Observation of $^{54}$Ni: Cross-Conjugate Symmetry in $f_{7/2}$ Mirror Energy Differences


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Gamma decays from excited states up to $J^e = 6^+$ in the $N = Z - 2$ nucleus $^{54}$Ni have been identified for the first time. Level energies are compared with those of the isobars $^{54}$Co and $^{54}$Fe and of the cross-conjugate nuclei of mass $A = 42$. The good but puzzling $f_{7/2}$ cross-conjugate symmetry in mirror and triplet energy differences is analyzed. Shell model calculations reproduce the new data but the necessary nuclear charge-dependent phenomenology is not fully explained by modern nucleon-nucleon potentials.

Isospin symmetry is a consequence of the fact that the strong interaction is essentially charge independent. It has long been known that, in addition to the Coulomb force, there exist nuclear isospin breaking (NIB) effects [1,2] that have a significant influence on the Coulomb displacement energies (CDE) between the ground states of isobaric analogues [3]. Recent experimental developments have made it possible to extend these studies to excited states, which demand the identification of spectra in proton-rich species. Most of the recent data come from the $f_{7/2}$ region ($40 < A < 56$) where excitation energy differences between mirrors (MED) and second differences in the $T = 1$ triplets (TED) have been studied up to high spin. One of the best examples is the $T = 1$ $A = 50$ mirror pair $^{50}$Cr and $^{50}$Fe where the level schemes can now be compared up to spin and parity $J^e = 11^+$ [4].

While the CDE are large (several MeV) and dominated by the Coulomb force, the MED and TED are small (at most some 200 keV) and here NIB effects are as important as Coulomb ones. This was established by Zuker et al. [5] by assuming that the $A = 42$ yrast bands were dominantly of $f_{7/2}$ character and by ascribing the differences between observed and Coulomb-calculated spectra to NIB terms. The outcome was a single term parametrization through $J = 0$ and 2 pairing operators for TED and MED, respectively, leading to very good results for $A = 47, 49, 50,$ and $51,$ based on precise wave functions obtained by shell model calculations [6]. Further work in $A = 53$ confirmed the crucial importance of the “anomalous” $J = 2$ MED contribution [7].

The fact that $f_{7/2}$ dominance plays an important role in guessing the NIB terms seems to be at odds with the observation that $f_{7/2}$ calculations often yield quite good energetics but poor wave functions, in general quite inadequate to deal with MED and TED subtleties. A good place to resolve this paradox is $A = 54$: in a pure $f_{7/2}$ model, the two-particle spectra in $A = 42$ should be identical to the two-hole spectra in $A = 54$. To within some 150 keV discrepancies, they are, but in $A = 54$ the mixing occurs within the $pf$ shell, while in $A = 42$ it is due to $sd$ shell degrees of freedom. Therefore, this is a case of good energetics and significant departures—of different origin—from $f_{7/2}$ dominance. Two interesting things can happen, depending on whether the MED and TED are very much the same in both cases: (a) if they are, we
may learn in which circumstances departures from $f_{7/2}$ dominance do not matter; (b) if they are not, it will mean that something masquerading as NIB is in fact due to configuration mixing. The spectra of two members of the $T = 1$ triplet in $A = 54$ are known (odd-odd $^{54}$Co and even-even $^{54}$Fe [8]). This Letter reports the first experimental observation of the excited states in $^{54}$Ni up to $J^\pi = 6^+$, which allows to derive MED and TED for $A = 54$ nuclei. These results, compared with those of $A = 42$ [9], reveal a remarkable cross-conjugate symmetry between the two extremes of the $f_{7/2}$ shell.

The present experiment was performed at the Vivitron facility of the IReS-Strasbourg Laboratory. The $^{54}$Ni nucleus was populated using the $^{24}$Mg($^{32}$S, $2n$)$^{54}$Ni reaction with a $^{32}$S beam at an energy of 75 MeV and a 0.5 mg/cm$^2$ $^{24}$Mg (99.92% isotopically enriched) target on an 8 mg/cm$^2$ $^{90}$Zr backing. The incident energy was chosen to favor the two-particle evaporation channels and well below the Coulomb barrier for the reaction with the $^{90}$Zr backing. The gamma rays emitted in the reaction were detected using the EUROBALL IV array in a configuration consisting of 26 Clover and 15 Cluster composite (Compton suppressed) detectors. The peak efficiency at 1.3 MeV for this configuration was close to 7%. The forward 1 $\pi$ solid angle was covered by the neutron wall [10], consisting of 50 liquid scintillator neutron detectors. The charged particle detector EUCLIDES [11], composed of 40 $\Delta E-E$ Si telescopes (130 $\mu$m $+$ 1000 $\mu$m thick), covering 80% of the 4$\pi$ solid angle, was also used in the measurement. Events were recorded when either one Ge detector and one neutron wall detector were in coincidence or at least two Ge detectors were in coincidence. Unambiguous identification of $\gamma$-ray transitions belonging to $^{54}$Ni has been achieved by comparing the $\gamma$-ray spectra in coincidence with two neutrons only and with two neutrons and any charged particle. The construction of the two-neutron gated spectrum is a complex procedure which takes into account the exclusion of neighboring neutron wall detectors and the time-of-flight between them, in order to avoid hits in two or more detector elements produced by a single neutron [12]. The high energy portions of the two spectra are compared in Fig. 1(a). The peaks at 1227.1 keV and 1391.8 keV, clearly seen in the spectrum in coincidence with two neutrons, disappear when a coincidence with a charged particle is required. The low energy peak at 451.4 keV partially overlaps with the $^{46}$V 451.9-keV transition populated in the $^{16}$O($^{32}$S, $pn$) reaction, and therefore, at this energy, a peak remains in coincidence with charged particles. The three new $\gamma$ rays are in coincidence with each other as shown in the analysis of a $\gamma-\gamma$ matrix with the condition of at least one neutron detected. This can be seen in Fig. 1(b), where a sum of gates on the 1227.1-keV and 1391.8-keV transitions is shown. Possible contaminants confounding the identification could come from the 0.08% presence of the other Mg isotopes in the enriched material, from oxygen due to the oxidation of the target after preparation and from the carbon deposited on the target surface during the experiment. The two-neutron evaporation channel from the reaction of the $^{32}$S beam with $^{25,26}$Mg and $^{12}$C are nuclei with well-known structure. Gamma rays from these nuclei are not present in the spectra above the sensitivity limit. Two weak lines (less than 1/10 of the $^{54}$Ni intensity) with

![Image of gamma-ray spectrum](image-url)
energies of \( \approx 892.4 \) keV and \( \approx 1095.4 \) keV were observed in the two-neutron gated singles-gamma spectrum. They may be the known \( 2^{+} \rightarrow 0^{+} \) and \( 4^{+} \rightarrow 2^{+} \) transitions from the deexcitation of \( ^{46}\text{Cr} \) populated in this case by the \( ^{16}\text{O}(^{32}\text{S}, 2n) \) reaction. Channels evaporating three or more neutrons, leading to unknown neutron-deficient nuclei, are excluded on the ground of cross-section systematics and calculations with the statistical model codes CASCADE [14] and HIVAP [15]. We conclude that the three lines in Fig. 1(b) must come from the \( 2n \) channel of the \( ^{24}\text{Mg} \) and therefore correspond to the \( ^{54}\text{Ni} \) deexcitation spectrum. The transitions have been placed in the level scheme, shown in Fig. 2, on the basis of coincidence relationship and relative intensities. The corresponding levels of \( ^{54}\text{Co} \) and \( ^{54}\text{Fe} \) [8] are also shown in Fig. 2. The stretched quadrupole character of the three transitions is consistent with the measured anisotropy between the Clover (\( \approx 90^\circ \)) and Clusters (centered around \( \approx 145^\circ \)) detectors [see Fig. 1(c)]. The three transitions assigned to \( ^{54}\text{Ni} \) have, as expected, a close similarity to the first three transitions in the mirror nucleus \( ^{54}\text{Fe} \). Preliminary results of this work have been reported in Ref. [16]. Our identification of the \( 2^{+} \) level at 1392 keV in \( ^{54}\text{Ni} \) has been confirmed in Coulomb excitation experiments using a radioactive \( ^{54}\text{Ni} \) beam produced by fragmentation of \( ^{58}\text{Ni} \) [17,18].

In Fig. 3(a) we compare the experimental \( \text{MED}(J) = E_{J}^{(54}\text{Ni}) - E_{J}^{(54}\text{Fe}) \) with \( -\text{MED}(J) = E_{J}^{(42}\text{Ca}) - E_{J}^{(42}\text{Ti}) \) (signs change upon cross conjugation). The \( f_{7/2}^{2} \) strength in \( A = 42 \) is known to be fragmented. This is of little consequence except for \( J = 2 \) where it splitting evenly between the two lowest states [19]. Hence the corresponding values are given for their average and for the lowest state. For both, the agreement with \( A = 54 \) is quite good. Which brings us back to the question: why should \( f_{7/2} \) cross conjugation work so well in this case? The answer will come by examining the full \( pf \) shell model calculation.

In Fig. 3(b) the experimental MED for \( A = 54 \) are compared with calculations performed as described in Ref. [5]; i.e., the eigenfunctions are obtained with the KB3G [20] residual interaction in the \( pf \) shell, with a \( J = 8 \) truncation. The MED are given as the corresponding differences (\( \Delta_{\text{MED}} \)) of expectation values of the Coulomb and NIB terms:

\[ \text{MED}(\text{keV}) \]

\[ \begin{array}{cccc}
A=42 & -\text{MED} = E_{J}^{(42}\text{Ca}) - E_{J}^{(42}\text{Ti}) \\
A=54 & -\text{MED} \text{ (AVERAGE)} \\
A=54 & \text{MED} = E_{J}^{(54}\text{Ni}) - E_{J}^{(54}\text{Fe}) \\
\end{array} \]

\[ \text{MED(keV)} \]

\[ \begin{array}{cccc}
A=42 & \text{MED (AVERAGE)} \\
A=54 & \text{MED} \text{ (AVERAGE)} \\
\end{array} \]

\[ \text{MED(keV)} \]

\[ \begin{array}{cccc}
A=54 & \text{Experimental MED} \\
A=42 & \text{MED (AVERAGE)} \\
\end{array} \]

\[ \text{MED(keV)} \]

\[ \begin{array}{cccc}
A=54 & \text{Experimental MED} \\
A=42 & \text{MED (AVERAGE)} \\
\end{array} \]

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A=54 & \text{Experimental MED} \\
A=42 & \text{MED (AVERAGE)} \\
\end{array} \]

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A=54 & \text{Experimental MED} \\
A=42 & \text{MED (AVERAGE)} \\
\end{array} \]

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\[ \begin{array}{cccc}
A=54 & \text{Experimental MED} \\
A=42 & \text{MED (AVERAGE)} \\
\end{array} \]

FIG. 3. (a) MED for the \( A = 54 \) mirror nuclei compared with two variants of those of the \( A = 42 \) mirrors (see text). (b) Measured and calculated MED for \( A = 54 \). (c) Measured and calculated TED. Contributions in Eqs. (1) and (2) are shown separately in (b) and (c).
MED \( (J) = \Delta_{\text{MED}} (V^J_{CM} + V^J_{B(1)} + V^J_{C} + C_{ll}^J + C_{ls}^J), \) \( (1) \)

where \( V_{CM} \) is the Coulomb interaction in the \( pf \) shell, \( V^J_{B(1)} \) is the isovector quadrupole pairing NIB, and \( V_{C} \) reflects the difference in Coulomb repulsion due to the \( J \) dependence of the charge radii. These three terms are those present in Ref. [5]. \( C_l \) and \( C_s \) [21,22] are single-particle contributions of electromagnetic origin, ignored in [5] and added here for consistency. It is apparent from Fig. 3(b) that the overall pattern is due to the interplay of \( V^J_{CM} \) and \( V^J_{B(1)} \), while the contribution to MED of \( V_{C} \), \( C_{ll} \), and \( C_{ls} \)—which depends on the occupancies of the different orbits—is small. It means that the occupancies must be very much independent of \( J \). Configuration mixing may be large: in our case all the \( f_{7/2} \) components amount to about 60\% of the wave functions, but the other configurations have weights that are almost constant with \( J \). That explains why a pure \( f_{7/2} \) description works well: the terms that may break cross conjugation symmetry do exist but they have \( J \)-independent effects that vanish for the MED and TED.

Concerning the parameters: the \( V_{B(1)} \) strength is as in [5], while the \( a_m \) coefficient affecting \( V_{C} \) has been reduced from 300 to 200 keV following [23]. As for \( T = 1 \) it doubles with respect to \( T = 1/2 \); it is actually 400 keV.

Turning now to the experimental triplet energy differences, \( TED(J) = E^J_{54Ni} + E^J_{54Co} - 2E^J_{54Fe} \), we are limited by the fact that the \( J^\pi = 6^+ \) state in \( 54Co \) is not known and we can therefore compare data [see Fig. 3(c)] up to \( J^\pi = 4^+ \) [8]. The corresponding \( A = 42 \) values are seen to follow exactly the same pattern within a factor of about 1.35. As a consequence, when the theoretical TED values are calculated:

\[
TED \( (J) = \Delta_{\text{TED}} (V^J_{CM} + V^J_{B(2)}), \)
\]

the monopole pairing strength of \( V^J_{B(2)} \) that fits \( A = 54 \) is 50 keV, half the 100 keV needed for \( A = 42 \) and adopted in [5]. To try to shed some light on this apparent anomaly we recall that \( V^J_{B(1)} \) and \( V^J_{B(2)} \) are each represented by a single \( f_{7/2} \) matrix element. As it was shown that, for the Coulomb force, the four \( f_{7/2} \) matrix elements reproduce the full \( pf \) results within a multiplicative factor, we try to do the same within the \( V_{lowk} \) formulation, using a modern charge-dependent potential that reproduces the nucleon-nucleon scattering data up to 350 MeV. Here we have chosen the AV18 potential with \( \hbar\omega = 10 \) MeV [24], but similar results are obtained for any such potential. The results are shown in Table I. It is clear that for the MED the phenomenologically essential \( J = 2 \) quadrupole will be missed, whatever the multiplicative factor. For the TED—though the trend is well reproduced—the effect of AV18 amounts to trebling the Coulomb force, while we need a bit less than doubling (and a bit more in \( A = 42 \), also shown for comparison). We can therefore here conclude that we understand why cross conjugation works, but we do not understand why it works better for MED than for TED. The subject deserves further study.

Summarizing, the \( f_{7/2} \) region is unique in that it makes possible to analyze high quality MED and TED data with theoretical tools that range from a very simple \( f_{7/2} \) model to the most rigorous microscopic calculations available. In particular, it is the only region where NIB effects have been detected at a spectroscopic level (i.e., beyond displacement energies). The \( A = 54 \) triplet is in turn special in that the simple model works best, where it is expected to do poorly. The study of the MED reveals why this is so, and confirms the existence of an important NIB mechanism. Comparison with modern charge-dependent potentials reveals that they fail to account for the quadrupole pairing responsible for the MED patterns across the region, while reproducing but overshooting the observed TED trends.

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| Table I. Experimental MED and TED (MeV) in \( A = 54 \) compared with \( f_{7/2} \) values for matrix elements for the AV18 potential [24], scaled by a factor, and for the Coulomb potential. |
| --- | --- | --- | --- | --- |
| \( J = 0 \) | \( J = 2 \) | \( J = 4 \) | \( J = 6 \) |
| MED (experiment) | 0.000 | -0.016 | 0.082 | 0.122 |
| TED (experiment) | 0.000 | -0.090 | -0.144 | -0.162 |
| Coulomb | 0.000 | -0.056 | -0.087 | -0.092 |
| TED(expA = 42) | 0.000 | -0.122 | -0.202 | -0.252 |