Iterative Multi-User Detection and Decoding for DS-CDMA System with Space-Time Linear Dispersion

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***Abstract*—This paper considers a Q-ary orthogonal DS-CDMA system with high rate space-time linear dispersion codes (LDC) in time-varying Rayleigh fading MIMO channels. We propose a joint multi-user detection, LDC decoding, Q-ary demodulation and channel decoding algorithm and apply turbo processing principle to improve the system performance in an iterative fashion. The proposed iterative scheme demonstrates faster convergence and superior performance compared to the V-BLAST based DS-CDMA system, and is shown to approach the single-user performance bound. We also show that the CDMA system is able to exploit the time diversity offered by the LDCs in rapid fading channels.**

Index Terms: code division multiple access, MIMO, linear dispersion codes, iterative detection and decoding, multi-user detection.

I. Introduction

In this paper, we study a Q-ary orthogonally modulated DS-CDMA system which employ high-rate space-time linear dispersion code and an iterative detection and decoding receiver. The orthogonal modulation of the DS-CDMA system is accomplished by Walsh codes which combines the advantages of spreading and coding to achieve improved performance for spread spectrum (CDMA) systems. Multi-user detection (MUD) [1] is an effective tool to increase the capacity of interference-limited CDMA systems while reducing some technical requirements such as power control. Several iterative MUD schemes were proposed, e.g., in [2] for uncoded Q-ary orthogonal systems with affordable complexity which is significantly less than that of an optimum receiver. It has been shown that the performance of an iterative MUD receiver is far better than the conventional receiver, especially in high-load networks in which the interference from other users is severe [2].

Channel coding is usually employed in practical systems to improve the error detection and correcting capability and power efficiency. The CDMA systems exhibit their full potential when combined with forward error correction (FEC) coding [3]. In this context, the joint multi-user detection and decoding including soft interference cancellation, linear MMSE filtering, trellis based Log-MAP multi-user detector, and blind Bayesian multi-user detection are some of the schemes studied in [4–7] to reduce the deteriorative effect of interference. In [8], we provide a thorough treatment of joint multi-user detection, channel estimation, demodulation and decoding for the serially concatenated CDMA system (orthogonal modulation concatenated with outer convolutional code) over multipath fading channels. Decision-directed interference cancellation/suppression and channel estimation were proposed to combat the effect of multiple access interference (MAI) and to improve the reliability of the demodulation process. For a general reference to iterative methods for joint detection, readers are referred to [9].

In recent years, multiple transmit and multiple receive (MIMO) antenna systems have attracted extensive interest and research. In particular, space-time block coding (STBC) has emerged as one of the most promising technologies for meeting the high data rate and high service quality requirements for wireless communications [10–12]. A critical issue for high data rate transmission is the STBC designs for large MIMO arrays. In particular, full rate STBCs cannot be found for complex constellations when the number of transmit antennas is greater than two. Hassibi and Hochwald proposed a high-rate space-time coding framework, called linear dispersion codes (LDC) [13]. The principle is to transmit sub-streams of data in linear combinations over space and time. LDCs are designed to optimize the mutual information between the transmitted and received signals, and at the same time, retain decoding simplicity due to their linear structure. They provide a powerful means to combat fading by dispersing the transmitted signals over time and space, which is equivalent to creating better effective channels and improving effective signal to interference plus noise ratio (SINR), leading to an improved system performance. It is shown in [13] that LDC may achieve a coding rate of up to one and outperform the well-known full-rate uncoded V-BLAST [14] scheme.

Applications of low-rate orthogonal space-time block codes to CDMA systems have been studied, e.g., in [15–17]. In order to support future high data rate CDMA systems, the use of high-rate space-time block codes, e.g, LDCs, may be desirable. However, very limited efforts have been made in investigating the application of LDCs to CDMA systems. In [18], a LDC decoder combined with a blind subspace-based multi-user detector is studied for the downlink of a DS-CDMA system, and a subspace-based sphere decoding algorithm is proposed in order to further improve the performance. The iterative decoding of LDC codes in a frequency-selective channel is considered in [19], where only a single-user approach is studied and multi-user scenarios are not investigated. To the best of our knowledge, the issue of iterative multi-user detection and LDC de-

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coding has not been addressed for the orthogonally modulated DS-CDMA systems in the existing literature.

In this paper, we propose a joint multi-user detection, LDC decoding, and Q-ary demodulation algorithm for the system under investigation. Turbo processing principle is employed to improve the system performance in an iterative manner, while maintaining a reasonable computational complexity. The role of LDC in this work is to exploit space and time diversity for multiple users in DS-CDMA systems. The performance of the LDC coded DS-CDMA system is compared to that of the V-BLAST based system, and time diversity gains obtained by the LDC codes are investigated for the case when the channels gains change within one LDC codeword.

The remainder of this paper is organized as follows. The system model is introduced in Section II. Different LDC codes suitable for the orthogonally modulated CDMA system are discussed in Section III. Iterative multi-user detection, demodulation and decoding schemes are introduced in Section IV. Different algorithms are examined and compared numerically in Section V. Conclusions are drawn in Section VI.

II. System Model

Fig. 1 illustrates a block diagram of the baseband received signal due to the kth user. The kth user's lth information bit is denoted as $b_l^k \in \{+1,-1\}$ ($k=1,\ldots,K,\ \ l=1,\ldots,L_b$, where L_b is the block length). The information bits are convolutionally encoded into code bits $\{u_{n,l}^k\} \in \{+1,-1\}$, where $u_{n,l}^k$ denotes the nth code bit due to b_l^k. For example, in the case of a rate $1/3$ code, b_l^k is encoded into $u_{0,l}^k, u_{1,l}^k, u_{2,l}^k$. Code bits are subsequently interleaved and each block of $\log_2 Q$ coded and interleaved bits $\{u_{n,l}^{\prime k}\} \in \{+1,-1\}$ is mapped into $\mathbf{w}_k(j) \in \{\mathbf{w}_0,\ldots,\mathbf{w}_{Q-1}\}$, which is one of the Q Walsh symbol (j is the symbol index). The interleaver and deinterleaver are denoted as Π and Π^{-1}, respectively, in Figs. 1 and 2. The Walsh codeword $\mathbf{w}_k(j) \in \{+1,-1\}^Q$, is then encoded with a LDC as follows

$$\mathbf{W}_k(j) = \sum_{q=1}^{Q} w_k^q(j)\mathbf{A}_q, \qquad (1)$$

where the $w_k^q(j)$ denotes the qth bit of the Walsh codeword $\mathbf{w}_k(j)$, and $\mathbf{W}_k(j) \in \mathbb{C}^{T\times N_t}$ is the LDC encoded matrix (T is the number of time slots or channel uses needed to transmit Q LDC symbols, and N_t is the number of transmit antennas, and the symbol $\mathbb{C}$ denotes the complex field). The matrices $\mathbf{A}_q \in \mathbb{C}^{T\times N_t}, q=1,\ldots,Q$ are called dispersion matrices, which transform data symbols (Walsh codeword in this context) into a space-time matrix.

In a more general case, when the data sequence is modulated using complex-valued symbols $\gamma_q = \alpha_q + i\beta_q$, chosen from an arbitrary constellation (e.g., r-PSK or r-QAM), a linear dispersion code, $\mathbf{S}_{LDC}$, is defined as [13]

$$\mathbf{S}_{LDC} = \sum_{q=1}^{Q} (\alpha_q\mathbf{A}_q + i\beta_q\mathbf{B}_q), \qquad (2)$$

where $i=\sqrt{-1}$. Note that w_k^q in (1) is real-valued, therefore $\mathbf{B}_q$ becomes irrelevant for the system under investigation.

In order to facilitate linear LDC decoding in the receiver, it is desirable to reorder $\mathbf{W}_k(j)$ column by column in a vector form. Define $\mathrm{vec}(\cdot)$ operation of $m\times n$ matrix $\mathbf{K}$ as $\mathrm{vec}(\mathbf{K}) = \begin{bmatrix}\mathbf{K}_{.1}^{\mathcal{T}} & \mathbf{K}_{.2}^{\mathcal{T}} & \cdots & \mathbf{K}_{.n}^{\mathcal{T}}\end{bmatrix}^{\mathcal{T}}$, where $()^{\mathcal{T}}$ denotes the transpose operation, and $\mathbf{K}_{.i}$ is the i^{th} column of $\mathbf{K}$. Denoting

$$\mathbf{G}_{\mathrm{vec}} = \begin{bmatrix}\mathrm{vec}(\mathbf{A}_1^{\mathcal{T}}) & \mathrm{vec}(\mathbf{A}_2^{\mathcal{T}}) & \cdots & \mathrm{vec}(\mathbf{A}_Q^{\mathcal{T}})\end{bmatrix};$$
$$\mathbf{w}_k(j) = \begin{bmatrix}w_k^1(j) & w_k^2(j) & \cdots & w_k^Q(j)\end{bmatrix}^{\mathcal{T}},$$

we can now express the jth LDC codeword of the kth user's in vector form as

$$\mathbf{x}_k(j) = \mathrm{vec}(\mathbf{W}_k(j)^{\mathcal{T}}) = \mathbf{G}_{\mathrm{vec}}\mathbf{w}_k(j), \qquad (3)$$

where the vectors $\mathbf{x}_k(j)$, $\mathbf{w}_k(j)$ and the LDC generator matrix $\mathbf{G}_{\mathrm{vec}}$ are of sizes $TN_t\times 1$, $Q\times 1$, and $TN_t\times Q$, respectively. The LDC codeword $\mathbf{x}_k(j)$ is then repetition encoded into symbol sequence $\mathbf{s}_k(j) = \mathrm{rep}\{\mathbf{x}_k(j),\ \ N_c\}$ where $\mathrm{rep}\{\cdot,\cdot\}$ denotes the repetition encoding operation, its first argument is the input bits and the second one is the repetition factor. Therefore, each LDC symbol per channel use is spread (repetition coded) into N_c symbols, which causes bandwidth expansion (signal spreading in frequency domain). The spread sequence $\mathbf{s}_k(j)$ is then scrambled (randomized) by a scrambling code unique to each user to form the transmitted symbol sequence $\mathbf{a}_k(j) = \mathbf{C}_k(j)\mathbf{s}_k(j)$, where $\mathbf{C}_k(j) \in \{-1,+1\}^{N_cTN_t\times N_cTN_t}$ is a diagonal matrix whose diagonal elements correspond to the scrambling code for the k^{th} user's j^{th} symbol. The purpose of scrambling is to separate users. In this paper, we focus on the use of long codes, e.g., the scrambling code differs from symbol to symbol. The scrambled sequence $\mathbf{a}_k(j)$ is transmitted over MIMO channel via multiple transmit antennas. For simplicity, we only consider flat-fading channel here. The received signal is the sum of K users' signals plus the additive white complex Gaussian noise. After descrambling and despreading, the received signal due to the kth user's jth transmitted sequence can be written in a vector form as $\mathbf{y}_k(j) = N_c\mathbf{H}_k(j)\mathbf{x}_k(j) = N_c\mathbf{H}_k(j)\mathbf{G}_{\mathrm{vec}}\mathbf{w}_k(j)$. Denoting the channel gain of the path between the mth transmit antenna and nth receive antenna for the tth channel use of $\mathbf{W}_k(j)$ as $h_{m,n}^{k,t}$, the MIMO channel matrix corresponding to the kth user's jth symbol can be expressed as

$$\mathbf{H}_k(j) = \begin{bmatrix}\mathbf{H}_k^{(1)}(j) & \cdots & \mathbf{0}\\ \vdots & \ddots & \vdots\\ \mathbf{0} & \cdots & \mathbf{H}_k^{(T)}(j)\end{bmatrix}, \qquad (4)$$

where

$$\mathbf{H}_k^{(t)}(j) = \begin{bmatrix}h_{1,1}^{k,t}(j) & \cdots & h_{N_t,1}^{k,t}(j)\\ \vdots & \ddots & \vdots\\ h_{1,N_r}^{k,t}(j) & \cdots & h_{N_t,N_r}^{k,t}(j)\end{bmatrix}.$$

In this work, we focus on asynchronous transmission. Without loss of generality, we assume $\tau_1 < \tau_2 < \ldots < \tau_k \ldots < \tau_K$, where τ_k is the propagation delay for user k, and is assumed to be multiple of chip intervals. The maximum delay spread is assumed to be less than or equal to a symbol interval (N_c chip intervals). After descrambling and despreading, the received signal corresponding to the kth user's jth transmitted sequence

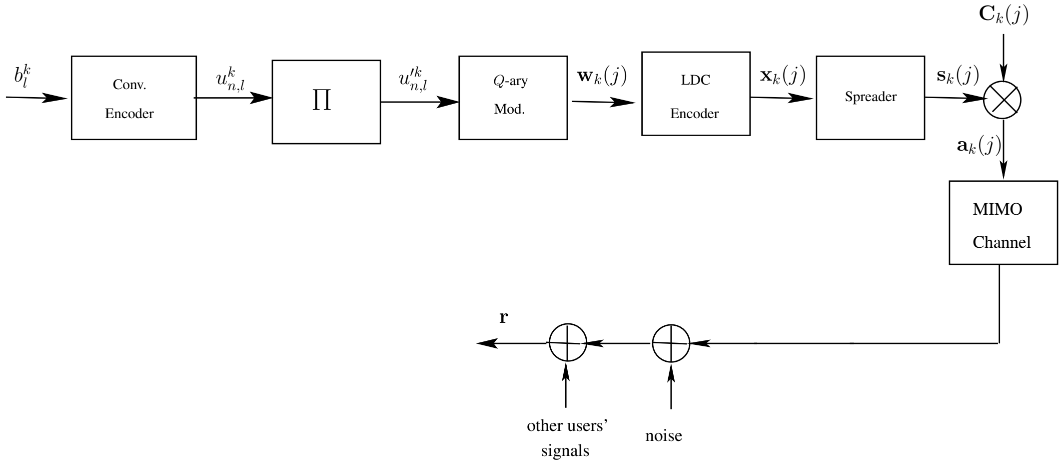

Fig. 1. Block diagram of the transmitter for the LDC coded DS-CDMA system for the kth user.

can now be expressed as

$$\mathbf{r}(j) = \sum_{k=1}^{K} \mathbf{y}_k(j) + \mathbf{n}(j) = \underbrace{N_c \mathbf{H}_k(j) \mathbf{G}_{\text{vec}} \mathbf{w}_k(j)}_{\text{desired signal}} + \underbrace{\mathbf{n}(j)}_{\text{noise}}$$
$$+ \underbrace{\sum_{s=1}^{k-1} (N_c - \tau_k + \tau_s) \mathbf{H}_s(j) \mathbf{G}_{\text{vec}} \mathbf{w}_s(j)}_{\text{MAI}}$$
$$+ \underbrace{\sum_{s=1}^{k-1} (\tau_k - \tau_s) \mathbf{H}_s(j+1) \mathbf{G}_{\text{vec}} \mathbf{w}_s(j+1)}_{\text{MAI}}$$
$$+ \underbrace{\sum_{s=k+1}^{K} (\tau_s - \tau_k) \mathbf{H}_s(j-1) \mathbf{G}_{\text{vec}} \mathbf{w}_s(j-1)}_{\text{MAI}}$$
$$+ \underbrace{\sum_{s=k+1}^{K} (N_c - \tau_s + \tau_k) \mathbf{H}_s(j) \mathbf{G}_{\text{vec}} \mathbf{w}_s(j)}_{\text{MAI}}, \quad (5)$$

where $\mathbf{n}(j) \in \mathbb{C}^{TN_t}$ is a vector of iid complex Gaussian noise samples with zero mean and variance matrix N_0, i.e., $\mathbf{n}(j) \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$.

III. Linear Dispersion Codes

In this section, we discuss two classes of LDC codes that are well suited for the DS-CDMA system with orthogonal signalling under study.

A. Rectangular U-LDC

As an extension of rate-one square LDC matrices of size $N_t \times N_t$ defined in Eq. (31) of [13], a class of algebraically designed rate-one rectangular linear dispersion codes of arbitrary size $T \times N_t$, called uniform linear dispersion codes (U-LDC), have been reported in [22]. They are particularly suited to the DS-CDMA application under investigation due to their flexible choice of space-time dimensions.

1) The case of $T \le N_t$: The $T \times N_t$ LDC dispersion matrices are defined as

$$\mathbf{A}_{N_t(k-1)+l} = \frac{1}{\sqrt{T}} \mathbf{D}^{k-1} \mathbf{\Gamma} \mathbf{\Pi}^{l-1}, \quad (6)$$

where $k = 1, ..., T$, $l = 1, ..., N_t$, $\mathbf{D} = diag(1, e^{i\frac{2\pi}{T}}, ..., e^{i\frac{2\pi(T-1)}{T}})$, $\mathbf{\Pi} = \begin{bmatrix} \mathbf{0}_{1\times(N_t-1)} & 1 \\ \mathbf{I}_{N_t-1} & \mathbf{0}_{(N_t-1)\times 1} \end{bmatrix}$, $\mathbf{\Gamma} = \begin{bmatrix} \mathbf{I}_T & \mathbf{0}_{T\times(N_t-T)} \end{bmatrix}$, and $diag(\mathbf{a})$ denotes a diagonal matrix with vector $\mathbf{a}$ on the main diagonal, $i = \sqrt{-1}$, $\mathbf{D}$ is of size $T \times T$, $\mathbf{\Pi}$ is of size $N_t \times N_t$, and $\mathbf{\Gamma}$ is of size $T \times N_t$, $\mathbf{I}_X$ denotes identity matrix of size $X \times X$, $\mathbf{0}_{X\times Y}$ denotes all-zero matrix of size $X \times Y$.

2) The case of $T > N_t$: The $T \times N_t$ LDC dispersion matrices are defined as

$$\mathbf{A}_{N_t(k-1)+l} = \frac{1}{\sqrt{N_t}} \mathbf{\Pi}^{k-1} \mathbf{\Gamma} \mathbf{D}^{l-1}, \quad (7)$$

where $k = 1, ..., T$, $l = 1, ..., N_t$, $\mathbf{D} = diag(1, e^{i\frac{2\pi}{N_t}}, ..., e^{i\frac{2\pi(N_t-1)}{N_t}})$, $\mathbf{\Pi} = \begin{bmatrix} \mathbf{0}_{1\times(T-1)} & 1 \\ \mathbf{I}_{T-1} & \mathbf{0}_{(T-1)\times 1} \end{bmatrix}$, and $\mathbf{\Gamma} = \begin{bmatrix} \mathbf{I}_{N_t} \\ \mathbf{0}_{(T-N_t)\times N_t} \end{bmatrix}$, where $\mathbf{D}$ is of size $N_t \times N_t$, and $\mathbf{\Pi}$ as previously defined, is of size $T \times T$, and $\mathbf{\Gamma}$ is of size $T \times N_t$.

B. Full-diversity, full-rate LDC

In general, space-time linear dispersion codes [13] do not necessarily reach any guaranteed space diversity order. LDC was applied to space-time CDMA systems in [18], however, the selected LDC (see Equ. (31) in [13]) does not offer full-diversity. Damen, et al., proposed a class of high-rate full-transmit diversity based space-time CDMA systems in [23]. However, they actually use space codes instead of space-time codes. Note that in [23], N_t source data symbols are transformed using a signal space diversity rotation to obtain N_t coded symbols, and each of coded symbols is spread through a signature sequence. Although these spread symbols are over time, the signatures are not spread over multiple data symbols. The approach in [23] actually limits the potentials to exploit time diversity in signal levels.

In order to obtain full diversity over multiple symbol time channel uses, we adopt the full-diversity full-rate (FDFR) complex-field code proposed in [24, 25] in this work. It is a class of full space diversity linear dispersion codes with equal real and imaginary dispersion matrices in each source data symbol. The generator matrix of FDFR codes is calculated as

$$\mathbf{G}_{vec} = \begin{bmatrix} (\mathbf{P}_1 \mathbf{D}_\beta) \otimes [\theta_1]^{\mathcal{T}} & \dots & (\mathbf{P}_{N_t} \mathbf{D}_\beta) \otimes [\theta_{JN_t}]^{\mathcal{T}} \end{bmatrix}^{\mathcal{T}}, \quad (8)$$

where $[\theta_k]^{\mathcal{T}}$ is the k-th row vector of Θ, and

$$\Theta = \frac{1}{JN_t}\left[\mathbf{F}_{JN_t}\right]^{\mathcal{H}} diag\left(1, \alpha, ..., \alpha^{JN_t-1}\right) = \left[\theta_1, ..., \theta_{JN_t}\right]^{\mathcal{T}};$$
$$\mathbf{D}_\beta = diag\left(1, \beta, ..., \beta^{N_t-1}\right);$$
$$\mathbf{P}_m = \begin{bmatrix} \mathbf{0}_{(N_t-m+1)\times(m-1)} & \mathbf{I}_{m-1} \\ \mathbf{I}_{N_t-m+1} & \mathbf{0}_{(m-1)\times(N_t-m+1)} \end{bmatrix}, \quad (9)$$

where the superscript operator $(\ )^{\mathcal{H}}$ denotes the conjugate transpose operation, and $\mathbf{F}_Q$ denotes discrete Fourier transform matrix of dimension $Q \times Q$. The constants α and β for $N_t = 2^k$ (k is a positive integer) are specified as

$$\alpha = \begin{cases} e^{i\pi/(2N_t)}, & \text{for Design A;} \\ e^{i\pi/(N_t)^3}, & \text{for Design B;} \end{cases}$$
$$\beta = \begin{cases} e^{i\pi/(4(N_t)^3)}, & \text{for Design A;} \\ \alpha^{1/N_t}, & \text{for Design B;} \end{cases} \quad (10)$$

In [24], FDFR codes of size $N_t \times N_t$ were proposed as space-time codes; FDFR codes of size $JN_t \times N_t$ (J is an integer number) were proposed as space-frequency codes and space Doppler-codes. In this paper, we consider FDFR codes of size $JN_t \times N_t$ as a class of full space diversity rectangular space-time linear dispersion codes.

Defining LDC coding rate as $R_{LDC}^{sym} = Q/(N_tT)$ [20], one can see that both U-LDC and FDFR codes mentioned above have rate one ($Q = N_tT$). This is in contrast to all orthogonal space-time codes, for which the rate is less than one (even the so-called full-rate Alamouti code [11] has a rate of $1/2$ by this definition).

IV. Joint LDC decoding and Q-ary symbol demodulation

The proposed iterative detection and decoding scheme is illustrated in Fig. 2. The soft metric $\lambda(u'^k_{n,l}; O)$ from the inner soft-input, soft-output (SISO) block is deinterleaved to $\lambda(u^k_{n,l}; I)$. The k^{th} user's Log-MAP decoder computes an a posteriori log-likelihood ratio (LLR) of each information bit $\lambda(b^k_l; O)$ and each code bit $\lambda(u^k_{n,l}; O)$ based on the soft input $\lambda(u^k_{n,l}; I)$ and the trellis structure of the convolutional code. The former is used to make a decision on the transmitted information bit at the final iteration, while the latter is used for multi-user detection in the inner SISO block at the next iteration. We use the notations $\lambda(\cdot; I)$ and $\lambda(\cdot; O)$ to denote the input and output ports of a SISO device.

The algorithms discussed above require the design of an inner SISO block that can produce soft reliability values for each bit $u'^k_{n,l}$ from the received signal in order to enable soft input channel decoding. To this end, we propose an integrated multi-user detection, LDC decoding, and symbol demodulation scheme, which will be described next. To simplify the notation, we sometimes suppress the index j from $\mathbf{x}_k(j)$, $\mathbf{w}_k(j)$ and $\mathbf{H}_k(j)$, etc., whenever no ambiguity arises.

A. Single-user approach

Let $\mathbf{r}_k$ denote the delay aligned version of the received vector due to the transmission of kth user's jth symbol. The single-user detection approach is based on the assumption that different users' scrambling codes are (nearly) orthogonal to each other (their cross-correlations are approximately zero) and their auto-correlation approximates the delta function. The received vector corresponding to the kth user's jth symbol after descrambling and despreading can be expressed as

$$\mathbf{z}_k = N_c\mathbf{H}_k\mathbf{x}_k + \mathbf{v}_k = N_c\mathbf{H}_k\mathbf{G}_{\text{vec}}\mathbf{w}_k + \mathbf{v}_k, \quad (11)$$

where $\mathbf{z}_k$ is descrambled and despread version of $\mathbf{r}_k$, and $\mathbf{v}_k$ denotes the combined MAI and noise, which is a complex Gaussian random vector, i.e., $\mathbf{v}_k \sim \mathcal{CN}(\mathbf{0}, N_v\mathbf{I})$ [26], where N_v is the combined MAI and noise variance.

The Walsh codeword $\mathbf{w}_k$ (or equivalently the jth Walsh symbol for user k) can be estimated by a linear MMSE algorithm, i.e.,

$$\hat{\mathbf{w}}_k = \mathbf{\Phi}^{\mathcal{H}}\mathbf{z}_k = \mathbf{\Phi}^{\mathcal{H}}(N_c\mathbf{H}_k\mathbf{G}_{\text{vec}}\mathbf{w}_k + \mathbf{v}_k) = \mathbf{U}\mathbf{w}_k + \xi_k, \quad (12)$$

where the matrix $\mathbf{\Phi}$ is designed to minimize $\text{E}[\|\hat{\mathbf{w}}_k - \mathbf{w}_k\|^2]$, leading to the solution $\mathbf{\Phi} = \mathbf{R}^{-1}\mathbf{P}$, where

$$\mathbf{R} = \text{E}[\mathbf{z}_k\mathbf{z}_k^{\mathcal{H}}] = \text{E}[(N_c\mathbf{H}_k\mathbf{G}_{\text{vec}}\mathbf{w}_k + \mathbf{v}_k)(N_c\mathbf{w}_k^{\mathcal{H}}\mathbf{G}_{\text{vec}}^{\mathcal{H}}\mathbf{H}_k^{\mathcal{H}} + \mathbf{v}_k^{\mathcal{H}})]$$
$$= N_c^2\mathbf{H}_k\mathbf{H}_k^{\mathcal{H}} + N_v\mathbf{I}; \quad (13)$$
$$\mathbf{P} = \text{E}[\mathbf{z}_k\mathbf{w}_k^{\mathcal{H}}] = \text{E}[(N_c\mathbf{H}_k\mathbf{G}_{\text{vec}}\mathbf{w}_k + \mathbf{v}_k)\mathbf{w}_k^{\mathcal{H}}] = N_c\mathbf{H}_k\mathbf{G}_{\text{vec}};$$
$$\mathbf{U} = \mathbf{\Phi}^{\mathcal{H}}N_c\mathbf{H}_k\mathbf{G}_{\text{vec}} = \mathbf{\Phi}^{\mathcal{H}}\mathbf{P}. \quad (14)$$

Equations (13) and (14) are derived utilizing the fact that for Walsh codewords, $\text{E}[\mathbf{w}_k\mathbf{w}_k^{\mathcal{H}}] = \mathbf{I}_Q$, and $\mathbf{G}_{\text{vec}}$ is a unitary matrix for both U-LDC and FDFR codes (proof for U-LDC is given in [22], and can be derived similarly for FDFR). Therefore, $\mathbf{G}_{\text{vec}}\mathbf{G}_{\text{vec}}^{\mathcal{H}} = \mathbf{I}_Q$. The noise term $\xi_k = \mathbf{\Phi}^{\mathcal{H}}\mathbf{v}_k$ is Gaussian, since it is a linear transformation of a Gaussian random vector, with zero mean and covariance $\text{E}[\xi_k\xi_k^{\mathcal{H}}] = N_v\mathbf{\Phi}^{\mathcal{H}}\mathbf{\Phi}$. The probability density function (PDF) of the MMSE filter output $\hat{\mathbf{w}}_k$, conditioned on that the mth Walsh symbol is transmitted, can be expressed as

$$f(\hat{\mathbf{w}}_k|\mathbf{w}_m) = \frac{\exp[-(\hat{\mathbf{w}}_k - \mathbf{U}\mathbf{w}_m)^{\mathcal{H}}(N_v\mathbf{\Phi}^{\mathcal{H}}\mathbf{\Phi})^{-1}(\hat{\mathbf{w}}_k - \mathbf{U}\mathbf{w}_m)]}{\pi^Q\det(N_v\mathbf{\Phi}^{\mathcal{H}}\mathbf{\Phi})}$$
$$= \frac{1}{\pi^Q\det(N_v\mathbf{\Phi}^{\mathcal{H}}\mathbf{\Phi})}\exp\left[-\frac{\|\mathbf{\Phi}^{-\mathcal{H}}\hat{\mathbf{w}}_k - \mathbf{P}\mathbf{w}_m\|^2}{N_v}\right].$$

The soft metric for the bit $u'^k_{n,l}$ can thus be computed in terms of LLR as

$$\lambda(u'^k_{n,l}; O) = \ln\frac{\sum_{m:u'^k_{n,l}=+1} f(\hat{\mathbf{w}}_k|\mathbf{w}_m)}{\sum_{m:u'^k_{n,l}=-1} f(\hat{\mathbf{w}}_k|\mathbf{w}_m)}$$
$$\approx \ln\frac{\max_{m:u'^k_{n,l}=+1} f(\hat{\mathbf{w}}_k|\mathbf{w}_m)}{\max_{m:u'^k_{n,l}=-1} f(\hat{\mathbf{w}}_k|\mathbf{w}_m)}$$
$$= \ln\frac{\max_{m:u'^k_{n,l}=+1}\exp(-\|\mathbf{\Phi}^{-\mathcal{H}}\hat{\mathbf{w}}_k - \mathbf{P}\mathbf{w}_m\|^2/N_v)}{\max_{m:u'^k_{n,l}=-1}\exp(-\|\mathbf{\Phi}^{-\mathcal{H}}\hat{\mathbf{w}}_k - \mathbf{P}\mathbf{w}_m\|^2/N_v)}$$
$$= \frac{1}{N_v}\text{Re}\{[2(\mathbf{\Phi}^{-1}\mathbf{P}\mathbf{w}^+)^{\mathcal{H}}\hat{\mathbf{w}}_k - \|\mathbf{P}\mathbf{w}^+\|^2] - [2(\mathbf{\Phi}^{-1}\mathbf{P}\mathbf{w}^-)^{\mathcal{H}}\hat{\mathbf{w}}_k - \|\mathbf{P}\mathbf{w}^-\|^2]\}, \quad (15)$$

where $m : u'^k_{n,l} = \pm 1$ denotes the set of Walsh codewords $\{\mathbf{w}_m\}$ that correspond to the code bit $u'^k_{n,l} = \pm 1$, and $\mathbf{w}^+$ denotes the Walsh codeword $\mathbf{w}_m$ that corresponds to $\max_{m:u'^k_{n,l}=+1} f(\hat{\mathbf{w}}_k|\mathbf{w}_m)$; $\mathbf{w}^-$ denotes the Walsh codeword $\mathbf{w}_m$ that corresponds to $\max_{m:u'^k_{n,l}=-1} f(\hat{\mathbf{w}}_k|\mathbf{w}_m)$.

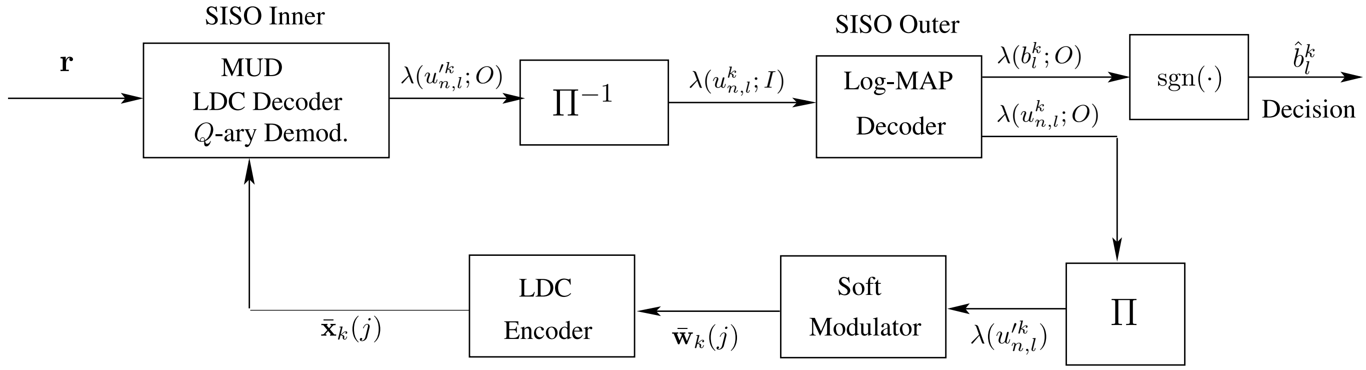

Fig. 2. Iterative receiver structure.

TABLE I
MAPPING BETWEEN INPUT BITS, SYMBOL INDICES, AND WALSH CODEWORDS

Code bits $u'^k_{0,l}$ $u'^k_{1,l}$ $u'^k_{2,l}$	Symbol index m	Walsh codeword $\mathbf{w}_m$
$+1\ +1\ +1$	0	$\mathbf{w}_0: +1\ +1\ +1\ +1\ +1\ +1\ +1\ +1$
$+1\ +1\ -1$	1	$\mathbf{w}_1: +1\ +1\ +1\ +1\ -1\ -1\ -1\ -1$
$+1\ -1\ +1$	2	$\mathbf{w}_2: +1\ +1\ -1\ -1\ +1\ +1\ -1\ -1$
$+1\ -1\ -1$	3	$\mathbf{w}_3: +1\ +1\ -1\ -1\ -1\ -1\ +1\ +1$
$-1\ +1\ +1$	4	$\mathbf{w}_4: +1\ -1\ +1\ -1\ +1\ -1\ +1\ -1$
$-1\ +1\ -1$	5	$\mathbf{w}_5: +1\ -1\ +1\ -1\ -1\ +1\ -1\ +1$
$-1\ -1\ +1$	6	$\mathbf{w}_6: +1\ -1\ -1\ +1\ +1\ -1\ -1\ +1$
$-1\ -1\ -1$	7	$\mathbf{w}_7: +1\ -1\ -1\ +1\ -1\ +1\ +1\ -1$

In the case $Q = 8$, the k^{th} user's j^{th} Walsh codeword $\mathbf{w}_k(j)$ corresponds to 3 coded and interleaved bits: $u'^k_{0,l}$, $u'^k_{1,l}$, $u'^k_{2,l}$. We know from Table I that $u'^k_{0,l} = +1$ holds for $m = 0, 1, 2, 3$ and $u'^k_{0,l} = -1$ holds for $m = 4, 5, 6, 7$. According to (15)

$$\begin{aligned}\lambda(u'^k_{0,l}; O) \approx \max\{z_k(0), z_k(1), z_k(2), z_k(3)\} \\ - \max\{z_k(4), z_k(5), z_k(6), z_k(7)\},\end{aligned}$$

where $z_k(m) = \frac{1}{N_v} \operatorname{Re}\{2(\mathbf{\Phi}^{-1}\mathbf{P}\mathbf{w}_m)^{\mathcal{H}}\hat{\mathbf{w}}_k - \|\mathbf{P}\mathbf{w}_m\|^2\}$. Similarly,

$$\begin{aligned}\lambda(u'^k_{1,l}; O) \approx \max\{z_k(0), z_k(1), z_k(4), z_k(5)\} \\ - \max\{z_k(2), z_k(3), z_k(6), z_k(7)\}; \\ \lambda(u'^k_{2,l}; O) \approx \max\{z_k(0), z_k(2), z_k(4), z_k(6)\} \\ - \max\{z_k(1), z_k(3), z_k(5), z_k(7)\}.\end{aligned}$$

From (15), we can see that LDC decoding and symbol demodulation, as well as symbol-to-LLR mapping are all integrated into one step, the complexity of the receiver is thus greatly reduced.

The above scheme is suboptimal because the orthogonal condition is hardly satisfied in uplink transmission where each user transmits asynchronously. However, this kind of single-user approach has been adopted in practical CDMA systems, e.g., IS-95 due to its low computational complexity. Next, we introduce a multi-user detection (MUD) technique to increase the capacity of interference-limited CDMA systems. Among different MUD techniques, the multistage parallel interference cancellation (PIC) scheme [26] is known to be simple and effective for mitigation of MAI in long-code DS-CDMA systems. In what follows, we shall explain how this PIC based MUD detection technique can be incorporated into the demodulation and decoding process to mitigate the effect of MAI.

B. Multi-user approach

Once the transmitted signals are estimated for all the users at the previous iteration, interference can be removed by subtracting the estimated signals of the interfering users from the received signal $\mathbf{r}$ to form a new signal vector $\mathbf{r}'_k$ for demodulating the signal transmitted from user k. The descrambled and despread signal corresponding to the kth user's jth symbol after interference cancellation can now be expressed as

$$\begin{aligned}\mathbf{r}'_k(j) = \mathbf{r}(j) - \sum_{s=1}^{k-1}(N_c - \tau_k + \tau_s)\mathbf{H}_s(j)\mathbf{G}_{\text{vec}}\hat{\mathbf{w}}_s(j) \\ - \sum_{s=1}^{k-1}(\tau_k - \tau_s)\mathbf{H}_s(j+1)\mathbf{G}_{\text{vec}}\hat{\mathbf{w}}_s(j+1) \\ - \sum_{s=k+1}^{K}(\tau_s - \tau_k)\mathbf{H}_s(j-1)\mathbf{G}_{\text{vec}}\hat{\mathbf{w}}_s(j-1) \\ - \sum_{s=k+1}^{K}(N_c - \tau_s + \tau_k)\mathbf{H}_s(j)\mathbf{G}_{\text{vec}}\hat{\mathbf{w}}_s(j) \\ = N_c\mathbf{H}_k(j)\mathbf{G}_{\text{vec}}\mathbf{w}_k(j) + \mathbf{v}'_k(j), \quad (16)\end{aligned}$$

where $\hat{\mathbf{w}}_s(j)$ is an estimate of $\mathbf{w}_s(j)$ using hard decision, and $\mathbf{v}'_k(j)$ denotes the combined cancellation residual and noise. The vector $\mathbf{r}'_k$ is the interference canceled version of $\mathbf{r}$ after subtracting the contributions from all the other users using decision feedback. The symbol index j is sometimes omitted for simplicity. With interference cancellation technique, $\mathbf{v}'_k$ contains much less MAI compared to $\mathbf{v}_k$ in (11), leading to significant performance improvement. In case of perfect interference cancellation, the cancellation residual vanishes, and $\mathbf{v}'_k \sim \mathcal{CN}(\mathbf{0}, N_c N_0 \mathbf{I})$. As will become apparent in Section V, the assumption of perfect cancellation can be approached by proper design of the iterative receiver and by proper choice of full diversity LDC codes. The rest of LDC decoding and derivation of LLR values for $u'^k_{n,l}$ is the same as described in the previous section, only with $\mathbf{z}_k$ replaced by $\mathbf{r}'_k$, and N_v replaced by $N_c N_0$ in equations (12) – (15).

The conventional interference cancellation is subject to performance degradation due to incorrect decisions on interference that are subtracted from the received signal. To prevent error propagation from the decision feedback, soft interference cancellation was proposed, e.g., in [27] for uncoded Q-ary DS-CDMA systems. The rationale is that the cancellation with hard decisions tends to propagate errors and increase the interference with incorrect decision feedback; while with soft cancellation, an erroneously estimated symbol usually has small LLR and does not make much contribution to the feedback, and therefore the error propagation problem is alleviated. In our case, the interference cancellation scheme using soft symbol estimates can be reformed as (16) with $\hat{\mathbf{w}}_s(j)$ replaced by $\bar{\mathbf{w}}_s(j)$ (the soft estimate of $\mathbf{w}_s(j)$).

This MUD-based iterative scheme is shown in Fig. 2. It uses soft information of $\mathbf{x}_k(j)$, denoted as $\bar{\mathbf{x}}_k(j)$, for interference cancellation in order to reduce the error propagation. To this end, we compute $\bar{\mathbf{w}}_k(j) = [\bar{w}_k^0(j)\ \bar{w}_k^1(j)\ \cdots\ \bar{w}_k^{Q-1}(j)]^{\mathcal{T}}$, the soft estimate of the codeword $\mathbf{w}_k(j)$, from its LLR $\lambda(\mathbf{w}_k(j))$, which is derived by feeding $\lambda(u'^k_{n,l}) = \Pi\{\lambda(u^k_{n,l}; O)\}$ into a soft modulator. The soft estimate $\bar{\mathbf{w}}_k(j)$ is then LDC encoded to produce the LDC codeword $\bar{\mathbf{x}}_k(j)$.

When soft information is to be used for interference cancellation, a serially concatenated system would be rather different from the uncoded or non-concatenated systems in that the soft values are not directly available for all the inner code bits from the outer decoder. In particular, in our case, only the soft information can be extracted for the systematic bits of the Walsh codewords from a SISO channel decoder. To tackle this problem, we design a soft modulator to derive the soft estimates for parity bits, which will be explained next.

In the case $Q = 8$, the code bits to Walsh codeword (Walsh symbol) mapping rule is given in Table I. The three systematic bits are $w_k^1(j), w_k^2(j)$, and $w_k^4(j)$, where $w_k^q(j)$ denotes the q^{th} bit of the codeword. The columns corresponding to the systematic bits are highlighted in the table. To ease understanding, we use $Q = 8$ as an example. However, the extension of the proposed algorithms to other values of Q is straightforward. From Table I, we can see that the parity bits are formed by systematic bits $w_k^1(j), w_k^2(j), w_k^4(j)$ as

$$w_k^0(j) = +1;\ \ w_k^3(j) = w_k^1(j) \oplus w_k^2(j);\ \ w_k^5(j) = w_k^1(j) \oplus w_k^4(j);$$
$$w_k^6(j) = w_k^2(j) \oplus w_k^4(j);\ \ w_k^7(j) = w_k^1(j) \oplus w_k^2(j) \oplus w_k^4(j).$$

The LLRs for systematic bits are

$$\lambda(w_k^1(j)) = \lambda(u'^k_{0,l});\quad \lambda(w_k^2(j)) = \lambda(u'^k_{1,l});\quad \lambda(w_k^4(j)) = \lambda(u'^k_{2,l}).$$

Considering the fact that the interleaver breaks the memory of the convolutional encoding process, the bits $u'^k_{0,l}, u'^k_{1,l}, u'^k_{2,l}$ can be modeled as statistically independent random variables. Also assuming that they are independent conditioned on the received signal, then the LLRs for parity bits can thus be computed according to [28] by

$$\begin{aligned}
\lambda(w_k^3(j)) &= \lambda(w_k^1(j) \oplus w_k^2(j)) \\
&= 2\text{arctanh}\left\{\tanh(\lambda(u'^k_{0,l})/2) \cdot \tanh(\lambda(u'^k_{1,l})/2)\right\} \\
\lambda(w_k^5(j)) &= \lambda(w_k^1(j) \oplus w_k^4(j)) \\
&= 2\text{arctanh}\left\{\tanh(\lambda(u'^k_{0,l})/2) \cdot \tanh(u'^k_{2,l})/2)\right\} \\
\lambda(w_k^6(j)) &= \lambda(w_k^2(j) \oplus w_k^4(j)) \\
&= 2\text{arctanh}\left\{\tanh(\lambda(u'^k_{1,l})/2) \cdot \tanh(\lambda(u'^k_{2,l})/2)\right\} \\
\lambda(w_k^7(j)) &= \lambda(w_k^1(j) \oplus w_k^2(j) \oplus w_k^4(j)) \\
&= 2\text{arctanh}\left\{\prod_{n=0}^{2} \tanh(\lambda(u'^k_{n,l})/2)\right\} \qquad (17)
\end{aligned}$$

Finally, the soft estimate (expected value given the received observation) for each bit of the Walsh codeword $w_k^p(j)$ is computed based on $\lambda(w_k^p(j))$ as

$$\begin{aligned}
\bar{w}_k^p(j) &= \text{E}[w_k^p(j)|\mathbf{z}'_k] = (+1) \times P\{w_k^p(j) = +1|\mathbf{z}'_k\} + \\
&\quad (-1) \times P\{w_k^p(j) = -1|\mathbf{z}'_k\} \\
&= (+1)\frac{e^{\lambda(w_k^p(j))}}{1 + e^{\lambda(w_k^p(j))}} + (-1)\frac{e^{-\lambda(w_k^p(j))}}{1 + e^{-\lambda(w_k^p(j))}} \\
&= \tanh\{\lambda(w_k^p(j))/2\}.
\end{aligned}$$

V. Numerical Results

In the simulations, we employ a rate $R_c = 1/3$ convolutional code with constraint length $L_c = 5$ and generator polynomials $(25, 33, 37)$ in octal form for all the users, unless otherwise stated. Each block of 3 interleaved bits from each user is then converted into one of $Q = 8$ Walsh symbols, which is subsequently encoded with a LDC code. For this study, we use the U-LDC code expressed by (7) and the FDFR code (design A) expressed by (8) – (10). The parameter setting is chosen to be $N_t = 2, N_r = 2$ and 4, $T = 4$. In this case, 8-bit Walsh codeword is dispersed into 2 transmit antennas, each accommodates 4 LDC symbols ($T = 4$ channel uses before spreading). Each LDC symbol is spread (repetition encoded) to $N_c = 8$ symbols. The effective spreading factor of the system is given as the reciprocal of the overall code rate divided by N_t, i.e., $N_c Q/(N_t R_c \log_2 Q) = 32$ LDC symbols/information bit/transmit antenna. Hence, the spectral efficiency of a single link is equal to $1/32$ times the spectral efficiency of an uncoded, unspread system. The number of users is chosen to be $K = 18$, which represents a fairly heavily loaded system, as $K = 32$ would be a fully loaded system. The long scrambling codes $\mathbf{C}_k$ are generated randomly, and all the transmit antennas of a specific user are assigned the same scrambling code. The noise variance N_0, and $\mathbf{C}_k$ as well as path delays $\tau_1, \tau_2, \ldots, \tau_K$ are assumed to be known to the receiver, and the different paths of the MIMO channels for the same user have the same delay. We assume uplink asynchronous transmission, and the delays are therefore different for different users.

We compare the performance of the proposed LDC iterative scheme with that of the soft demodulation and decoding algorithm [8] for Q-ary orthogonally modulated DS-CDMA system without LDC codes in a 2×2 or 2×4 V-BLAST system. In this case, information bearing signals are divided into multiple substreams, each encoded and modulated independently. Unlike in the LDC systems, each user's two transmit antennas are assigned with different scrambling codes and transmit the data simultaneously. The employed convolution code and spreading factor are the same as in the LDC system. To transmit one Walsh symbol, the V-BLAST system needs $Q = 8$ channel uses (time slots), whereas the LDC system only needs $T = 4$ channel uses at both transmit antennas. However, due to simultaneous transmission from two transmit antennas in the V-BLAST system, the two systems have the same data rate.

The channel gain $h_{m,n}^{k,t}$ is a complex circular Gaussian process with autocorrelation function $\text{E}[h_{m,n}^{k,t*} h_{m,n}^{k,t+\tau}] = P_{m,n}^k J_0(2\pi f_D \tau)$ where f_D is the maximum Doppler frequency, $J_0(x)$ is the zero-th order Bessel function of the first kind, and $P_{m,n}^k$ is the average power of $h_{m,n}^{k,t}$. The amplitude of $h_{m,n}^{k,t}$ follows a Rayleigh distribution. The Doppler shifts are due to the relative motion between the base station and mobile units. Perfect slow power control is assumed in the sense that $P_k = \sum_{m,n} \text{E}[|h_{m,n}^{k,t}|^2]$, the average received power, is equal for all users, and it is normalized such that $P_k = 1$ for all k. However,

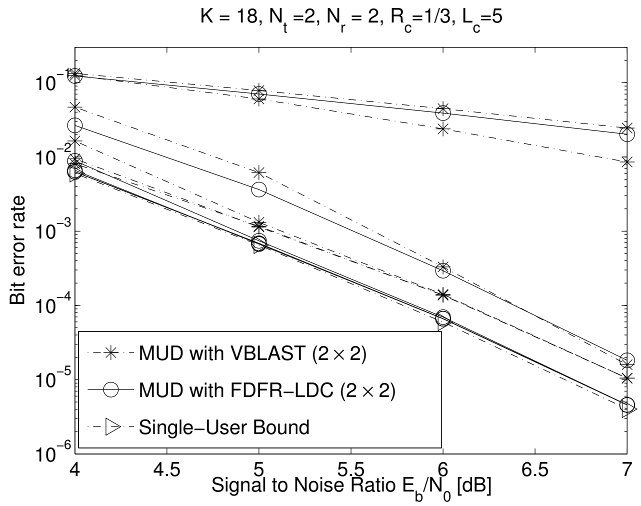

Fig. 3. Comparison of different MUD schemes: LDC vs. V-BLAST. Curves with the same marker represent the performance of the same scheme at different iterations (including the first single user detection stage).

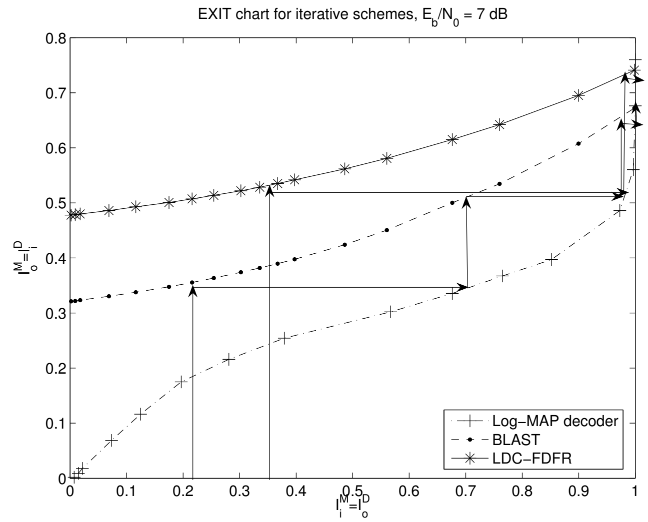

Fig. 4. Convergence property of the iterative schemes. $\rightarrow$ represents the decoding process and $\uparrow$ represents the multiuser detection process.

the instantaneous power $|h_{m,n}^{k,t}|^2$ may vary from one user to another. Perfect knowledge of the channel state information (CSI) is assumed in our simulations. In Figs. 3 – 7, we assume channel gains remain constant during the transmission of one LDC codeword, i.e., $h_{m,n}^{k,t}(j) = h_{m,n}^{k,t+1}(j) = h_{m,n}^{k,t+2}(j) = h_{m,n}^{k,t+3}(j)$, and vary from one codeword to another. The normalized Doppler frequency is assumed to be $f_D T_s = 0.01$, where T_s is the symbol (LDC codeword) duration. During each Monte-Carlo run, the block size is set to 1526 information bits followed by 4 tails bits to terminate the trellis. The coded bits are passed through a random interleaver. The simulation results are averaged over random fading, noise, delays, and scrambling codes with minimum of 50 blocks of data transmitted and at least 100 bit errors generated.

In Fig. 3, we compare the performance of the iterative MUD algorithms at different iterations. The PIC-based soft demodulation scheme [8] is used for the V-BLAST system. Here, E_b is defined as received bit energy, and is normalized with the number of receive antennas. The single-user scheme is used in the beginning of the iterative process to obtain an initial estimate of data, which are needed for MUD at subsequent iterations. We notice that the V-BLAST system has to iterate 3 times before reaching convergence (excluding the single user detection stage), while the LDC system only needs to iterate 2 times to converge. The use of LDC leads to faster convergence for the iterative receiver, and also superior performance compared to the V-BLAST system. By comparison, the performance of the U-LDC code is not as good as that of the FDFR codes, especially at high SNR (the curves for the U-LDC code are not included to conserve space). In order to fully exploit the space diversity, FDFR codes should be used.

Their convergence behavior is further examined at $E_b/N_0 =$ 7 dB in Fig. 4 using the extrinsic information transfer (EXIT) chart which traces the evolution of the mutual information $I_i^M/I_o^M \in [0,1]$ between input/output LLR and $u'^k_{n,l}$ for multiuser detector; and the mutual information I_i^D/I_o^D between input/output LLR and $u^k_{n,l}$ for the Log-MAP decoder. Refer to [29] for detailed discussion of this analytical method and [30, 31] for its application to the analysis of iterative schemes in CDMA and MIMO systems. It should be noted that in our case, $I_i^M > 0$ at the starting point. This is due to the SUD stage applied in the beginning of the iterative process to yield an initial estimate of data, therefore, the a priori information is not zero when the MUD process starts. The output LLR of the detector I_o^M is forwarded to the decoder as input, i.e., $I_i^D = I_o^M$; the output LLR of the decoder I_o^D is fed back to the detector, i.e., $I_i^M = I_o^D$, and so on. As indicated by the transfer curves in Fig. 4, the output LLR I_o^M becomes more reliable (its value increases) as the input LLR I_i^M becomes more reliable in the detector. The iterative detection and decoding process is depicted by a staircase trace between the transfer curves of the detector and decoder. The trace shows that 2 (3) iterations of detection/decoding are needed for the LDC (V-BLAST) system to converge (reach the maximum I_o^D). This is in close agreement with the results presented in Fig. 3. The initial value of I_i^M is obtained through simulations for both LDC and V-BLAST systems.

In Figs. 5–7 , we compare the two systems with different channel codes and diversity orders. Two convolutional codes are tested: i) a weak code with rate $R_c = 1/2$, constraint length $L_c = 3$ and generator polynomials $(5,7)$; ii) a strong code with rate $R_c = 1/3$, constraint length $L_c = 9$ and generator polynomials $(575, 623, 727)$, which is an optimum distance spectrum code [32]. Comparing Fig. 5 with Fig. 6, one can see that the advantages of LDCs over VBLAST become smaller when a stronger code is used. For example, with the $(5,7)$ code, the LDC system outperforms the V-BLAST system by 1.0 dB upon reaching convergence at target BER $=10^{-4}$; whereas the difference is only 0.5 dB when the $(575, 623, 727)$ code is used. Obviously, it is more advantageous to apply LDC with a weak code. The single user bounds for the FDFR-LDC coded systems are shown in Figs. 3 and 5. They are obtained by the proposed scheme in a single user environment, no interference mitigation is needed in this case, and they give lower bounds on the best achievable performance by applying MUD technique. It can be seen that the performance of the proposed iterative MUD approach upon reaching convergence is very close to the single user bound, meaning that MAI can be effectively eliminated by proper design of iterative MUD schemes.

Fig. 7 shows the performance comparison between the two systems with the (5,7) code in a 2×4 antenna setup. Comparing to Fig. 5, it is obvious that the VBLAST system converges

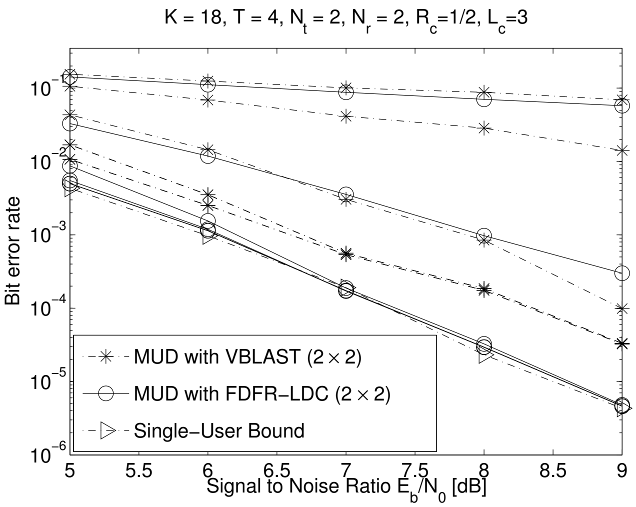

Fig. 5. Performance comparison with (5,7) convolutional code in 2Tx-2Rx systems. Curves with the same marker represent the performance of the same scheme at different iterations.

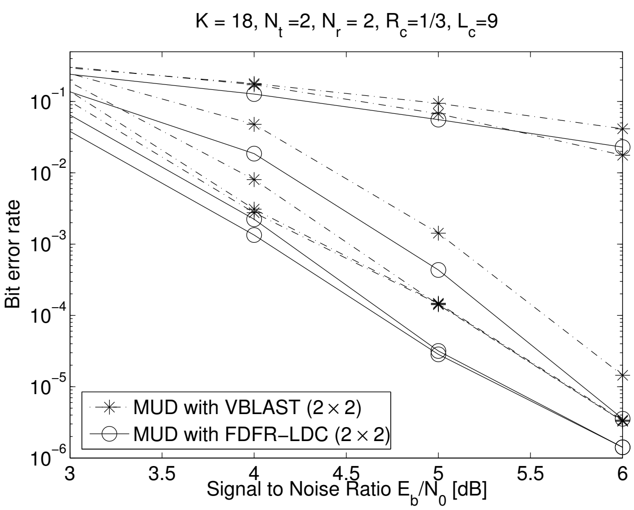

Fig. 6. Performance comparison with (575,623,727) convolutional code in 2Tx-2Rx systems.

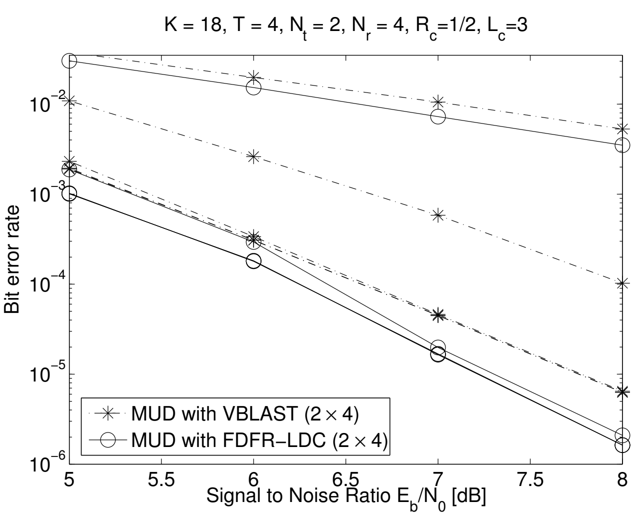

Fig. 7. Performance comparison with (5,7) convolutional code in 2Tx-4Rx systems.

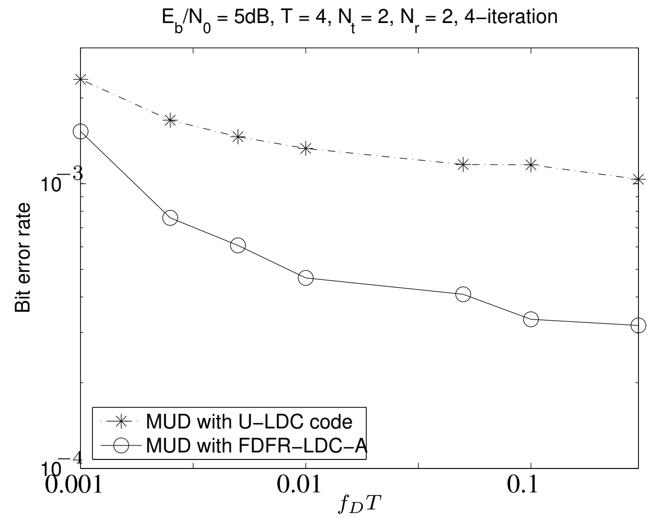

Fig. 8. BER performance vs. normalized Doppler frequency for the LDC coded MIMO CDMA system.

faster and the performance gap between the two systems becomes smaller as the number of receive antennas (spatial diversity order) increases.

Note that the proposed system does not require constant channel during the transmission of one LDC codeword, channel gains can vary from one time slot to another, i.e., $h_{m,n}^{k,t}(j) \neq h_{m,n}^{k,t+1}(j) \neq h_{m,n}^{k,t+2}(j) \neq h_{m,n}^{k,t+3}(j)$. This is in contrast to previous work on LDC for CDMA systems, which were designed for static channel over the whole LDC codeword. In Fig. 8, we examine the performance of the LDC coded system using the proposed iterative MUD approach in faster fading channels when the channel gains keep changing at each time slot within one LDC codeword. To this end, we re-define the normalized Doppler frequency as $f_D T$, where T is the duration of one time slot (channel use). The SNR is set to be $E_b/N_0 = 5$ dB. Slow power control is assumed so that the average received power is equal for all users. The BER curve is plotted at the 4th iteration when the system reaches convergence. Fig. 8 shows that the system performance improves as the Doppler frequency increases, which clearly indicates the time diversity obtained by the LDC codes, and the diversity gain is more obvious by applying the FDFR-LDC code than the U-LDC code. However, this time diversity is not exploited for slow fading channels as in the previous cases, where the normalized Doppler frequency is set to be $f_D T_s = 0.01$ and code level channel stationarity is assumed. It was shown in [8], that for non-LDC systems, the performance degrades as the normalized Doppler frequency increases.

The complexity of the iterative MUD approach for the LDC coded system and the VBLAST system is compared in Table II, which shows the required number of complex multiplications/divisions, and additions/subtractions for one user's one symbol estimate corresponding to the calculation of LLRs for $\log_2 Q$ code bits. One can see from the table that the proposed LDC system increases the complexity from $O(Q^2)$ to $O(Q^3)$ compared to the VBLAST system, mainly due to the matrix inverse operation (see (15)) at symbol rate in the LDC decoding process. However, the complexity increase is partly compensated by faster convergence achieved by the LDC system. The time diversity shown in Fig. 8 also justifies the use of LDC codes in fast fading channels.

TABLE II

COMPLEXITY FOR ONE USER'S ONE SYMBOL ESTIMATE AT ONE ITERATION FOR THE ITERATIVE MUD ALGORITHMS CONSIDERED.

operations	$\times/\div$	$+/-$
VBLAST system	$3Q^2N_c + 2QN_c$	$QN_c(K+1)/K + Q^2N_c - Q + \log_2 Q + 2$
LDC system	$8Q^3 + 6Q^2 + 4QN_c$	$8Q^3 - 3Q^2 + (4N_c + N_c/K - 1)Q + \log_2 Q$

VI. CONCLUSIONS

Iterative detection and decoding for orthogonally modulated and LDC coded MIMO DS-CDMA system has been investigated in this paper. We propose an integrated design of MUD, LDC decoding, symbol demodulation, and symbol-to-LLR mapping in order to reduce the complexity of the iterative receiver. Numerical results show that the choice of LDC codes is important for the system performance, and the use of full diversity, full rate LDC codes leads to high transmission bandwidth efficiency and significantly improved BER performance. Under the assumption of perfect CSI, it enables the system to approach the single-user bound even in heavily loaded systems. Compared to the V-BLAST system with the same transmission rate, the proposed LDC system shows superior performance and faster convergence. However, we have observed that the advantages of applying LDC become smaller when a strong channel code is used or the diversity order increases. Furthermore, by exploiting time diversity, LDCs also provide us a powerful means to combat impairment caused by rapid fading channels. The extension of the current work to frequency-selective channels by incorporating orthogonal frequency division multiplexing (OFDM) technique or using turbo equalization method is a future research topic for the authors.

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