Power Allocation for the Downlink of Nonregenerative Cooperative Multi-User MIMO Communication System

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***Abstract*— Amplify-and-forward (AF) is one of the most popular and simple approaches to transmit information over a cooperative multi-input multi-output (MIMO) relay channel. In this paper, we propose three novel power allocation methods for the downlink of cooperative multi-user MIMO AF system, which are designed to maximize the weighted sum-rate (WSR) of the cooperative system, by considering two different levels of channel state information (CSI). We design efficient precoding algorithms at the relay by assuming that either only the receive CSI or both receive and transmit CSI is available at the relay. Results show the performance improvement that our schemes can achieve in terms of sum-rate and WSR metrics.**

I. Introduction

Cooperative communication has recently attracted considerable research interests [1]–[6]. In cooperative communication, relays are employed for improving the coverage and enhancing the spectral efficiency. Relay node (RN) can be either utilized in a regenerative, e.g., decode and forward (DF), or in a nonregenerative way, e.g., amplify-and-forward (AF). In DF, the full decoding of the source message followed by the forwarding of the whole message to the destination node (DN) via the RN are performed. Whereas in AF, the RN amplifies and forwards the signal received from the source node (SN).

In cooperative single user multi-input multi-output (MIMO) scenario, the RN was first employed as a simple equal gain (EG) amplifier, i.e., original AF scheme [6]. However, it has recently been shown in [7]–[9] that it can also be utilized as a smart precoder for fine-tuning the power allocation over the relay channel and, thus, improving the spectral efficiency of the cooperative system. For the downlink (DL) of nonregenerative cooperative multi-user (MU) MIMO system, some methods have first been proposed in [10] and [11] to efficiently perform the precoding at the RN but only for the single antenna per user case. Recently in [12], a method for the MIMO case has been designed by assuming that the full channel state information (CSI) of the relay channel is available at the SN and that dirty paper coding (DPC) [13] is employed. In this paper, we develop three power allocation methods for the DL of nonregenerative cooperative MU MIMO system where all the nodes of the system have multiple antennas. Moreover, our methods are designed to maximize the weighted sum-rate (WSR), instead of the sum-rate, as it is the case in [10]–[12]. Contrarily to [12], we aim at designing an efficient precoder at the RN by considering two more realistic CSI assumptions, i.e., only the receive CSI or both receive and transmit CSI is available at the RN, and without relying on DPC at the SN.

Our novel power allocation methods are designed according to the DL cooperative MU MIMO system model, which is introduced in Section II. In Section III, we first introduce some existing methods that can be used for designing the precoder at the RN when only the SN-RN link CSI is available. We then present a novel method that uses statistical knowledge about the RN-DN links to improve WSR performance. We investigate in Section IV the case where SN-RN and RN-DN link CSI is available at the RN for performing the precoding and design two novel schemes for maximizing the WSR of the cooperative system. The performances of our three novel power allocation schemes in terms of sum-rate and WSR are presented in Section V. The results indicate that our scheme of Section III-B outperforms the other existing schemes when only receive CSI is available at the RN, but none of these schemes can mitigate MU interference. However, it can be mitigated when receive and transmit CSI is available at the RN, without employing DPC at the SN, by utilizing our two novel schemes of Section IV. Moreover, these two schemes can also be used to improve WSR performance. Finally, conclusions are drawn in Section VI.

II. DL of the Cooperative MU MIMO System

We consider a cooperative MU MIMO system that is composed of K+2 nodes, where a SN, which is equipped with n antennas, cooperates with a nonregenerative RN, which is equipped with q antennas, to transmit data to K DNs, which are equipped with r_k antennas, as it is depicted in Fig. 1.

For the simplicity of the introduction, we assume a half duplex relaying scenario with two equal duration phases as in [7] and [8], where in the first phase the SN broadcasts the signal $\mathbf{x} = \sum_{k=1}^{K} \mathbf{R}_k \mathbf{s}_k$ to the DN and RN, and in the second phase only the RN transmits to the DN. Note that $\mathbf{R}_k \in \mathbb{C}^{n\times n}$ is the k-th user precoding matrix at the SN. The signal $\mathbf{x}$ is received by each DN as $\mathbf{y}_{0,k} = \mathbf{H}_{0,k}\mathbf{x} + \mathbf{n}_{0,k}$ and by the RN as $\mathbf{y}_1 = \mathbf{H}_1\mathbf{x} + \mathbf{n}_1$ at the end of the first phase, where $\mathbf{H}_{0,k} \in \mathbb{C}^{r_k\times n}$ as well as $\mathbf{H}_1 \in \mathbb{C}^{q\times n}$ characterize the MIMO channel of each SN-DN link and of the SN-RN link, respectively. During the second phase, the signal $\mathbf{y}_1$ is amplified by using the precoding matrix $\mathbf{G} \in \mathbb{C}^{q\times q}$, is then transmitted towards the DNs and is

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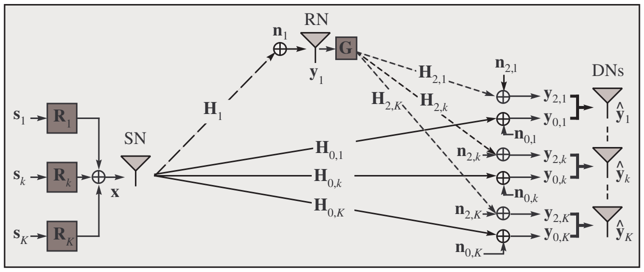

Fig. 1. DL of nonregenerative cooperative MU MIMO system.

received as $\mathbf{y}_{2,k} = \mathbf{H}_{2,k}\mathbf{G}\mathbf{y}_1 + \mathbf{n}_{2,k}$ by each DN, where $\mathbf{H}_{2,k} \in \mathbb{C}^{r_k \times q}$ characterizes the MIMO channel of each RN-DN link. Moreover, each of the channel matrices $\mathbf{H}_{0,k}$, $\mathbf{H}_1$ $\mathbf{H}_{2,k}$ is a random matrix having independent and identically distributed (i.i.d.) complex Gaussian entries with zero-mean and unit variance. Furthermore, $\mathbf{n}_{0,k} \in \mathbb{C}^{r_k \times 1}$, $\mathbf{n}_1 \in \mathbb{C}^{q \times 1}$ and $\mathbf{n}_{2,k} \in \mathbb{C}^{r_k \times 1}$ are vectors of independent zero-mean complex Gaussian noise entries with a variance of σ^2. The system model for the DL of the nonregenerative cooperative MU MIMO communication system can be summarized as

$$\widehat{\mathbf{y}} = \begin{bmatrix} \mathbf{y}_0 \\ \mathbf{y}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{H}_0 \\ \mathbf{H}_2\mathbf{G}\mathbf{H}_1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{I}_r & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_2\mathbf{G} & \mathbf{I}_r \end{bmatrix} \begin{bmatrix} \mathbf{n}_0 \\ \mathbf{n}_1 \\ \mathbf{n}_2 \end{bmatrix}, \quad (1)$$

where $\widehat{\mathbf{y}} = [\widehat{\mathbf{y}}_1^\dagger, \ldots, \widehat{\mathbf{y}}_K^\dagger]^\dagger \in \mathbb{C}^{2r \times 1}$, $\mathbf{y}_i = [\mathbf{y}_{i,1}^\dagger, \ldots, \mathbf{y}_{i,K}^\dagger]^\dagger \in \mathbb{C}^{r \times 1}$, $\mathbf{n}_i = [\mathbf{n}_{i,1}^\dagger, \ldots, \mathbf{n}_{i,K}^\dagger]^\dagger \in \mathbb{C}^{r \times 1}$, $\mathbf{H}_2 = [\mathbf{H}_{2,1}^\dagger, \ldots, \mathbf{H}_{2,K}^\dagger]^\dagger \in \mathbb{C}^{r \times q}$, $\mathbf{H}_0 = [\mathbf{H}_{0,1}^\dagger, \ldots, \mathbf{H}_{0,K}^\dagger]^\dagger \in \mathbb{C}^{r \times n}$, $\mathbf{I}_r$ is a $r \times r$ identity matrix, $r = \sum_{k=1}^K r_k$, and $(.)^\dagger$ denotes the conjugate transpose operator. The cooperative mutual information (MI) of each user can then be expressed as [14]

$$I(\widehat{\mathbf{y}}_k; \mathbf{s}_k) = \frac{1}{2}\log_2 \left| \mathbf{I}_{2r_k} + \mathbf{H}_k \mathbf{R}_k \mathbf{R}_k^\dagger \mathbf{H}_k^\dagger \mathbf{R}_{\mathbf{n},k}^{-1} \right|, \quad (2)$$

where $\mathbf{H}_k = \left[\mathbf{H}_{0,k}^\dagger \quad (\mathbf{H}_{2,k}\mathbf{G}\mathbf{H}_1)^\dagger \right]^\dagger$,

$$\mathbf{R}_{\mathbf{n},k} = \begin{bmatrix} \mathbf{R}_{\mathbf{n}_0,k} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{2,k}\mathbf{G}\mathbf{R}_{\mathbf{n}_1,k}\mathbf{G}^\dagger\mathbf{H}_{2,k}^\dagger + \sigma^2\mathbf{I}_{r_k} \end{bmatrix},$$

the factor 1/2 accounts for the two-phase transmission, $\mathbf{R}_{\mathbf{n}_0,k} = \sigma^2\mathbf{I}_{r_k} + \mathbf{H}_{0,k}\left(\mathbf{R}_\mathbf{x} - \mathbf{R}_k\mathbf{R}_k^\dagger\right)\mathbf{H}_{0,k}^\dagger$ and $\mathbf{R}_{\mathbf{n}_1,k} = \sigma^2\mathbf{I}_q + \mathbf{H}_1\left(\mathbf{R}_\mathbf{x} - \mathbf{R}_k\mathbf{R}_k^\dagger\right)\mathbf{H}_1^\dagger$ are noise plus residual interference covariance matrices, where $\mathbf{R}_\mathbf{x} = \sum_{k=1}^K \mathbf{R}_k\mathbf{R}_k^\dagger$. The direct link and relay link MI of each user, i.e., $I(\mathbf{y}_{0,k}; \mathbf{s}_k)$ and $I(\mathbf{y}_{2,k}; \mathbf{s}_k)$, can also be computed by employing (2) for $\mathbf{H}_k = \mathbf{H}_{0,k}$, $\mathbf{R}_{\mathbf{n},k} = \mathbf{R}_{\mathbf{n}_0,k}$ and $\mathbf{H}_k = \mathbf{H}_{2,k}\mathbf{G}\mathbf{H}_1$, $\mathbf{R}_{\mathbf{n},k} = \mathbf{H}_{2,k}\mathbf{G}\mathbf{R}_{\mathbf{n}_1,k}\mathbf{G}^\dagger\mathbf{H}_{2,k}^\dagger + \sigma^2\mathbf{I}_{r_k}$, respectively, such that

$$\begin{aligned} I(\mathbf{y}_{0,k}; \mathbf{s}_k) &= \frac{1}{2}\log_2 \left| \mathbf{I}_{r_k} + \mathbf{H}_{0,k}\mathbf{R}_k\mathbf{R}_k^\dagger\mathbf{H}_{0,k}^\dagger\mathbf{R}_{\mathbf{n}_0,k}^{-1} \right|, \\ I(\mathbf{y}_{2,k}; \mathbf{s}_k) &= \frac{1}{2}\log_2 \Big| \mathbf{I}_{r_k} + \mathbf{H}_{2,k}\mathbf{G}\mathbf{H}_1\mathbf{R}_k\mathbf{R}_k^\dagger\mathbf{H}_1^\dagger\mathbf{G}^\dagger\mathbf{H}_{2,k}^\dagger \\ &\quad \times (\mathbf{H}_{2,k}\mathbf{G}\mathbf{R}_{\mathbf{n}_1,k}\mathbf{G}^\dagger\mathbf{H}_{2,k}^\dagger + \sigma^2\mathbf{I}_{r_k})^{-1} \Big|. \end{aligned} \quad (3)$$

In addition, $I(\widehat{\mathbf{y}}_k; \mathbf{s}_k)$ can be simplified and re-expressed as

$$\begin{aligned} I(\widehat{\mathbf{y}}_k; \mathbf{x}_k) = & I(\mathbf{y}_{0,k}; \mathbf{x}_k) + \frac{1}{2}\log_2 \Big| \mathbf{I}_{r_k} + \mathbf{H}_{2,k}\mathbf{G}\mathbf{H}_1\mathbf{R}_k\mathbf{A}_k^{-1}\mathbf{R}_k^\dagger \\ & \times \mathbf{H}_1^\dagger\mathbf{G}^\dagger\mathbf{H}_{2,k}^\dagger(\mathbf{H}_{2,k}\mathbf{G}\mathbf{R}_{\mathbf{n}_1,k}\mathbf{G}^\dagger\mathbf{H}_{2,k}^\dagger + \sigma^2\mathbf{I}_{r_k})^{-1} \Big| \end{aligned} \quad (4)$$

by using [15], where $\mathbf{A}_k = \mathbf{I}_n + \mathbf{R}_k^\dagger\mathbf{H}_{0,k}^\dagger\mathbf{R}_{\mathbf{n}_0,k}^{-1}\mathbf{H}_{0,k}\mathbf{R}_k \succeq \mathbf{I}_n$ is a positive definite matrix. Hence, $I(\widehat{\mathbf{y}}_k; \mathbf{s}_k) \leq I(\mathbf{y}_{0,k}; \mathbf{s}_k) + I(\mathbf{y}_{2,k}; \mathbf{s}_k)$ according to (3) and (4). Moreover, it can easily be proved that $I(\widehat{\mathbf{y}}_k; \mathbf{s}_k) \geq \min\{I(\mathbf{y}_{0,k}; \mathbf{s}_k), I(\mathbf{y}_{2,k}; \mathbf{s}_k)\}$. Thus, $\min\{I(\mathbf{y}_{0,k}; \mathbf{s}_k), I(\mathbf{y}_{2,k}; \mathbf{s}_k)\} \leq I(\widehat{\mathbf{y}}_k; \mathbf{s}_k) \leq I(\mathbf{y}_{0,k}; \mathbf{s}_k) + I(\mathbf{y}_{2,k}; \mathbf{s}_k)$ and $I(\widehat{\mathbf{y}}_k; \mathbf{s}_k)$ can be increased by maximising $I(\mathbf{y}_{2,k}; \mathbf{s}_k)$, or equivalently by optimizing $\mathbf{G}$ at the RN, as it has recently been shown in [8] for the single user case.

In the MU case, instead of maximising independently the MI of each user, we aim at finding a $\mathbf{G}$ matrix that maximizes the weighted sum of the users' MI. The relay link MI that can be achieved by the weighted sum of the users is given according to (3) by

$$\widehat{\Sigma}_{\mathbf{y}_2} = \frac{1}{2}\sum_{k=1}^K w_k \log_2 \left| \frac{\sigma^2\mathbf{I}_{r_k} + \mathbf{H}_{2,k}\mathbf{G}\mathbf{R}_{\mathbf{y}_1}\mathbf{G}^\dagger\mathbf{H}_{2,k}^\dagger}{\sigma^2\mathbf{I}_{r_k} + \mathbf{H}_{2,k}\mathbf{G}\mathbf{R}_{\mathbf{n}_1,k}\mathbf{G}^\dagger\mathbf{H}_{2,k}^\dagger} \right|, \quad (5)$$

where w_k is the k-th weight, $w_k \geq 0, \forall k \in [1, \ldots, K]$ and $\mathbf{R}_{\mathbf{y}_1} = \mathrm{E}\left\{\mathbf{y}_1\mathbf{y}_1^\dagger\right\} = \sigma^2\mathbf{I}_q + \mathbf{H}_1\mathbf{R}_\mathbf{x}\mathbf{H}_1^\dagger$ is the transmit covariance matrix. The problem of maximizing the weighted sum MI, or WSR, under the constraint that the transmit power at the RN should not exceed P_2 is such that

$$\max_{\mathbf{G}} \widehat{\Sigma}_{\mathbf{y}_2} \text{ s.t. } \mathbf{G} \succeq 0; \mathrm{tr}\left(\mathbf{G}\mathbf{R}_{\mathbf{y}_1}\mathbf{G}^\dagger\right) \leq P_2, \quad (6)$$

where $\mathrm{tr}\,(.)$ denotes the trace of a matrix and P_2 is the total transmit power of the RN. Note that the sum MI $\Sigma_{\mathbf{y}_2}$, or sum-rate, is a specific case of the WSR, such that $\Sigma_{\mathbf{y}_2} = \widehat{\Sigma}_{\mathbf{y}_2}$ when $w_k = 1$, $\forall k \in [1, \ldots, K]$. In the following, we design three methods for solving the problem in (5) when only receive CSI and both receive and transmit CSI is available at the RN, but without relying on DPC at the SN. In the rest of the paper, we consider that $\sigma = 1$ and $\mathrm{tr}\,(\mathbf{R}_\mathbf{x}) \leq P_1$, where P_1 is the total transmit power of the SN.

III. Relay receive CSI only power allocation

In this section, schemes that rely only on the knowledge of $\mathbf{R}_{\mathbf{y}_1}$ or $\mathbf{H}_1$ at the RN for designing the precoding matrix $\mathbf{G}$ are considered.

A. Traditional AF power allocation methods

In the original single user AF EG approach, the precoding matrix $\mathbf{G}$ is designed such that the power at the RN is evenly distributed amongst the available eigenmodes of the RN-DN link and such that the total transmit power of the RN is constrained as

$$\mathrm{P_c} : \mathrm{E}\{\|\mathbf{G}\mathbf{y}_1\|_F^2\} \leq P_2 \Leftrightarrow \mathrm{P_c} : \mathrm{tr}\left(\mathbf{G}\mathbf{R}_{\mathbf{y}_1}\mathbf{G}^\dagger\right) \leq P_2, \quad (7)$$

where $\|.\|_F^2$ denote the Frobenius norm. In the MU case, the same constraint holds and the $\mathbf{G}$ matrix can be expressed as follows

$$\mathbf{G} = \mathbf{J}\sqrt{P_2/\,\mathrm{tr}\left(\mathbf{J}\mathbf{R}_{\mathbf{y}_1}\mathbf{J}^\dagger\right)}, \quad (8)$$

with $\mathbf{J} = \mathbf{I}_q$ when an AF EG approach is followed. Note that the AF EG approach does not require the explicit knowledge of $\mathbf{H}_1$, but only of $\mathbf{R}_{\mathbf{y}_1}$. Lately in [7], other AF based approaches, which are denoted matched filter based relaying (MFR) and minimum mean square error filtering (MMSEF), have been proposed for the single user case. They can also be applied for the MU case and their $\mathbf{G}$ matrix formulations are the same as in (8), but where $\mathbf{J} = \mathbf{H}_1^\dagger$ and $\mathbf{J} = \mathbf{R}_{\mathbf{x}}\mathbf{H}_1^\dagger\mathbf{R}_{\mathbf{y}_1}^{-1}$, respectively.

B. Power allocation based on statistical knowledge about $\mathbf{H}_{2,k}$

We assume here that the CSI of the various RN-DN links is not available at the RN, but that at least statistical knowledge is known about these links. Thus, instead of finding a $\mathbf{G}$ that maximizes $\widehat{\Sigma}_{\mathbf{y}_2}$, we look for a $\mathbf{G}$ that maximizes the expectation of $\widehat{\Sigma}_{\mathbf{y}_2}$ over the various $\mathbf{H}_{2,k}$ channels. We refer this scheme as AF-SKRDL, i.e., AF-statistical knowledge of the relay-destination links.

Let $\overline{\mathbf{H}} \in \mathbb{C}^{a\times b}$ be a random matrix having i.i.d. complex Gaussian entries with zero-mean and unit variance and $\mathbf{\Delta} = \mathrm{diag}(\mathbf{d}_b)$ be a $b\times b$ diagonal matrix with $\mathbf{d}_b = \{\delta_1, \delta_2, \ldots, \delta_b\}$. The expectation of $\log_2\left|\mathbf{I}_a + \overline{\mathbf{H}}\mathbf{\Delta}\overline{\mathbf{H}}^\dagger\right|$ over $\overline{\mathbf{H}}$ is equivalent to the expression of the MI of the MIMO semi-correlated Rayleigh fading channel that is given in [16]. However, the complexity of the formulation of this expression makes it impractical for power allocation purpose. On the other hand, we have recently shown in [17] that $\mathrm{E}_{\overline{\mathbf{H}}}\left\{\log_2\left|\mathbf{I}_a + \overline{\mathbf{H}}\mathbf{\Delta}\overline{\mathbf{H}}^\dagger\right|\right\}$ is asymptotically equivalent to

$$\chi_a(\mathbf{d}_b) = \frac{1}{\ln(2)}\left[-a\ln\left(\frac{d_{0,a}(\mathbf{d}_b)}{a}\right) + \sum_{i=1}^{b}\ln(d_{0,a}(\mathbf{d}_b)\delta_i + 1)\right.$$
$$\left. -b + \sum_{i=1}^{b}\frac{1}{d_{0,a}(\mathbf{d}_b)\delta_i + 1}\right], \tag{9}$$

where $d_{0,a}(\mathbf{d}_b)$ is the largest nonnegative root of the polynomial given by $P_a(d, \mathbf{d}_b) = (d-a)\prod_{i=1}^{b}\left(\frac{1}{\delta_i} + d\right) + d\sum_{i=1}^{b}\prod_{j=1 j\neq i}^{b}\left(\frac{1}{\delta_j} + d\right)$. Notice that (9) and (9) in [17] are similar, when inserting $n = 1$, $m = b$, $\alpha = 1$, $\beta = a$, $\omega = 1$ and $\upsilon_i = 1/\delta_i$ in (9) of [17].

In order to apply the result of [17] in (5), we need $\mathbf{G}\mathbf{R}_{\mathbf{y}_1}\mathbf{G}^\dagger$ and $\mathbf{G}\mathbf{R}_{\mathbf{n}_1,k}\mathbf{G}^\dagger$ to be diagonal matrices for any k values. In the case that the channel $\mathbf{H}_1$ is known, $\mathbf{H}_1$ can be decomposed via singular valued decomposition (SVD) as $\mathbf{H}_1 = \mathbf{U}\widehat{\mathbf{\Lambda}}_1^{\frac{1}{2}}\mathbf{V}^\dagger$ where $\mathbf{U} \in \mathbb{C}^{q\times q}$ and $\mathbf{V} \in \mathbb{C}^{n\times n}$ are unitary matrices, and $\widehat{\mathbf{\Lambda}}_1$ is a $q\times n$ rectangular diagonal matrix. Moreover, $\mathbf{\Lambda}_1 = \widehat{\mathbf{\Lambda}}_1^{\frac{1}{2}}\widehat{\mathbf{\Lambda}}_1^{\frac{1}{2}\dagger}$ is a $q\times q$ diagonal matrix with diagonal elements $\lambda_{1,i} \in \mathbb{R}_+$. Then, we can define $\mathbf{G} = \widetilde{\mathbf{G}}\mathbf{U}^\dagger$ and $\mathbf{R}_k = \widetilde{\mathbf{R}}_k\mathbf{V}$, where $\widetilde{\mathbf{G}} = \mathrm{diag}(\sqrt{g_1}, \sqrt{g_2}, \ldots, \sqrt{g_q})$ and $\widetilde{\mathbf{R}}_k = \mathrm{diag}(\sqrt{p_{k,1}}, \sqrt{p_{k,2}}, \ldots, \sqrt{p_{k,n}})$ are a $q\times q$ and a $n\times n$ diagonal matrices, respectively. Using these $\mathbf{G}$ and $\mathbf{R}_k$ matrices, it implies that $\mathbf{G}\mathbf{R}_{\mathbf{y}_1}\mathbf{G}^\dagger = \mathrm{diag}(\mathbf{g}\cdot\mathbf{z}) = \mathrm{diag}(g_1(1 + \lambda_{1,1}\sum_{i=1}^{K}p_{i,1}), \ldots, g_q(1 + \lambda_{1,q}\sum_{i=1}^{K}p_{i,q}))$ and $\mathbf{G}\mathbf{R}_{\mathbf{n}_1,k}\mathbf{G}^\dagger = \mathrm{diag}(\mathbf{g}\cdot\overline{\mathbf{z}}_{\mathbf{k}}) = \mathrm{diag}(g_1(1 + \lambda_{1,1}[\sum_{i=1}^{K}p_{i,1} - p_{k,1}]), \ldots, g_q(1 + \lambda_{1,q}[\sum_{i=1}^{K}p_{i,q} - p_{k,q}]))$ are diagonal matrices for any k values, where $\mathbf{g}\cdot\mathbf{z}$ is the entrywise product between vectors $\mathbf{g}$ and $\mathbf{z}$. Consequently, $\mathrm{E}_{\mathbf{H}_2}\left\{\widehat{\Sigma}_{\mathbf{y}_2}\right\}$ can be asymptotically approximated by using (9) and the problem of maximizing $\mathrm{E}_{\mathbf{H}_2}\left\{\widehat{\Sigma}_{\mathbf{y}_2}\right\}$ can be defined as

$$\max_{\mathbf{g}} f(\mathbf{g}) = \sum_{k=1}^{K}\frac{w_k}{2\ln(2)}\left[\sum_{i=1}^{q}\ln\left(\frac{d_{0,r_k}(\mathbf{g}\cdot\mathbf{z})g_i z_i + 1}{d_{0,r_k}(\mathbf{g}\cdot\overline{\mathbf{z}}_k)g_i\overline{z}_{k,i} + 1}\right)\right.$$
$$+\sum_{i=1}^{q}\frac{1}{d_{0,r_k}(\mathbf{g}\cdot\mathbf{z})g_i z_i + 1} - \sum_{i=1}^{q}\frac{1}{d_{0,r_k}(\mathbf{g}\cdot\overline{\mathbf{z}}_k)g_i\overline{z}_{k,i} + 1}$$
$$\left.-r_k\ln\left(\frac{d_{0,r_k}(\mathbf{g}\cdot\mathbf{z})}{d_{0,r_k}(\mathbf{g}\cdot\overline{\mathbf{z}}_k)}\right)\right] \text{ s.t. } g_i \geq 0; \mathrm{P_c}: \sum_{i=1}^{q}g_i z_i \leq P_2. \tag{10}$$

In order to solve the problem in (10), $d_{0,a}(\mathbf{d}_b)$ must be evaluated for any vector $\mathbf{d}_q$ of length b and any value of a. Two methods for evaluating $d_{0,a}(\mathbf{d}_b)$ can be found in [18] along with an algorithm to solve any problem similar to (10).

IV. Relay receive and transmit CSI power allocation

In this section, the case where $\mathbf{H}_1$ and all $\mathbf{H}_{2,k}$ are known at the RN is studied. We introduce two novel methods for maximizing $\widehat{\Sigma}_{\mathbf{y}_2}$ in (5); the first one relies on channel block-diagonalization (CBD) [19] for eliminating the other user interference without requiring DPC at the SN; the second one involves a constrained gradient search (CGS) on the $\mathbf{G}$ matrix itself.

A. Channel block-diagonalization based method

The aim of the block-diagonalization process is to design for each user k a matrix $\mathbf{W}_k \in \mathbb{C}^{q\times l_r}$, which is part of the RN precoding matrix $\mathbf{G}$, such that $\mathbf{H}_{2,j}\mathbf{W}_k = \mathbf{0}$ for $j \neq k$. Let $\overline{\mathbf{H}}_{2,k} = [\mathbf{H}_{2,1}^\dagger, \ldots, \mathbf{H}_{2,k-1}^\dagger, \mathbf{H}_{2,k+1}^\dagger, \ldots, \mathbf{H}_{2,K}^\dagger]^\dagger$ be the complementary channel of user k, $\mathbf{Y}_k$ be a matrix of rank ρ_k that contains the q right-singular vectors of $\overline{\mathbf{H}}_{2,k}$ and $\mathbf{Y}_{k,[\rho_k+1:q]}$ contains the last $q-\rho_k$ columns of $\mathbf{Y}_k$. In addition, let $\mathbf{Z}_k$ be a matrix that contains the $q-\rho_k$ right-singular vectors of $\mathbf{H}_{2,k}\mathbf{Y}_{k,[\rho_k+1:q]}$ and $\mathbf{Z}_{k,[1:l_k]}$ contains the first l_k columns of $\mathbf{Z}_k$ such that $\sum_{k=1}^{K}l_k \leq q$. Then, we can define $\mathbf{W}_k = \mathbf{Y}_{k,[\rho_k+1:q]}\mathbf{Z}_{k,[1:l_k]}$ to ensure that $\mathbf{H}_{2,j}\mathbf{W}_k = \mathbf{0}$ for $j \neq k$. Moreover, we define the $\mathbf{G}$ matrix as

$$\mathbf{G} = \mathbf{W}\widetilde{\mathbf{G}}\mathbf{U}^\dagger, \tag{11}$$

where $\mathbf{W} = [\mathbf{W}_1, \mathbf{W}_2, \ldots, \mathbf{W}_K]$. Consequently, (5) can be re-expressed as $\widehat{\Sigma}_{\mathbf{y}_2}(\mathbf{g}) =$

$$\frac{1}{2}\sum_{k=1}^{K}w_k\prod_{i=1}^{r_k}\log_2\left(\frac{1 + g_u\omega_u\left(1 + \lambda_u\sum_{j=1}^{K}p_{j,u}\right)}{1 + g_u\omega_u\left(1 + \lambda_u\left[\sum_{j=1}^{K}p_{j,u} - p_{k,u}\right]\right)}\right) \tag{12}$$

by applying (11) into (5) and assuming that $r_k = l_k$ and $\sum_{k=1}^{K}r_k = q$. Note that in (12), ω_u is the u-th nonnegative eigenvalues of $\mathbf{H}_{2,k}\mathbf{Y}_{k,[\rho_k+1:q]}\mathbf{Y}_{k,[\rho_k+1:q]}^\dagger\mathbf{H}_{2,k}^\dagger$ that are sorted in descending order, $u = \alpha_k + i$ and $\alpha_k = \sum_{j=1}^{k-1}r_j$. This expression can be further simplified by considering that each user can only transmit over r_k eigenmodes at the SN such that

$$\begin{cases} p_{k,i} \neq 0, \text{ if } i \in [1+\alpha_k, \ldots, \alpha_{k+1}] \\ p_{k,i} = 0, \text{ else} \end{cases}, \tag{13}$$

and then (12) can finally be re-expressed as

$$\widehat{\Sigma}_{\mathbf{y}_2}(\mathbf{g}) = \frac{1}{2}\sum_{k=1}^{K} w_k \prod_{i=1}^{r_k} \log_2\left(\frac{1+g_u\omega_u(1+\lambda_u p_{k,u})}{1+g_u\omega_u}\right), \tag{14}$$

which is equivalent to the expression of the relay link MI in the single user case [8] for $K=1$, $r_1=r$ and $w_k=1$. Therefore, the MU MIMO relay channel has been transformed into K independent MIMO relay channels by using the knowledge on $\mathbf{H}_2$ at the RN for block-diagonalizing the MU relay channel and by adequately defining each user precoding matrix $\mathbf{R}_k$ at the SN. Inserting (14) into (6), The optimization problem simplifies as

$$\max_{\mathbf{g}} \widehat{\Sigma}_{\mathbf{y}_2}(\mathbf{g}) \text{ s.t. } g_i \geq 0; \mathrm{P_c}: \sum_{k=1}^{K}\sum_{i=1}^{r_k} g_u\omega_u(1+\lambda_u p_{k,u}) \leq P_2. \tag{15}$$

This problem can be directly solved by slightly modifying the low-complexity algorithm for the single user case in [7] and [8]. Note that this technique can be extended for the case where $r > q$ by resorting to eigenmode selection per user or user selection [19].

B. Constrained gradient search based method

The $\mathbf{G}$ matrix structure in (11) is optimal in the single user case, as it has been proved in [8]. However, it has recently been reported in [12] that it is no more the case in the MU context. In the two previous methods that we proposed in Section III-B and IV-A, $\mathbf{G}$ is first decomposed into the product of a diagonal matrix with some unitary matrices. Then, the problem is to find the vector $\mathbf{g}$ that maximizes a particular criterion. Instead of sticking to a particular $\mathbf{G}$ matrix structure, we can use a CGS algorithm for finding a $\mathbf{G}$ matrix that maximizes (5). The gradient of $\widehat{\Sigma}_{\mathbf{y}_2}$ is given by

$$\frac{\partial\widehat{\Sigma}_{\mathbf{y}_2}}{\partial\mathbf{G}} = \sum_{k=1}^{K}\frac{w_k}{\ln(2)}\Big[\mathbf{H}_{2,k}^{\dagger}(\mathbf{I}_{r_k}+\mathbf{H}_{2,k}\mathbf{G}\mathbf{R}_{\mathbf{y}_1}\mathbf{G}^{\dagger}\mathbf{H}_{2,k}^{\dagger})^{-1}\mathbf{H}_{2,k} \times\mathbf{G}\mathbf{R}_{\mathbf{y}_1} - \mathbf{H}_{2,k}^{\dagger}(\mathbf{I}_{r_k}+\mathbf{H}_{2,k}\mathbf{G}\mathbf{R}_{\mathbf{n}_1,k}\mathbf{G}^{\dagger}\mathbf{H}_{2,k}^{\dagger})^{-1}\mathbf{H}_{2,k}\mathbf{G}\mathbf{R}_{\mathbf{n}_1,k}\Big] \tag{16}$$

since $\partial\ln\left|\mathbf{I}+\mathbf{X}\mathbf{Y}\mathbf{X}^{\dagger}\right|/\partial\mathbf{Y} = 2\left(\mathbf{I}+\mathbf{X}\mathbf{Y}\mathbf{X}^{\dagger}\right)^{-1}\mathbf{X}\mathbf{Y}$ if $\mathbf{Y}$ is an Hermitian matrix. The algorithm of this method is summarized in **Algorithm 1**.

This algorithm has a greater computational complexity than the previous method in Section IV-B, especially as the size of the matrices increase, since it deals with matrices instead of vectors. In comparison with a standard gradient search algorithm, extra computation is needed to ensure that the searched $\mathbf{G}$ matrices are always within the search space, which slightly increases the complexity. Notice that the trade-off between accuracy and complexity can be fine-tuned by appropriately modifying the value of ϵ.

Algorithm 1 : AF-CGSDL

```
Input: P_2, ε, R_y1, H_2,k, R_n1,k, r_k and w_k, ∀k ∈ [1, K];
Set m = 1, t = 2 and G = G_init;
Compute f = Σ̂_y2(G) in (5);
repeat
   Evaluate δG = (∂Σ̂_y2/∂G) R_y1^{-1} by using (16);
   Set δG = δG sqrt(P_2 / tr(δG R_y1 δG†));
   Set Ĝ = G + t^{-1} δG;
   Set Ĝ = Ĝ sqrt(P_2 / tr(Ĝ R_y1 Ĝ†));
   Compute f̂ = Σ̂_y2(G = Ĝ)
   Set a = f̂ − f;
   Set m = m + 1;
   if (a < ε) then
      Set t = t + 1;
   else
      Set G = Ĝ and f = f̂;
   end if
until (|a| < ε or m > 1/ε)
Output: G.
```

V. Results

Our novel AF power allocation schemes, i.e., AF-SKRDL, AF-CBD and AF-CGSDL which have been introduced in Sections III-B, IV-A and IV-B, respectively, are compared against each others and the schemes in Sections III-A in terms of sum-rate and WSR performances.

In our simulations, we define SNR$_1$(dB)$=\log_{10}(P_1)$ as the SNR of the SN-RN link and SNR$_2$(dB)$=\log_{10}(P_2)$ as the SNR of the all RN-DN links. In addition, we set SNR$_0$= 0 (dB), i.e., the SNR of all the SN-DN links is zero dB. A single-tap i.i.d. Rayleigh fading channel is assumed between the various links, SN-RN, SN-DNs and RN-DNs. We considered 5×10^3 realisations of each channel for our sum-rate and WSR simulations. Note that the parameter ϵ, which is used to fine-tune the accuracy of **Algorithm 1**, has been set to $\epsilon = 10^{-4}$ and $\mathbf{G}_{\text{init}} = \mathbf{G}$ in (8) for $\mathbf{J} = \mathbf{I}_q$.

As explained in Section III-B, we consider that $\mathbf{R}_k = \widetilde{\mathbf{R}}_k\mathbf{V}$, where $\widetilde{\mathbf{R}}_k = \text{diag}(\sqrt{p_{k,1}}, \sqrt{p_{k,2}}, \ldots, \sqrt{p_{k,n}})$ and the $p_{k,i}$ for each user k follows (13). Two types of power allocation are considered at the SN. First in SN-EG, $p_{k,i} = P_1/n$ is set for any $k \in [1,\ldots,K]$ and $i \in [1+\alpha_k,\ldots,\alpha_{k+1}]$. Secondly in SN-water filling (WF), the CSI of the SN-RN is assumed to be known at the SN and $p_{k,i}$ is obtained by WF according to the eigenmodes of $\mathbf{H}_1$. Let $\overline{\lambda}_1$ be the vectors of elements $\lambda_{1,i}$ that are sorted in descending order such that $\overline{\lambda}_{1,1} \geq \overline{\lambda}_{1,2} \geq \ldots \geq \overline{\lambda}_{1,q}$. Moreover, let $\mathbf{ind} = [1, K+1, \ldots, (r_1-1)K+1, 2, K+2, \ldots, (r_2-1)K+2, \ldots, K, 2K, \ldots, r_kK]$ be a set of indices, then we set $\lambda_{1,[1:q]} = \overline{\lambda}_{1,\mathbf{ind}}$ in our simulations.

In Fig. 2, we compare the sum relay MI of our AF-SKRDL scheme against the AF-EG, AF-MFR and AF-MMSEF schemes for SNR$_2$ = 10 dB, $K=2$, $n=q=4$, $w_k=1, r_k=2, \forall k \in [1,\ldots,K]$. The direct transmission (DT) performance is also displayed to show that cooperative communication is efficient in the considered SNR settings. The curves plotted with black and light grey lines have been obtained by using SN-EG and SN-WF power allocations at the

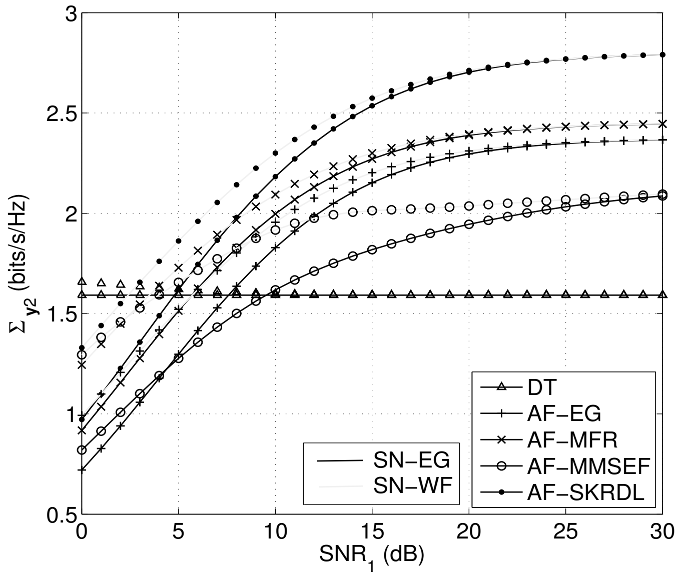

Fig. 2. Sum relay MI performance of various power allocation methods for $\text{SNR}_2 = 10$ dB, $K = 2$, $n = q = 4$, $w_k = 1, r_k = 2, \forall k \in [1, \ldots, K]$.

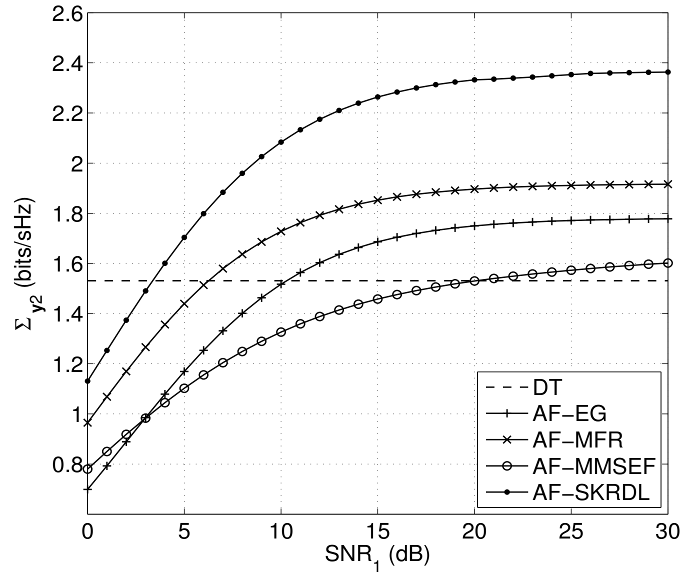

Fig. 3. Sum relay MI performance of various power allocation methods for $\text{SNR}_2 = 10$ dB, $K = 4$, $n = q = 8$, $w_k = 1, r_k = 2, \forall k \in [1, \ldots, K]$.

SN, respectively. The results clearly show that our AF-SKRDL scheme outperforms the other receive only CSI schemes. It indicates that the incorporation of statistical knowledge about the RN-DN channel in the precoding optimization process at the RN is beneficial for the performance of the system. This graph also depicts the performance enhancement that is obtained for each scheme, especially at low SNRs, when relying on WF instead of EG power allocation at the SN. In Fig. 3, we compare the sum relay MI of the same schemes but for $K = 4$, $n = q = 8$, $w_k = 1, r_k = 2, \forall k \in [1, \ldots, K]$. The results show again that our scheme outperforms all the other schemes. Moreover by comparing Fig. 2 with Fig. 3, it can be seen that the sum relay MI of each scheme is lower in the 4-user scenario than in the 2-user scenario. Indeed, by considering only the receive CSI at the RN, the schemes of Section III are not able to mitigate the MU interference. As the number of user increases as the interference increases and the overall relay MI performance decreases.

In Fig. 4, we compare the sum relay MI of our three novel schemes for $\text{SNR}_2 = 10$ dB and three different user settings, namely $K = 1$, $w_1 = 1$, $n = q = r_1 = 4$, $K = 2$, $n = q = 4$, $w_k = 1, r_k = 2, \forall k \in [1, \ldots, K]$ and $K = 4$, $n = q = 8$, $w_k = 1, r_k = 2, \forall k \in [1, \ldots, K]$, and employ SN-EG at the SN. Notice that we used the scheme in [8] to obtain the result for AF-CBD in the single user case, since CBD is irrelevant in this case. Comparing the results of the AF-SKRDL scheme for the three different settings show the performance degradation due to the MU interference. On the contrary, our AF-CBD scheme, which utilizes the RN-DN CSI knowledge to eliminate the MU interference at the RN, exhibits performance improvement when K goes from 2 to 4. The performance degradation due to the mitigation of the MU interference can be quantified, i.e., 1.5 bits/s/Hz at SNR_1 (dB), by comparing the performance of the AF-CBD scheme for $K = 1$ and $K = 2$ users. Even though the AF-CBD scheme transforms the MU MIMO relay into K independent MIMO relay channels, its performance cannot be as good as the one in the single user case, because of the CBD process. Finally, the graph points out that the AF-CGSDL is the best of the three proposed techniques. In the single user case, it performs almost as good as the optimal methods of [7] and [8]. In the MU case, it provides a gain of 0.7 and 2.5 bits/s/Hz in comparison with AF-CBD for $K = 2$ and 4, respectively. These results indicate that the AF-CGSDL scheme can also decouple the user channels by not relying on any specific $\mathbf{G}$ matrix structure. The single user way of defining the $\mathbf{G}$ matrix structure does not guarantee optimality in the MU case since AF-CGSDL outperforms AF-CBD, which confirms the statement of [12] on this matter.

We display in Fig. 5 the relay link sum-rate and WSR performances, i.e., $\Sigma_{\mathbf{y}_2}$ and $\widehat{\Sigma}_{\mathbf{y}_2}$, respectively, of the AF-CBD and AF-CGSDL schemes for $\text{SNR}_2 = 10$ dB, $K = 2$, $n = q = 4$, $r_k = 2, \forall k \in [1, \ldots, K]$. We consider the $\mathbf{G}$ matrices that are obtained when $w_1 = w_2 = 1$ and $w_1 = 1.5, w_2 = 0.5$ for each algorithm and insert them in (5) for evaluating the sum-rate, i.e., $w_1 = w_2 = 1$ and the WSR when $w_1 = 1.5$ and $w_2 = 0.5$. Thus, we expect AF-CBD and AF-CGSDL for $w_1 = 1.5, w_2 = 0.5$ to outperform AF-CBD and AF-CGSDL for $w_1 = w_2 = 1$, respectively, in terms of $\widehat{\Sigma}_{\mathbf{y}_2}$ but at the expense of lower $\Sigma_{\mathbf{y}_2}$ performances. The results in Fig. 5 confirm our expectations and clearly indicate that our AF-CBD and AF-CGSDL methods can be utilised to improve the WSR performance, and consequently increase the fairness of the system, but at the expense of a reduced sum-rate.

VI. Conclusions

In this paper, we have introduced three novel power allocation methods for the DL of nonregenerative cooperative MU

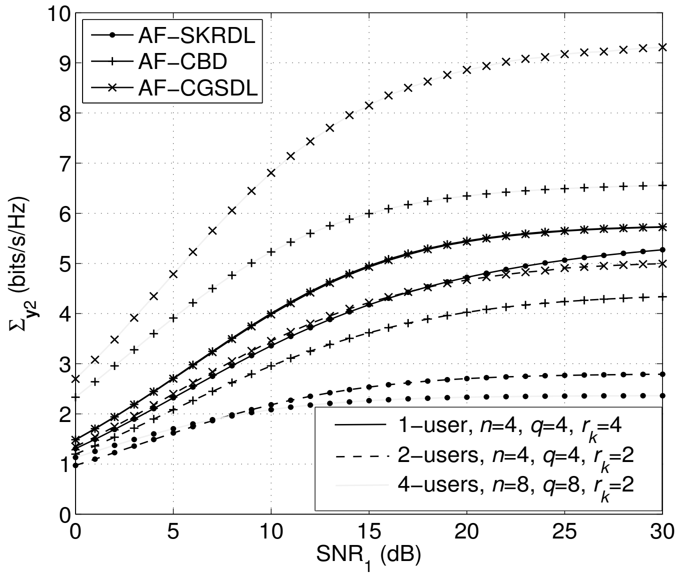

Fig. 4. Sum relay MI of our three novel schemes for $\mathrm{SNR}_2 = 10$ dB and three different user settings.

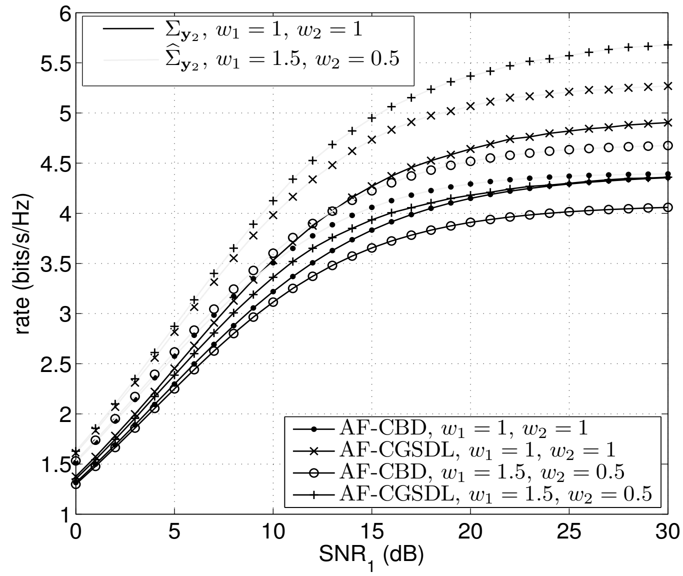

Fig. 5. Relay link sum-rate and WSR performances of our AF-CBD and AF-CGSDL schemes for $\mathrm{SNR}_2 = 10$ dB, $K = 2$, $n = q = 4$, $r_k = 2, \forall k \in [1, \ldots, K]$ and different w_k.

MIMO communication system, which are designed to maximize the weighted sum of the users' MI when only receive CSI or when both receive and transmit CSI is available at the RN. The performances of our three novel power allocation schemes have been assessed in terms of sum-rate and WSR. The results have proved that our AF-SKRDL outperforms the other existing schemes when only receive CSI is employed, however, receive CSI on its own is not sufficient to reduce MU interference. They have also shown that our AF-CBD and AF-CGSDL schemes can mitigate the MU interference by taking advantage of the transmit CSI at the RN and without employing DPC at the SN. Furthermore, it has been indicated that these two schemes can be utilized for fine-tuning the precoder at the RN in order to improve the WSR performances of the cooperative MU MIMO system. Future work could be carried out by considering the joint power allocation at SN and RN, when the full CSI of the relay and direct channels would be available at the SN and RN.

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