Low-Complexity Time-Domain SNR Estimation for OFDM Systems

A. Ijaz, A.B. Awoseyila and B.G. Evans

A low-complexity SNR estimation algorithm for OFDM systems in frequency-selective fading channels is proposed. The estimator is based on a conventional preamble having two identical parts. It uses the correlation of the received signal samples to estimate signal power while noise power is estimated using the difference between received samples. Simulation results show that the proposed estimator is robust to the channel's frequency selectivity and its attained accuracy and reduced complexity make it an attractive choice for current wireless OFDM systems.

Introduction: Orthogonal frequency division multiplexing (OFDM) supports both time-division and frequency-division multiple access and enables high data rate transmission in frequency-selective fading channels. Adaptable transmission parameters such as modulation and coding can play an important role in making an efficient use of OFDM system resources. The parameters are adapted according to channel conditions indicated by a knowledge of the signal-to-noise ratio (SNR). SNR estimation is also used in soft decoding algorithms and other applications [1].

Several SNR estimators have been proposed for the additive white Gaussian noise (AWGN) channel [2], [3]. However they cannot be applied in frequency-selective fading channels since the channel coefficients are varying across the bandwidth. In this paper we focus on preamble-based estimators for wireless OFDM systems due to their improved performance as compared to non-data-aided techniques. Boumard proposed a method to estimate SNR in OFDM systems assuming that the channel frequency response varies slowly [4]. However, its performance is significantly degraded in highly frequency selective channels. Ren [4] proposed a method to estimate the noise variance by using the least square (LS) channel estimates obtained from two identical OFDM training symbols while the signal power is estimated from their second order moment. In [5], Ren used a similar preamble structure to estimate noise variance using the difference between the received OFDM subcarriers, thereby improving performance. In [6], Zivkovic proposed a low complexity estimator, based on periodically-used subcarriers (named PS estimator). It estimates signal and noise power from active and inactive OFDM subcarriers respectively. However, its accuracy decreases as the number of used subcarriers reduces.

We propose a time-domain algorithm for SNR estimation in OFDM systems based on conventional preamble structure with two identical parts [7], [8]. It has a low computational complexity while maintaining performance close to Cramer-Rao bound (CRB). Furthermore, the proposed algorithm is applicable to CP-based SNR estimation.

System Model: In an OFDM system, the transmitted signal samples are given by:

$$x(k)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N_{used}-1}X(n)e^{j2\pi kn/N};\quad k=0,1,2,...,N-1 \tag{1}$$

where $X(n)$ is the transmitted symbol on n^{th} subcarrier and N is the total number of subcarriers of which N_{used} are used. Assuming perfect synchronization and sampling precision, the received signal samples of an OFDM symbol, after the removal of the cyclic prefix (CP), can be represented as:

$$r(k)=y(k)+\omega(k);\quad k=0,1,2,...,N-1 \tag{2}$$

where $\omega(k)=\omega_I(k)+j\omega_Q(k)$ is the zero-mean complex AWGN, $\omega_{I/Q}(k)$ are independent and identically distributed random variables with $E\{|\omega_{I/Q}(k)|^2\}=W/2$ and $y(k)=\sum_{l=0}^{L-1}h(l)x(k-l)$ is the received noise-free signal. $h(l)$ is the impulse response of the wideband (frequency-selective) channel whose memory order is *L*-1 samples.

Since we consider SNR estimation algorithms for the purpose of adaptive transmission, it is assumed that the channel is coherent over several OFDM symbols. The average SNR is given as:

$$\rho_{avg} = \frac{E\{|\, y(k)\,|^2\}}{E\{|\, \omega(k)\,|^2\}} = \frac{S}{W} \tag{3}$$

where *S* is the average signal power.

SNR in n^{th} subcarrier is given as:

$$\rho(n) = \frac{|H(n)|^2}{W} \tag{4}$$

where $H(n)$ is the channel frequency response on n^{th} subcarrier.

Proposed Estimator: The proposed method employs a preamble consisting of one OFDM symbol with two identical parts in the time-domain. This structure is suitable for robust timing and frequency synchronisation [7], [8] and also conforms to existing OFDM-based standards such as WiFi and WiMAX.

Provided that a CP is used, the received noiseless preamble from a wideband channel will also consist of two identical parts, i.e., $y(k) = y(k + N/2)$. Therefore average signal power can be represented as follows:

$$S = E\{|\, y(k)\,|^2\} = E\{|\, y(k + N/2)\,|^2\}, \quad \forall k = 0,1,2,\ldots,N/2-1 \tag{5}$$

Given an estimate of the noise power $\hat{W}$, conventional approaches estimate the signal power $\hat{S}$, using the second order moment of the received signal as follows:

$$\hat{S} = \frac{1}{N}\sum_{k=0}^{N-1} |r(k)|^2 - \hat{W} \tag{6}$$

Rather than taking this approach, we propose a computationally-efficient technique based on correlation. The correlation of in-phase and quadrature components of the two parts of received preamble can be written as:

$$R_I = E\{r_I(k) r_I(k+N/2)\} \tag{7}$$

$$R_Q = E\{r_Q(k) r_Q(k+N/2)\} \tag{8}$$

Since the noise is a zero-mean process and the noise samples are independent to each other and to the signal samples, the above equations simplify to:

$$R_I = E\{y_I(k) y_I(k)\} \tag{9}$$

$$R_Q = E\{y_Q(k) y_Q(k)\} \tag{10}$$

From (5), (9) and (10), it can be observed that the signal power can be expressed as:

$$S = R_I + R_Q \tag{11}$$

Therefore, we propose to estimate signal power from the samples of received preamble as:

$$\hat{S} = \frac{1}{N/2} \sum_{k=0}^{N/2-1} \left[r_I(k) r_I(k+N/2) + r_Q(k) r_Q(k+N/2) \right] \tag{12}$$

Based on the conventional approach, noise power can be estimated by subtracting the estimated signal power from total received power as follows:

$$\hat{W} = \frac{1}{N} \sum_{k=0}^{N-1} |r(k)|^2 - \hat{S} \tag{13}$$

Substituting (12) in (13) we get:

$$\hat{W} = \frac{1}{2N/2} \sum_{k=0}^{N/2-1} \left[(\omega_I(k) - \omega_I(k+N/2))^2 + (\omega_Q(k) - \omega_Q(k+N/2))^2 \right] \tag{14}$$

However, the difference in noise samples can be obtained by subtracting one half of the received preamble from the other. Hence noise power can also be estimated as:

$$\hat{W} = \frac{1}{2N/2} \sum_{k=0}^{N/2-1} \left[(r_I(k) - r_I(k+N/2))^2 + (r_Q(k) - r_Q(k+N/2))^2 \right] \tag{15}$$

Comparing (13) and (15), it can be seen that we now need *N* times less multiplications for noise power estimation. Hence, average SNR can be estimated as:

$$\hat{\rho}_{avg} = \frac{2 \sum_{k=0}^{N/2-1} \left[r_I(k) r_I(k+N/2) + r_Q(k) r_Q(k+N/2) \right]}{\sum_{k=0}^{N/2-1} \left[(r_I(k) - r_I(k+N/2))^2 + (r_Q(k) - r_Q(k+N/2))^2 \right]} \tag{16}$$

SNR per subcarrier can be estimated using (4), (15) and the sub-channel estimate. Proposed estimator can also be used for CP-based SNR estimation, provided that the channel memory order (*L*) is known, wherein:

$$\hat{\rho}_{avg,CP} = \frac{2\sum_{k=\hat{L}}^{G}\left[r_I(k)r_I(k+N)+r_Q(k)r_Q(k+N)\right]}{\sum_{k=\hat{L}}^{G}\left[\left(r_I(k)-r_I(k+N)\right)^2+\left(r_Q(k)-r_Q(k+N)\right)^2\right]} \qquad (17)$$

where *G* is the CP length in samples.

Simulation Results: Computer simulations were performed to verify the performance of the proposed estimator in comparison with Ren's estimator [5] and the PS estimator [6] using QPSK modulation and a preamble structure with identical parts. We assume an OFDM system similar to fixed WiMAX with *N*=256 subcarriers, N_{used}=200 and *G*=16 samples [7]. Performance is evaluated in a channel consisting of *L*=8 independent Rayleigh fading paths with delays of $\tau_l = 0,1,\ldots,L-1$ samples and an exponential power delay profile having average power of $e^{\left(-\tau_l/L\right)}$ as used in [7], [8]. Performance is evaluated in terms of mean square error (MSE) of the estimated SNR normalized to actual SNR i.e. NMSE, and also compared with the CRB [2].

Fig. 1 and 2 show the NMSE performance of average SNR estimation and SNR estimation per subcarrier, respectively. The number of OFDM symbols in the preamble is indicated within brackets next to the investigated techniques. It can be observed that the accuracy of the proposed estimator is better than Ren's and PS estimators. It has a lower complexity than Ren's estimator, as shown in the complexity comparison in Table 1. Although the PS estimator has lower complexity, overall performance of the proposed estimator in terms of accuracy and complexity makes it a more robust option. Another advantage of the proposed estimator is its applicability to any CP-based OFDM system as shown in (17).

Conclusion: In this letter, a low-complexity SNR estimation algorithm for wireless OFDM systems has been presented. The estimator uses an OFDM training symbol with identical parts to estimate SNR more efficiently than the existing preamble-based estimators. Its robust accuracy and reduced complexity in frequency-selective channels make it an attractive choice for modern wireless OFDM systems. Furthermore, it can be used for CP-based SNR estimation if the channel delay spread is known.

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Authors' affiliations:
A. Ijaz, A.B. Awoseyila and B.G. Evans (Centre for Communications Systems Research, University of Surrey, Guildford, GU2 7XH, United Kingdom).

E-mail: a.ijaz@surrey.ac.uk

Figure\Table captions:

Fig. 1 NMSE of the average SNR

Fig. 2 NMSE of SNR per subcarrier

Table 1 Complexity comparison of estimators

Figure 1

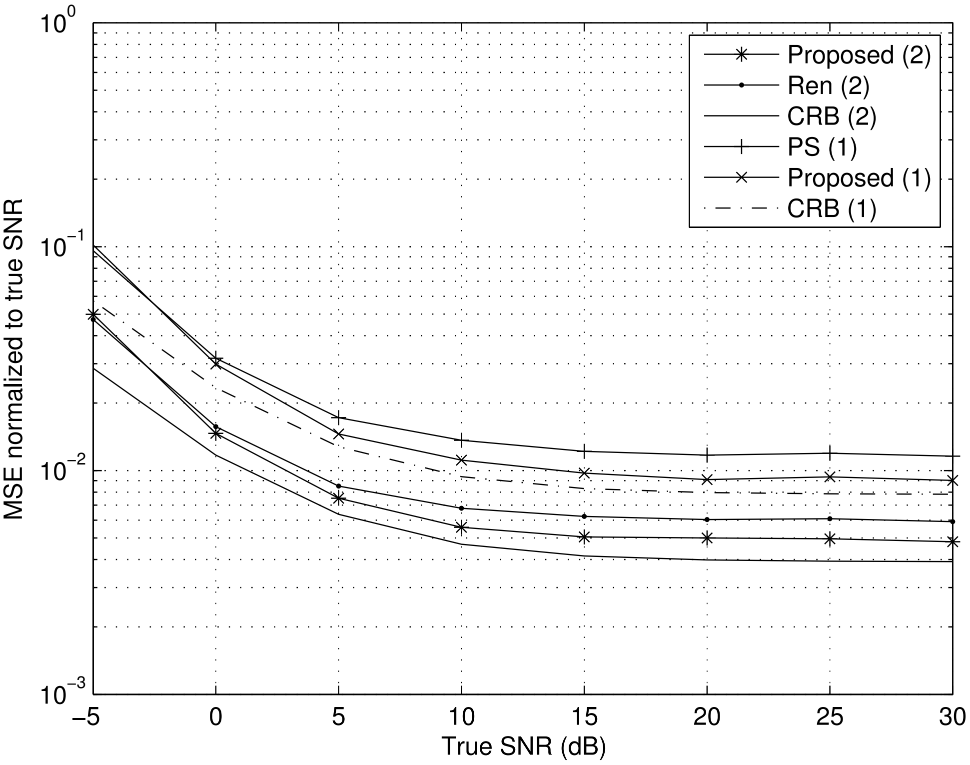
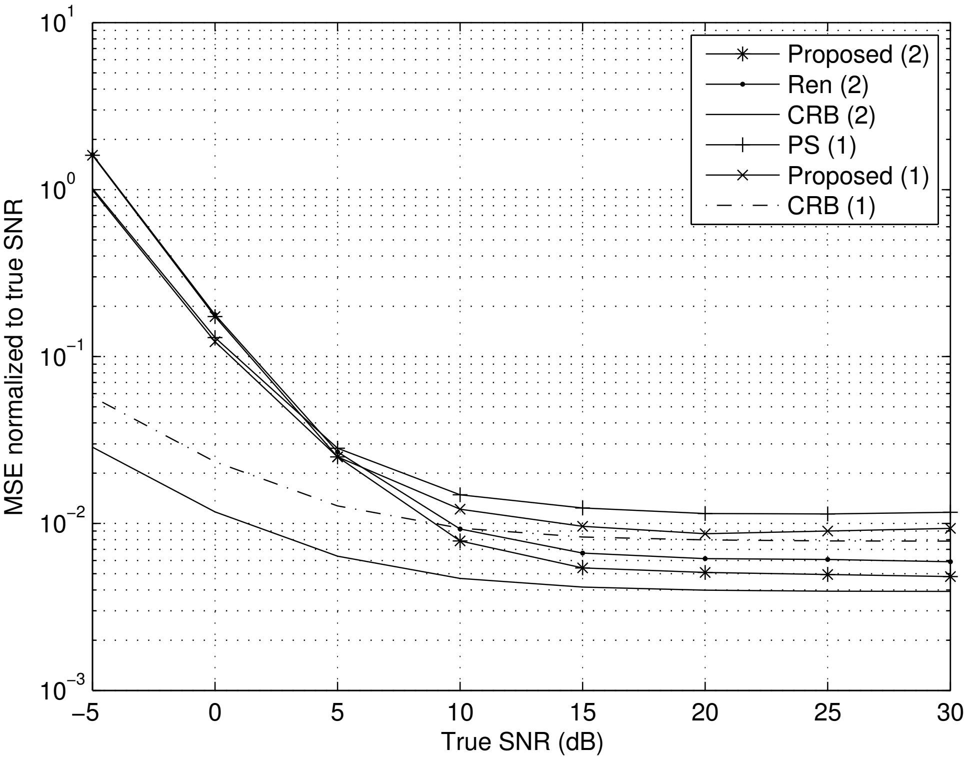

Proposed (2)
Ren (2)
CRB (2)
PS (1)
Proposed (1)
CRB (1)
MSE normalized to true SNR
True SNR (dB)
10^{0}
10^{-1}
10^{-2}
10^{-3}
−5
0
5
10
15
20
25
30

Figure 2



10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
MSE normalized to true SNR
–5
0
5
10
15
20
25
30
True SNR (dB)
Proposed (2)
Ren (2)
CRB (2)
PS (1)
Proposed (1)
CRB (1)

Table 1

Operation	2 OFDM Symbols		1 OFDM Symbol	
	Ren	**Proposed**	**PS**	**Proposed**
Additions	$8N_{used}-1$	$6N-2$	$2N_{used}-1$	$3N-2$
Multiplications	$6N_{used}$	$4N$	$2N_{used}+2$	$2N+2$