decays of $\eta_c \to \omega\phi$ via (a) $K^*\bar{K}^*$ and (b) $\rho\rho$.