HEAT TRANSFER AND PRESSURE DROP DURING SINGLE AND TWO-PHASE FLOW THROUGH UNCONSOLIDATED POROUS MEDIA

BY

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I dedicate this work to

my late brother, ESSA,

who achieved such a great deal in his relatively short life.
ABSTRACT

Single and two-phase flow and heat transfer through unconsolidated porous media composed of stationary granular particles frequently find applications in chemical catalytic reactors, building thermal insulation, heat exchangers, petroleum reservoirs, geothermal operations, packed beds and many others.

The aim of the present study is to investigate single and two-phase flow and heat transfer in cylindrical porous media. The study particularly concentrates on the areas where literature information is not sufficient.

To perform experimental work, a suitable experimental set-up is designed and built which enables the investigation of single and two-phase flow and of heat transfer in a cylindrical bed made of unconsolidated particles under constant heat flux. In single-phase flow experiments, distilled water is used as the saturating medium. The ascending air/water flow provide the moving phases during two-phase flow experiments.

Single-phase pressure drop is investigated and the experimental results are compared with existing correlations. The results show the Darcian regime is predominant and that the effects of non-Darcian terms are unimportant. Mathematical models for the prediction of heat transfer coefficients and temperature distribution in a cylindrical saturated porous medium are developed. The comparison of the results obtained from the models and the experimental data shows excellent agreement. The co-existence of natural and forced convection is also studied and the results demonstrate that the contribution of natural convection to the heat transfer coefficient is insignificant within the experimental operating conditions.

Experimental data on hold-up and pressure drop with ascending air/water flow through the cylindrical porous medium are collected and correlations are presented for regions where the published data suffer insufficiency. The models developed to predict pressure drop and gas hold-up are verified against experimental data, showing a good accuracy.

For the first time experimental studies of heat transfer coefficients in a cylindrical porous medium made of granular particles with ascending air/water flow under
constant heat flux are carried out. A mathematical model is presented and the predicted values are compared with experimental data. The results show good agreement.
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NOMENCLATURE

A cross-sectional area, m²
a_p specific surface area of the particle
a_s specific surface area of the bed
C inertia coefficient, -
c_f dimensionless drag constant in Forchheimer equation
c_p heat capacity, J/kg °C
\(c_{p,g}\) gas heat capacity, J/kg °C
\(c_{p,l}\) liquid heat capacity, J/kg °C
d_b bubble diameter, m
d_c test section diameter, m
d_p particle diameter, m
d_{p,h} diameter of a sphere having the same diameter as the particle, m
dp pressure drop, N/m²
\(dp/dz\) pressure gradient in z direction, N/m³
D duct diameter, m
\(D_a\) characteristic length defined by Turpin and Huntington
Da Darcy number, K/ft²
f friction factor, (dp/du)(dp/du²), -
f* dimensionless friction factor, f* = f(\(g \cdot \delta \cdot (\rho_l - \rho_g)\))
g gravitational acceleration, m²/sec
g_c conversion factor
G gas mass flux, kg/m²·sec.
h enthalpy
I electric current
K permeability, m²
L liquid mass flux, kg/m²·sec.
l_h heated length, m
l_l pore length, m
l length, m
m mass fraction
\(N_T\) number of pores, -
\(N_b\) number of bubbles, -
p pressure, N/m²
\(\Delta p/l\) pressure gradient, Pa/m
\(\Delta p\) pressure drop, Pa
\(p^*\) dimensionless pressure drop, \(p^* = \Delta p \cdot (g \cdot \delta \cdot (\rho_l - \rho_g))\)
q'' heat flux, W/m²
r radial distance, m
R electrical resistance, Ω
R_o radius, m
\(Re_m\) modified Reynolds number defined as dp . ρ . u / μ . (1-δ), -
\(Re_l\) Reynolds number defined as dp . ρ . u / μ, -
\(Re_o\) Reynolds number defined as D . ρ . u / μ, -
S_b surface area per unit volume of the bed, m
S_o surface areas per unit volume of particles, m
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<thead>
<tr>
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<th>Description</th>
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<tr>
<td>T</td>
<td>temperature, °C or K</td>
</tr>
<tr>
<td>T_b</td>
<td>bulk temperature, °C or K</td>
</tr>
<tr>
<td>T_c</td>
<td>center temperature, °C or K</td>
</tr>
<tr>
<td>T_s</td>
<td>inside surface temperature of the test section, °C or K</td>
</tr>
<tr>
<td>T_w</td>
<td>wall temperature, °C or K</td>
</tr>
<tr>
<td>u</td>
<td>velocity, m/sec.</td>
</tr>
<tr>
<td>u_b</td>
<td>bubble terminal rising velocity, m/sec.</td>
</tr>
<tr>
<td>u_C</td>
<td>characteristic velocity, m/sec.</td>
</tr>
<tr>
<td>u_D</td>
<td>Darcian velocity, m/sec.</td>
</tr>
<tr>
<td>u_s</td>
<td>slip velocity, m/sec.</td>
</tr>
<tr>
<td>u_l</td>
<td>superficial liquid velocity, m/sec.</td>
</tr>
<tr>
<td>u_g</td>
<td>superficial gas velocity, m/sec.</td>
</tr>
<tr>
<td>V</td>
<td>volume, m³</td>
</tr>
<tr>
<td>V_p or V_pore</td>
<td>volume of pores, m³</td>
</tr>
<tr>
<td>V_T</td>
<td>total volume, m³</td>
</tr>
<tr>
<td>X</td>
<td>dimensionless length; x/l_h, x=0 at inlet and x=1 at outlet of the test section</td>
</tr>
<tr>
<td>x</td>
<td>distance, m</td>
</tr>
<tr>
<td>y</td>
<td>distance, m</td>
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**Greek Symbols**

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<thead>
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<tbody>
<tr>
<td>α</td>
<td>heat transfer coefficient, W/m² K</td>
</tr>
<tr>
<td>β</td>
<td>coefficient of thermal expansion, K⁻¹</td>
</tr>
<tr>
<td>χ</td>
<td>Lockhart-Martinelli parameter</td>
</tr>
<tr>
<td>χ'</td>
<td>parameter employed by Goto and Gaspillo (1991)</td>
</tr>
<tr>
<td>δ</td>
<td>porosity, (-)</td>
</tr>
<tr>
<td>δ∞</td>
<td>porosity at infinity</td>
</tr>
<tr>
<td>ε</td>
<td>hold up</td>
</tr>
<tr>
<td>φ</td>
<td>sphericity</td>
</tr>
<tr>
<td>κ</td>
<td>constant in the Carman-Kozeny equation</td>
</tr>
<tr>
<td>λ</td>
<td>thermal conductivity, W/m K</td>
</tr>
<tr>
<td>λ_ss</td>
<td>thermal conductivity of stainless steel, W/m K</td>
</tr>
<tr>
<td>μ</td>
<td>dynamic viscosity, kg / m . sec</td>
</tr>
<tr>
<td>ν</td>
<td>kinematic viscosity, m²/sec</td>
</tr>
<tr>
<td>ρ</td>
<td>density, kg/m³</td>
</tr>
<tr>
<td>σ</td>
<td>surface tension</td>
</tr>
<tr>
<td>τ</td>
<td>shear stress, N/m²</td>
</tr>
<tr>
<td>A</td>
<td>thermal diffusivity, m²/sec.</td>
</tr>
<tr>
<td>Λ</td>
<td>separation constant</td>
</tr>
<tr>
<td>Ω</td>
<td>averaging volume defined by equation (2.8)</td>
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<tr>
<td>Ψ</td>
<td>correction factor</td>
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**Subscripts**

<table>
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<tr>
<td>ac</td>
<td>active</td>
</tr>
<tr>
<td>D</td>
<td>based on diameter of an unpacked tube</td>
</tr>
<tr>
<td>d</td>
<td>dispersion</td>
</tr>
<tr>
<td>ds</td>
<td>dispersion, single-phase</td>
</tr>
<tr>
<td>dt</td>
<td>dispersion, two-phase</td>
</tr>
<tr>
<td>e</td>
<td>effective</td>
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<tr>
<td>Symbol</td>
<td>Definition</td>
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<td>--------</td>
<td>----------------------------------</td>
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<tr>
<td>es</td>
<td>effective, single-phase</td>
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<tr>
<td>et</td>
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<tr>
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<tr>
<td>et</td>
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<tr>
<td>f</td>
<td>fluid, frictional</td>
</tr>
<tr>
<td>ft</td>
<td>frictional, two-phase</td>
</tr>
<tr>
<td>g</td>
<td>gas</td>
</tr>
<tr>
<td>i</td>
<td>interfacial</td>
</tr>
<tr>
<td>l</td>
<td>liquid</td>
</tr>
<tr>
<td>m</td>
<td>mean value</td>
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<tr>
<td>o</td>
<td>stagnant</td>
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<tr>
<td>os</td>
<td>stagnant, single-phase</td>
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<tr>
<td>s</td>
<td>single-phase</td>
</tr>
<tr>
<td>s</td>
<td>solid</td>
</tr>
<tr>
<td>sph</td>
<td>sphere</td>
</tr>
<tr>
<td>p</td>
<td>particle</td>
</tr>
<tr>
<td>pg</td>
<td>pore-gas</td>
</tr>
<tr>
<td>T</td>
<td>tube, pore</td>
</tr>
<tr>
<td>TC</td>
<td>thermocouple</td>
</tr>
<tr>
<td>t</td>
<td>two-phase</td>
</tr>
<tr>
<td>w</td>
<td>wall</td>
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**Superscripts**

* dimensionless
- vector notation

**Other**

<> denotes the 'local volume average of a quantity'
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1. INTRODUCTION

Over the past decades, porous media have been the focus of immense studies both experimentally and theoretically. Precise instrumentation and new experimental procedures and techniques have paved the way to investigate the physical properties, the flow and transport processes in porous media. New computational procedures have allowed us to model and simulate various features of porous media and thus a deep understanding of this has been gained. However, one can claim that in spite of the bulky and voluminous research work in this area, there are parts that still need to be exploited both experimentally and theoretically. Moreover, some previous work needs to be revised and the results ought to be updated if more efficient design procedures are to be developed.

Application of transport and flow phenomena in porous media and in industrial synthetic porous matrices are very important for many diverse fields of science and engineering, ranging from agricultural, biomedical, construction, ceramic, chemical, and petroleum engineering to food and soil sciences, and powder technology.

More than 50% of the original oil-in-place is left in a typical oil reservoir by traditional (primary and secondary) recovery techniques. This remaining oil is the main target for enhanced or tertiary oil recovery methods now being developed. However, oil recovery processes constitute only a small fraction of an enormous, and still rapidly growing, literature on porous media. In addition to oil recovery processes, the closely related areas of soil science and hydrology are perhaps the best-established topics. The study of groundwater flow and the restoration of aquifers that have been contaminated by various pollutants are important current areas of research on porous media problems.

Classical research areas of chemical engineers dealing with porous media include filtration, centrifugation, drying, multiphase flow in packed columns, and diffusion and reaction in porous catalysts. Lesser-known, but equally important, phenomena involving porous media are also numerous. For example, for the construction industry, transmission of water by building materials (bricks and concrete) is an important problem to consider when designing a new building. Various properties of wood, an interesting and unusual
porous medium, have been studied for a long time. Some of the phenomena involving wood include drying and impregnation by preservatives. Civil engineers have long studied asphalts as water-resistant binders for aggregates, protection of various type of porous material from frost heave, and the properties of road beds and dams with respect to water retention.

Some porous media whose pore space morphology and wetting behavior are of physiological interest are skin, hair, feathers, teeth and lungs. Other types of porous media that are widely used are ceramics, pharmaceuticals, contact lenses, explosives, and various kinds of membranes. In any phenomenon that involves a porous system one has to deal with the complex pore structure of the porous medium and how it affects the distribution, flow, displacement of one or more fluids, or dispersion (i.e. mixing) of one fluid in another. Each process can, by itself, be very complex. For example, displacement of one fluid by another can be carried out by many different mechanisms, which may involve heat and mass transfer, thermodynamic phase change, and the interaction of various forces such as viscous, buoyancy, and capillary forces. If the solid matrix of the porous medium is deformable, its porous structure may change during flow or by some other transport phenomenon. If the fluid is reactive, or if it carries solid particles of various shapes, sizes, and electrical charges, the pore structure of the medium may change due to the reaction of the fluid with the pore surface, or physicochemical interaction between the particles and the pore surface.

In all the above cases, the void fraction of the media might be saturated with one or more fluids, which gives rise to single and multi-phase flow respectively. The simplest situation of the multi-phase flow is two-phase flow where the void volume is occupied with two miscible or immiscible fluids.

Fluid flow in cylindrical tubes packed with a porous medium, which is saturated with a single fluid has been studied by a number of investigators, who have proposed different models and correlations. Although the presented models can describe the hydrodynamics of single-phase flow, there are nevertheless some disagreements in the predictions. However, there are models which have been accepted by many investigators in the field
and as long as they are used in their specified range, the predictions are acceptable. One of the accepted models is Darcy's phenomenological law which is used at low fluid flow rates. There have been many modifications to Darcy's law over the years to extend its range of applicability; i.e. Forchheimer's and Brinkman's equations. Moreover, there have been efforts to include non-Darcian effects, solid boundaries; inertial forces, and variable porosity, so that a wider range of applications can be covered.

The studies on forced convection heat transfer in cylindrical porous media saturated with a single fluid cover both Darcian and non-Darcian ranges. The investigations have mostly dealt with a gaseous medium (commonly air) as the saturating fluid and less information has been published with liquid as the saturating fluid. Most of the studies in the Darcian range have been carried out at constant wall temperature. The available information on heat transfer under constant heat flux is inadequate leaving a research area with a reasonably high potential.

Considerable interest has been shown in studying the hydrodynamics of two-phase flow in porous media. It is readily recognized that the two-phase frictional pressure drop represents one of the most important design variables in various applications. Consequently over the years, a large body of information has become available on the prediction of the two-phase pressure drop in porous media. However, a careful inspection of the pertinent literature reveals that the majority of research efforts have been directed at developing predictive methods for two-phase pressure drop associated with the co-current flow of gas/liquid downward through packed beds of different shapes and sizes of particles. These investigations mostly deal with media made of particles with sizes usually encountered in packed beds. In contrast, very little information is available on the two-phase frictional pressure drop in packed beds operating in ascending movement of gas/liquid which offers specific advantages over the descending mode:

1. The liquid distribution is readily uniform; this helps in efficient distribution of heat and its transfer to and from the wall when desired and prevents the formation of hot spots.

2. The larger liquid hold-up gives higher production rates for a given size of reactor,
3. The liquid-side mass transfer coefficient is higher.

Typical examples of gas liquid co-current up-flow through packed beds are

- hydration of olefins.
- alcohol amination.
- hydro-desulfurization of crude oil and heavy oil.
- in processes where liquid phase reactant interact with relatively small amounts of the gas which is characteristic of the reaction of ammonolysis of alcohol and the hydration of nitrogen compounds and olefins.
- and in processes where large residence time is needed to achieve the necessary conversion specially where \( d_v/d_p \) is relatively low and therefore the contacting efficiency of the liquid with the solid granules is higher as compared to descending co-current flow.

The information on the liquid and gas hold-up is equally important in studying the hydrodynamics of the two-phase flow in packed beds. These have been investigated along with the research on pressure drop in packed beds. The available information however, does not cover a wide range of operational variables; i.e. liquid and gas flow rates. Since the experimental procedures used to determine the values of gas and liquid hold-up are variable, the empirical models often show significant differences in their predictions.

Few, if any, serious attempts have been made to study two-phase heat transfer in packed beds. Therefore, in applications where there is a need for large amounts of heat to be transferred due to exothermic or endothermic processes to immiscible fluids flowing in the void space of the bed in order to maintain a suitable temperature profile, references are scarce. Moreover, information on heat transfer in equipment with ascending gas/liquid flow is practically nonexistent.

The need for further investigation regarding fluid flow and heat transfer in saturated as well as unsaturated porous media has been outlined above.
The present work studies systematically the mechanism of fluid flow and heat transfer in a cylindrical porous medium saturated with a single fluid and with two immiscible fluids under constant heat flux.

The aim of the investigations in single-phase operation is to carry out experiments in media made of different materials with varying particle diameters, which are saturated with water as the fluid filling the pore volume. The effect of parameters such as particle diameter and fluid flow rate on pressure drop in the medium at low flow rates of fluid is investigated. The influence of the operational variables such as heat flux, fluid flow rate, axial and radial conduction, medium thermal conductivity and particle size on the amount of heat transferred to the fluid flowing in the porous media are studied.

The objective of the studies on two-phase operation includes the examination of fluid flow and heat transfer as well as fluid hold-up and the flow regimes in media made of different materials with varying particle diameter. The investigations employ water and air as the fluids, which flow co-currently upward through the pore volume. The influence of fluid flow rates and particle diameter on pressure drop, fluid hold-up and flow regimes is observed. In addition, the effect of different parameters such as fluid flow rates, particle diameters and material on the heat transfer coefficient is investigated.
2. LITERATURE REVIEW

2.1 Single-phase flow

2.1.1 Fluid flow and pressure drop

The fluid flow through porous media is not a new topic. There has been extensive theoretical analysis and experimental studies to investigate the fluid flow and transport processes through saturated porous media as well as columns packed with granular material. The published literature covers nearly all aspects of flow mechanisms which have been critically reviewed by different authors.

The first experimental work on the subject of fluid flow through porous media was carried out by Darcy in 1830 in Dijon when he examined the rate of flow of water from the local fountains through beds of sand of various thickness. He showed that the average velocity, as measured over the whole area of the bed, was directly proportional to the driving pressure and inversely proportional to the thickness of the bed. This relation, often termed Darcy’s Law, has subsequently been confirmed by a number of workers and can be written as follow:

\[
\frac{u_D}{\mu} = K (-\nabla p \pm \rho g)
\]

(2.1)

The coefficient K is independent of the nature of the fluid and depends on the geometry of the medium. It is called the specific or absolute permeability of the medium. In the case of single-phase flow, it is abbreviated as permeability. For the case of an isotropic medium, the permeability is a scalar and when the medium is an-isotropic, it is generally a second-order tensor. It has the dimension of \((\text{length})^2\) and it is measured in \(\text{m}^2\) or 'Darcy'.

Values of K for different materials vary widely. Some typical values can be found in Table 2.1.
Table 2.1 Properties of some common porous media (based on data compiled by Kaviany, 1991 and Nield and Bejan, 1992)

<table>
<thead>
<tr>
<th>Material</th>
<th>Porosity</th>
<th>Permeability ($m^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandstone (oil sand)</td>
<td>0.08 – 0.38</td>
<td>5.0E-16 – 3.0E-12</td>
</tr>
<tr>
<td>Brick</td>
<td>0.12 – 0.34</td>
<td>4.8E-15 – 2.2E-13</td>
</tr>
<tr>
<td>Limestone, dolomite</td>
<td>0.04 – 0.10</td>
<td>2.0E-15 – 4.5E-14</td>
</tr>
<tr>
<td>Leather</td>
<td>0.56 – 0.59</td>
<td>9.5E-14 – 1.2E-13</td>
</tr>
<tr>
<td>Black slate powder</td>
<td>0.57 – 0.66</td>
<td>4.9E-14 – 1.2E-13</td>
</tr>
<tr>
<td>Agar – agar</td>
<td>0.57 – 0.66</td>
<td>2.0E-14 – 4.4E-13</td>
</tr>
<tr>
<td>Soils</td>
<td>0.43 – 0.54</td>
<td>1.3E-14 – 5.1E-14</td>
</tr>
<tr>
<td>Silica powder</td>
<td>0.37 – 0.49</td>
<td>2.9E-13 – 1.4E-11</td>
</tr>
<tr>
<td>Bituminous concrete</td>
<td>-</td>
<td>1.0E-13 – 2.3E-11</td>
</tr>
<tr>
<td>Fiberglass</td>
<td>0.88 – 0.93</td>
<td>2.4E-11 – 5.1E-11</td>
</tr>
<tr>
<td>Sand (loose beds)</td>
<td>0.37 – 0.50</td>
<td>2.0E-11 – 1.8E-10</td>
</tr>
<tr>
<td>Hair felt</td>
<td>-</td>
<td>8.3E-10 – 1.2E-09</td>
</tr>
<tr>
<td>Cork board</td>
<td>-</td>
<td>3.3E-10 – 1.5E-09</td>
</tr>
<tr>
<td>Wire crimps</td>
<td>0.68 – 0.76</td>
<td>3.8E-09 – 1.0E-08</td>
</tr>
<tr>
<td>Cigarette</td>
<td>0.17 – 0.49</td>
<td>1.1E-07</td>
</tr>
<tr>
<td>Berl saddles</td>
<td>0.68 – 0.83</td>
<td>1.3E-07 – 3.9E-07</td>
</tr>
</tbody>
</table>

A great deal of effort has been spent to predict the value of $K$ empirically. For the case of simple geometry and unconsolidated porous media, it can be estimated in terms of the geometrical parameters and bed porosity with reasonable accuracy. Various methods for estimation of permeability are documented by Dullien (1979).

For example, in the case of beds made of particles or fibers one can introduce an effective average particle or fiber diameter $d_p$. The hydraulic radius theory of Kozeny-Carman (Kozeny, 1927; Carman, 1937) leads to the relationship

$$ K = (K_0^2 \alpha)^{-1} ; \quad \alpha = \frac{(1-\delta)^2}{\delta^3} \quad (2.2) $$

Or

$$ K = \frac{d_p^2}{36\kappa \alpha} \quad (2.3) $$

The average effective particle diameter is given by
\[ dp_2 = \frac{\int_0^\infty dp^3 h(dp) dp}{\int_0^\infty dp^2 h(dp) dp} \]  

(2.4)

with \( h(dp) \) being the density function for the distribution of the diameter \( dp \).

The porosity of a packed bed, \( \delta \), in addition to particle size, is strongly influenced by the particle size distribution and the particle shape which are the most important parameters in determining the voidage of the bed, (Chhabra, 1993). It is readily recognized that particle shape and orientation are difficult to be characterized. One widely used measure of shape is sphericity (\( \phi \)), which is defined as the ratio of the surface area of a sphere (of the same volume) to that of the particle. The porosity of a randomly packed bed, in turn, correlates rather well with the sphericity, (Chhabra, 1993). Then particle size for nonspherical particles is defined as;

\[ dp = \phi \cdot d_{sph} \]  

(2.5)

where \( d_{sph} \) is the diameter of the sphere having the same volume as the particle.

Originally the constant \( \kappa \) in equation (2.3) was found to be 5, but other investigators such as Fand et al. (1987) reported that the value of the Kozeny-Carman constant may be found in terms of the measured experimental quantities \( dp, \Delta p/L, \mu, \) and \( \delta \) for different media. They also reported that the measured value of \( \kappa \) is subjected to unavoidable experimental error due to the so-called wall effect. This effect is the consequence of the fact that spheres which contact the wall of a test section do so to discrete points of tangency, and hence the porosity of the medium near the container wall is not uniform. However, they concluded that the wall effect is nearly negligible for spheres whose diameters are less than 2 mm. In their later publication Fand and Thinakaran (1990) reported that for cylindrical ducts packed with spheres, the wall effect becomes significant for \( d_c/d_p < 40 \).

Generally speaking, equation (2.3) has been reported to give satisfactory results for media that consist of particles of approximately spherical shape and whose diameters fall within
a narrow range. The equation is often not valid in the case of particles that deviate strongly from spherical shape, broad particle size distributions, and consolidated media. Nevertheless, it is reported to predict the permeability of packed beds reasonably well.

For packed beds of spherical particles with narrow range of distribution in size, Rumpf and Gupte (1971) show that

\[
K = \frac{\delta^{5.5}}{5.6} d_p^2
\]  

(2.6)

gives a better agreement with the experimental results providing that the considered range of porosity is between 0.35 and 0.67.

Fig. 2.1 Comparison of different models for prediction of the permeability.

A comparison of the predictions made by equation (2.3) and equation (2.6) is illustrated in Fig. 2.1. It can be seen that the predictions due to both correlations are in close agreement at lower porosity, however, the correlations deviate notably in their predictions for larger values of bed porosity.
As in any fluid flow problem, the range of validity of Darcy’s law is expressed in terms of a Reynolds number, Re. The Reynolds number is normally defined in terms of a characteristic length of the system in which fluid flow occurs (Nield and Bejan, 1992).

However, for flow through unconsolidated porous media it is customary to express Re in terms of a characteristic grain size, and for flow in consolidated porous media in terms of a mean pore size. Some authors have suggested $\sqrt{(K/\delta)}$ as the characteristic length, as K has the units (length)$^2$, while other have used $\sqrt{K}$. In any event it is generally accepted that Darcy’s law is applicable to the region where viscous forces are predominant.

Dybbs and Edwards (1975), studied the flow of liquids in porous media using laser anemometry visualization and indicated the existence of four regimes of flow in porous media. (1) The Darcy regime where the flow is dominated by viscous forces. This type of flow occurs at Re < 1, it is the region where Darcy’s law holds. (2) The inertial flow regime. This is the region where the boundary layer begins to build up and causes a non-linear relationship between the pressure gradient and the flow rate. The transition Reynolds number is between 1 to 10 and persists to Re = 200. (3) An unsteady laminar flow regime in the range 200 < Re < 350, which is characterized by laminar wake oscillations for 200 < Re < 300 followed by the formation of vortices in the range 300 < Re < 350. (4) A highly unsteady and chaotic regime for Re > 350 that quantitively resembles turbulent flow. However, Bear (1972), in addition to the flow regimes described by Dybbs and Edwards (1975), reported a pre-Darcy flow in which Darcy’s law does not hold. The Reynolds number for such a flow is reported to be below the Darcian Reynolds number. The existence of the pre-Darcy flow is attributed to non-Newtonian behavior of fluids and the fact that the streaming potential generated by the flow, particularly in fine grain media, can produce small countercurrents along the pore walls in a direction opposite to that of the main flow, Bear (1972). An important aspect of the pre-Darcy flow is that within this region a finite value of the pressure gradient exists below which the Darcian velocity is zero. There is no precise information available concerning the magnitude of the Reynolds number at which this type of flow occurs, (Fand et al., 1987).
However, Fand and Thinakaran (1990) studied the flow properties through a packed bed made of spherical particles and found the upper bound of Reynolds number at which Darcian flow vanishes to be 2.3. The lower and upper bounds of the Reynolds numbers at which Forchheimer (inertia) flow starts and ends are 5 and 80 respectively. Turbulent flow was found to take effect above a Reynolds number of 120.

![Fig. 2.2 Schematic representation of friction factor as a function of $Re_m$.](image)

Fig. 2.2 Schematic representation of friction factor as a function of $Re_m$.

Fig. 2.2 schematically displays the range of different regimes, which may exist at a fixed bed porosity.

Darcy's law may also break down for flow of gases at low pressures through porous media. Since many gases such as air are used for determining permeability of porous media, the deviation from Darcy's law may be important (Sahimi, 1995).

However, many authors have given various derivations of equation (2.1) (Bear, 1972; Gray and O'Neil, 1976; Neumann, 1977; Keller, 1980; Larson, 1981; Whitaker, 1966; Rubinstein and Torquato, 1989). Whitaker (1966) derived equation (2.1), which was based on the volume averaging method developed by him and his co-workers. The porous
media which he considered is shown in Fig. 2.3. In Whitaker's (1966) work, the equations of motion and continuity are developed for a finite volume, $\varsigma$, which is necessarily small compared to the gross dimensions of the porous media, but large compared to the characteristic volume of the voids. These equations are termed macroscopic while the equations of motion and continuity, which apply at a point in the fluid, are considered microscopic.

Considering the most general case in which the medium is an-isotropic and solving the boundary value problem which is expressed by the continuity and momentum equations and after performing a rigorous mathematical procedure Whitaker (1986a) showed that;

$$
\langle u_f \rangle = \frac{K}{\mu_f} (\nabla \langle p_f \rangle - \rho_f g)
$$

Where $\langle \cdot \rangle$ indicates the quantity is an averaged quantity and $K$ is the permeability tensor. For an isotropic medium it reduces to equation (2.1).

Fig 2.3 Schematic view of the finite volume, $\varsigma$, used in the volume averaging method (Whitaker, 1966).
The volume-averaging method used to arrive at equation (2.7) is defined for any quantity \( \langle \phi \rangle \), where \( \phi \) is a scalar, vector or tensor, as follow

\[
\langle \phi \rangle = \frac{1}{\Omega} \int \phi \, d\Omega \quad (2.8)
\]

where \( \Omega \) is the averaging volume including both fluid and solid. If the fluid velocity is high then, as it has already been stated, additional terms are necessary to be incorporated in the equation (2.1) in order to take care of the effect of the deviations.

The increase in fluid velocity, as mentioned above, produces the breakdown in linearity which is due to the fact that the form drag due to the solid obstacles is now comparable to the surface drag due to friction. Hence, another correlation must be sought to predict the pressure drop.

According to Joseph et al. (1982), the appropriate modification to Darcy’s equation is to replace equation (2.1) by

\[
\nabla p = -\frac{\mu}{K} u - c_F K^{-1/2} \rho_f |u|^2 
\]

Equation (2.9) is a modification of an equation associated with the names of Forchheimer who, in 1901, proposed a second order and later a third order equation to fit experimental data. Hence the last term in equation (2.9) is called the Forchheimer term, but in fact the dependence on \( \rho_f K^{-1/2} \) is a modern discovery (Ward, 1964). Ward thought that \( c_F \) might be a universal constant but later Beavers et al. (1973) found that \( c_F \) does vary with the nature of the porous medium, and can be as small as 0.1 in the case of foam metal fibers.

A major shortcoming of Forchheimer’s equations is that none of them can adequately account for the combined effects of geometry and viscosity and hence the empirical constants contained therein must be predetermined for each specific porous medium.

An alternative to Darcy’s equation is what is commonly known as Brinkman’s equation which takes the form of

\[
\nabla p = -\frac{\mu}{K} u - c_F K^{-1/2} \rho_f |u|^2
\]
\[ \nabla p = -\frac{\mu}{K} u + \mu' \nabla^2 u \]  

As it can be seen there are two viscous terms appearing in Brinkman’s equation. The first is the usual Darcy term and the second is analogous to the Laplacian term that appears in the Navier-Stokes equation. The coefficient \( \mu' \) is an effective viscosity. Brinkman (1948) set \( \mu \) and \( \mu' \) equal to each other, but in general, they are only approximately equal.

Experimental verifications of the theory proposed by Brinkman (1948) have been indirect and few in number. The basic drawback is the fact that it is not possible to rigorously justify the model suggested by Brinkman (1948), except when the porosity is close to unity. For many practical purposes there is a need to include the Laplacian term. Its use only becomes important when a no-slip condition is encountered, but its effect is significant only in a thin boundary layer whose thickness is of the order \( (\mu'K/\mu)^{1/2} \), the layer being thin since the continuum hypothesis requires that \( K^{1/2} \ll l \) where \( l \) is a characteristic macroscopic length scale of the problem being considered. When Brinkman’s equation is employed, it will usually be necessary to also account for the effects of porosity variation near the wall.

Several recent authors have added a Laplacian term to equation (2.10) to form a “Forchheimer-Brinkman” equation. The validity of this is not clear since Brinkman’s equation is valid when the porosity is large and there is some uncertainty about the validity of Forchheimer’s model at such large porosity (Nield and Bejan, 1992).

If \( l \) is the appropriate characteristic length scale, the ratio of the last term in equation (2.10) to the previous term in the same equation is of the order of magnitude of the Darcy number, i.e. \( (\mu'/\mu)K/l^2 \). Authors who assume that \( \mu' = \mu \) define the Darcy number to be \( K/l^2 \) (Nield and Bejan, 1992).

The contemporary literature to remedy the shortcomings associated with the forgoing models on the flow of fluids in porous media is indeed voluminous. The objective of nearly all models is to develop means for estimating the pressure gradient required to maintain a fixed flow rate through the porous medium in an envisaged application or
conversely to predict the throughput for an available pressure gradient. A variety of model approaches have been employed in the literature. Depending upon one's taste and viewpoint, these may be characterized in a number of ways. Broadly speaking, one can discern four distinct strategies:

(1) Dimensional approach together with empirical considerations (also known as phenomenological models).

(2) Conduit or capillary flow analogy.

(3) Submerged object or drag theories.

(4) The use of field equations

**Dimensional approach and empirical considerations**

In this category, dimensionless empirical correlations, is the simplest and perhaps the oldest class of descriptions of the flow of Newtonian liquids through beds of particles. Perhaps the most complete and general dimensional analysis of this flow problem is that of Rumpf and Gupte (1971). They established a relationship between the friction factor and the Reynolds number in the form of;

\[
f \cdot \text{Re} = A' \cdot F(\delta)
\]

Indeed, most of the correlations available in the literature are of this form. However the numerical values of \(A'\) and the choice of \(F(\delta)\) continues to be a matter of disagreement in this area. These uncertainties coupled with the additional possible dependence of the constant \(A'\) on particle shape, size distribution, structure of the bed etc. have been the main difficulties in developing a universally applicable form of equation. When the friction factor-Reynolds number data for flow through unconsolidated porous media is obtained over wide ranges of kinematic conditions, the dependence of \(f\) on \(\text{Re}\) changes gradually from linear to non-linear (Chhabra, 1993).

At low values of the Reynolds number, the Darcy model characterizes the regime of flow where the viscous forces dominate the flow. At higher Reynolds numbers or for the flow
conditions outside the range of applicability of Darcy's law, the literature abounds with numerous empirical formulae, which purport to provide satisfactory means of estimating pressure drop. Perhaps the two best known correlations are suggested by Ergun (1952) for Re < ~ 1000 (1- δ) and by Burke-Plummer (1928) for Re > 1000 (1- δ). Both are based on data obtained in columns packed with uniform packing and with no severe channeling.

Conduit or capillary flow analogy

In conduit or capillary models, the interstitial void space present in a porous medium is envisioned to form tortuous conduits of complicated cross-section but with a constant cross-sectional area on the average. Thus, the flow in porous media is equivalent to that in conduits whose length and diameter are so chosen that the same resistance to flow is offered as in the actual porous medium. Undoubtedly, the formed conduit or capillaries are interconnected in an irregular manner but the simplest models of this class do not take into account this complexity. There are essentially three models available, which fall in this category: the Blake (1922), Blake-Kozeny (1927), and Kozney-Carman (1937) models. In the Blake model the bed is replaced by a bundle of straight tubes of complicated cross-section. The model proposed by Blake-Kozney (1927), on the other hand, postulates that the effective length of the tangled capillaries is greater than that of the porous medium thereby introducing the so-called tortuosity factor. Finally, the model suggested by Kozney-Carman (1937) is exactly identical to the Blake-Kozeny model except that it also corrects the average velocity for the tortuous nature of the flow path. It must, however, be emphasized that in spite of the great degree of similarities between the end results of the Kozney-Carman (1937) and Ergun (1952) models, the latter is purely an empirical development whereas the Kozney-Carman development is based on a presupposed model of the porous medium.

Submerged object or drag theories

In submerged object models or drag theories, the flow through porous media or packed beds is viewed as being equivalent to the flow around an assembly of submerged objects, and the resulting fluid dynamic drag manifests itself as the frictional pressure drop across
the bed. Thus, the central problem here is essentially that of calculating the drag force on a typical particle of the assembly. A variety of ideas have been employed to achieve this objective; all of which however involve the modification of the drag on a single particle to account for the additional resistance arising from the presence of neighboring particles. However, extensive comparison between drag theories and experimental data has revealed that the submerged object models describe the flow at high values of the bed voidage, whereas the capillary approach provides a good description of the flow at low values of $\delta$ with a gray area in between the two (Chhabra, 1993).

The use of field equations

This last strategy which has been employed by various investigators to take into account the excess pressure loss resulting from the convergent-divergent nature of flow in porous media is the use of the field equations to describe the flow. Simple models assume a flat radial velocity profile which does not take care of the radial component of the velocity in convergent-divergent type of flow. Numerous investigators have attempted to estimate the extent of this contribution to the overall pressure drop by solving the Navier-Stokes equations for a variety of constricted tube configurations whereas others have averaged the field equations over a representative region of the matrix. The techniques for volume averaging of the Navier-Stokes equations over a representative portion of a rigid porous mass have been dealt with extensively by Whitaker (1986), Slattery (1969), and others. Dullien and Azzam (1977) employed the volume averaging technique, due to Slattery, to obtain a suitable form of the Navier-Stokes equation and they obtained the well-known Forchheimer equation. In their analytical work, they showed that the pressure drop in porous media is due to viscous and inertia forces. At low velocities, the inertial force is negligibly small and Darcy's model describes the flow whereas at high flow rates, the viscous term drops out and the inertial force dominates the flow and the condition is known as turbulent flow.

Many investigators have followed one of the above approaches and presented models to predict the pressure drop in porous media. Among the earliest researchers, Blake (1922) made a significant contribution. Assuming the resistance to flow in a packed bed to be
analogous to that in simple conduits and by means of dimensional analysis, Blake obtained the following equation for the laminar flow:

\[
\frac{\Delta p}{1} \left( g_c \rho / G^2 \right) \left( \delta^3 / a \right) = k \left( \mu a / g \right)
\]

(2.12)

where \( a \) is the total surface area of the particles per unit volume of the bed and \( k \) is a constant taken to be equal to 5.

Kozney (1927) derived independently an equation, which proved identical with that of Blake. The first systematic method of expressing the behavior of non-spherical shapes and of irregular particles was introduced by Carman (1937) who defined a shape factor, \( \phi_s \), which is now, called the particle sphericity. In the streamline range, which he defined as corresponding to values of \( Re < 20 \), Carman's equation is identical with those of Blake and Kozney.

The experimental study of Oman and Watson (1944) is of considerable interest due to the wide range of packing geometry they employed. They modified Blake's equation by using \( \delta^{1.7} \) instead of \( \delta^3 \). Their correlation applies to a random dense bed of particles of uniform size and shape.

Leva (1947) carried out an extensive investigation on flow through packed beds. Starting with dimensional analysis considerations of the general flow equation through a simple conduit, and by replacing the particle diameter by a modified hydraulic radius of the interstices, he obtained the following equation for turbulent flow:

\[
\frac{\Delta p}{1} = \left( k / g_c \right) \left( 1 - \delta / \delta^3 \right) (d_p G / \mu)^{1.9} \left( \mu^2 \lambda_{1.1} / \rho d_p^3 \right)
\]

(2.13)

Where \( \lambda \) is a shape factor and is equal to \( \phi^{-1} \) as defined by Carman (1937). The value of the constant \( k \) is equal to 3.5.

The work of Ergun (1952) has been widely quoted in the literature. From theoretical considerations, he generalized Forchheimer's equation by examining the phenomenon from the point of view of its dependence upon the flow rate and properties of the fluid,
and upon the fractional void volume, orientation, size and shape of the porous matrix. He concluded that the pressure gradient could be equated to the sum of two terms as follows:

$$\frac{\Delta p}{l} = \left( \frac{A\alpha \mu}{d_p^2} \right) \frac{d_p}{u} + \left( \frac{B\beta \rho}{d_p} \right) \frac{d_p}{u^2};$$ \hspace{1cm} (2.14)

$$\alpha = \frac{(1-\delta)^2}{\delta^3} \quad \text{and} \quad \beta = \frac{(1-\delta)}{\delta^3};$$ \hspace{1cm} (2.15)

Where $A$ and $B$ are dimensionless constants and they are referred to as the first and second Ergun constants for Forchheimer flow. He concluded that the values of the introduced constants are 150 and 1.75 respectively. In 1958, Irmay derived equation (2.14) for steady flow by averaging the Navier-Stokes equations. If equation (2.14) is divided by $(\mu u d_p)$ the following convenient form results:

$$\frac{\Delta p}{l} \cdot \frac{d_p}{\mu u} = C_1 d_p + C_2 Re$$ \hspace{1cm} (2.16)

Where

$$C_1 d_p = \frac{A\alpha}{d_p}, \quad C_2 = \frac{B\beta}{d_p} \quad \text{and} \quad Re = \frac{d_p u p}{\mu}$$ \hspace{1cm} (2.17)

Equation (2.14) is valid for the Forchheimer regime and several authors have investigated its applicability. McDonald et al. (1979), have compared equation (2.14) with data obtained by Rumpf and Gupte (1971) and found that these data indicate a weak functional dependence of $A$ and $B$ upon the porosity. Furthermore, they claimed that a better fit to the data can be obtained by using $\delta^{-5.5}$ instead of $(1-\delta)^2 / \delta^3$. However, this dependence upon porosity was found to be inaccurate over the wide range of porosity from 37 to 64 percent. McDonald et al. (1979) recommend that for engineering applications the values of $A$ and $B$ should be taken as 180 and 1.8 for smooth particles respectively.
Fig. 2.4 is the illustration of some available correlations for prediction of the single-phase pressure drop in porous media.

![Diagram of pressure gradient as a function of modified Reynolds number predicted by different authors.](image)

**Fig. 2.4 Comparison of pressure gradient as a function of modified Reynolds number predicted by different authors.**

As it can be seen in Fig. 2.4, Darcy’s law predicts the flow of fluids in porous media at low fluid flow rates and at higher flows it deviates appreciably. A comparison of Darcy’s law and the Ergun model reveals that both predicts the pressure drop at lower fluids flow rates rather well and at higher flow rates the predictions made by the Ergun model tend to be more accurate as it takes into account the effect of the inertia forces.

The linear terms in equation (2.14) and the unidirectional case of equation (2.9) can be made identical by defining:

\[
K = \frac{d_p^2}{\xi \alpha}
\]  

(2.18)

Where \( \xi \) is a constant and the equation is known as Kozeny-Carman equation, (equation (2.3)), but it is not possible to make the quadratic terms identical.
Symbol, $\delta$, in the proceeding discussion represents the porosity, a property of the medium, which is a measure of the pore volume per unit of the matrix. It is defined in mathematical form as:

$$
\delta = \frac{V_{\text{pore}}}{V_c}
$$

(2.19)

Depending on the nature of the porous medium, the porosity may vary from near zero to almost unity. Table 2.1 the gives the porosity of some of the commonly encountered porous media in nature.

The fractional voidage, porosity, and the specific surface area of the bed, $s_B$, characterize the general structure of a bed of particles. If a point contact occurs between particles and that overlapping loses only a very small fraction of surface area, then Coulson and Richardson (1978) suggest;

$$
s_B = s_0 (1 - \delta)
$$

(2.20)

In a porous medium filling a channel or pipe with rigid or impermeable walls there is, in general, an increase in porosity as one approaches the walls because the solid particles are unable to pack together as efficiently as elsewhere because of the presence of the wall. Some researchers have studied the influence of this so-called wall effect on flow characteristics in cylindrical packed tubes. When a porous medium whose matrix is composed of discrete solid particles is confined in a duct, the wall of the duct affects the local magnitude of the porosity, because the spatial distribution of the particles must conform with the shape of the wall. Benenati and Brosilow (1962) have reported that the zone, within which variations in local porosity occur, extends inward from a cylindrical wall a distance of approximately five spherical particle diameters. Hence, in the presence of a uniform pressure gradient, the average velocity of the flow is higher within this zone. This effect is commonly known as channeling, see Fig. 2.5. Clearly, the annular zone of the wall where the porosity is high comprises an increasing fraction of the cross-sectional area of a cylinder as $d_0/d_p$ decreases.
Many investigators have studied the wall effect. Especially noteworthy among them are Carman (1956), Metha and Hawley (1969), and Riechelt (1972).

Carman (1956) included in his report, as an appendix, a discussion of the porosity in pipes packed with spheres that is interesting and relevant to the present review. Clearly, since the effect of the wall upon porosity propagates inward several sphere diameters, the packing diverges progressively from being random with decreasing value of \(\frac{d_v}{d_p}\). In fact, for \(\frac{d_v}{d_p} \leq 2\), the porosity is completely determined by the geometry. Carman (1956) derived two formulas for \(\delta\) for \(1 < \frac{d_v}{d_p} \leq 1.866\) and \(\frac{d_v}{d_p} = 2\) and he recommended that interpolation based on his equations be used to determine porosity for \(1.886 < \frac{d_v}{d_p} \leq 2\).

To summarize, the packing is completely deterministic for sufficiently low values of \(\frac{d_v}{d_p}\) and it only approaches a condition that is called random as \(\frac{d_v}{d_p}\) increases beyond 40.

Equation (2.14) can be written in the following form:

\[
f = \frac{A}{Re_m} + B
\]  

(2.21)

Where \(f = \frac{d_p \Delta p}{\rho u^2} \) and \(Re_m = Re/(1 - \delta)\) are called the modified friction factor and the modified Reynolds number respectively.

In order to account for the wall effect, Metha and Hawley (1969) defined a hydraulic radius, \(R_H\), as follows

\[
R_H = \frac{\delta d_p}{6(1 - \delta)M} \quad M = 1 + 2 \left[ \frac{\frac{d_p}{d_c(1 - \delta)}}{3} \right]
\]  

(2.22)

Based on this definition, they derived the following modification of equation (2.21)

\[
f = \frac{A}{Re_m} M^2 + BM \quad ; \quad A = 150 \quad B = 1.75
\]  

(2.23)

Metha and Hawley (1969) concluded that wall effects are not significant if the diameter ratio is greater than 50. Fand and Thinakaran (1990), after careful scrutiny of the
experimental data obtained by Metha and Hawley (1969) indicated that the last conclusion is somewhat overly conservative, and that, in fact, wall effects are not significant if the diameter ratio is greater than 40.

Riechelt (1972) further modified equation (2.23). He defined a wall modified hydraulic radius,

$$R_{hw} = \frac{R_h}{M} \quad (2.24)$$

which, when inserted into the defining equation for $f$ and $Re_m$ yields corresponding wall modified parameters;

$$f_w = \frac{f}{M}, \quad Re_w = \frac{Re_m}{M} \quad (2.25)$$

with which he obtained the following modification of equation (2.23);

$$f_w = \frac{A_w}{Re_w} + B_w \quad (2.26)$$

Riechelt (1972) experimentally found the value of $A_w$ to be equal to 150 and concluded that the value of $B_w$ can be calculated from the following empirical formula:

$$\frac{1}{\sqrt{B_w}} = \frac{1.5}{(D/d_p)^2} + 0.88 \quad (2.27)$$

The quantities $A_w$ and $B_w$ are called the first and the second Ergun-Riechelt parameters.

Fand and Thinakaran (1990), have studied the wall effect and reported that for ducts packed with spheres, the wall effect becomes significant for $d_w/d_p < 40$, and consequently the flow parameters become functionally dependent on $d_w/d_p$ for $d_w/d_p < 40$. They also presented a simple empirical equation that expresses the porosity and the flow parameter as functions of $d_w/d_p$ for $1.4 \leq d_w/d_p < 40$.

The variable porosity close to the wall leads to a number of important effects such as flow mal-distribution and channeling. Channeling has also been reported by a number of
other investigators including Schwartz and Smith (1958), and Schertz and Bischoff (1969). Furthermore, the measurements of Benenati and Brosilow (1962), show a distinct porosity variation in packed beds. The functional dependence of the porosity on the distance from the boundary can be found from the experimental results of Benenati and Brosilow (1962). Their results can be represented very well by an exponential function of the following form (Vafai et al., 1985):

\[ \delta = \delta_e \left[ 1 + b \cdot \exp \left( -c y/d_p \right) \right] \]  

(2.28)

The empirical constant \( b \) and \( c \) are dependent on the ratio of the bed to particle diameter. The particle diameters used in their experimental study were 5 and 8 mm. These correspond to experimental bed to particle diameter ratios of 10 and 6.25 receptively. From the results of Benenati and Brosilow (1962) the porosity variation as a function of \( y/d_p \) is found to be almost identical for these bed to particle diameter ratios. The constants chosen to represent the porosity variation were \( b=1; \ c=2 \) for the 5 mm beads and \( b=0.9 \) for the 8 mm beads. These are similar to the constants used by Chandrasekhara and Vortmeyer (1979).

Having discussed the non-linearity caused by the deviation from Darcy’s flow model, other workers have tried to incorporate the non-Darcian terms and therefore arrived at relations to take account of these deviations.

Tien and Hunt (1987) derived the continuity and momentum equations by volume averaging the Navier-Stoks equations and equating the additional terms to empirical relations. The steady governing equations for the average velocity, \( u \), are

\[ \nabla \cdot \langle u \rangle = 0 \]  

(2.29)

\[ \frac{\rho}{\delta^2} \nabla \cdot \langle u \rangle = -\frac{\mu}{K} \langle u \rangle - \rho C |\langle u \rangle| \nabla \langle u \rangle - \nabla \langle p \rangle + \rho g + \frac{\mu}{\delta} \nabla^2 \langle u \rangle \]  

(2.30)

In the above equation, \( K \) and \( C \) are the permeability and inertia coefficient which are determined from the relations developed by Ergun (1952).
In the momentum equation, equation (2.30), the term on the left-hand side is the convective term, which is important in examining the developing region within a porous medium. This term is important only for downstream distances smaller than $K_u D / v$ and therefore is neglected after a short entrance region (Vafai and Tien, 1981). On the right-hand side of the equation, the first term is the Darcian force representing the large pressure loss due to the presence of the solid particles. The second term is the inertial effect which accounts for the additional pressure drop due to the inter-pore mixing found at higher Reynolds numbers. The last term on the right hand side of equation (2.30), the boundary effect, accounts for the shear stress along the solid boundaries and is significant in the near-wall region (Vafai and Tien, 1981). The pressure loss caused by the boundary is generally quite small, but the inclusion of this term can greatly affect the heat transfer by requiring the velocity to be zero at the wall surface.

This volume averaged momentum equation is very different from Darcy's model. Fig. 2.5 depicts a Darcian velocity profile along with the profiles determined by two of the models including non-Darcian effects. The variation in the second profile results from the viscous shear force and the no-slip boundary condition (Vafai and Tien, 1981). The third profile is the channelling profile caused by the non-homogeneous porosity variation near the wall. The variables $C$, $K$ and $\delta$ contained in the governing equations vary with distance from the wall. Close to the wall, as shown in Fig. 2.5, the porosity is high, increasing the permeability and the local velocity. Beyond approximately two particle diameters, the porosity equals the free-stream value. Though the porosity variation oscillates, an exponential decrease is usually assumed, as depicted in the Fig. 2.5 by the broken line. With the porosity variation, the Darcian profile is modified and resembles the third profile (Tien and Hunt, 1987).
The currently available literature on the flow of fluids in porous media is indeed voluminous. Although the research efforts have been directed to elucidating the pressure loss, wall effect, the effect of particle shape and size, and porosity variation on pressure drop there still exist wide discrepancies between the predictions obtained by different models (Fig. 2.4).

The existing literature typically presents correlations which are available for the design and operation of the packed beds with particle sizes larger than 3 mm and higher fluid flow rates. Therefore, the range of the operating variables, i.e. large particle sizes and high flow rates limits their usefulness.

![Porosity Variation, Boundary effect, Darcian flow](image)

**Fig. 2.5** Porosity variation, non-Darcian and Darcian velocity profiles.
2.1.2 Heat transfer

The ultimate goal of studies in convective heat transfer in porous media is to determine the dimensionless heat transfer coefficient, the Nusselt number. A considerable amount of research has been carried out to accomplish this, and empirical correlation for the Nusselt number for a variety of configurations and boundary conditions have been established.

The reviews by Cheng and Hsu (1987) and Combarnous and Bories (1975) provide some well-thought views on a number of different issues in heat transfer in fluid saturated porous media. The work presented by Cheng et al. (1990), Tien and Vafai (1990) aims at providing a comprehensive review that covers most of the aspects of interest in convective heat transfer and multiphase transport in porous media.

Several studies relating to heat transfer by forced convection in packed tubes have been published to date. But most of these studies have dealt mainly with gaseous media, (usually air), as the saturating fluid and very few experimental data with liquids have heretofore been available, (Tsotsas and Schlunder, 1990; and Varahasamy and Fand, 1996). Moreover, most of these investigations concentrate on heat transfer in spaces where the confining walls are subjected to constant temperature.

Forced convection in confined spaces like tubes in the presence of a porous medium is influenced mainly by the flow regime. There are two main areas in which the previous investigations have been carried out; Darcian and the non-Darcian flow. While the latter takes into account the effect of a solid boundary, inertia forces and variable porosity, the former neglects these effects.

One of the earliest investigators in the field was Colburn (1931) who found that the heat transfer rate for forced convection to air in a packed tube is about eight times higher than that of an unpacked tube. Later several investigators, namely Leva (1948), Verschoor and Schuit (1952), Plautz and Jhonstone (1955), and Quinton and storrow (1956) have studied the heat transfer in such media and published empirical correlations for the average heat transfer coefficient for air flow in packed beds. These correlations exhibit discrepancies
as large as 100% from one study to the next, (Varahasamy and Fand, 1996). However, Li and Finlayson (1977) suggested that these discrepancies can be attributed to the dependence of the heat transfer coefficient on the length of the packed bed. Dixon and Cresswell (1979) speculated that the discrepancies are due to the failure to recognize the influence of additional parameters such as the ratio of particle to tube diameter, the ratio of fluid to particle thermal conductivity and the fluid Prandtl number.

Considering the forced convection heat transfer in channels or ducts packed with a porous material in the Darcian regime, the longitudinal volume-averaged velocity $u$ is uniform over the channel cross section. For this reason, when the temperature field is fully developed the relationship between the wall heat flux and the local temperature difference is analogous to the formula for fully developed heat transfer to plug flow through a channel without a porous matrix. In cases where a tube with diameter $d_c$, see Fig. 2.6, is packed with a porous medium, the relation for fully developed heat transfer in the porous medium can be presented as a constant Nusselt number, (Nield and Bejan, 1992).

$$\text{Nu}_{d_c} = \frac{q'(x)}{T_w - T_b(x)} \frac{d_c}{\lambda_m} = 5.78$$  \quad (T_w = \text{constant}) \quad (2.33)$$

$$\text{Nu}_{d_c} = \frac{q''}{T_w(x) - T_b(x)} \frac{d_c}{\lambda_m} = 8$$ \quad (q''=\text{constant}) \quad (2.34)$$

where,

$$\lambda_m = (1-\delta)\lambda_u + \delta\lambda_f$$  \quad (2.35)$$

The forced-convection results given in equations (2.33) and (2.34) are valid when the temperature profile across the channel is fully developed, i.e., sufficiently far from the entrance $x = 0$, see Fig. 2.6.

Kaviany (1985) adopted the governing energy equation for the Darcian regime in a tube with constant wall heat flux and derived a correlation for the fully developed Nusselt
number in terms of the Darcy number, Da. He obtained the same results as those provided by equation (2.34).

In addition to studies which have empirically correlated the observed heat transfer data in porous media, some studies have employed the solution of the governing energy equations to establish a theoretical model to predict the heat transfer coefficient. These equations have been derived and presented in their general form and after the necessary simplifications, with regards to the assumptions made, are solved with respect to the prescribed boundary conditions.

A number of authors have presented these equations. Bejan (1976) presented the Darcian versions in their generalized form, while Vafai and Tien (1981) and Tien and Hunt (1987) established these equations by local volume-averaging techniques and incorporated solid boundary and inertial force effects on forced convection through constant porosity media.

For most flow conditions, an energy equation which regards the porous medium as homogeneous can be derived to satisfactorily represent the transport through the solid-fluid system. To derive such an equation, first the energy equation is volume-averaged (Carbonell and Whitaker, 1984; Whitaker, 1986). Then the steady, volume-averaged energy equation for the solid phase as well as the liquid phase can be written as follows (Tien and Hunt, 1987);
\[ 0 = \nabla \cdot \lambda_s \langle \nabla T_s \rangle + \frac{1}{\Omega_s} \int q_s \cdot n_{sf} \, dA \]  

(2.36)

where \( q_s \) the interfacial solid-phase heat flux, \( \Omega \) is the averaging volume and \( n_{sf} \) is a unit normal to the interfacial area \( A_{sf} \). The equation for the fluid contains both conduction and convection,

\[ \rho_f c_{pf} \left( \langle u_p \rangle \cdot \nabla \langle T_f \rangle + \nabla \langle u_p \cdot T^* \rangle \right) = \nabla \cdot \lambda_f \langle \nabla T_f \rangle + \frac{1}{\Omega_f} \int q_f \cdot n_{fs} \, dA \]  

(2.37)

where \( u_p \) is the pore velocity. All quantities in equation (2.37) correspond to the fluid phase. The last term relates to the interfacial heat flux between the fluid and the solid. The second term on the left-hand side is the dispersion term, resulting from local variations in velocity and temperature (Whitaker, 1986).

By defining the spatially averaged temperature as

\[ \langle T \rangle = \delta \langle T_f \rangle + (1 - \delta) \langle T_s \rangle \]  

(2.38)

the effective stagnant thermal conductivity (based on no flow condition), and the dispersion conductivity are defined as follows:

\[ \lambda_o \nabla \langle T \rangle = \delta \lambda_f \langle \nabla T_f \rangle + (1 - \delta) \lambda_s \langle \nabla T_s \rangle \]  

(2.39)

and,

\[ \lambda_d \nabla \langle T \rangle = -\delta \rho_f c_{pf} \langle u_p \cdot T^* \rangle \]  

(2.40)

The fluid and solid equations are combined to yield the homogeneous energy equation:

\[ \rho_f c_{pf} \langle u \rangle \cdot \nabla \langle T \rangle = \nabla \cdot \left( \lambda_e \nabla \langle T \rangle \right) \]  

(2.41)

where \( \lambda_e \) is the effective conductivity defined as;
\[ \lambda_e = \lambda_o + \lambda_d \] (2.42)

The model assumes the existence of local thermal equilibrium between the two phases, \((T_s - T_f \equiv 0)\), which is valid under most flow conditions, (DeWasch and Froment, 1972; Whitaker, 1986; Tien and Hunt, 1987), or equivalent local temperature gradients \((\nabla T_s \equiv \nabla T_f)\). Under these conditions, a homogeneous model accurately represents the thermal transport and existing empirical correlations can be used to evaluate the stagnant and dispersion conductivities.

Hence, the volume-averaged steady energy equation in a cylindrical porous medium assuming Darcian flow and neglecting the third dimension (axi-symmetric flow) becomes:

\[
\frac{\partial^2 T}{\partial t^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) = \frac{\mu \partial T}{\alpha_e \partial x} + \frac{\partial^2 T}{\partial x^2} \tag{2.43}
\]

In the past, a one-dimensional model has been used to describe the heat exchange process between the packed bed and its walls. The continuum model assumes a uniform fluid temperature for a section perpendicular to the flow direction. However, it was soon realized that the mean temperature predicted by the one-dimensional model deviates significantly from the actual radial mean temperature (De Wasch and Froment, 1972). These considerations lead to the use of a two-dimensional model that accounts for the transverse heat transfer as well as the longitudinal heat transfer by convection. In such a model, the heat transfer in transverse direction can be characterized by the effective thermal conductivity and the wall heat transfer coefficient. The latter parameters were estimated from the temperature profile measurements performed in fluid flowing through packed beds, heated or cooled from the wall (Lerou and Froment, 1977). However, caution must be exercised while extrapolating into regions where measurements are not available. The results of different investigations show many discrepancies, the causes for which have been attributed to the experimental techniques and the various definitions of the parameters employed by different researchers (Balakrishnan and Pei, 1974). This type of analysis was shown to be valid after the introduction of the entrance effect, which necessitates the inclusion of the length effect in these studies. This approach requires
extensive experimental investigations (Li and Finlayson, 1977; Wakao and Kaguei, 1982).

Effective thermal conductivity

The effective thermal conductivity, \( \lambda_e \), appearing in the energy equation, (equation (2.41)) is one of the most important parameters in determining the rate of heat transfer in porous media. Different authors have presented several methods for estimating the effective thermal conductivity. The presented models will be discussed in the following sections.

The effective thermal conductivity of the medium has two components; the stagnant, \( \lambda_o \), and dispersion, \( \lambda_d \), conductivity.

The stagnant conductivity is a function of porosity and conductivity of fluid and solid (Nield and Bejan, 1992). If the heat conduction in the solid and fluid phases occurs in parallel, then the overall conductivity, \( \lambda_A \) is the weighted arithmetic mean of the conductivities \( \lambda_s \) and \( \lambda_f \),

\[
\lambda_A = (1 - \delta)\lambda_s + \delta \cdot \lambda_f
\]  

(2.44)

On the other hand, if the structure and orientation of the porous medium is such that heat conduction takes place in series, with all the heat flux passing through both the solid and the fluid, then the overall conductivity \( \lambda_H \) is the weighted harmonic mean of \( \lambda_s \) and \( \lambda_f \):

\[
\frac{1}{\lambda_H} = \frac{1 - \delta}{\lambda_s} + \frac{\delta}{\lambda_f}
\]  

(2.45)

In general, \( \lambda_A \) and \( \lambda_H \) will provide upper and lower bounds respectively, on the actual stagnant conductivity \( \lambda_o \). We always have \( \lambda_H \leq \lambda_A \), with equality if and only if \( \lambda_s = \lambda_f \).

For practical purposes, Nield (1991b) suggested that a rough and ready estimate correlation for \( \lambda_o \) is provided by \( \lambda_G \), the weighted geometric mean of \( \lambda_s \) and \( \lambda_f \) which is defined by

\[
\lambda_G = \lambda_s^{1 - \delta} \lambda_f^\delta
\]  

(2.46)
This provides a good estimate as long as \( \lambda_s \) and \( \lambda_f \) are not too different from each other (Nield and Bejan, 1992).

Based on the one dimensional heat conduction model, Kunii and Smith (1960) presented the following equation for prediction of stagnant thermal conductivity in unconsolidated packed beds;

\[
\lambda_o = \lambda_f \left[ \frac{\delta + \frac{a_1(1-\delta)}{a_3 + a_2 \lambda_f / \lambda_s}}{a_3 + a_2 \lambda_f / \lambda_s} \right] \quad (2.47)
\]

The values of the constants are given in the reference.

Employing the tube bundle theory, Krupiczka et al. (1958) solved numerically a set of two heat conduction equations in two dimensions and arrived at the following correlation;

\[
\lambda_o = \lambda_f \left( \frac{\lambda_f}{\lambda_s} \right)^n \quad (2.48)
\]

where

\[
n = 0.280 - 0.757 \log_{10} \delta + 0.057 \log_{10} \delta \quad (2.49)
\]

Zehner and Schlünder (1970), based on one dimensional heat flow, presented the following correlation for prediction of the stagnant conductivity:

\[
\lambda_o = \lambda_f \left[ 1 - \sqrt{1 - \delta} + \frac{2 \sqrt{(1-\delta)}}{1-\zeta} \left( \frac{1-\zeta}{1-\zeta B} \right) \ln \left( \frac{1}{\zeta B} \right) - \frac{B+1}{2} - \frac{B-1}{1-\zeta B} \right] \quad (2.50)
\]

where

\[
\zeta = \frac{\lambda_f}{\lambda_s} \quad \text{and} \quad B = 1.25 \left( \frac{1-\delta}{\delta} \right)^{10.9} \quad (2.51)
\]

Prasad et al. (1989) evaluated the correlations of Kunii and Smith (1960), Krupiczka et al. (1958) and Zehner and Schlünder (1970) against their own experimental data as well as other experimental data. Prasad et al. (1989) concluded that the predictions made by the
correlation of the above authors agree reasonably with experimental data as long as the conductivity ratio, $\zeta$, is not higher than unity. Fig. 2.7 compares the different models for prediction of the stagnant thermal conductivity.

Several other authors have also presented empirical models to estimate the value of the stagnant thermal conductivity (Froment and Bischoff, 1979; and Wakao and Kaguei, 1982) which show, according to Prasad et al. (1989), a reasonable agreement.

The effective conductivity also includes thermal transport due to dispersion. Dispersion results from the mixing of local fluid streams as the fluid follows tortuous paths around the solid particles. If a temperature gradient exists across the solid particles, the mixing of the fluid streams yields a net transport of heat. Therefore the process is assumed to be macroscopically diffusive and dependent on the temperature gradient. Dispersion effects become dominant for the case of high Peclet numbers and may already play a reasonably significant role for the cases of low Peclet numbers.
Hunt and Tien (1988a) have performed experiments, which explicitly investigated the effect of thermal dispersion in fibrous media. They were able to correlate their Nusselt number data, for high Reynolds number flows, in terms of a parameter \( u_a L^{1/2} \alpha^{1/2} / \alpha_0 \) where \( u_a \) is the average stream-wise Darcy velocity and \( L \) is the characteristic length. Since this parameter does not depend explicitly on the thermal conductivity, they concluded that dispersion overwhelmed transport due to stagnant conduction. Hunt and Tien were able to explain this behavior using a dispersion conductivity of the form

\[
\frac{\lambda_d}{\lambda_f} = C_t \text{Pe}_K
\]  

(2.52)

In the above equation the Peclet number, \( \text{Pe}_K \), is defined in terms of the velocity of the bed and uses the square root of the permeability of the porous medium as the characteristics length. The constant, \( C_t \), is a numerical dispersion coefficient and its value is determined experimentally.

Among the several investigators who have considered the thermal dispersion in packed beds with large particles (> 2 mm) and have presented empirical correlations for dispersion conductivity, Ranz (1952), Yagi and Wakao (1959), Yagi and Kunii (1960) have shown that the average radial or transverse thermal dispersion conductivity at high Reynolds numbers can be correlated as a linear function of Reynolds number, i.e.

\[
\frac{\lambda_d}{\lambda_f} = C_t \text{Pe}_m
\]  

(2.53)

where the value of constant \( C_t \) lies between 0.09 and 0.1. The mean Peclet number, \( \text{Pe}_m \), is defined in terms of the mean velocity of the liquid and the particle diameter of the bed.

From the early experiments by some of the aforementioned authors, it has been observed that steep radial temperature gradients exist near the heated or cooled wall in the packed columns. These steep temperature gradients were attributed to the channeling effect.

In a series of reports Cheng et al. (1986 – 1990) have analyzed the phenomena of steep temperature gradients in forced convection in a packed channel by taking into account the
effect of thermal dispersion, porosity and non-uniform velocity distribution. They assumed that the local transverse thermal dispersion conductivity, $\lambda_d$ is:

$$\frac{\lambda_d}{\lambda_f} = D_t \text{Re}_p \text{Pr}_f \frac{I(u/u_m)}{1 - \exp\left[- \frac{y}{\omega d_p}\right]} \tag{2.54}$$

where the factor $u/u_m$ is introduced to account for the local velocity variation. In equation (2.54) $l$ is a dimensionless dispersive length (normalized with respect to $d_p$) which can be expressed as follows;

$$l = 1 - \exp\left[- \frac{y}{\omega d_p}\right] \tag{2.55}$$

The empirical constants $D_t$ in equation (2.54) and $\omega$ in equation (2.55) were obtained by comparing the predicted heat transfer characteristics with experimental data. Thus, Cheng et al. (1986 – 1990) reported that the values of $D_t$ and $\omega$ depended on the values of $b$ and $c$ used in equation (2.28). Therefore, they recommended the following values of $D_t$ and $\omega$ for the best match between theory and experiments.

(i) $D_t = 0.17$ and $\omega = 1.5$ if $b = 1.0$ and $c = 2$

(ii) $D_t = 0.12$ and $\omega = 1.0$ if $b = 1.4$ and $c = 5$

Effect of deviation from Darcian flow

Non-Darcian transport describes the non-uniform flow and thermal anomalies often found in flow through packed beds. These effects include the high flow rate inertial pressure loss, near-wall porosity variation, solid-boundary shear, and thermal dispersion. Inclusion of these effects significantly alters the velocity and temperature profiles from those predicted by models using uniform or Darcian flow.

Vafai and Tien (1981) analyzed the effects of a solid boundary and of inertial forces on flow and heat transfer. They solved the related volume-averaged governing equations and concluded that the inclusion of these terms affects appreciably the flow and temperature field as compared with the Darcy flow model.
Poulikakos and Kazmierczak (1987) obtained closed form analytical solutions of the Brinkman equation for parallel plates and for a circular pipe with constant heat flux on the walls, for the case where there is a layer of porous medium adjacent to the wall and clear fluid interior. They also obtained numerical results for the case when the walls were at constant temperature. For all values of Darcy number, the Nusselt number goes through a minimum as the relative thickness of the porous region $s$, varies from 0 to 1. The minimum deepens and attains at a smaller value of $s$, as the Darcy number increases.

Hunt and Tien (1987) also investigated the non-Darcian convection in cylindrical packed beds on the basis of a Brinkman-Forchheimer-extended Darcy equation. To account for the transverse thermal dispersion, they used the effective thermal conductivity varying with radial direction in the volume-averaged steady state energy equation. Their results agree well with experimental data for chemical catalytic reactors and are independent of the extensive experimental relations needed in conventional reactor models.

A re-evaluation of non-Darcian forced and mixed convection in cylindrical packed tubes was carried out by Chu et al. (1994) who claimed that there were great discrepancies between the values of non-Darcian Nusselt number reported by different workers. Chu et al. (1994) concluded that the discrepancies between the existing models are mainly due to the modeling of channeling and thermal dispersion effects for the cases of low and high Peclet number, respectively. Chu et al. (1994) used the flow model reported by Chandrasekhara and Vortmeyer (1979) which was also used by Poulikakos and Renken (1987); the predicted values of Nusselt number were found to agree well with data of Quinton and Storrow (1956) for flow of air. By adopting a non-linear dependence of $\lambda_d$ and $\text{Pe}$ they showed a good agreement between their own experimental data and those reported in the literature.

Recently published research work emphasizes the importance of porosity changes and the corresponding velocity changes near the wall, and has shown, by comparison with experiments, that the theoretical prediction of chemical reactor performance can be improved by assuming a non-uniform velocity distribution within the packed beds (Kalthoff and Vortmeyer, 1980). Vortmeyer and Schuster (1983) solved the Brickman-
extended Darcy equation by a variational method and explained the large deviations between calculated and measured profiles.

Vafai (1986) performed a boundary layer analysis for a variable porosity medium using the method of matched asymptotic expansion on the Brinkman-Forchheimer-extended Darcy equation and the energy equation. The concept of the triple momentum boundary layer to incorporate the channeling effect in variable porosity media was introduced in this work. This type of analysis relies directly on the fundamental principles, thus enabling a more thorough analysis of the problem. Cheng and Hsu (1986) performed a numerical analysis for a fully developed forced convective flow through a packed bed, confined by concentric walls maintained at different temperatures. In their work, Cheng and Hsu (1986) based their analysis on the Brinkman model with variable permeability and the mixing length theory proposed by Cheng and Vortmeyer (1988) for transverse thermal dispersion.

Variable porosity effects in a circular pipe were also examined numerically by Poulikakos and Renken (1987), for the case of a fully developed velocity field. They assumed that the porosity variation had negligible effects on the thermal conductivity, an assumption which breaks down when there is a large difference between the thermal conductivities of the two phases (David et al., 1991). Poulikakos and Renken (1987) found that in the fully developed region the effect of channeling was to produce a Nusselt number increase of 22% (above the value based on the Darcy model) for a circular pipe.

Varahasamy and Fand (1996), after studying the available literature on heat transfer in porous media composed of spheres, commented that every such study had dealt with a gaseous medium (commonly air) as the saturating medium and no experimental data with liquids had heretofore been available. They also stated that the available numerical models used to study heat transfer in pipes packed with spheres did not predict heat transfer coefficients accurately for potentially useful ranges of the variable $d_c / d_p$. Therefore, Varahasamy and Fand (1996) systematically investigated heat transfer in two horizontal test sections of different diameter, hence providing different characteristic
dimensions. They extended their study from the Darcy regime through the Forchheimer regime to the turbulent flow regime.

In analyzing their data, they presented a correlation for the Nusselt number which includes the wall effect due to spatial variation of the porosity near the wall as follows;

\[ \text{Nu} = C \text{Re}^m \text{D}^n \left( \text{Pr}_{w} \text{Re}_{w} \right)^q \left[ \arctan \left( \frac{d_c}{d_p} \right) \right]^r \]  

where \( m, n, p, q, \) and \( r \) are constants provided by the authors for different flow regimes.

Varahasamy and Fand (1996) demonstrated that equation (2.56) with appropriate constants for different flow regimes represents the mean values of \( \text{Nu} \) obtained in their study for \( 3 < \frac{d_c}{d_p} > 15, \) and; \( x/d_c \geq 8 \) (where the flow is thermally fully developed) with a degree of accuracy that is deemed acceptable for design purposes.

The foregoing review of the literature on heat transfer in saturated porous media reveals that, in spite of the completion of numerous research programs, there are areas which need to be explored in depth. Most of the studies are aimed at predicting the temperature profiles and consequently the heat transfer coefficients in the non-Darcian region either empirically or by theoretical means. Some of these correlations can be simplified to predict heat transfer in the Darcian region, however, since they are designed for larger particles, the results will contain large errors.

Most of the empirical correlation presented by different authors to cater for lower Reynolds number are based on air as the saturating fluid, and the information on water as a saturating fluid especially in beds whose matrices are composed of spheres is rare. Nearly all the studies employ constant wall temperature as one of the boundary conditions to investigate the heat transfer coefficient. Therefore, the prediction of the heat transfer coefficient at low fluid flow rates in media made of small particle diameters (specially spheres) saturated by water under constant heat flux has not been investigated sufficiently.
2.2 Two-phase flow phenomena in packed beds

2.2.1 Hydrodynamics

Wide ranging experimental and theoretical studies have been made to investigate the hydrodynamics of two-phase flow through packed beds. However, a careful inspection of the relevant literature reveals that a majority of research work in this field has been directed at developing predictive methods for descending co-current gas-liquid flow. (Specchia and Baldi, 1977; Charpentier and Favier, 1975; Kan and Greenfield, 1979; Sai and Varma, 1988; Ellman et. al., 1990; Larchi et. al., 1991; Shiny and Varma, 1995). Moreover, these investigations vary considerably in the representation of the correlated results and they do not cover descending co-current gas-liquid flow systems comprehensively.

In comparison with descending co-current gas-liquid flow, less information is available on hydrodynamics of beds involving the ascending movement of a gas-liquid two-phase flow. The pervious investigations hence do not provide a general correlation for the hydrodynamics of two-phase flow. Furthermore, data from research on pressure drop of ascending a gas-liquid flow under steady-state conditions are few in numbers. The predictions of pressure drop and liquid/gas hold-up resulting from previous investigations vary significantly as different operating variables were employed.

In the following sections the main features of two-phase flow through porous media will be reviewed.

Flow regimes

Many attempts have been made by different investigators to identify the flow regimes encountered when two immiscible fluids flow co-currently upwards through porous beds. With some exceptions, all studies have been made through observations of the flow patterns in packed columns and the results are mostly reported in the form of a map which identifies the occurrence of different regimes with respect to fluid flow rates.
Larkins et al., (1961) observed different flow regimes by establishing a constant liquid flow rate through the test bed and then increasing the gas flow rate so that the observation of transitions in the flow pattern could be observed. As the gas rate was increased, small bubbles of gas were seen in the liquid until a point, where the liquid layer on the particles started to get thinner and slugging occurred. Up to this point, the flow was termed homogeneous. As the gas rate was increased further, a fully developed slugging flow appeared. At higher gas flow rates a situation was reached where the liquid was carried through as a mist, with very little liquid clinging to the surface of the packing material. Hence in reporting the data on the mode of flow a code was used to designate four types of patterns as follows: homogeneous, transition, slug, and close slug or blur.

Eisenklam and Ford (1962) first noted the existence of various hydrodynamic regimes during co-current movement of a two-phase gas-liquid flow through a stationary bed of finely divided granules. At low liquid velocities and high gas velocities, a regime of “true” two-phase flow occurs in the free volume of the packed bed. At low gas velocities and high liquid velocities, particularly in beds of fine particles, the gas has a tendency to move through channels, i.e., the regime of so-called single-phase flow through the pores is observed (Turpin and Huntington, 1967; Eisenklam and Ford, 1962). Saada (1972), using the data obtained by Eisenklam and Ford (1962), constructed a diagram of two hydrodynamic regimes – a single-phase regime and a regime of two-phase flow – for flow through beds of finely divided particles. He also proposed an empirical relationship, which gives a gas Reynolds number at which the transition from one regime to the other occurs.

Turpin and Huntington (1967) reported the observation of three distinct flow patterns. These were termed bubble flow, slug flow, and spray flow. The relative location of each of these flow regimes in terms of the mass flow rates of the respective phases was given graphically. In order to visualize each of the flow regimes, the liquid flow rate was kept constant and the gas flow rate varied from zero to its maximum value. With single-phase liquid flow established in the bed, the bubble regime was encountered with the introduction of very low gas flows. The range of this regime was reported to be wider at higher liquid flow rates. As the gas rate was increased a non-homogeneous regime was
observed and termed as slug flow, which was characterized by alternate portions of denser and less dense mixtures of the two phases passing through the column. A third pattern named as spray flow was observed at higher gas flow rates. The onset of this regime was reported to be characterized by the disappearance of the difference in densities between the alternate slugs. Turpin and Huntington (1967) also reported that data regarding the pressure gradient and liquid hold-up could be correlated independent of the flow regimes. Since no abrupt change of pressure and liquid hold-up was noted for any transitions between flow types, they claimed that the graphical representation (Fig. 2.8) of the flow regimes could be utilized for up-ward as well as downward flow.

![Flow map and regime transitions, Turpin and Huntington (1967).](image)

Khan and Varma (1997) identified bubble, pulse, and spray flow by increasing the gas flow rate. They presented experimental data in the form of a flow map and compared them with those reported by Saada (1974). They claimed that the map by Saada, which divides the flow into low and high interaction regimes was inadequate as the interaction between the phases is different in pulse flow from that in the spray regime. Khan and
Varma (1997) also presented empirical corelations for predicting the transition between the different regimes. Fig. 2.9 illustrates the flow regime transition suggested by Khan and Varma (1997).

![Flow regime transitions, Khan and Varma (1997)](image)

**Fig. 2.9 Flow regime transitions, Khan and Varma (1997)**

**Pressure drop**

The pressure drop investigations of two-phase flow have been approached from two different view points. The first method employs the usual friction factor and evaluates it as a function of all possible variables affecting the driving force inside the medium in terms of dimensionless groups. The second approach to the correlation of data employs the well known Lockhart-Martinelli parameter which is defined by Larkins et al. (1961).

One of the early investigations on hydrodynamics of two phase flow, was performed by Larkins et al. (1961) who correlated the frictional pressure drop in co-current downward flow through packed towers in Lockhart-Martinelli fashion, i. e. equation (2.57). He based his correlation of the experimental data on the knowledge of single-phase friction losses for the gas, \( \Delta p_g/L \) and the liquid, \( \Delta p_l/L \) when they flow alone in the bed. He
claimed that his model can hold for any orientation of the bed as long as co-current flow is maintained. His experimental data were correlated with a standard deviation of 13% on friction loss and 16% on liquid saturation.

\[
\log_{10}\left(\frac{Ap_{fl}}{L} \left(\frac{\Delta p_{l}}{1} + \frac{\Delta p_{g}}{1}\right)\right) = \frac{0.416}{(\log_{10} \chi)^2 + 0.666}
\]  

(2.57)

where

\[\chi = \left(\frac{\Delta p_{l}}{1} / \frac{\Delta p_{g}}{1}\right)^{0.5}\]  

(2.58)

\(\Delta p_{l}/1\) and \(\Delta p_{g}/1\) are the frictional pressure drop of liquid and gas when flowing alone through the bed. In his analysis Larkins et al. used the well-known Ergun equation to predict the pressure drop in single-phase flow. They also used their own experimental data to determine the coefficients in the Ergun equation. Therefore,

\[
\frac{\Delta p}{1} \cdot \left(\frac{g_{cd}d_{p}^{3}}{\mu^{2}}\right) \cdot \left(\frac{\delta}{(1 - \delta)}\right) = Re \cdot (150 + 1.75 Re)
\]  

(2.59)

Turpin and Huntington (1967) achieved a basis for correlation of their experimental data by introducing a friction factor, \(f_{fl}\), as given by

\[
\ln f_{fl} = 8.0 - 1.12 (\ln Z) - 0.0769 (\ln Z)^{2} + 0.0152 (\ln Z)^{3}
\]  

(2.60)

which was found to correlate rather well with parameter, \(Z\), where

\[Z = \frac{Re_{g}^{1.167}}{Re_{l}^{0.767}}\]  

(2.61)

The frictional pressure drop was then calculated using;
\[
f_{fl} = \frac{\Delta P_{fl}}{1} \cdot \frac{D_e \varepsilon_c}{2 \rho_g u_g^2}
\] (2.62)

In equation (2.62) \( u_g \) is the velocity which the gas would have if it were in single-phase flow through the unpacked conduit at the entering density \( \rho_g \).

Turpin and Huntington (1967) presented the same correlations to cater for downward flow with different coefficients to suit. However, the comparison of their correlation with Larkins et al. (1961) showed differences of 30% to 50% for small values of \( Z \), which was attributed to the wide range of experimental variables employed. A comparison of upward and downward flow revealed differences of up to 30% between the two flow directions over the entire range of \( Z \) values.

Chu et al. (1983) studied the pressure drop of two-phase flow through porous media. They used the Ergun (1952) equation for each phase separately. The correlation for two-phase frictional pressure drop is presented as the sum of the single-phase frictional pressure drop with variables incorporated to take into account the contribution of each phase to the two-phase frictional pressure drop. The variables were presented in terms of the active void fraction, \( \varepsilon_{ac} \). In this investigation, beds containing uniformly sized particles as well as mixtures of different sizes, ranging from 1 to 6 mm, were used. Chu et al. (1983) presented their correlation in the following form:

\[
\frac{\Delta P_{fl}}{1} = \frac{A_{\mu_1} u_1^2}{\eta_l} + \frac{A_{\mu_g} u_g^2}{\eta_g} + \frac{B_{\rho_l} u_1^2}{\eta_l} + \frac{B_{\rho_g} u_g^2}{\eta_g}
\] (2.63)

where the variables \( k_l, k_g, \eta_l, \eta_g \), are given by the following empirical correlations:

\[
k_l = (1 - \varepsilon_{ac})^3 \left[ \frac{(1 - \varepsilon_{ac}) + (1 - \varepsilon_{ac})^2}{2} \right]
\] (2.64)

\[
k_g = 0.2 \varepsilon_{ac} + 0.8 \varepsilon_{ac}^2
\] (2.65)
\[ \eta_l = (1 - \varepsilon_{ac})^3 \] (2.66)

\[ \eta_g = 0.4 \varepsilon_{ac}^2 + 0.6 \varepsilon_{ac}^3 \] (2.67)

A comparison of their experimental data with the presented correlation showed that the predicted values for uniform particles were within ±35% and for mixtures of particles within +40% to −35%. They concluded that the agreement was reasonable considering the sensitivity of the presented correlation to uncertainties in mean particle diameter, bed porosity and void fraction.

Tung and Dhir (1988) developed a hydrodynamic model to predict the pressure drop for one-dimensional flow through porous media.

\[ p^* = \frac{(1-\varepsilon + \rho^* \varepsilon)}{(1-\rho^*)} + f_{pl}^* + f_{pg}^* \] (2.68)

The model considers flow configuration and takes into account the particle-gas drag, particle-liquid drag, and liquid-gas interfacial drag which were then evaluated in conjunction with force balances on the two phases to obtain the pressure gradient as a function of liquid and gas superficial velocities. In their analysis, pore sizes much larger than the bubble diameters were encountered because of the very large particles used. Therefore, the bubble diameter was correlated in terms of surface tension and used in the analysis. They found the results to agree very favorably with data of Chu et al. (1983) for both co-current and counter-current flow conditions.

Ford (1960) studied co-current flow in beds made up of particles with approximately 1 mm in diameter. He identified two regimes of flow that took place inside the pores, namely "single-phase pore flow" and "two-phase pore flow", for which he introduced two dimensionless correlations (equations (2.69) and (2.70)). The selection of the correct correlation for each of the two flow regimes was made possible by employing the suggested prediction for liquid hold-up. He reported that the value of liquid hold-up at the transition point between the two flow regimes had a constant value of 0.43 ± 0.03% for
all gas and liquid Reynolds numbers. Saada (1974) showed that Ford’s transition value was a variable, instead suggested a maximum Reynolds number of 11.3. So he found it necessary to repeat part of Ford’s work.

Single phase pore flow;

\[
\frac{1}{g \rho_l} \frac{\Delta p_{ft}}{l} = 0.0485 \text{Re}_l^{0.67} \text{Re}_g^{0.3} \left(\frac{\mu_l}{\mu_g}\right)^{0.8}
\]

(2.69)

Two phase pore flow;

\[
\frac{1}{g \rho_l} \frac{\Delta p_{ft}}{l} = 0.0407 \text{Re}_l^{0.29} \text{Re}_g^{0.57} \left(\frac{\mu_l}{\mu_g}\right)
\]

(2.70)

Saada (1974) applied the Buckingham \(\Pi\)-theorem taking into account the variables; \(G, L, d_p, d_c, z, \rho_g, \rho_l, \mu_g, \mu, \sigma, \) and \(\delta\). He also introduced two correlations based on the gas and liquid Reynolds numbers for the two regimes previously identified by Ford (1960). The transition between the two regimes was given a Reynolds number, \(\text{Re}^*\), which was developed by equating the correlations suggested for the two regimes. The correlations by Saada took the following form;

For single-phase pore flow;

\[
\frac{1}{g \rho_l} \frac{\Delta p_{ft}}{l} = 0.024 \text{Re}_l^{0.6} \text{Re}_g^{0.39} \left(\frac{d_p}{d_c}\right)^{-1.1}
\]

(2.71)

and for two-phase pore flow;

\[
\frac{1}{g \rho_l} \frac{\Delta p_{ft}}{l} = 0.027 \text{Re}_l^{0.35} \text{Re}_g^{0.51} \left(\frac{d_p}{d_c}\right)^{-1.15}
\]

(2.72)
Khan and Varma (1997) identified three distinct regimes, which they observed in two-phase flow through porous beds made up of particles with 6 mm in diameter. They therefore, presented the correlated experimental frictional pressure for each regime. Khan and Varma (1997) argued that separate correlations for the different regimes were advantageous when compared to the correlation of Turpin and Huntington (1967). The experimental data were satisfactorily correlated if the following correlations were used for bubble, pulse and spray flow respectively.

\[
\text{Re}_g^* = 0.44 \text{Re}_l \left( \frac{d_p}{d_c} \right)^{0.38}
\]

(2.73)

\[
f = 3 \times 10^7 \text{Re}_g^{0.18} \text{Re}_l^{-1.7} (d_p/d_c)^{1.5}
\]

(2.74)

\[
f = 2.36 \times 10^7 \text{Re}_g^{0.26} \text{Re}_l^{-1.7} (d_p/d_c)^{1.5}
\]

(2.75)

\[
f = 3.91 \times 10^5 \text{Re}_g^{1.12} \text{Re}_l^{-1.82} (d_p/d_c)^{1.5}
\]

(2.76)

where

\[
f = \frac{\Delta p_{fl}}{2d_p} \frac{2d_p}{u_l^2 \rho_l}
\]

(2.77)

The transitions between the flow regimes were given by the following empirical correlations:

Bubble to pulse flow

\[
G = 0.1 + \frac{\exp(0.2L)}{200 + \exp(0.2L)}
\]

(2.78)

and for pulse to spray flow

\[
G = 0.06L + 0.39
\]

(2.79)
Goto and Gaspillo (1991) studied gas-liquid flow through beds of small packing and presented correlations to predict frictional pressure drop in downward as well as upward flow in a manner similar to that of Lockhart and Martinelli. In their investigation, the Ergun (1952) equation was employed to estimate the single phase pressure drop. The correlation, which predicts pressure drop in terms of single-phase pressure drop due to each fluid in the system, is given by

$$\ln y = \frac{0.55}{\ln(\chi'/1.2)^2 + 0.666} \quad (2.80)$$

$$y = \frac{\Delta p_{fi}}{1} \left( \frac{\Delta p}{1} + \frac{\Delta p}{1} \right)^{-1} \quad (2.81)$$

$$\chi' = \frac{\Delta p_{li}}{1} / \frac{\Delta p_{gi}}{1} \quad (2.82)$$

Fig. 2.10 depicts a comparison between some of the aforementioned correlations. As it can be seen from this figure, the discrepancies between the predictions of these models are high.

**Liquid and gas hold-up**

An important parameter for ascending gas-liquid flow calculations is the gas hold-up. In general, the gas hold-up in packed beds is greater than that in empty sparged columns. The gas hold-up can be divided into two components: a dynamic, $\varepsilon_g$ and a static, $\varepsilon_{gs}$. The dynamic component is determined experimentally at zero gas velocity. The porosity of the medium, the overall liquid hold-up, $\varepsilon_i$, and the gas hold-up, $\varepsilon_g$, are related to one another as follows

$$\delta = \varepsilon_i + \varepsilon_g \quad (2.83)$$

As indicated by the published data, the overall gas hold-up increases with increasing velocity of the gas stream and is dependent on the physical properties of both moving phases, the geometry of the stationary bed, and the diameter of the column. The values of
gas hold-up obtained for different packing are substantially different (Zhukova et al., 1990).

Most of the investigations which have dealt with the hydrodynamics of flow have studied the effect of gas hold-up on the flow mechanism. These studies, in addition to providing the prediction of hold-up in porous beds, are also used for predicting pressure drop.

![Comparison of different models for prediction of the pressure drop.](image)

Fig. 2.10 comparison of different models for prediction of the pressure drop.

Larkins et al. (1960) established the following relationship between liquid hold-up and the Lockhart-Martinelli parameter in packed beds with packing porosity between 0.375-0.52,

\[
\log_{10} \varepsilon_l = -0.774 + 0.525(\log_{10} \chi) - 0.192(\log_{10} \chi)^2
\]  

(2.84)

Using their correlation, 80% of the experimental data for all flow conditions fell within 20% of the correlating curve with a standard deviation of 16%.
Turpin and Huntington (1967) correlated the liquid saturation in terms of mass flow rates of fluids. The comparison of their model with experimental data for upward flow was quite variable; but essentially all data points were within ±25% of the suggested correlation. The best curve fit was found to be a fourth-degree polynominal. However, it was observed that this curve was closely approximated by the linear least square curve shown below

\[ \varepsilon_l = -0.035 + 0.182 \left( \frac{L}{G} \right)^{1.04} \text{ for } 1.0 \leq \left( \frac{G_l}{G_g} \right)^{0.24} \leq 6.0 \] (2.85)

Chu et al. (1983) observed trapping of some bubbles in the pores between particles which gave rise to inactive void fraction. They found this inactive void fraction to decrease with increasing superficial velocity of the fluids in the system and the maximum value was associated with 3 mm diameter particles and decreased with smaller or larger particles. The value of the inactive void fraction was found to be the difference between the total and the active void fraction. In their investigation, no effort was made to correlate the inactive void fraction. Chu et al. (1983) presented the following semi-empirical drift flux model to predict void fraction.

\[ \varepsilon_g = u_g \cdot \left\{ \sqrt{2(u_g + u_l)} + 4.6 \left( \frac{\delta^3}{1 - \delta} \right)^{1/2} \left[ \frac{d_p (\rho_l - \rho_g) g}{\rho_l} \right]^{1/2} \left[ \frac{\sigma_g (\rho_l - \rho_g)}{d_p^2} \right] \right\}^{-1} \] (2.86)

They reported that most of the observed void fractions for all particle sizes were predicted within +10% to −27% error. However, these error limits rose to +27% and −23% for mixtures of particles.

Tung and Dhir (1988) developed a one-dimensional model for flow in porous media. Their model took account of drag forces between the particles and the gas-liquid system as well as drag forces on the gas. This yielded

\[ \varepsilon (1 - \varepsilon) + \varepsilon f_{pl}^* - (1 - \varepsilon) f_{pg}^* - f_i^* = 0 \] (2.87)

They claim that their predicted results agreed well with experimental results obtained by Chu et al. (1983).
Saada (1974) presented a dimensional analysis procedure to arrive at a correlation to predict the liquid hold-up. He revised Ford’s work and presented two correlations for hold-up in each of the flow regimes previously identified by Ford (1960). He also suggested a Reynolds number, $Re^*$, by which a particular flow regime could be predicted. The results of his investigation are summarized as the following correlation

$$\varepsilon = K \left( \frac{Re_l}{Re_g} \right)^a$$  \hspace{1cm} (2.88)

where the values of $(K \text{ and } a)$ are $(0.48, 1.25)$ and $(0.32, 0.07)$ for flow below and above the transition Reynolds number, respectively. The Reynolds number at which the transition occurs is given by;

$$Re_g^* = 0.44 Re_l \left( \frac{d_p}{d_c} \right)^{0.38}$$  \hspace{1cm} (2.89)

Goto and Gaspillo (1991) introduced a correlation for dynamic hold-up for both upward and downward flow. Their correlation employed a Lockhart-Martinelli type parameter, which is defined as the ratio of single phase pressure gradients when each fluid flows alone in the system. They reported that a good agreement between the calculated results and experimental data was achieved, using the following correlation.

$$\ln \left( \frac{\varepsilon_l}{\delta} \right) = -0.442 (\ln \chi')^2 + 0.386 \ln \chi' - 0.178$$  \hspace{1cm} (2.90)

The comparison between different correlations is illustrated in Fig. 2.11.

The preceding review of the literature on hydrodynamics of two phase flow shows that in spite of its importance in many engineering application, the existing research does not cover all aspects of the fluid flow especially when it comes to ascending two-phase flow of gas/liquid systems.
The observations of flow patterns reported by different investigators suggest that there are two or three different regimes. Some investigators have presented different correlations for every type of flow regime encountered while others are of the opinion that one correlation is adequate to describe the flow mechanism. The presented correlations predict different results for hold-up, which is mainly due to the experimental procedure as well as the correlating procedure of the experimental data, (Fig. 2.11). Pressure drop values predicted by different investigators show wide discrepancies, see Fig. 2.10. As far as the supporting data for these models are concerned, they are confined to a very narrow range of system geometry as well as operating range, and that is why the results are quiet scattered. The correlations due to Goto and Gaspillo (1991) and Larkins et al. (1961) show a relatively close agreement when used to predict a system geometry other than that used by the authors, but even these models produce diverging results when used to predict different particle diameters, see Fig. 2.12. Evidently, further work should be performed to highlight the theoretical understanding of porous media composed of smaller particle diameters.
Fig. 2.12 Comparison of different models for prediction of the pressure drop.
2.2.2 Heat transfer

The subject of two-phase heat transfer in porous media has gained considerable attention during the past two decades. The studies mostly concentrate on drying of different porous materials, condensation in porous materials, heat pipe application and geothermal applications. Dybbs and Schweitzer (1973) formulated the problem of non-isothermal flow in porous media using the volume-averaging technique for low Reynolds number where the inertia term is negligible. Dinulscscu and Eckert (1980) studied the problem of moisture migration due to a temperature gradient in porous media. They performed a one-dimensional analysis of this problem and produced an analytical solution. The first set of boundary conditions used were constant temperature and impermeable wall boundaries at the two sides of the porous medium.

Huang et al. (1978) established a model for heat and moisture transfer in concrete slabs. In their work the diffusive and convective transport were analyzed and the effect of the temperature gradient was emphasized. Baladi et al. (1981) studied the transport processes in soil around a buried heat source. They presented a one-dimensional model to describe the phenomena of moisture migration and energy transport. In the numerical model of the porous system, distinct wet and dry regions were assumed to exist. Harmathy (1969) was one of the first authors who studied this problem in the context of drying. An important assumption used in this work was to consider all moisture transfer to take place in the gaseous phase only.

Lyczkowski and Chao (1984) also reported studies on two-phase drying. In their work, the authors give a comparison of the two phase drying model developed by Lyczkowski and the Stefan (1983) model for coal drying. In the model discussion non-condensables are not taken into account. Plumb et al. (1985) studied the problem of drying in softwood. They formulated a model for multiphase transport processes taking into account both diffusion and capillary action in the liquid phase.

The phenomena of condensation in porous media have been investigated by several research groups. Ogniewicz and Tien (1981) reported an analytical work on condensation in porous wall insulation. The model developed is one-dimensional and steady state, and
it accounts for convective and diffusive transport. The problem studied consists of a porous slab subjected to different environments on two sides. Nilson and Romero (1980) investigated another condensation problem in a porous matrix and formulated the one-dimensional case of a porous matrix with an inflow of saturated vapor at the boundary.

Vafai and Whitaker (1986) studied a different problem using a two-dimensional transient model. In this study, the multiphase transport processes with phase change in porous slabs were also modeled by using the volume averaging technique.

In the recent investigation by White and Tien (1987a), laminar film condensation in a porous medium was analyzed. In their analysis, the no-slip boundary condition was employed for the velocity at the wall, using an exponential function for the porosity. Modified boundary layer equations were employed for the region near the wall. White and Tien (1987b) also reported experimental studies on the same topic.

Chuah and Carey (1985) presented an analytical model for the two-phase transport, which results from heating of a liquid-saturated porous medium with variable porosity from below. In this study the porosity of the medium is expressed in exponential form. Darcy’s law is used for liquid and vapor transport. In this study, empirical correlations are used extensively for some variables based on previous experimental data. Permeability is expressed by the Kozney-Carman formula in terms of porosity. Relative permeabilities of liquid and vapor are expressed in terms of liquid saturation based on experimental data. Capillary pressure is described by correlation of Leverett (1941) which expresses capillary pressure in terms of porosity, permeability and surface tension.

Almost all the models developed for analyzing problems of multi-phase transport processes and phase change utilize the assumption of local thermodynamic equilibrium between the phases in the porous system. While this is a good assumption for many practical applications, it might not be a realistic assumption for cases in which the fluid velocity is high and / or where the system is subjected to an intense heat flux.

Wong and Dybbs (1978) used this assumption to obtain the volume-averaged energy equation in single-phase flow in a saturated porous medium. The heat transfer between
the solid and fluid phases is formulated by a convection term. A relationship is also established for the effective thermal conductivity of the saturated porous medium in terms of the effective thermal conductivities of the individual phases.

Limited experimental work has been performed for obtaining transport coefficients for two-phase flow through porous material. Singh and Dybbs (1979) reported one investigation. Besides forming the conservation equations for multiphase systems, they also carried out experimental work for determining the permeability of the porous matrix and the effective thermal conductivity of the fluid saturated porous system.

Naik and Dhir (1982) studied forced flow evaporative cooling of a volumetrically heated porous layer. In this investigation the temperature rise and pressure drop experienced by an evaporating coolant flowing through an inductively heated porous layer was studied experimentally. A theoretical model for the temperature profile in the liquid region and in the two-phase region was formulated and found to compare well with experimental results.

The above review of the literature indicates that the available studies mainly deal with the fluid flow through porous media accompanied by phase change. However, Zhukova et al. (1990) did a thorough review of the literature on two-phase gas-liquid flow in stationary beds and concluded that research on the heat transfer in equipment with stationary beds with ascending gas-liquid flow were practically non-existent. They pointed out that under any given condition, the heat transfer coefficient at the wall in such beds should be higher than in beds with descending flow of gas and liquid streams.

However, a comprehensive search of the present author revealed that the first related study was made by Weekman and Myers (1965) on heat transfer characteristics of co-current gas-liquid flow in packed beds. In their report they cited only one paper (Larkin et al., 1961) which had been concerned with pressure drop and liquid holdup over a wide range of co-current, two-phase conditions in packed beds. They carried out experiments over a wide range of air-water flow rates, 0-1.35 and 0-270 kg/m².s respectively using particles 3.5 – 6.5 mm in diameter. The thermal conductivity of the bed material ranged
from 0.173 – 0.52 W/m K. Weekman and Myers (1965) modeled the heat transfer coefficient using the energy equation as follows

\[
(Gc_{p,g} + Lc_{p,l}) \frac{\partial T}{\partial z} = \lambda_e \left( \frac{1}{r} \frac{\partial}{\partial r} \frac{\partial T}{\partial r} \right)
\]

(2.91)

They solved the above equation for constant temperature at the walls of the bed and reported the following expression for the equivalent heat transfer coefficient

\[
h_o = \frac{2.839}{R_o} \lambda_e + \frac{0.183 R_o (Lc_{p,l} + Gc_{p,g})}{z}
\]

(2.92)

To find a model which could predict the effective thermal conductivity used in equation (2.92), they assumed that all the various heat transfer mechanisms could be lumped into two terms; eddy diffusion in radial direction plus molecular conduction in the stagnant bed. Based on their assumptions they reported the following expression for the effective thermal conductivity

\[
\frac{\lambda_e}{\lambda_f} = \frac{\lambda_e}{\lambda_f} + \gamma Re_l \cdot Pr_l + \beta \left( \frac{\lambda_g}{\lambda_l} \right) \cdot Re_g \cdot Pr_g
\]

(2.93)

The values of \( \lambda_e \), \( \gamma \), and \( \beta \) were found by the method of least squares to be 7.71, 0.00174, and 0.174 respectively. In their investigation, Weekman and Myers (1965) reported that the primary effect of the gas is to impart a greater velocity to the liquid phase, thus, while the fraction of the liquid velocity in the radial direction is the same, the radial component will be larger for two-phase flow. Based on the work of Larkins et al. (1961), they were able to calculate the liquid mass velocity based only on the liquid flow area and thus account for the increased liquid velocity due to the presence of the air. They then defined a modified liquid Reynolds number (Reynolds number / porosity × liquid holdup) in which the effect of gas rate was accounted for, since as the gas rate increases liquid hold-up decreases and consequently the modified liquid Reynolds number increases, therefore equation (2.93) was changed to give a single correlation for all gas flow rates with the coefficients replaced by 7.03, 0.000285 and 0 respectively. Weekman
and Myers (1965) claimed that equation (2.93) with the new values for constants \( \lambda_c \), \( \gamma \), and \( \beta \) satisfactorily accounts for the increase in the radial liquid velocity due to the presence of the gas phase.

Hashimoto et al. (1976) acknowledge the work of Weekman and Myers (1965) to be the only research that was available and commented that Weekman and Myers (1965) disregarded the heat transfer resistance near the wall and assumed that the only heat transfer resistance is in the inner sections of the packed bed. Therefore, they suggested that the effective thermal conductivity in two-phase flow of gas and liquid through a packed bed can be represented as the sum of three terms to avoid the deficiency constituted with the Weekman and Myers model. They solved the basic equation for temperature distribution as represented by Weekman and Myers (1965) with the proper boundary conditions. The effective thermal conductivity which was used, was presented in the following form

\[
\frac{\lambda_e}{\lambda_1} = \left( \frac{\lambda_c^0}{\lambda_1} \right) + (\alpha \beta)_g \left( \frac{d_p G}{\mu_g} \right) \left( \frac{C_{p,g} \mu_g}{\lambda_g} \right) \frac{\lambda_g}{\lambda_1} + (\alpha \beta)_l \left( \frac{d_p L}{\mu_l} \right) \left( \frac{C_{p,l} \mu_l}{\lambda_l} \right) \tag{2.94}
\]

The terms on the right hand side of the above equation represent the effective thermal conductivity of the wet packed bed and the effective thermal conductivity of the fluid mixture in the radial direction. The values of \( \left( \lambda_c^0 \right)_g \) and \( (\alpha \beta)_g \) were taken as 0.4 kcal/ m. hr. °C, (Yamakawa et al., 1972) and 0.104, (Yagi et al., 1957) respectively. The authors also presented an expression from which the value of \( (\alpha \beta)_l \) can be calculated. They concluded that under the investigated operating conditions experimental and calculated results agreed to within ± 20%.

Freire et al. (1991) studied the heat transfer in a fixed bed with air-water co-current upflow at low rates. The particles used were glass spheres with diameters of 1.8 mm, establishing in this way a bed to particle diameter ratio, \( d_b/d_p \), of 26.7, sufficient to minimize the wall effects on the flow of the fluids. They used two formulations. The first formulation used the pseudo-homogeneous (equation (2.91)) model neglecting the axial conduction for a constant wall temperature boundary condition. The second formulation
used the correlation of Kwong and Smith (1957) which assumes a variation in the effective thermal conductivity at a fixed radial position.

They concluded that the first formulation which considers velocity of the mixture and effective thermal conductivity to be constant in the radial direction led to good fits between the measured and predicted temperatures. However, the formulation proposed by Kwong and Smith (1957) which admits the occurrence of a radial velocity and effective thermal conductivity distribution did not agree well with experimental data. Nevertheless, Freire et al. (1991) suggested more work was needed to evaluate the potential of the Kwong and Smith (1957) formulation.

Wang and Beckerman (1993) developed a new formulation for two-phase flow through capillary porous media. In their model, the two phases were regarded as constituents of a binary mixture. In this definition, phases were assumed to be distinct and separable components. In this way, they derived the conservation equations with defining the mean mixture properties in terms of liquid saturation as follows;

(a) Conservation of mass

\[ \delta \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \]  

(b) Conservation of momentum

\[ u_D = -\frac{K}{\mu(e_i)}[\nabla P - \rho_k(e_i)g] \]  

(c) Conservation of energy

\[ \frac{\partial}{\partial t} \left[ (1 - \delta) \rho_S h_S + \delta \rho_l h_l + \delta (1 - \epsilon_1) \rho_g h_g \right] + \nabla \left( \rho_l \vec{u}_l h_l + \rho_g \vec{u}_g h_g \right) = \nabla (\lambda_c \nabla T) + Q \]  

In the above equations, all properties are the mean mixture properties, which have been evaluated in terms of liquid saturation, \( e_i \) by the authors. Since the definitions of different
properties are too voluminous to be presented here, they are introduced in the Appendix A1.

The review of the literature on two-phase flow heat transfer in porous media and packed beds reveals that nearly all the research which has been carried out in this area is directed towards phase change in porous media where the input is usually a liquid (usually water). Most of the correlations are based on the energy equation, which describes the energy transfer in both liquid, and the gas. The energy equation is then solved with regards to the boundary conditions either analytically (where possible) or in most cases numerically. There are cases where an empirical or a semi-empirical correlation is presented to describe the heat transfer in porous media. However, there is little information on heat transfer in porous media with two-phase flow as the input. Nevertheless the presented review suggests that in order to predict heat transfer in two-phase flow through porous media, one has to start with the solution of the two-phase energy equation as recommended by some of the published articles, under a specified condition. One of the difficulties is the derivation of relationships for the mean properties of the fluids and the solid.

The presented literature review on single and two-phase flow and heat transfer through porous media reveals that

- there are available correlations which can predict the single-phase pressure drop in saturated porous media in both Darcian and non-Darcian flow regime. However, models presented for the prediction of the permeability of beds with different geometry differ in their predictions, which affects the predicted values of the pressure drop.

- as far as single-phase heat transfer through saturated porous media is concerned, most of the previous work has been directed at developing correlations for predicting the heat transfer coefficient in beds with large particles (like those which are routinely used in packed beds) at constant wall temperature and the information on convective heat transfer under constant wall heat flux is particularly insufficient especially at low fluid flow rates.
the hydrodynamics of two-phase flow have been studied by a number of investigators. The presented correlations vary in their end results significantly. The studies mostly investigate the hydrodynamics of two-phase flow in packed beds with large particles in which fluids with relatively high mass fluxes flow.

two-phase heat transfer has not received much attention and the information on the subject is rare.

Appreciating the areas where design information is not available, the following framework is contemplated to bring about a novel and comprehensive study of multi-phase flow and heat transfer in porous media.

- a test rig which facilitates the study of the effect of various parameters on fluid flow and heat transfer in porous media at low liquid flow rates, is designed and built.

- suitable experimental procedures are devised so that the effect of various operating parameters such as bed materials and geometry, fluid flow rates and heat flux on hydrodynamics and heat transfer of multi-phase flow can be investigated.

- Theoretical models for the prediction of single/two-phase pressure drop and heat transfer are derived. The predictions of these models are compared with experimental observations and the range of their applicability is determined.
3. EXPERIMENTAL ARRANGEMENTS

3.1 Experimental Set-up

Experiments were carried out using the test rig which is schematically shown in Fig. 3.1. The integral parts of the rig are discussed in the following sub-sections. (Photographs of the test rig and its components can be found in Appendix B-1)

3.1.1 Holding tank

The temperature controlled holding tank is made of stainless steel and can maintain 10 l. of liquid at temperatures up to approximately 50 °C using four band heaters. Each band heater is 40 mm wide and has a power rating of 500 W at 240 Volts. The band heaters are heating against a small cooling coil which is placed inside the tank, to allow a better control of the liquid temperature in the tank. The cooling coil is made from 20 s. w. g stainless steel tubing with an OD of 6.35 mm, and a length of 380 mm. A rotameter is installed to control the cooling water flow rate. A cooling unit supplies the cooling water at the desired temperature (Fig. 3.2).

The temperature of the liquid inside the tank is determined using a thermocouple which is connected to a controller which in turn will control the band heater power output. This will hold the temperature at a desired set point. A stirrer is also placed inside the tank to agitate the liquid to provide a uniform temperature. The stirrer can rotate between 0-2500 rev/min.

3.1.2 Peristaltic pump

A peristaltic pump is chosen over other pump designs, since it is the only available pump capable of delivering the liquid with minimum and maximum flows of 1 cm³/min to 200 cm³/min up to a pressure of 3 bar. Another feature of this pump is that it is self-primed therefore does not need to be primed before it can be started. Varying the pump speed, which is displayed on a digital screen, can accurately control the liquid flow. The maximum design pressure of the pump is achieved by utilising a tubing with an internal diameter of 6.35 mm and a wall thickness of 1.6 mm. The tubing is made from Marprene II, a material which is resistant to water and mineral oil.
Fig. 3.1 Schematic diagram of the test rig.
3.1.3 Test section

The test section, which is holding the porous medium, is made of stainless steel. The tube has an internal diameter of 32 mm and a wall thickness of 5 mm. The total length of the tube is 580 mm. Spiral and longitudinal grooves to accommodate heating wire and thermocouples measuring the wall temperature have been milled on the outside of the tube. Bores to insert thermocouples for measuring the sand bed temperature and on the opposite side, bores for pressure tappings have been drilled as shown in Fig. 3.3. The heating wire is a single core, cold ended heater, which is 6m long and 2 mm in diameter. It is wrapped around the tube at an 8 mm pitch giving a heated area of 0.03488 m². The uniformity of the heating wire is given in Appendix B3.

The heating wire is placed inside the 2 mm-hemispherical groove and silver soldered to the tube to keep it in position. The whole heated section is covered in silver solder and lagged to minimise heat losses to the surroundings. The heating wire has a hot length of 600 mm two cold ends each 100 mm long and two connectors. It has a maximum power output of 2000 Watt and a total resistance of 18.6 Ω. The power is controlled to maintain the wall temperature below the maximum operating temperature of around 200 °C. The quantity of heat per unit area produced by the heating wire can be calculated using:

\[ q^" = \frac{\psi \cdot I^2 \cdot R}{A_h} \]  \hspace{1cm} (3.1)
Along the tube length are six pressure transducers and six thermocouples placed at distances shown in Fig. 3.3. At the same cross section as the third and the forth thermocouple from the inlet, there are four thermocouples, which are placed inside the longitudinal grooves at a spacing of $90^\circ$ (see Fig. 3.4). The average value of thermocouples will be read to calculate the wall temperature. Using the average readings from these thermocouples, it is possible to calculate the local heat transfer coefficient at two positions along the test section.

The heat transfer coefficient at the wall, $\alpha_w$, can be determined using

$$\alpha_w = \frac{q''}{(T_s - T_b)}$$

(3.2)
$T_b$ is measured but $T_s$ in Fig. 3.5 will have to be calculated to account for the temperature difference between thermocouple location and the pipe inside wall. The value of surface temperature, $T_s$, can therefore be calculated using the values of $\lambda_{ss}/x$. The values of $\lambda_{ss}/x$ for each thermocouple are determined by the calibration method outlined in Appendix B-2. Table 3.1 gives the values of $\lambda_{ss}/x$ for each thermocouple.

**Table 3.1** Values of $\lambda_{ss}/x$ for different thermocouples.

<table>
<thead>
<tr>
<th>TC</th>
<th>TC11</th>
<th>TC12</th>
<th>TC13</th>
<th>TC14</th>
<th>TC21</th>
<th>TC22</th>
<th>TC23</th>
<th>TC24</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{ss}/x$ (W/m² K)</td>
<td>4673</td>
<td>4710</td>
<td>5935</td>
<td>7035</td>
<td>5086</td>
<td>5668</td>
<td>6050</td>
<td>5412</td>
</tr>
</tbody>
</table>

To facilitate the observation of the flow patterns, a perspex test section of the same size is connected in parallel to the stainless steel test section. In normal operation, the transparent...
Stainless steel test section

**Fig. 3.5 Theoretical temperature profile for the cross-section of the test section.**

test section is isolated from the setup and it can easily form a part of the circuit when needed, by isolating the stainless steel test section.

3.1.4 Sintered screens

At the inlet and outlet of the test section two sintered screens with holes of approximately 100 micrometers are used to hold the bed in position. The size is chosen so that they prevent the particles from leaving the test section. The non-uniformity caused by the presence of the screens is small for most practical applications, Vafai and Tien (1981).

3.1.5 Temperature measurement

Stainless steel sheathed K type insulated junction thermocouples 0.5 mm in diameter, 200 mm in length and 1.5 mm in diameter, 150 mm long are used to measure the wall and the bed temperatures, respectively. These thermocouples can measure temperatures ranging from 0 to 1100 °C.

3.1.6 Pressure measurement

Six high sensitivity pressure transducers which are placed at equal distances along the test section measure the pressure drop along the test section. They can measure pressures up to 3.5 bar.
3.1.7 Flow circuit

Gas and liquid flow in two separate circuits, hence providing single and two-phase flow through the porous medium. The arrangements of the valves in the circuits are designed to allow the measurements to be carried out in upward or downward mode. When operating in two-phase flow mode, the two fluids are mixed at a short distance below the inlet of the test section. There are two one-way valves in each of the circuits to prevent undesired flow. The liquid is supplied from the tank at a fixed temperature and pumped to the test section using the peristaltic pump. After flowing through the circuit, the liquid returns to the tank, hence forming a closed-loop circuit. The gas (air) is supplied from the outlet of a compressor capable of delivering 170 m$^3$ of air per hour at a pressure of 14 bar. Before entering the circuit, the gas is passed through a filter and a pressure regulator valve, which reduces the gas pressure to about 2.5 bar. The gas flow is measured by a rotameter ranging from 0.05 l/min to 25 l/min. The fluid leaving the bed is fed to a separator where the liquid flow rate is precisely measured. In two-phase flow operations, the gas is fed from the separator to a gasometer where the precise amount of the gas and liquid can be measured at atmospheric pressure. Both circuits use 20 s. w. g. stainless steel tubing with an outside diameter of 6.35 mm and fittings of the same size. All tubings are well insulated to reduce heat losses.

To facilitate the measurements of bed porosity and gas hold-up, a burette is connected to the inlet of the test section so that filling the void volume of the porous medium is made possible. The displaced liquid volume from the burette can be used to calculate the bed porosity as well as the gas hold-up.

3.1.8 Data acquisition system

The data acquisition system is outlined in Fig. 3.6. It consists of two boards, namely the CIO-DAS08 and the CIO-EXP32 cards that are connected to a desktop computer. The CIO-EXP32 analogue input multiplexer expands the number of analogue input channels of any A/D board to 32 channels. Two banks of 16 analogue inputs are multiplexed into two of the A/D boards analogue channels. Therefore, one bank will be used for inputting thermocouple voltages from the test section, while the other bank will be used for
inputting pressure transducer measurements from the test section. The thermocouples and pressure transducers are directly wired into the appropriate banks.

The analogue signals produced by the thermocouples and pressure transducers are taken to an analogue multiplexer in the CIO-DAS08 card by a standard 37 pin ‘D’ type connector. This multiplexer provides 8 channels of single ended input and is protected against 30 volts maximum. Two of these channels will be taken up by the analogue signals coming from the CIO-EXP32 card. One channel will handle input signals coming from the bank to which the thermocouples are connected, while the second channel will handle input signals coming from the bank to which the pressure transducers are connected. Two more channels in the CIO-DAS08 card are used for the flow-meter (if there is one) and for the heating wire control signals. The flow-meter will send input signals proportional to the measured flow, while the control box will send input signals proportional to the power rating of the heating wire. The CIO-DAS08 card turns the PC into a medium speed data acquisition and control
A menu driven Quick Basic program is used to process the acquired raw data. The wall and bed temperatures, pressures, heat fluxes, flow rates and the heat transfer coefficients are the parameters which are saved in a file for each test.

3.1.9 Porous medium

The packing materials for the test section used in this investigation are mineral casting sand and glass beads. To prepare the porous medium, a suitable amount of the packing material is introduced into the test section, the liquid (water) is allowed to flow upward in the bed in order to force the entrapped air out of the pore volume, hence saturating the medium. When the test section is fully packed, the porous medium fills the space between two screens at the ends of the test section. Care is taken to follow the same bed preparation procedure each time so that reproducible bed properties are ensured. The properties of the packing materials and those of the fluids are given in Table 3.2.

3.2 Experimental Procedure

Preliminary tests were performed to obtain the time at which the bed was stabilised. To achieve this, the tank was filled with distilled water and the band heaters were switched on to raise the temperature of the liquid to a desired value. Liquid was then pumped through the bed for about one hour to obtain a homogenous condition. The power to the heater was then switched on and kept constant at a predetermined value. While all operational variables of the system were kept constant, readings were taken at short time intervals until the steady state was reached. This time, which was determined to be 20 min, was taken as the time at which the steady state condition is reached and was observed in all experiments.

3.2.1 Porosity measurement

The porous media were formed in the same way as explained in section 3.1.9, then the pore volume of the dry porous medium was filled with liquid supplied from the burette connected
Table 3.2 Physical properties of fluids and packing materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>( \phi_p (\mu m) )</th>
<th>( \lambda ) (W/m °C)</th>
<th>( \rho ) (kg/m³)</th>
<th>( C_p ) (J/kg °C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>180-250</td>
<td>5.345</td>
<td>2640</td>
<td>0.820</td>
</tr>
<tr>
<td></td>
<td>250-425</td>
<td>5.345</td>
<td>2640</td>
<td>0.820</td>
</tr>
<tr>
<td></td>
<td>425-600</td>
<td>5.345</td>
<td>2640</td>
<td>0.820</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>5.345</td>
<td>2640</td>
<td>0.820</td>
</tr>
<tr>
<td>Glass beads</td>
<td>180-300</td>
<td>1.050</td>
<td>2550</td>
<td>0.770</td>
</tr>
<tr>
<td></td>
<td>250-425</td>
<td>1.050</td>
<td>2550</td>
<td>0.770</td>
</tr>
<tr>
<td></td>
<td>400-600</td>
<td>1.050</td>
<td>2550</td>
<td>0.770</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>1.050</td>
<td>2550</td>
<td>0.770</td>
</tr>
<tr>
<td></td>
<td>1200</td>
<td>1.050</td>
<td>2550</td>
<td>0.770</td>
</tr>
<tr>
<td></td>
<td>3000</td>
<td>1.050</td>
<td>2550</td>
<td>0.770</td>
</tr>
<tr>
<td>Polyethylene</td>
<td>3000</td>
<td>0.400</td>
<td>800</td>
<td>2300</td>
</tr>
<tr>
<td>Distilled water</td>
<td>-</td>
<td>0.6151</td>
<td>995.7</td>
<td>4185</td>
</tr>
<tr>
<td>Air</td>
<td>-</td>
<td>0.02624</td>
<td>1.1774</td>
<td>1.0057</td>
</tr>
</tbody>
</table>

To the inlet of the bed. Taking into account the necessary corrections, the porosity of the medium was then calculated as the ratio of the amount used to fill the pore volume divided by the total volume of the bed, as given in equation (3.3).

\[
\delta = \frac{V_l}{V_c}
\]  

(3.3)

The same procedure was repeated several times for each medium and the mean value was taken to represent the porosity of the medium, see Table 3.2.

3.2.2 Single-phase flow experiments

After the temperature of the distilled water in the tank reached the desired value, a predetermined flow of the liquid was established through the bed until the system reached a
steady state. Then the power to the heater was switched on and kept constant at a predetermined value. The system was again left to reach steady state and then the data acquisition system was switched on to record fluid flow rate, temperatures, pressures and the heat flux. The above procedure was repeated for different fluid flow rates. The experiments were repeated at various heat fluxes while the inlet temperature was kept constant. The range of operating variables is given in Table 3.3.

Table 3.3 Range of operating parameters in single-phase experiments.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat flux</td>
<td>1000 - 7000 W/m²</td>
</tr>
<tr>
<td>Bulk temperature</td>
<td>25 - 70 °C</td>
</tr>
<tr>
<td>Inlet temperature</td>
<td>25.30 °C</td>
</tr>
<tr>
<td>Flow rates</td>
<td>25 - 200 cc/min.</td>
</tr>
<tr>
<td>System pressure</td>
<td>1.2 bar</td>
</tr>
</tbody>
</table>

The local heat transfer coefficients were obtained from equation (3.4).

\[ \alpha = \frac{q^*}{T_s - T_c} \]  

(3.4)

The heat flux, is calculated from equation (3.5).

\[ q^* = \frac{\psi \cdot I^2 \cdot R}{A_h} \]  

(3.5)

where I and R are the electric current and the electric resistance of the heating wire respectively. The quantity \( \psi \) represent the cold-end correction which has been calculated as 0.9917, (Krüger, 1995). The heated area, \( A_h \), is 0.0382 m².

3.2.3 Two-phase flow experiments

A predetermined flow of liquid was established through the bed. While keeping the liquid flow rate constant, gas was then gradually added to the test section until a prearranged gas flow rate was reached. The system was then left to homogenize for one hour and then
the power to the heater was switched on and kept constant at a predetermined value. The system was again left to reach steady state before the data acquisition system was switched on to record flow rate, temperatures, pressures and heat flux. This procedure was repeated for different flow rates of the fluids. The preceding program was repeated for different heat fluxes. For each test the heat transfer coefficient was calculated using equation (3.4).

To measure the gas hold-up, the flows to the test section was cut off simultaneously using quick acting isolating valves. Then the part of the bed occupied by the gas was filled with liquid supplied from the burette connected to the inlet of the bed. The gas hold-up was then calculated by dividing the volume of the liquid displaced from the burette to the bed by the total volume of the bed plus any necessary corrections to cater for the volume of extra tubing which might have to be taken into account.

To investigate the flow patterns, the perspex test section was used.

All experiments were carried out at constant inlet temperature and in an arbitrary sequence. Some experiments were repeated to check the reproducibility of the results, which proved to be good. The range of operating variables is given in Table 3.4.

3.3 Experimental program

With the packing material in place, heat transfer and pressure drop experiments were conducted as described in the preceding section for single-phase flow mode. Experiments were carried out in the same manner to investigate fluid flow and heat transfer through the medium in two-phase flow mode. The devised experiments permit a thorough study of flow patterns, pressure drop, hold-up and heat transfer to be made.
Table 3.4 Range of operating parameters in two-phase flow experiments

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat flux</td>
<td>1000 - 7000 W/m²</td>
</tr>
<tr>
<td>Bulk temperature</td>
<td>25 - 70 °C</td>
</tr>
<tr>
<td>Inlet temperature</td>
<td>25-30 °C</td>
</tr>
<tr>
<td>Air flow rate</td>
<td>0.050 – 15.0 l/min.</td>
</tr>
<tr>
<td>Water flow rates</td>
<td>25 – 200 cc/min.</td>
</tr>
<tr>
<td>Maximum inlet pressure</td>
<td>2.5 bar</td>
</tr>
</tbody>
</table>
4. RESULTS AND DISCUSSION

4.1 Single phase pressure drop and velocity profile

When a fluid flows through a porous medium, the pressure drop which develops along the bed in the direction of flow is a function of system geometry, bed voidage and physical properties of bed and fluid. In order to consider the heat transfer phenomenon in a circular test section packed with a saturated porous medium, it is first necessary to have adequate information regarding the flow mechanism in the medium.

The operating conditions can result in four distinct flow regimes (Dybbs and Edwards, 1984): Darcy or creeping flow, inertial flow, unsteady laminar flow and chaotic (or turbulent) flow. In the Darcian region the pressure gradient is proportional to the flow rate and it is mathematically expressed by

\[ u_D = \frac{K}{\mu} \left( - \frac{\Delta p}{\Delta x} \right) \]  

(4.1)

If the relationship represented by equation (4.1) exists between the liquid flow rate and the pressure gradient, then the slope of the straight line can be taken as the ratio of permeability of the medium to the viscosity of the liquid saturating the porous medium.

The Darcy model persists at low fluid flow rates where the inertia forces dominate. To determine the transition points a Reynolds number is usually defined, \( Re = u d_p \rho / \mu \). In spite of the minor differences between the transition regions reported by different investigators, Fand et al. (1987) reported the higher and lower bounds for each regime. According to their reported data the upper Reynolds number at which the deviation from Darcy law occurs is \( 2.3 \pm 0.1 \). Considering the size of the particles used in the present investigation, the Darcian regime prevails. Nevertheless, many experiments were performed and the results were analysed in order to ascertain the type of flow which existed in the test section, taking into account the system geometry as well as the physical properties of bed material and liquid.
In the present study, the pressure drop is measured at six different positions along the porous medium in the direction of flow, over a wide range of liquid velocities for different sizes of particles.

The porous media used in the present analysis consisted of glass spheres and mineral sand saturated with distilled water. Knowing the density of the particles, quoted by the manufacturers, a statistically large number of each size was weighed. The particles were then screened to find out the proportion of different sizes retained by each screen size. An average equivalent mean diameter, \( d_p \), of the three sizes were found using

\[
d_p = \sum_i m_i \cdot d_{p_i}
\]  

(4.2)

The average equivalent mean diameter, \( d_p \), of different sizes are given in Table 4.1. With non-spherical sand particles, a sphericity factor in terms of porosity was calculated from the data given by Chhabra (1993). By multiplying this factor with the average equivalent mean diameter calculated from equation (4.2), the new average equivalent mean diameter was determined.

The porosity, \( \delta \), of each porous bed was measured by filling the pores of the medium with distilled water using a precise measuring burette. Knowing the volume of water required to fill the pores, the porosity of the bed was then taken as the ratio of the volume of water displaced to the total column volume. Since there were some discrepancies in the measured values of porosity, the procedure was repeated each time the test section was filled with a particular particle size and the mean value was taken to represent the porosity of the medium. The values of porosity for different sizes of particles are given in Table 4.1.

The measurements of pressure drop as a function of axial distance of different media at several liquid flow rates are presented in Figs. 4.1 – 4.4. The results show a linear relationship between the pressure drop and the axial distance in the direction of flow. The slope of these lines increases with liquid flow rate and similarly the pressure drop increases monotonously with increase of the liquid flow rate.
**Fig. 4.1** Variation of pressure drop as a function of axial distance.

**Fig. 4.2** Variation of pressure drop as a function of axial distance.
Fig. 4.3 Variation of pressure drop as a function of axial distance.

Fig. 4.4 Variation of pressure drop as a function of axial distance for non-spherical particles.
Table 4.1 Physical properties of the packing materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>Particle size range</th>
<th>Mean particle diameter</th>
<th>$\delta$</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(µm)</td>
<td>(µm)</td>
<td></td>
<td>(m²)</td>
</tr>
<tr>
<td>Sand</td>
<td>180-250</td>
<td>172.0</td>
<td>0.3662</td>
<td>1.89E-11</td>
</tr>
<tr>
<td></td>
<td>250-425</td>
<td>265.4</td>
<td>0.3750</td>
<td>5.64E-11</td>
</tr>
<tr>
<td></td>
<td>425-600</td>
<td>410.0</td>
<td>0.3679</td>
<td>1.09E-10</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>920.0</td>
<td>0.3684</td>
<td>5.36E-10</td>
</tr>
<tr>
<td>180-300</td>
<td>245.0</td>
<td>0.36114</td>
<td>4.45E-11</td>
<td></td>
</tr>
<tr>
<td>250-425</td>
<td>337.5</td>
<td>0.3616</td>
<td>7.96E-11</td>
<td></td>
</tr>
<tr>
<td>400-600</td>
<td>480.0</td>
<td>0.3624</td>
<td>1.53E-10</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>1000</td>
<td>0.3648</td>
<td>6.62E-10</td>
<td></td>
</tr>
<tr>
<td>1200</td>
<td>1200</td>
<td>0.3670</td>
<td>1.86E-09</td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td>3000</td>
<td>0.3756</td>
<td>6.69E-09</td>
<td></td>
</tr>
<tr>
<td>Glass beads</td>
<td>1200</td>
<td>0.3670</td>
<td>1.86E-09</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3000</td>
<td>0.3756</td>
<td>6.69E-09</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 4.5 shows the observed values of pressure drop as a function of axial distance for a bed with a particle diameter of 0.245 mm at heat flux of 5000 W/m² K. The results also show a linear relationship exists between the pressure drop and the axial distance. Fig. 4.6 compares the measured pressure drop at two different heat fluxes for a constant liquid flow rate. It is evident from the plots on this graph, the pressure drop decreases as the temperature of the fluid increases. This agrees with the fact that when temperature of the liquid (distilled water) saturating the porous medium increases, its viscosity decreases, hence reducing the pressure drop.

Knowing the viscosity of the liquid saturating the medium, the permeability of each medium can be calculated using the slope of the best-fit straight line through the data presented in Figs. 4.1 – 4.4 as the ratio of $\mu u_D / K$ in equation (4.1). The mean calculated permeabilities are given in Table 4.1.
Fig. 4.5 Variation of pressure drop as a function of axial distance at higher heat flux.

Fig. 4.6 Effect of heat flux on pressure drop
Table 4.2 Predicted values of permeability for different media

<table>
<thead>
<tr>
<th>Mean particle diameter (µm)</th>
<th>Porosity (-)</th>
<th>Kozney-Carman $K$, (m$^2$)</th>
<th>Rumpf and Gupte $K$, (m$^2$)</th>
<th>Darcy $K$, (m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>172.0</td>
<td>0.3662</td>
<td>2.01E-11</td>
<td>2.11E-11</td>
<td>1.89E-11</td>
</tr>
<tr>
<td>245.0</td>
<td>0.36114</td>
<td>3.85E-11</td>
<td>4.00E-11</td>
<td>4.45E-11</td>
</tr>
<tr>
<td>265.4</td>
<td>0.3750</td>
<td>5.28E-11</td>
<td>5.71E-11</td>
<td>5.64E-11</td>
</tr>
<tr>
<td>337.5</td>
<td>0.3616</td>
<td>7.34E-11</td>
<td>7.56E-11</td>
<td>7.96E-11</td>
</tr>
<tr>
<td>410.0</td>
<td>0.3679</td>
<td>1.16E-10</td>
<td>1.23E-10</td>
<td>1.09E-10</td>
</tr>
<tr>
<td>480.0</td>
<td>0.3624</td>
<td>1.50E-10</td>
<td>1.55E-10</td>
<td>1.53E-10</td>
</tr>
<tr>
<td>920.0</td>
<td>0.3684</td>
<td>5.89E-10</td>
<td>6.23E-10</td>
<td>5.36E-10</td>
</tr>
<tr>
<td>1000.0</td>
<td>0.3648</td>
<td>6.68E-10</td>
<td>6.97E-10</td>
<td>6.62E-10</td>
</tr>
<tr>
<td>1200.0</td>
<td>0.3670</td>
<td>1.02E-09</td>
<td>1.08E-09</td>
<td>1.86E-09</td>
</tr>
<tr>
<td>3000.0</td>
<td>0.3756</td>
<td>6.79E-09</td>
<td>7.36E-09</td>
<td>6.69E-09</td>
</tr>
</tbody>
</table>

Mean average error over the Darcy model 11.6% 10% -

Although this is the most accepted method of calculating the permeability of a porous medium, available empirical correlations can also be used to calculate the permeability of a medium. These empirical correlations predict the value of permeability of a medium in terms of particle diameter and the porosity. The values of permeability predicted by different correlations are given in Table 4.2.

The influence of particle diameter on pressure drop has been reported in the literature, (Ergun and Orning (1949), and Gauvin and Katta (1973)). To investigate the effect of the particle size, data of pressure gradient as a function of superficial liquid velocity were collected for various particle diameters. The data are presented in Fig. 4.7. As can be observed the pressure gradient increases as the particle diameter decreases. These plots also emphasises the linear relationship between the pressure gradient and the superficial liquid velocity.

While the Darcy model describes the viscosity-dominated flow at low Reynolds numbers (or according to Fand et al.(1990), $d_p/d_\text{d}_\text{p}> 40$), however, at higher liquid flow rates where both viscous and inertial forces are significant, the pressure drop is given by the well-known Ergun equation:
\[
\Delta p = \left( \frac{A\alpha \mu}{d_p^2} \right) \cdot u + \left( \frac{B\beta \rho}{d_p} \right) \cdot u^2
\]

where

\[
\alpha = \frac{(1-\delta)^2}{\delta^3} \quad \text{and} \quad \beta = \frac{(1-\delta)}{\delta^3}
\]

A and B are dimensionless constants, called the first and second Ergun constants for Forchheimer flow respectively. Equation (4.3) also predicts the flow at relatively higher Reynolds numbers, where the flow is turbulent. For turbulent flow the constants A and B adopt different values, Riechelt (1972).

Equation (4.3) can be transformed into a more useful form as follows:
\[ f = \frac{A}{R_{em}} + B \]  

(4.5)

While the values of the constants \( A \) and \( B \) were suggested to be 150 and 1.75 by Ergun (1952) respectively, however, subsequent experimental data have shown that these values are definitive (McDonald, 1979 and Rumpf and Gupte, 1971). However, Fand et al. (1987) reported the values of \( A \) and \( B \) for inertial flow to be 182 and 1.73 respectively.

Fig. 4.8 has been prepared to show the variations of friction factor with the modified Reynolds number. Different plots on this diagram represent; (1) the observed values of \( f \), (2) values calculated from equation (2.3) using \( \kappa = 5.34 \) as suggested by Fand et al. (1987) and \( \kappa = 5 \) as concluded from the present study, and (3) calculated values of \( f \) using equation (4.5) with the constants \( A \) and \( B \) taken as 182 and 1.92 as suggested by Fand et al. (1987).

Fig. 4.8 Comparison of the friction factor, as a function of modified Reynolds number, using different correlations.
It is evident from the results shown in Fig. 4.8 that the observed values are in excellent agreement with the predicted values using the Kozney-Carman model with $\kappa = 5$ and in good agreement with the Ergun model taking the values of the constants as suggested by Fand and Thinakaran (1990), whereas the Kozney-Carman model with $\kappa = 5.34$ slightly overpredicts the observed values. Therefore, it is worthwhile to note that for the media used in the present study, the viscous forces predominate the flow and both the Ergun and Darcy model can be used to predict the pressure drop.

The above presentation of the results agrees well with the values cited in the literature. Fand et al., (1987), Fand and Thinakaran (1990) and Varahasamy et al. (1996) reported the upper value of the Reynolds number at which the transition occurs to be 2.3. In their studies on the flow mechanism in porous media, they also pointed out that for relatively low Reynolds numbers and for $d_{c}/d_{p} > 40$ viscosity dominated Darcy flow occurs and could be represented by the well known Darcy equation.

When a porous medium whose matrix consists of discrete solid particles is confined in a duct, the wall of the duct affects the local distribution of the porosity. For the case of spherical particles contained in a circular cylinder, the porosity tends toward unity upon approach to the cylinder wall. Further, it was reported by several investigators that the local porosity near to the wall varies cyclically in a zone extending to a few particle diameters from the wall into a bed of spheres of uniform diameter. This variation in porosity sometimes gives rise to channelling which results in higher liquid velocity near the wall. Clearly the annular zone of the wall comprises an increasing fraction of the cross-sectional area of the cylinder as $d_{c}/d_{p}$ decreases. Therefore, the influence of the wall upon the flow (viz. channelling) becomes more significant as $d_{c}/d_{p}$ progressively decreases. Fand et al. (1987), have reported that for cylindrical ducts packed with spheres, the wall effect becomes significant for $d_{c}/d_{p} < 40$, and consequently the flow parameters become functionally dependent upon $d_{c}/d_{p}$ for $d_{c}/d_{p} < 40$.

Although the above discussion indicates that the wall effect is negligible, the effects of solid boundary and inertial forces (non-Darcian effects) have to be taken into consideration as they affect the fluid flow and heat transfer through porous media. The following
mathematical treatment is carried out to find the effect (if any) of non-Darcian terms on the pressure drop.

Using the volume average of a divergence theorem, Whitaker (1969) and Slattery (1978) established the local volume averages of the differential balance laws for an incompressible, steady flow through a porous medium as follows

\[ \rho \nabla \cdot \langle u \rangle = 0 \]  (4.6)

\[ \rho_f \langle (u \cdot \nabla)u \rangle = -\nabla \langle p \rangle + \mu_f \nabla^2 \langle u \rangle + X \]  (4.7)

Equation (4.6) and equation (4.7) are macroscopic conservation equations for fluid mass and momentum respectively and X in equation (4.7) represents the body force. Due to volume averaging some information is lost, thus requiring a supplementary empirical relation for X.

The pressure drop caused by frictional drag is directly proportional to velocity for low speed flow and at higher velocities, the inertial effect becomes appreciable therefore, the total pressure drop would be the sum of the two terms. Joseph et al. (1982) expressed the total pressure drop as the following:

\[ \nabla p = \frac{\mu_f}{K} u_D + c_F K^{-1/2} \rho_f u_D^2 \]  (4.8)

Consider the body force in equation (4.7), which is in effect a measure of the flow resistance offered by the solid matrix. Since the pressure gradient as given in equation (4.8) can also be interpreted as a measure of the same resistance to flow in the bulk of the porous media (Vafai and Tien, 1981) then

\[ X = \nabla p = \frac{\mu_f}{K} u_D + c_F K^{-1/2} \rho_f u_D^2 \]  (4.9)

substitution of equation (4.9) into equation (4.7) yields
\[
\rho_f ((u \cdot \nabla)u) = -\nabla(p) + \mu_f \nabla^2(u) + \frac{\mu_f}{K} u_D - c_F K u_D^2
\] (4.10)

In equation (4.10), the first term on the left hand side is the convective term which is usually neglected after a short entrance (Vafai and Tien, 1981). The first and the third terms on the right hand side are pressure drop and viscous drag caused by porous medium. The second term on the right hand side is the boundary effect which takes into account the no-slip conditions at the solid boundaries. The forth term on the same side is the Ergun term which takes care of the pressure losses due to the presence of inertial forces at high velocities.

Neglecting the convection term and after some mathematical manipulation, equation (4.10) for a cylindrical porous medium becomes (Tien and Hunt, 1987):

\[
\frac{K}{\delta} \frac{d}{r \, dr} (r \frac{du}{dr}) - u + \frac{K}{\mu} (\rho g - \nabla p) - \frac{K \rho_f}{\mu} C u^2 = 0 ; \quad C = \frac{1.75(1 - \delta)}{d_p \delta^3}
\] (4.11)

Equation (4.11) can be non-dimensionalized by introducing the following variables:

\[
R = \frac{r}{R_0} , \quad f = \frac{K}{R_0 \delta} , \quad Re_c = \frac{KC u_D \rho}{\nu} , \quad U = \frac{u}{u_D} , \quad u_D = \frac{K}{\mu} (\rho g - \nabla p)
\] (4.12)

Substitution of the above equation into equation (4.11) and dividing by \(u_D\) yields:

\[
1 - U + Re_c U^2 + \frac{f}{R} \frac{d}{dR} (R \frac{dU}{dR}) = 0
\] (4.13)

In this investigation, the inertia term can be neglected because \(Re_c < 0.05\) (Tien and Hunt, 1987). Thus equation (4.13) reduces to:

\[
1 - U + \frac{f}{R} \frac{d}{dR} (R \frac{dU}{dR}) = 0
\] (4.14)

Equation (4.14) can be solved using finite difference methods. If \(n\) nodes are taken along the radius of the cylinder, then:
The nodal equation can be obtained from the substitution of equation (4.15) into equation (4.14):

\[
\left( \frac{f}{\Delta R^2} + \frac{f}{2R_n\Delta R} \right) U_{n+1} - \left( 1 + \frac{2f}{\Delta R^2} \right) U_n + \left( \frac{f}{\Delta R^2} - \frac{f}{2R_n\Delta R} \right) U_{n-1} + 1 = 0 \quad (4.16)
\]

Boundary conditions for the solution of equation (4.14) are:

\[
U = 0 \text{ at } R = 1 \quad \text{and} \quad \frac{dU}{dR} = 0 \text{ at } R = 0 \quad (4.17)
\]

In Fig. 4.9 results obtained from the solution of the above finite difference equation are compared with experimental results. Excellent agreement exists between the predictions of equation (4.11) and the experimental data. Thus it can be assumed that the non-Darcian flow effects are negligible in this investigation. The program code is given in Appendix C-1.
4.2 Single-phase heat transfer coefficient

The local heat transfer coefficient at the two cross-sections is calculated using:

$$\alpha_x = \frac{q''}{(T_s - T_c)}$$  \hspace{1cm} (4.18)

Where $T_s$ and $T_c$ are the pipe inside surface and centre temperatures corresponding to the two axial locations. Then the local Nusselt number was evaluated from

$$N_u_x = \frac{\alpha_x D}{\lambda_{es}}$$  \hspace{1cm} (4.19)

$\lambda_{es}$ is the effective conductivity composed of the stagnant and dispersion conductivities,

$$\lambda_{es} = \lambda_{os} + \lambda_{ds}$$  \hspace{1cm} (4.20)

Effective thermal conductivity

The stagnant thermal conductivity $\lambda_{os}$ is a function of porosity and the conductivities of the fluid and the solid, and thus accounts for the molecular conduction between the phases. Its bulk value is determined from various empirical models such those presented in section 2.1.2. The discrepancy between the presented correlations considered is 2-3%. Nevertheless, the value of the stagnant conductivity is taken from equation (4.21) which provides a good estimate (Nield, 1991b).

$$\lambda_{os} = \lambda_s^{(1-\delta)} \cdot \lambda_f^\delta$$  \hspace{1cm} (4.21)

The effective conductivity also includes thermal transport due to dispersion. Dispersion results from the mixing of local fluid streams as the fluid follows tortuous paths around the particles, the mixing of the fluid streams yields a net transport of heat. The effect of this mixing increases with Reynolds number. If a temperature gradient occurs across the particles in the medium, then the recirculation or dispersion mixes the fluid and increases the net transport. Therefore the effect of dispersion must be taken into consideration where its
contribution to the heat transfer becomes significant. Several investigators have presented empirical relations to predict the value of thermal transport due to dispersion. The experimental results indicate that the dispersion conductivity is a function of local velocity, particle diameter and the properties of the fluid saturating the medium, see section 2.1.2.

Hunt and Tien (1988a) suggested an empirical relationship in terms of the square root of the permeability of media with the value of the constant taken as 0.025. The influence of thermal dispersion for the two different media employed in this study is illustrated in Fig. 4.10 as a function of the superficial liquid velocity. As can be seen, the influence of this quantity on the value of effective thermal conductivity and consequently the heat transfer coefficient is negligible.

Therefore, the effective thermal conductivity is taken as equal to the stagnant thermal conductivity for the forthcoming calculations. However, it is worthwhile mentioning that as the velocity of the liquid increases, the effect of dispersion become appreciable and that this effect is more pronounced with an increase in particle diameter.
Heat transfer coefficient

As several investigators have reported, the value of the heat transfer coefficient is enhanced in packed tubes as compared to unpacked tubes. The enhancement achieved is about two to seven times the value for unpacked tubes for laminar flow and two to two and a half times for turbulent flow. Moreover, the enhancement ratio increases as $d_o/d_p$ decreases under the same pumping power (Varahasmay and Fand, 1996). Therefore, the heat transfer coefficient is expected to increase with the liquid flow rate with higher values associated with beds made of larger particles.

To investigate the heat transfer mechanism and other related phenomena a series of tests were performed and the results are discussed in the following.

Fig. 4.11 represents a typical variation of the heat transfer coefficient with liquid superficial velocity at two different locations along the bed at constant heat flux of 3000 W/m². For comparison, the calculated values, using equation (4.22) of Sieder and Tate (1936), is also included.

\[
\text{Nu}_D = 1.86\left(\text{Re}_D \text{ Pr}\right)^{1/3} \left(\frac{D}{x}\right)^{1/3} \left(\frac{\mu}{\mu_w}\right)^{0.14}
\]  

As indicated in Fig. 4.11, the heat transfer coefficient in porous media is higher than those values usually incurred in unpacked tubes. The results also show that the convective heat transfer coefficient in the porous medium increases with increasing liquid flow rate and that this dependency is more pronounced at higher liquid flow rates. Furthermore, the variation of forced convective heat transfer with flow rate is more pronounced in the porous medium than in the unpacked tube.

Effect of heat flux

The variation of the heat transfer coefficient as a function of heat flux at two different liquid flow rates is represented in Fig. 4.12. The graph shows that a constant heat transfer coefficient exists at the various heat fluxes. Therefore, to perform all the experiments under
Fig. 4.11 Variation of measured heat transfer coefficient as a function of superficial liquid velocity.

Fig. 4.12 Variation of measured heat transfer coefficient with heat flux.
identical operational conditions in this investigation, the heat flux can be kept constant during the study of the effect of various operating parameters on convective heat transfer in the porous medium.

Effect of particle diameter

The effect of particle diameter for three particle sizes is presented in Fig. 4.13. The general trend of heat transfer coefficient with flow velocity remains the same for the different media. It is worthwhile mentioning for larger particles different rates of heat transfer are expected because the effect of Non-Darcian terms and variable porosity will become appreciable, Vafai and Tien (1981).

![Graph showing variation of measured heat transfer coefficient as a function of superficial liquid velocity.]

**Fig. 4.13 Variation of measured heat transfer coefficient as a function of superficial liquid velocity.**

Effect of particle material

The influence of different particle materials is illustrated in Fig. 4.14. Inspection of this figure reveals that the heat transfer rate for media with higher thermal conductivity is higher than that of media with lower thermal conductivity, due to the decrease in thermal
resistance. This increase in thermal conductivity will, therefore, result in an increase in the value of the stagnant thermal conductivity which in turn increases the effective thermal conductivity of the medium and therefore the heat transfer coefficient.

![Graph showing the effect of particle material on the heat transfer coefficient.](image)

**Fig. 4.14 Effect of particle material on the heat transfer coefficient.**

**Temperature profiles**

Liquid enters the porous medium at a uniform temperature, which is different from the wall temperature of the bed. Therefore, convective heat transfer occurs from the wall to the porous medium, and radial and axial temperature profiles begin to develop. Typical radial temperature profiles at two different locations along the bed are shown in Fig. 4.15, obtained at a constant heat flux of 3000 W/m² and a liquid flow rate of 200 cm³/min. It shows a parabolic variation with the maximum and minimum values occurring at the inside surface and in the centre of the bed respectively. It suggests the heat transfer coefficient that is defined with the difference between wall and bulk temperature is always lower than that defined with the difference between wall and the average mean bulk temperature.
The temperature gradient in flow direction along the central axis of the bed has also been measured. A typical axial temperature profile is depicted in Fig. 4.16. The heated length covers a distance of 38 cm along the test section as shown in Fig. 3.3.

![Graph showing measured temperature distribution as a function of dimensionless radial distance.](image)

Fig. 4.15 Measured temperature distribution as a function of dimensionless radial distance.

The flat part of the temperature-distance relationship is due to the developing temperature profile, see appendix C-5 for identification of fully developed region.

**4.2.1 Formulation of model**

The volume-averaged homogeneous energy equation is;

\[ \rho_f c_p \langle u \rangle \cdot \nabla \langle T \rangle = \lambda_{es} \nabla^2 \langle T \rangle \]  

(4.23)

Where \( \lambda_{es} \) is the effective conductivity, \( \lambda_{es} = \lambda_{os} + \lambda_{ds} \). The model assumes the existence of local thermal equilibrium between the solid-liquid phases, \( (T_s - T_f = 0) \), which is valid under most flow conditions, (DeWasch and Froment, 1972; Whitaker, 1986), or equivalent local temperature gradients \( (\nabla T_s \equiv \nabla T_f) \). Under these conditions, a
homogeneous model accurately represents the thermal transport and existing empirical correlations can be used to evaluate the stagnant and dispersion conductivities. However,

\[
q' = 3000 \text{ W/m}^2 \\
\lambda_s = 1.05 \text{ W/m K} \\
L_s = 200 \text{ cm}^3/\text{min.} \\
d_p = 0.245 \text{ mm}
\]

![Graph showing temperature profile](image)

**Fig. 4.16 Measured temperature profile in axial direction.**

if the particle size is large or the flow rate is high the homogeneous model may no longer be valid.

The volume-averaged, steady energy equation in a cylindrical porous medium assuming Darcian flow and neglecting the third dimension (axi-symmetric flow), is therefore;

\[
\frac{\partial^2 T}{\partial t^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{u_D}{A_{es}} \right) \frac{\partial T}{\partial r} - \frac{\partial^2 T}{\partial x^2} = 0 
\]  \hspace{1cm} (4.24)

where \( A_{es} \) is the effective thermal diffusivity and defined as follows

\[
A_{es} = \frac{\lambda_{es}}{\rho_f c_{p,f}} 
\]  \hspace{1cm} (4.25)

Equation (4.24) can be non-dimensionalised using the following variables:
\[
\theta = \frac{T_w - T}{T_w - T_0}, \quad r^+ = \frac{r}{r_o}, \quad x^+ = \frac{x/r_o}{Re_o Pr_e}
\]  

(4.26)

Thus

\[
\frac{\partial^2 \theta}{\partial r^+^2} + \frac{1}{r^+ \partial r^+} \frac{1}{2 \partial x^+} \left( \frac{1}{(Re_o Pr_e)^2} \right) \frac{\partial^2 \theta}{\partial x^+^2} = 0
\]

(4.27)

In the above nondimensionalized energy equation, the last term on the right hand side is negligible providing \(Re_o Pr_e\) is large (Kays and Crawford, 1980). In the present investigation this condition prevails (the minimum value of \(Re_o Pr_e\) is 42 which will form a numerical coefficient of 2E-5 for the second term in equation (4.27)) and therefore the energy equation (4.24) reduces to:

\[
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r \partial r} \frac{u_D \partial T}{A_{es} \partial x} = 0
\]

(4.28)

or

\[
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r \partial r} \beta_s \frac{\partial T}{\partial x} = 0
\]

(4.29)

where

\[
\beta_s = \frac{u_D}{A_{es}}
\]

(4.30)

The initial and boundary conditions applicable to the solution of the above partial differential equation with respect to the test bed under consideration are as follows;

\[
T_{(r,0)} = T_0
\]

(4.31)

Boundary conditions:

\[
\left. \frac{\partial T}{\partial r} \right|_{r=0} = 0
\]

(4.32)

\[
\left. \frac{\partial T}{\partial r} \right|_{r=R_e} = \frac{q''}{\lambda_{es}}
\]

(4.33)
Taking

\[ \theta(r, x) = T(r, x) - T_0 \]  

then

\[ \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} - \beta_s \frac{\partial \theta}{\partial x} = 0 \]  

Equation (4.35) can be solved using the following initial and boundary conditions

Initial condition:

\[ \theta(r, 0) = 0 \]  

Boundary conditions

\[ \left. \frac{\partial \theta}{\partial r} \right|_{r=0} = 0 \]  

\[ \left. \frac{\partial \theta}{\partial r} \right|_{r=R_s} = \frac{q''}{\lambda_{es}} \]  

The analytical solution of equation (4.35) yields the following general solution for the temperature distribution in the test section (the complete solution for equation (4.35) is given in Appendix C-2.)

\[ \theta(r, x) = C_0 + \sum_{n=1}^{\infty} C_n \exp\left(-\frac{\Lambda_n^2 x}{\beta_s}\right) J_0(\Lambda_n r) + \frac{q''}{2 \rho_0 \lambda_{es}} r^2 + \frac{2q''}{\rho_0 \lambda_{es} \beta_s} x \]  

The expressions for \( C_0 \) and \( C_n \) with reference to Fourier-Bessel series are as follow

\[ C_0 = -\frac{q'' R_0}{4 \lambda_{es}} \]  

\[ C_n = \frac{\frac{2q''}{\lambda_{es} R_0 \Lambda_n^2} J_2(\Lambda_n R_0)}{J_0^2(\Lambda_n R_0)} \]
Therefore the temperatures at the wall, $T_s$ and at the centre, $T_c$ are given as follows:

$$T(R_o, x) = T_0 + \frac{q^*R_o}{4\lambda_{es}} + \sum_{n=1}^{\infty} C_n \exp\left(-\frac{\Lambda_n^2 A_{es}}{u_D} x\right) J_0(\Lambda_n R_o) + \frac{2q^* A_{es}}{R_o \lambda_{es} u_D} x$$ (4.42)

$$T(0, x) = T_0 - \frac{q^*R_o}{4\lambda_{es}} + \sum_{n=1}^{\infty} C_n \exp\left(-\frac{\Lambda_n^2 A_{es}}{u_D} x\right) + \frac{2q^* A_{es}}{R_o \lambda_{es} u_D} x$$ (4.43)

The heat transfer coefficient becomes:

$$\alpha = \left[ \frac{R_o}{4\lambda_{es}} + \sum_{n=1}^{\infty} C'_n \exp\left(-\frac{\Lambda_n^2 A_{es}}{u_D} x\right) J_0(\Lambda_n R_o) \right]^{-1}$$ (4.44)

and the functional dependence of the Nusselt number can be expressed as

$$\text{Nu} = \left[ \frac{2R_o}{\left\{ \frac{R_o}{4} + \sum_{n=1}^{\infty} \lambda_{es} C'_n \exp\left(-\frac{\Lambda_n^2 A_{es}}{u_D} x\right) J_0(\Lambda_n R_o) \right\}} \right]$$ (4.45)

Where $C'_n = C_n / q^*$.

The comparison of the measured and predicted heat transfer coefficients as a function of the superficial liquid velocity is given for a constant heat flux of 3000 W/m² in Fig. 4.17. The calculated lines according to equation (4.44) are in excellent agreement with the experimental results. The applicability of the model for heat transfer in porous media under constant heat transfer is demonstrated in Fig. 4.18 where all experimental data obtained for various conditions are compared with those predicted from equation (4.44). The absolute mean average error between measured and predicted values is 3.2%. When the results of Krüger (1995) are included, the mean average error increases to 5%.
Fig. 4.17 Variation of measured heat transfer coefficient with superficial liquid velocity.

Fig. 4.18 Comparison of measured heat transfer coefficients with theoretical values calculated from equation (4.44).
Fig. 4.19 compares the measured radial temperature distribution in the bed with the calculated values obtained from equation (4.39). It illustrates a good agreement with an absolute mean average error of 3.9%.

Fig. 4.19 Comparison of measured radial temperature distribution with theoretical values.

Fig. 4.20 compares the calculated axial temperature with the measured values. As can be seen the values are in satisfactory agreement. Therefore equation (4.39) can be used to predict the temperature distribution in porous media. The difference between measured and calculated values is due to dispersion effects which are not taken into account in the energy equation.

Predicted Nusselt numbers along the flow direction are illustrated in Fig. 4.21 for different liquid flow rates. The Nusselt number increases as the liquid flow rate increases, and the Nusselt number approaches a constant value at a certain distance from the entrance. It also follows from Fig. 4.21 that the measured and theoretical values are in good agreement, equation (4.45) therefore predicts the heat transfer coefficients reasonably well.
Fig. 4.20 Comparison of measured temperature distribution in axial direction with calculated values.

Fig. 4.21 Variation of predicted values of the Nusselt number as a function of axial distance.
The effect of axial conduction

The volume-averaged, steady energy equation (4.24) for a cylindrical porous medium is solved using the initial and the boundary conditions given by equations (4.31) – (4.33). The last term in equation (4.24) takes care of the axial conduction.

Substitution of equation (4.34) into equation (4.24) yields:

\[
\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} = \beta_s \frac{\partial \theta}{\partial x} - \frac{\partial^2 \theta}{\partial x^2}
\]  

(4.46)

A finite difference method was employed to solve numerically the above partial differential equation using the boundary conditions given by equations (4.36) – (4.38). Equation (4.46) was discretized using: a central difference for the first and second terms on the left hand side and also for the last term on the right hand side and an upward difference for the first term on the right hand side of the equation. The scheme also uses forward and backward differences for the first and second boundary conditions, respectively. Since the domain is parabolic, a three point backward difference is used for the axial conduction term. The algebraic equations obtained are solved simultaneously. The discretization procedure, the flow chart and the program code are given in Appendix C-3.

Fig. 4.22 compares the calculated heat transfer coefficients using the presented numerical scheme with those calculated from equation (4.44) as well as with the experimental values. It depicts an excellent agreement and, therefore, suggests that the axial conduction effect on the process of convective heat transfer is negligible. Hence, the analytical model can be comfortably used to predict the heat transfer coefficients in porous media.

Effect of natural convection

Since the present investigation is concerned with forced convection in porous media, it is necessary to identify those experimental data in which the natural convection effect is not
negligible. The effects of natural convection will have to be considered if the fluid density changes appreciably. Since the density of the fluid is a function of temperature, an equation of state is required to complement the equations of mass, momentum, and energy. The simplest equation of state is (Bejan, 1984):

$$\rho = \rho_0 [1 - \beta(T - T_0)] \quad (4.47)$$

In equation (4.47) $\rho_0$ is the fluid density at some reference temperature $T_0$, and $\beta$ is the coefficient of thermal expansion. In order to simplify the subsequent analysis, the Boussinesq approximation, which accounts for the effect of variable density in the buoyancy force, is used whenever it is valid. Applying this approximation to the Darcy model yields:

$$u = \frac{K}{\mu_f} \left[ -\frac{dp}{dx} + \rho_0 g + \rho_0 g\beta(T - T_0) \right] \quad (4.48)$$

Equation (4.48) can be transformed into the following form
\[ u = B' + A'(T - T_o) \]  \hspace{1cm} (4.49)

where

\[ A' = \frac{\rho_o g \beta K}{\mu f} \quad \text{and} \quad B' = u_D = -\frac{K}{\mu} \left( \frac{\Delta p}{L} + \rho_o g \right) \] \hspace{1cm} (4.50)

By employing the following variables, equation (4.48) can be non-dimensionalized.

\[ U = \frac{u}{u_D}, \quad Ra = \frac{Kg \beta D \Delta T}{A \nu}, \quad Pe = \frac{u_D D}{A} \] \hspace{1cm} (4.51)

Substitution into equation (4.47) reduces it to:

\[ U = 1 + \frac{\rho_o g \beta K}{\mu f u_D}(T - T_o) \] \hspace{1cm} (4.52)

or

\[ U = 1 + \frac{Ra}{Pe} \] \hspace{1cm} (4.53)

The steady energy equation in a cylindrical porous medium neglecting the axial conduction according to equation (4.29) is:

\[ \frac{u}{A_s} \frac{\partial T}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \] \hspace{1cm} (4.54)

Substitution of equation (4.49) into equation (4.54) yields:

\[ [B' + A'(T - T_o)] \frac{\partial T}{\partial x} = A_s \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \] \hspace{1cm} (4.55)

The boundary conditions for equation (4.55) are given

\[ T = T_o \hspace{1cm} \text{at} \hspace{1cm} x = 0 \] \hspace{1cm} (4.56)
Equation (4.55) with the boundary conditions given by equations (4.56) – (4.58) is solved numerically and the results are compared with experimental data in Figs. (4.23) and (4.24). The excellent agreement between the theoretical and the observed values indicates again that the effect of natural convection is negligible. For both cases, the local heat transfer coefficient is defined with the difference between wall and centre temperature.

\[
\frac{\partial T}{\partial r} \quad \text{at} \quad r = 0 \quad (4.57)
\]

\[
\frac{\partial T}{\partial r} = \frac{q^*}{\lambda_{es}} \quad \text{at} \quad r = R_0 \quad (4.58)
\]

Fig. 4.23 Comparison of measured heat transfer coefficients with theoretical values as a function of heat flux.

To qualitatively evaluate the effect of natural convection, equation (4.55) can also be non-dimensionalized using the following variables:

\[
X = \frac{2x}{r_0 Re_D} , \quad x = \frac{r_0 Re_D X}{2} \quad \text{hence} \quad \partial x = \frac{r_0 Re_D}{2} \partial X \quad (4.59)
\]

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\[ \theta = \frac{\lambda_{es}}{q^* r_o} (T - T_o), \quad T - T_o = \frac{q^* r_o \theta}{\lambda_{es}} \quad \text{hence} \quad \partial T = \frac{q^* r_o}{\lambda_{es}} \partial \theta \]  

(4.60)

\[ X = 0.375, \; d_p = 0.2450 \text{ mm} \]
\[ X = 0.625, \; d_p = 0.2450 \text{ mm} \]
\[ X = 0.375, \; d_p = 0.4800 \text{ mm} \]
\[ X = 0.625, \; d_p = 0.3375 \text{ mm} \]

\[ X = 0.375, \; \text{theoretical} \]
\[ X = 0.625, \; \text{theoretical} \]

Heat flux, W/m²

Heat transfer coefficient, W/m² K

Fig. 4.24 Comparison of measured heat transfer coefficients with values obtained from the numerical solution of equation (4.55) as a function of heat flux.

\[ R = \frac{r}{r_o}, \quad r = r_o R \quad \text{hence} \quad \partial r = r_o \partial R \]  

(4.61)

Consequently, equation (4.55) reduces to

\[ \left[ 1 + \frac{A'}{u_D} \left( \frac{q^* r_o}{\lambda_{es}} \frac{\partial \theta}{\lambda_{es}} \right) \right] \frac{\partial \theta}{\partial X} = \frac{1}{P_{res}} \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial \theta}{\partial R} \right) \]  

(4.62)

and with substitution of equations (4.50) to
\[ [1 + \frac{Ra^*}{2Pe_{es}}] \frac{\partial \theta}{\partial X} = \frac{1}{Pr_{es}} \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial \theta}{\partial R} \right) \]  

(4.63)

where

\[ Ra^* = \frac{g \beta KD^2 q^*}{A_{es} \nu \lambda_{es}} \quad \text{Pe}_{es} = \frac{u_{D} D}{A_{es}} \quad \text{Pr}_{es} = \frac{\nu}{A_{es}} \]  

(4.64)

Since the value of \( Ra^*/Pe_{es} \) is small in the present investigation the effect of natural convection can be neglected.

The improvement in the heat transfer coefficient for all operating conditions in this investigation is calculated to be 0.5% when natural convection given by equation (4.63) is taken into account.
4.3 Two-phase hydrodynamics

When two immiscible fluids flow through a porous medium simultaneously, the void space is occupied by the two fluids. In this case, the medium is partly saturated by each fluid and the pressure drop, which is established along the direction of the flow, is a function of the bed geometry as well as the fluid saturation and the fluid flow rates. Therefore, the hydrodynamics of two-phase flow are more complex those of single-phase flow. The particle diameter affects the pressure drop significantly. It alters the porosity of the medium, which in turn alters the void space, consequently changing the resistance to the flow.

The effect of different experimental variables, such as those already mentioned, on frictional pressure drop has been studied. To carry out the study, a number of experiments were carried out and the results are presented and discussed in the following section.

4.3.1 Flow regimes

To investigate the flow pattern, the perspex test section replaced the stainless steel test section. A constant liquid flow rate was established through the bed at a predetermined value. Then the gas flow rate was increased gradually from zero to the maximum value. The visual observations are as follow:

At very low gas flow rates, few bubbles of uniform size were formed at the bottom of the bed and ascend through the continuous liquid phase at nearly constant velocity. This flow pattern was termed as bubbly or homogeneous flow. By increasing the gas flow rate, the number of bubbles increased, with the liquid phase still being the continuous phase. However, at higher rates of bubble formation, bubbles seemed to move in clusters (swarms) with a lower velocity. This is due to the fact that when bubbles move through the continuous phase they interact with each other and affect each other's velocities. At higher gas flow rates when the number of bubbles increased, this effect on the neighboring bubbles becomes considerable (hindrance effect) and causes a decrease in the velocity of bubbles.
At even higher gas flow rates the interaction between the bubbles becomes so considerable that they start to coalesce. These large bubbles move faster and take irregular paths through the column and as they move up. Some may consequently break into smaller bubbles of various sizes, too. Since these bubbles transport a larger volume of the liquid in their wake, a higher level of turbulence is experienced in the continuous liquid phase. This is known as heterogeneous flow.

With further increase of the gas flow rate, a situation was observed in which larger bubbles are stabilized inside the medium. This is accomplished by alternate portions of dense mixtures of the two phases (plugs) passing through the medium characterize this. The ascending movement of these plugs caused a considerable portion of the liquid to be displaced in their wake and produce a significant amount of turbulence.

Finally, the frequency of formation of these slugs increased to a point where a thin layer of the liquid moved adjacent to the packing material in the bed and the gas traveled upward flowing next to the liquid layer. With further increase in gas flow rate the former situation prevailed and it was thought that some spraying of the water content of the bed occurred at the expense of a reduction in the thickness of the liquid layer. Photographs of the different observed flow regimes are given in Appendix C-4.

Attempts were made to observe these patterns with 1mm particles; however it was only possible to observe the first two patterns without noticing the onset of each flow pattern. The observations were compared with those reported previously by other authors (Turpin and Huntington, 1967; Larkins et al., 1961) and it was found that the observations were in close agreement with the published results.

4.3.2 Gas hold-up

Gas hold-up is defined as the ratio of the volume of the pore occupied by the gas phase in the form of bubbles to the total volume of the fluid in the bed.

Theoretical and empirical correlations available for the prediction of design parameters, such as heat and mass transfer coefficients, are strong functions of the gas hold-up in the
medium. Therefore accurate knowledge of gas hold-up in two-phase flow in porous media is critical for the reliable estimation of these transfer coefficients.

In the general case, the gas hold-up in packed beds at low gas velocities is substantially greater than that in empty sparged columns; the opposite is true at high gas velocities (Zhukova et al., 1990). The porosity of packed bed, liquid hold-up, and gas hold-up are related to one another as follows:

\[
\delta = \varepsilon_1 + \varepsilon_g
\]  

(4.65)

As indicated by published data (Zhukova et al., 1990), the gas hold-up increases with increasing velocity of the gas stream and is dependent on the physical properties of both moving phases, the geometry of the bed, and the diameter of the column. The values of \(\varepsilon_g\) obtained for different packings are substantially different from one another. However, the surface roughness of the packing has a negligible effect on gas hold-up while the type of distributing devices is insignificant, since the stream hydrodynamics are primarily determined by the bed itself, at least at a distance equal to 1-2 column diameters from the gas distributor.

To investigate the effect of various parameters on the gas hold-up a series of experiments were conducted over a wide range of liquid and gas flow rates using particles with different sizes. The measured experimental data are presented in the following sections and the importance of various parameters is discussed.

Fig. 4.25 illustrates the variation of gas hold-up as a function of superficial gas velocity at different values of superficial liquid velocity for a medium made of 1 mm particles. As can be seen from the graph, gas hold-up increases with increasing superficial gas velocity. At lower gas velocities the slope of the curve is steeper than at higher rates of gas flow. The reason for this difference may be described as follow: As the flow rate of the gas increases bubbles coalesce to form larger bubbles which travel in a scattered path with a higher velocity and therefore leave the void space of the bed earlier than in the previous situation, resulting in a reduction in the slope of the curve. The change in the slope of the curves in Fig. 4.25 demonstrates a change in flow regime.
Fig. 4.25 Variation of gas hold-up with superficial gas velocity.

It is worthwhile to note that the change in the slopes of the curves does not occur suddenly but is gradual and takes place over a range of gas flow rates. The range over which the change in flow pattern occurs is termed as the transition zone.

Fig. 4.25 also illustrates that as the liquid flow increases the gas hold-up decreases and consequently the liquid hold-up increases. To investigate the effect of liquid flow rate on the gas hold-up Fig. 4.26 is presented which shows the variation of the gas hold-up as a function of superficial liquid velocity for different values of superficial gas velocity. The sharp decrease of gas hold-up at lower liquid flow rates can be seen clearly. However, at higher liquid flow rates the changes slow down and are expected to level off at some higher liquid flow rates. The reason for these variations is that when liquid flow rate increases, it causes an increase in the velocity of the bubbles which then leave the bed after a shorter time, which results in a reduction in gas hold-up. Since for a definite particle size, porosity is constant and equal to the sum of gas and liquid hold-ups, one increases at the expense of the other.
Another factor affecting the gas hold-up is the size of particles, which form the medium. Fig. 4.27 presents the variation of gas hold-up as a function of superficial gas velocity for different particle sizes at constant superficial liquid velocity. It is evident from Fig. 4.27 and Fig. 4.28 that when the particle size increases, gas hold-up increases too. The reason for this trend is the increase in porosity with particle diameter. At a constant liquid flow rate, an increase in the porosity of the bed will result in higher values of gas hold-up. However, the increase in gas hold-up is more than the difference in the porosity of the bed for different particles. The reason is that a portion of the gas is entrapped in stagnation zones of the medium and hence the observed increase in gas hold-up is higher than the expected value.

**Correlation of experimental data**

In the foregoing discussion of the experimental data, the various parameters affecting the value of the gas hold-up were determined. In general, the gas hold-up is a function of particle size, and the gas and liquid phase velocities. In analogy to two-phase gas-liquid
Fig. 4.27 Variation of gas hold-up with superficial gas velocity for different particle diameters.

Fig. 4.28 Variation of gas hold-up with porosity.
flow in tubes the functionality between the gas hold-up and the various parameters affecting it can be expressed in the following form (Bach and Pilhofer, 1978):

\[
\frac{\varepsilon_g}{\delta - \varepsilon_g} = G(u_g, u_1, d_p, d_b)
\]  

(4.66)

Tung et al. (1988) showed that in unconsolidated porous media the following relationship exists between bubble diameter and particle size of the bed:

\[
\frac{d_b}{d_p} = \frac{\delta}{1 - \delta}
\]  

(4.67)

Experimental investigations show that gas hold-up depends on the ratio of the fluid velocities (Figs. 4.25-4.27). Therefore

\[
\frac{\varepsilon_g}{\delta - \varepsilon_g} = F\left(\frac{u_g}{u_1}, \frac{\delta}{1 - \delta}\right)
\]  

(4.68)

and mathematical rearrangement of equation (4.68) yields:

\[
\frac{\varepsilon_g}{\delta} = \frac{F\left(\frac{u_g}{u_1}, \frac{\delta}{1 - \delta}\right)}{1 + F\left(\frac{u_g}{u_1}, \frac{\delta}{1 - \delta}\right)}
\]  

(4.69)

In the above equation, the non-linear function F is unknown and should be determined. Let us assume that F is dependent on the ratio of fluid velocities as well as bed porosity. This functionality can be represented as the following:

\[
F\left(\frac{u_g}{u_1}, \delta\right) = P\left(\frac{u_g}{u_1}\right)_{|\delta} \cdot Q\left(\delta\right)_{u_g/u_1}
\]  

(4.70)

Now finding \(F\left(\frac{u_g}{u_1}, \delta\right)\) reduces to finding \(P\left(\frac{u_g}{u_1}\right)_{|\delta}\) and \(Q\left(\delta\right)_{u_g/u_1}\). Fig. 4.29 illustrates a graph of \(\varepsilon_g\) as a function of \(u_g/u_1\) at constant bed porosity. The best-fit
function through the data is in the form of \( y = ax^b/(1 + ax^b) \). Substituting these findings into equation (4.69) yields the following correlation for gas hold-up in two-phase gas-liquid flow through porous media:

\[
\frac{\varepsilon_g}{\delta} = \frac{\left( \frac{u_g}{u_1} \right)^{0.675} \left( \frac{\delta}{1 - \delta} \right)^{5.24}}{1 + \left( \frac{u_g}{u_1} \right)^{0.675} \left( \frac{\delta}{1 - \delta} \right)^{5.24}}
\]

Equation (4.71) is validated by comparing its predictions with experimental data. The result of this comparison is summarized in Figs. 4.30-4.32. The absolute mean average error between predicted and experimental data is less than 6.8%, which demonstrates the applicability of this correlation.
Fig. 4.30 Comparison of theoretical values with experimental data of gas hold-up

Fig. 4.31 Comparison of experimental gas hold-up with theoretical values at constant liquid flow rate.
4.3.3 Pressure drop

As has been pointed out in the literature, the pressure drop for gas-liquid co-current upward flow through porous media increases with an increase in gas and liquid flow rates, or a decrease in bed porosity or particle size. Pressure drop also increases with increasing viscosity, interfacial tension, and the density of the fluid phases.

The measured two-phase frictional pressure drop is presented in Fig. 4.33 as a function of gas Reynolds number for a constant liquid Reynolds number. The curve represents an individual experiment in which the liquid flow rate was held constant and the gas flow rate was increased. The observed trend conforms to what is commonly reported in the literature, (Saada, 1974). If the data are presented on a log-log scale, see Fig. 4.34, then two distinct zones with linear variation are observed. This suggests that with increasing gas flow rate a point is reached where the slope of the line shows a change. From this point onwards the pressure drop rises faster than before and this is where the transition
from one regime to the other occurs. Hence this point is termed the transition point, which can be estimated from the graph. Experimental observations indicate that below the transition point the flow is homogeneous and the extent of liquid turbulence is low. Therefore the gas flows in the form of bubbles resulting in an only moderate increase of the pressure drop in the direction of flow. At higher gas velocities where the non-homogeneous regime is encountered, the turbulence of the liquid is due to the passage of a large portion of the gas in the form of plugs becomes more significant and results in larger pressure drop across the bed.

Figs. 4.34 and 4.35 have been prepared to show the effect of liquid flow rate for the same range of gas Reynolds number. The graphs in Fig. 4.35 indicate the same trend as depicted by Fig. 4.33. Moreover, it points out that, at lower gas flow rates the effect of liquid flow rate is more significant and at higher gas flow rates, the increase in liquid flow rate loses its importance. However, the growth in the air flow rate is responsible for the sharp increase in the pressure drop along the direction of the flow. As it is indicated by all graphs on, Fig. 4.34 the transition from one regime to the other occurs when the
Modified gas Reynolds number, $Re_{m,g}$

**Fig. 4.34** Variation of pressure drop with modified gas Reynolds number.

Modified Gas Reynolds number, $Re_{m,g}$

**Fig. 4.35** Variation of pressure drop with modified gas Reynolds number.
slope of the curves changes. Fig. 4.35 confirms the aforementioned effects for a particle diameter of 1.5 mm.

Fig. 4.36 illustrates the effect of particle diameter on frictional pressure drop for a constant liquid flow rate over a wide range of gas Reynolds numbers. It can be noted from this figure that a reduction in particle size will result in higher-pressure gradients with a reduction in gas hold-up for a constant superficial liquid velocity. This is due to the fact that a reduction in particle diameter decreases the voidage of the bed through which the two phases are flowing. Therefore, the effect of the interaction between the phases increases, which offers more resistance to the flow.

The effects depicted in these figures reveal the fact that the pressure drop in porous media is highly dependent on the superficial velocities of the phases, the physical properties of the fluids and the system geometry, bed porosity and permeability.

Fig. 4.36 Comparison of pressure drop for different particle sizes as a function of modified gas Reynolds number.
Correlation of experimental data

Many approaches have been proposed to model the particle-fluid pressure drop in granular beds. Because of the complexity of the system, however, none of the models is completely satisfactory.

In the following section a fundamental approach is employed to first model the single-phase pressure drop. It is then extended to model two-phase flow in an unconsolidated porous medium.

Among the several methods, which have been recommended and used for correlating the experimental data, the capillary models have been most frequently employed to interpret the pressure drop for the flow of fluid flow through unconsolidated porous media.

Prediction of single/two-phase pressure drop using cylindrical pore model

Because the detailed description of fluid flow through porous media is highly complicated, it seems reasonable to substitute a fictitious but simpler porous medium for the actual one. Fluid flow through the model must on the mean, in some statistical sense, be equivalent to fluid flow in the actual medium. Naturally, conclusions drawn from the model must not conflict to a significant degree with actual observations. One of the simplest porous media models is the cylindrical pore model, in which the porous medium is visualized as a bundle of narrow cylindrical pores or tubes is idealized picture of a complex reality. According to this model, the interstitial void space in a porous medium is envisioned to form tortuous pores of complicated cross-section but a constant cross-sectional area on the average. Thus, the flow in porous media is equivalent to that in cylindrical pores (i.e. tubes) whose length and diameter are so chosen that the same resistance to flow is offered as in actual porous medium. Undoubtedly, the formed pores are interconnected in an irregular manner but the simple cylindrical pore model does not take into account this complexity. However, this model together with the solution of the flow through it can be used to develop simple correlations for the prediction of the friction factor, pressure drop and holdup for single and two-phase flow through unconsolidated porous media.
For the flow of a fluid in a differential length of pore \( dl_T \) of diameter \( d_T \), the total frictional force at the walls is the product of the shear stress \( \tau \) and the surface area of the pore \( \pi d_T \cdot dl_T \). This frictional force results in a change in pressure \( dp_f \) so that:

\[
\tau \cdot \pi d_T \cdot dl_T = dp_f \cdot \frac{\pi d_T^2}{4}
\]  

(4.72)

or

\[
dp_f = \frac{4\tau}{d_T} \cdot dl_T
\]  

(4.73)

On the other hand, the friction factor for flow of a fluid in a tube is defined as:

\[
\tau = \frac{f}{2} \rho u_p^2
\]  

(4.74)

Substituting equation (4.74) into equation (4.73) and solving for \( dp_f \) yields:

\[
dp_f = \frac{4f}{d_T} \cdot \frac{1}{2} \rho u_p^2 \cdot l_T
\]  

(4.75)

For an incompressible fluid flowing in a pore of constant cross section, the flow velocity is constant and equation (4.75) can be integrated over the entire length of the pore to give:

\[
\frac{\Delta p_f}{l_T} = \frac{4}{d_T} \left( \frac{f}{2} \right) \rho u_p^2
\]  

(4.76)

In equation (4.76) \( d_T \) and \( u_p \) are pore diameter and fluid velocity (pore velocity) in the single pore respectively. These two parameters should be calculated in terms of the particle diameter of the porous medium, \( d_p \), and the superficial velocity of the fluid in the bed, \( u_s \).
Pore diameter in terms of the particle diameter of the bed

The general structure of a porous medium can often be characterized by the specific surface area and the porosity of the porous media. The specific surface area of a particle is its surface area divided by its volume. For smooth spherical particles it becomes:

\[ s_o = \frac{\pi d_p^2}{\pi d_p^3 d_p} = \frac{6}{d_p} \quad (4.77) \]

Bed porosity is the fraction of the volume of porous medium not occupied by the solid particles. Assuming a pore length equal to that of the bed, hence:

\[ \delta = \frac{V_p}{V_c} = \frac{N_T \frac{\pi d_T^2 l_T}{4}}{\frac{\pi d_c^2 l_c}{4}} = \frac{N_T d_T^2}{d_c^2} \quad (4.78) \]

The specific surface area of the porous medium is the surface area presented to the fluid per unit volume of the system and is equal to:

\[ s_B = N_T \cdot \frac{\pi d_T l_T}{\frac{\pi d_c^2 l_c}{4}} = \frac{4N_T d_T^2}{d_c^2} \quad (4.79) \]

Dividing equations (4.78) and (4.79) and solving for the pore diameter gives:

\[ d_T = \frac{4\delta}{s_B} \quad (4.80) \]

If point contact occurs between particles such that only a small fraction of surface area is lost by overlapping, then (Coulson et al., 1983):

\[ s_B = (1 - \delta) s_o \quad (4.81) \]

Replacing equation (4.77) and (4.81) into equation (4.80), it reduces to:
\[ d_T = \frac{\delta d_p}{1.5(1-\delta)} \]  

(4.82)

For a bed packed with 0.5 mm particles equation (4.82) predicts a pore diameter of about 0.19 mm. In equation (4.82), \( \delta \) is calculated using the correlation present by Fand et al. (1990).

**Fluid velocity in a single pore as a function of the velocity in the porous medium**

Writing material balances on all cylindrical pores and on the porous medium leads to an expression between the average fluid velocity in a single pore and the superficial fluid velocity in the porous medium:

\[ N_T \left( \rho u_p \cdot \frac{\pi d_p^2}{4} \right) = \rho u_f \frac{\pi d_c^2}{4} \]  

(4.83)

or

\[ u_p = \frac{d_c^2}{N_T d_T^2} u_f \]  

(4.84)

Substitution of equation (4.78) into equation (4.84) yields:

\[ u_p = \frac{u_f}{\delta} \]  

(4.85)

Tien and Hunt (1987) have also reported equation (4.85).

Replacing equation (4.82) and (4.85) into equation (4.76), assuming a pore length equal to bed length, yields:

\[ \frac{\Delta p}{1} = \frac{f}{2} \cdot \frac{6(1-\delta)}{d_p \delta^3} \cdot \rho u_f^2 \]  

(4.86)
Equation (4.76) can now be extended to two-phase flow in unconsolidated porous media with the appropriate definition of $d_T$ and $u_p$ in terms of the fluid velocities in the bed. In two-phase flow, fluid velocity can be expressed in terms of superficial gas velocity, $u_g$, superficial liquid velocity, $u_l$, or mixed flow velocity ($u_s = u_g + u_l$). In this study, extensive experimental results are obtained in terms of gas velocity; hence, two-phase frictional pressure drop is defined in terms of this variable.

The pore cross-sectional area may be divided into two zones, the area occupied by the gas phase, $A_g$, and the area occupied by the liquid $A_l$ as being illustrated in Fig. 4.37. Referring to this figure, the following relationships can be written:

\[
d_l = d_T - d_g = d_T (1 - \xi) \quad (4.87)
\]

\[
A_l = A_T - A_g = A_T (1 - \xi^2) \quad (4.88)
\]

Where

\[
\xi = \frac{d_g}{d_T} \quad (4.89)
\]

\[
\xi^2 = \frac{A_g}{A_T} \quad (4.90)
\]

![Fig. 4.37 Nomenclature used in the derivation of equation (4.87) and (4.88)](image)

Therefore, the gas velocity in the pore can be related to the gas velocity in the bed as follows:
Substituting equations (4.90) and (4.78) into equation (4.91) yields:

\[ \rho_{pg} u_{pg} = \frac{1}{\delta_x^2} u_g \]  

(4.92)

Equation (4.76) for the gas phase in Fig. 4.37 can be modified as:

\[ \frac{\Delta P_f}{1} = \frac{f_s}{2} \cdot \frac{4}{d_g} \rho_g u_{pg} = \frac{f_s}{2} \cdot \frac{4}{\xi d_T} \rho_g u_{pg} \]  

(4.93)

Replacing equations (4.82) and (4.92) into equation (4.93) gives:

\[ \frac{\Delta P_t}{1} = \frac{f_t}{2} \cdot \frac{6(1-\delta)}{d_p \delta^3} \rho_g u_g^2 \]  

(4.94)

where

\[ \frac{f_t}{2} = \frac{1}{\xi^5} \left( \frac{f_s}{2} \right) \]  

(4.95)

Through dimensional analysis it can be easily shown that:

\[ \frac{f_t}{2} = F(Re_g, Re_l) \]  

(4.96)

This functionality can be determined from the experimental data in conjunction with equation (4.94). For this purpose a data bank containing 500 data points was compiled over a wide range of operating conditions. These data are used to develop the following correlation for the two-phase flow friction factor:
\[ f_\frac{t}{2} = 94.0 \left( \frac{Re_{l}^{1.11}}{Re_{g}^{1.8}} \right) + 4.40 \] (4.97)

Substituting equation (4.97) into equation (4.94) yields the following correlation for two-phase gas/liquid pressure drop across porous media:

\[ \frac{\Delta P_t}{l} = \left[ 94.0 \left( \frac{Re_{l}^{1.11}}{Re_{g}^{1.8}} \right) + 4.40 \right] \cdot \frac{6(1-\delta)}{d_p \delta^3} \cdot \rho_g u_g^2 \] (4.98)

The applicability of equation (4.98) for two-phase flow through porous media is demonstrated in Fig.4.38 where the experimental data of the present and of other investigators are compared with values predicted from the above correlation. The absolute mean average error between the measured and predicted data is about 6.7%. The prediction of equation (4.98) and of previously published correlations are compared with the experimental data in Table 4.3 in terms of the absolute mean average error of prediction. The best prediction is clearly obtained by using the correlation suggested as a result of the present investigation.

Fig. 4.38 Comparison of theoretical values with experimental data.
Table 4.3 Mean average error of different models.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Mean particle diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.00 mm</td>
</tr>
<tr>
<td>Larkins et al. 1961</td>
<td>59%</td>
</tr>
<tr>
<td>Goto and Gaspillo, 1991</td>
<td>28%</td>
</tr>
<tr>
<td>Turpin and Huntington, 1967</td>
<td>188%</td>
</tr>
<tr>
<td>Ford, 1960</td>
<td>70%</td>
</tr>
<tr>
<td>Saada, 1974</td>
<td>268%</td>
</tr>
<tr>
<td>Khan and Varma, 1997</td>
<td>707%</td>
</tr>
<tr>
<td>Present study</td>
<td>6.7%</td>
</tr>
</tbody>
</table>
4.4 Two-phase heat transfer

Contrary to single-phase heat transfer, the literature on research related to heat transfer in equipment with a stationary bed and ascending gas-liquid flow is practically non-existent. However, there may be industrial cases where substantial heat generation/removal due to exothermic or endothermic processes takes place. This results in a need to remove heat from or replenish heat to the bed in order to maintain a suitable temperature profile. Theoretical and experimental research is therefore required to ensure proper temperature control over the process.

A series of experiments were designed to investigate the heat transfer phenomena in beds with different thermal and physical properties over a wide range of fluid flow rates. The temperatures in the bed as well as at the inside wall were measured after steady state had been reached, and then the heat transfer coefficient was calculated as the ratio of the heat flux at the wall to the temperature difference between wall and bed.

Fig. 4.39 illustrates the variation of the heat transfer coefficient as a function of gas superficial velocity for a particle diameter of 1 mm at a liquid superficial velocity of 0.00417 m/s and a constant heat flux of 5000 W/m². The heat transfer coefficient varies as the velocity of the gas increases. At very low gas velocities, it quickly increases to a maximum value followed by a decrease to a minimum point. If the gas velocity increases further, the value of the heat transfer coefficient begins to rise again. The same trend is observed at both locations along the bed.

With the introduction of air at low flow rates into the system, distinct spherical bubbles are formed which ascend through the continuous phase (liquid) and leave the bed at the outlet. As they move through the bed they cause some radial movement of the liquid phase which causes turbulence. Therefore, the presence of the bubbles encourages radial dispersion leading to the initial rise in the heat transfer coefficient as depicted in Fig. 4.39. As the gas flow rate increases, more bubbles are formed which enhance dispersion and thus increase the heat transfer coefficient. The heat transfer coefficient continues to rise as more bubbles are formed until a maximum point is reached.
Beyond this point the heat transfer coefficient begins to decrease as more air is introduced into the bed. This change in the trend could be attributed to the fact that as the gas flow rate increases, the gas hold-up increases and therefore more void space is occupied by air hence reducing the wetted surface available to heat transfer process.

However, the reduction of wetted heat transfer surface could be the more prominent parameter for the observed reduction in the heat transfer coefficient depicted by Fig. 4.39.

The heat transfer coefficient continues to decline until a minimum point is reached beyond which it rises again.

Further increase in the gas flow rate increases the number of bubbles until a state is reached where the bubbles coalesce and larger bubbles are formed. The newly formed bubbles ascend faster in an irregular path through the liquid phase (heterogeneous regime). In this regime a high level of turbulence is produced in the bed that causes the rate of heat transfer to increase. This increase in heat transfer rate is illustrated in Fig. 4.39.
Nevertheless, the introduction of more air into the system causes an increasing level of turbulence on one hand, while at the same time lowering the effective thermal conductivity of the bed. Therefore, it is expected that the graph may level-off for some gas flow rate above the maximum flow rate shown on this graph. It is worthwhile to note that the point at which the graph shows a change in its trend coincides with the transition point of the gas hold-up graph that indicates the change in flow pattern (Fig. 4.25).

Fig. 4.40 and Fig. 4.41 have been prepared to demonstrate the above mentioned behavior for different liquid flow rates at two axial locations along the bed. It is evident that increasing the liquid flow rate causes the heat transfer coefficient to rise considerably. However, it is interesting to note that the change of flow regime occurs at higher gas flow rate, as the liquid flow rate increases. This is due to the rise in the liquid hold-up, which delays the change in the flow regime.

Fig. 4.42 presents the heat transfer coefficient as a function of superficial velocity of the gas phase. It has been prepared for a particle diameter of 1mm at constant superficial liquid velocity of 0.00417 m/s. On the same graph the variation of gas hold-up as a function of superficial gas velocity for the same particle diameter and liquid flow rate is plotted. It is noted from the figure that the change in the slope of both graphs happens around a superficial gas velocity of 0.05 m/sec. This shows that the explanation with respect to the effect of gas flow rate on the mechanism of heat transfer and hold-up is consistent.

The change in flow regime also agrees with the discussion about the hydrodynamics of the flow in the preceding section. Additional graphs for different gas and liquid velocities show the same phenomena.

Fig. 4.43 shows the effect of particle diameter on the rate of heat transfer in the bed. It is prepared to compare the effect of two particle diameters, 1 mm and 0.48 mm, for a constant liquid flow rate the larger particle show an increased heat transfer coefficient for the specified rage of gas flow rates. This is due to the increase in the gas hold-up resulting from the higher porosity. Although the change in porosity seems to be small, the
Fig. 4.40 Variation of heat transfer coefficient as a function of superficial gas velocity.

Fig. 4.41 Variation of heat transfer coefficient as a function of superficial gas velocity.
Fig. 4.42 Variation of heat transfer coefficient with superficial gas velocity.

The effect of this change on the gas hold-up is significant and, with reference to the discussion presented on Fig. 4.39, the rise in heat transfer associated with larger particles is higher. It should also be noted that larger dispersion coefficients are associated with larger particles which cause an increase in its own right.

Fig. 4.44 compares the heat transfer enhancement when two fluids replace the single fluid in the bed. The figure has been prepared at a constant superficial gas velocity of 0.2454 m/s for a constant particle diameter of 0.48 mm. The figure indicates a marked increase in heat transfer coefficient as a result of the introduction of the second fluid. However, it is expected that at higher gas flow rates when the heat transfer coefficient has approached its asymptotic value the enhancing effect of the second phase levels-off.

Fig. 4.45 illustrates the effect of the thermal conductivity of the porous medium on the heat transfer coefficient. The graphs have been prepared to show the variation of heat transfer coefficient over a range of superficial gas velocities for two different materials with the same size, but with a thermal conductivity 1.05 and 5.34 W/m K respectively. As has been discussed in the literature review section on two-phase flow heat transfer,
Fig. 4.43 Variation of heat transfer coefficient with particle size.

![Graph showing variation of heat transfer coefficient with particle size.]

Fig. 4.44 Comparison of two and single-phase heat transfer coefficients.

![Graph comparing two and single-phase heat transfer coefficients.]

---

$q'' = 5000 \text{ W/m}^2$

$\lambda_s = 1.05 \text{ W/m K}$

$u_l = 0.00417 \text{ m/s}$

$q'' = 3000 \text{ W/m}^2$

$\lambda_s = 1.05 \text{ W/m K}$

$u_l = 0.2454 \text{ m/s}$

$d_p = 0.480 \text{ mm}$
various empirical and semi-empirical correlations for the determination of the effective thermal conductivity have been presented.

The correlations suggest that the effective thermal conductivity of the bed consists of two parts, one relating to the extent of the dispersion caused by each of the fluids and the other depending on the material of which the porous medium is made. Therefore, it is very important to observe the effect of these changes on the heat transfer coefficient. As expected, the experimental results depicted in Fig. 4.45 suggest a direct relationship between the two variables. When the thermal conductivity of the medium rises it reduces the thermal resistance to heat transfer and hence increases the heat transfer coefficient.

The effect of different heat fluxes on the heat transfer coefficient is illustrated in Fig. 4.46. The graph is prepared at constant values of superficial gas and liquid velocities of 0.281 and 0.00417 m/s, respectively, and for a constant particle diameter of 0.48 mm. As the heat flux is increased, higher values of wall and the bed temperatures are reached, which consequently alter the physical properties of the fluids and of the bed. Since the
changes in the physical properties are small, the resulting changes in the heat transfer coefficient are also small and an almost constant heat transfer coefficient with heat flux is observed.

Fig. 4.47 shows the variation of center temperature as a function of length coordinates for different liquid flow rates, at a constant heat flux of 5000 W/m² and a constant gas flow rate of 0.00329 m/s. Heating starts at a distance of 1.75 cm downstream and extends for 38 cm along the bed. The graph shows a gradual increase in axial temperatures with higher values incurring down stream.

The preceding experimental investigation has revealed the effect of different process variables on the heat transfer coefficient in beds with ascending air/water flow under constant heat flux. Obviously, the effect of fluid flow rates and of the related gas/liquid hold-up are of prime consideration. This leads to the role of bed geometry which is characterized by particle size and hence the porosity of the bed. The thermal conductivity of the material forming the bed is also one of the major factors affecting heat transfers in the bed.
Fig. 4.47 Variation of center temperature as a function of heated length.

Modeling of the experimental data

One of the main problems encountered in the modeling of two-phase transport processes in porous media is the lack of information about the numerous transport coefficients required for formulation of the problem. These coefficients may be functions of temperature as well as of the composition of the fluids flowing through the porous medium. In the case of anisotropic media, some of these may be functions of more variables. To simplify the formulation of multiphase transport processes in porous media, a number of assumptions are commonly made such as isotropy and constant physical properties.

The differences between various models of two-phase heat transfer in porous media usually arise from the difference in the details of the physical phenomena considered and the different simplifying assumptions made according to the prevailing physical conditions. Moreover, the use of different empirical correlations for transport coefficients also gives rise to the difference in formulations.
Traditionally, the problem of two-phase flow in porous media has been approached by assuming plug flow of both gas and liquid. Then the partial differential equations for the conservation laws are written and solved according to the appropriate boundary conditions.

The partial differential equation for the cylindrical section of a packed column with co-current gas liquid flow and with equal gas, liquid, and solid temperature is given by (Weekman and Myers, 1965):

\[
\left( G \cdot c_{p,g} + L \cdot c_{p,l} \right) \frac{\partial T}{\partial x} = \lambda_{et} \left( \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} \right)
\]  

(4.99)

Where \( \lambda_{et} \) is the effective thermal conductivity which lumps all resistances to heat transfer. These resistances include thermal resistance at the wall and the thermal resistances between the phases.

The effect of various factors affecting the heat transfer coefficient was fully discussed in the preceding section. Among the operating variables affecting the heat transfer coefficient in two-phase flow, liquid and gas hold-up are important and therefore must be incorporated in the modeling of two-phase flow. Hence gas hold-up is included in equation (4.99) to take account of the amount of gas and liquid present in the pore volume of the bed. The inclusion of gas hold-up in equation (4.99) yields

\[
\left( G \cdot c_{g} \cdot c_{p,g} + L \cdot (1 - c_{g}) \cdot c_{p,l} \right) \frac{\partial T}{\partial x} = \lambda_{et} \left( \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} \right)
\]  

(4.100)

Noting that

\[
G = u_g \rho_g \quad \text{and} \quad L = u_l \rho_l
\]  

(4.101)

and dividing the left hand side of equation (4.100) by \( u_1 \) yields

\[
\left( c_{g} \cdot \frac{u_g}{u_1} + (1 - c_{g}) \cdot \frac{c_{p,l}}{u_1} \right) \frac{\partial T}{\partial x} = \lambda_{et} \left( \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} \right)
\]  

(4.102)

Dividing equation (4.102) by
we obtain

\[
\frac{u_1}{\lambda_{et}} \frac{\partial T}{\partial x} = \left( \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} \right) \left( \epsilon_g \rho_g \frac{u_g}{u_1} + (1 - \epsilon_g) \rho_l c_{p,l} \right)
\]

and taking

\[
\Gamma = \left( \frac{\lambda_{et}}{\epsilon_g \rho_g \frac{u_g}{u_1} + (1 - \epsilon_g) \rho_l c_{p,l}} \right)
\]

equation (4.104) reduces to

\[
\frac{u_1}{\lambda_{et}} \frac{\partial T}{\partial x} = \Gamma \left( \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} \right)
\]

and with

\[
\beta_t = \frac{u_1}{\Gamma}
\]

substitution of equation (4.107) into equation (4.106) yields

\[
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} - \beta_t \frac{\partial T}{\partial x} = 0
\]

Equation (4.108) is the same equation as equation (4.29) which was presented for single-phase heat transfer with \( \beta_t \) replacing \( \beta_s \). \( \beta_t \) in equation (4.108) can be treated in the same way as \( \beta_s \) was treated when solving equation (4.29). Therefore, equation (4.108) can be solved with the following initial and boundary conditions.

Initial condition: \( T(r, 0) = T_0 \) \hspace{1cm} (4.109)

Boundary conditions: \( \left. \frac{\partial T}{\partial r} \right|_{r=0} = 0 \) \hspace{1cm} (4.110)
Using the same procedure as for the solution of equation (4.29), the following expression for the two-phase heat transfer coefficient will result

\[
\alpha_t = \frac{1}{R_0/4\lambda_{et} + \sum_{n=1}^{\infty} C_n \exp \left( - \frac{\Lambda_n^2 \Gamma}{u_p} \right) J_0 \left( \Lambda_n R_0 \right)}
\]

(4.112)

By comparison with equation (4.44), \( \Gamma \) can be calculated by the following:

\[
\Gamma = \left( \lambda_{et} / \left( \varepsilon_p \rho_p C_p \varepsilon_g \frac{u_g}{u_l} + (1 - \varepsilon_g) \rho_l C_p \right) \right)
\]

(4.113)

The only factor, which needs evaluating in equation (4.112) is therefore \( \lambda_{et} \).

It has been proposed by different authors, that the effective thermal conductivity both in single and two-phase flow consists of two parts: one which is referred to stagnant thermal conductivity, \( \lambda_{ot} \) which takes account of the molecular conduction and the other is the conductivity due to radial mixing, \( \lambda_{dt} \). Therefore

\[
\lambda_{et} = \lambda_{ot} + \lambda_{dt}
\]

(4.114)

Dispersion thermal conductivity, \( \lambda_{ot} \), is the sum of dispersion conductivity arising from the movement of the two phases in the porous medium and can be presented in the following mathematical form (Weekman and Mayers, 1965; Hunt and Tien, 1988)

\[
\lambda_{dt} = \left( 1 - \varepsilon_g \right) \gamma_1 \rho_l C_p d_p + \varepsilon_g \gamma_g \rho_p C_p d_p
\]

(4.115)

The effective thermal conductivity is also a function of thermal conductivity of the solid matrix. The roughest estimation of \( \lambda_{ot} \) can be written as (Wang and Beckermann, 1992)

\[
\lambda_{ot} = \lambda_s (1 - \delta) + \lambda_l \delta (1 - \varepsilon_g) + \lambda_g \varepsilon_g
\]

(4.116)

Alternatively a more accurate expression can be sought to represent the stagnant thermal conductivity. A number of correlations for the prediction of thermal conductivity are available in the literature (Kelly et al., 1983). They are mostly the modifications of
correlations, which have been proposed for the prediction of the single-phase stagnant thermal conductivity. However they inherently have the same drawbacks as discussed in the literature review section. Therefore, equation (4.21) can be modified to account for the stagnant thermal conductivity of the bed when it is operating in two-phase flow. The modification of equation (4.21) for two-phase is preceded by the inclusion of the gas hold-up to yield:

\[ \lambda_{ot} = \lambda_s \left(1 - \varepsilon_g \right) \left[1 - \varepsilon_g \lambda _g \right] \delta \]  \hspace{1cm} (4.117)

Substituting equation (4.115) and equation (4.117) into equation (4.114) yields the following determining expression for two-phase thermal conductivity, \( \lambda_{et} \):

\[ \lambda_{ot} = \lambda_s \left(1 - \varepsilon_g \right) \left[1 - \varepsilon_g \lambda _g \right] \delta + \gamma_1 \left(1 - \varepsilon_g \right) \rho_l c_p g d_p + \gamma_g \varepsilon_g \rho_g c_p g d_p \]  \hspace{1cm} (4.118)

where the values of the constants \( \gamma_1 \) and \( \gamma_g \) are to be found experimentally. For this purpose, a large data bank with approximately 800 sets of data was compiled. It contains a large number of experimental data for heat transfer coefficients taken over a wide range of fluids flow rates with beds made of different particle diameter (Table 4.1) and thermal conductivity (Table 3.2). Then the values of the constants in equation (4.118) were calculated using non-linear regression. The results are tabulated in Table (4.4).

**Table 4.4 Coefficient to be used in equation (4.118)***

<table>
<thead>
<tr>
<th>( \gamma_1 )</th>
<th>( \gamma_g )</th>
<th>Mean average error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0596</td>
<td>3.96</td>
<td>8.6%</td>
</tr>
</tbody>
</table>

Fig. 4.48 presents the comparison of the theoretical values using the solution of equation (4.108) with experimental data for particles with different thermal conductivity and diameters. It can be seen, considering the nature of two-phase flow, that there is good agreement between the experimental data and the calculated values. Therefore, equation (4.112) can be used to predict the heat transfer coefficient over a wide range of operating conditions.
Fig. 4.48 Comparison of predicted and measured data.
5. CONCLUSIONS

Multi-phase flow and heat transfer through unconsolidated porous media composed of stationary granular particles frequently find applications in chemical catalytic reactors, building thermal insulation, heat exchangers, petroleum reservoir, geothermal operations, packed beds and many others. In all these applications it is necessary to calculate design parameters such as friction factors, heat and mass transfer coefficients in order to determine the size of the equipment required for a specific purpose.

Over the years, a large body of information has been published on single-phase flow and heat transfer in saturated porous media and there have been several reviews in the literature on the subject. However, a careful review of the literature reveals that there are areas where the available information is insufficient. The lack of research information is particularly felt in beds made of small particles operating under constant heat flux and using liquids as the saturating media.

Two-phase flow and heat transfer has also been the focus of several investigations both experimentally and theoretically in recent years. However, the presented correlations for predicting pressure drop have large discrepancies in their predictions plus the fact that the existing correlations neither cover a wide range of bed geometry nor a broad span of flow rates. Publications regarding heat transfer in beds with two-phase flow are few in number and they mainly deal with downward co-current / counter-current flow. Moreover, information on heat transfer with ascending gas / liquid flow is particularly rare.

The aim of this work has been to systematically study the multi-phase flow and heat transfer in a cylindrical porous medium where the results could be used to predict the pressure drop and the heat transfer coefficient in single/two-phase operations.

1- Single-phase flow and heat transfer

Experimental results have been compared with the existing available models to determine the flow regime. The comparison has revealed that, for the particle diameters and liquid flow rates which are employed in this investigation, Darcy's law is adequate to describe the flow. The potential effects of non-Darcian terms have also
been investigated. For this purpose, the momentum equation with the inclusion of non-Darcian terms has been solved numerically. The results showed a plug flow velocity profile which suggests that the effects due to these terms are insignificant. The permeability of different media is calculated using the Darcy model. The calculated values are compared with the predictions of other empirically available correlations. The discrepancies have been found to be approximately 10%. This is an acceptable error since permeability is a porosity dependent parameter and a large portion of the error could be attributed to the errors which are inherited through the experimental measurement of porosity.

The volume-averaged energy equation is employed to develop a model for the prediction of the heat transfer coefficient. The effect of the axial conduction term, \textit{a priori}, appearing in the energy equation is investigated. The results show that for beds with low thermal conductivity the effect of axial conduction is negligible. However, at relatively high values of bed thermal conductivity, larger than 10 W/m K, the effect becomes appreciable and must be taken into consideration.

A mathematical model based on the volume-averaged energy equation in which the axial conduction is neglected, has been developed to predict the heat transfer coefficient. Comparison of the predicted values with experimental data has exhibited a close agreement. The model can be used confidently to predict the heat transfer coefficient in cylindrical porous media under constant heat flux. The effective thermal conductivity which includes both stagnant and dispersion conductivity plays a significant role in predicting the heat transfer coefficient using the developed model. The dispersion part, which is highly dependent on fluid Reynolds number, has been proved to be insignificant for the present range of experimental variables. This argument is supported by the fact that at low Reynolds numbers, the heat transfer coefficient is independent of particle diameter. However when investigating heat transfer in beds of larger particles then the effect of dispersion must be taken into consideration. The model also demonstrates the non-dependency of the heat transfer coefficient on heat flux which is also supported by experimental confirmation.

The analytical examination of the effects of natural convection has suggested that it does not insignificantly contribute to heat transfer coefficient in the bed.
Nevertheless, when operating at low liquid flow rates this effect becomes appreciable and must be accounted for.

2- Two-phase flow and heat transfer

A large number of experiments is conducted to study different flow regimes. The experimental observations of the flow regimes suggest the existence of two distinct regions; homogeneous and non-homogeneous regime. While the homogeneous regime is characterised by the presence of spherical bubbles at low gas flow rate, the non-homogeneous regime is distinguished by large plugs of gas travelling at relatively higher velocities.

Gas/liquid hold-up plays a significant role in hydrodynamics of two-phase flow. Analysis of experimental data has shown the dependency of gas hold-up on fluids flow rates and particle geometry. Based on these finding gas hold-up experimental data are correlated. The comparison of the predicted values using the presented correlation with experimental data reveals the suitability of the model.

Based on a capillary flow model in porous media, single-phase pressure drop is modelled. The single-phase model is then extended to develop a model for the prediction of two-phase pressure drop. The new model is in good agreement with experimental data. The predictions of the model cover a wide rage of gas Reynolds numbers in media where the liquid flow rate is low and the bed geometry is characterised by a relatively high value of bed to particle diameter ratio.

The two-phase heat transfer coefficient is an important factor when the calculation of heat transfer to beds with ascending air/liquid flow is required. Analysis of experimental data shows that the two-phase heat transfer coefficient depends on fluids flow rates, properties of the fluids, and the material and geometry of the bed. With regards to these findings a new model emerges using the modified single-phase heat transfer model. The comparison of the predicted values with experimental data, considering the nature of two-phase flow, exhibits a good agreement. The mathematical model can therefore be used to predict two-phase heat transfer coefficients in unconsolidated porous media under constant heat flux. An expression is also suggested to replace the single-phase effective thermal conductivity. Due to the highly dispersed condition of the two-phase flow, the dispersion part of the
effective thermal conductivity is replaced by two terms due to the dispersion caused by the liquid and the gas.
6. FUTURE WORK

Introduction
The analysis of multi-phase transport processes in porous media has been approached using continuum equations in which the relevant variables are related to conserved quantities, such as local density, local momentum flux, etc. These equations are derived by defining averages of the corresponding local quantities (Whitaker, 1996; Bejan, 1984). The averages are then taken over a volume that is small compared to the volume of the system, but large enough for the transport equation to hold when applied to the volume. In this way the system is mathematically defined in the form of one or more non-linear partial differential equations. These equations may be solved numerically (i.e. by CFD) or, if simplifications are reasonable, analytically.

These continuum models capture the physics of the transport phenomena in terms of effective transport coefficients such as the permeability and effective thermal conductivity. These quantities have traditionally been determined experimentally or via phenomenological models (Yagi and Kunii, 1960).

Performing experiments is costly and time consuming plus the fact that they are subjected to errors namely; systematic and human. Experiments also rely on skilled operators and precise measuring instruments which may not be available.

The predicted quantity in correlations is usually expressed in terms of some dimensionless parameters. They are then fitted to experimental data and the constants are evaluated.

To elucidate the preceding argument we discuss the permeability of a media. Permeability is a measure of the ease with which fluid flows through porous media. Several theoretical and experimental studies have attempted to predict permeability from known rock properties, see literature review section. Despite extensive studies, theoretical estimates of permeability are often in error in as much as one order of magnitude or more.

The prediction of effective thermal conductivity is important to heat transfer in porous media. As it has been discussed in the previous sections this property of the
media is a function of the interstitial fluid characteristics as well as the conductivity of the phases. Therefore, again a precise knowledge of bed geometry is important in predicting the value of effective thermal conductivity.

Theoretical modelling has also been extended to two-phase flow and heat transfer in porous media, which is intrinsically complicated and difficult. One reason is the strongly non-linear and coupled nature of the governing equations for two-phase flow and transport. Another fundamental difficulty lies in the presence of moving and irregular interfaces between the single and two-phase sub-regions in the pore space. The location of such an internal interface is not known \textit{a priori} but must be determined by the coupled flows in adjacent regions.

Recently, Wang and Beckermam (1993) have removed these difficulties by developing an equivalent but reformulated version of the traditional separate-flow model for two-phase flow. In their formulation of a two-phase mixture model, Wang and Beckermam (1993) have introduced some definitions of the mean transport coefficients of the mixture, i.e. permeability and effective thermal conductivity. These are defined in terms of the saturation of each phase, i.e. gas/liquid hold-up. Hence the extent to which a particular phase saturates the pore volume of a porous medium is important in estimating the transport coefficient. This makes the evaluation of transport coefficients even more complicated.

However, continuum models have been widely applied because of their convenience and familiarity of the engineer. But they are not well suited for describing those phenomena in porous media in which the connectivity of the pore space plays a major role. Such models also breakdown if there are long-range correlations in the system.

In order for such limitations to be overcome one must establish a link between the macroscopic behaviour of a system and its micro/mesoscopic properties. So called discrete models can provide this link.

Discrete models replace the continuum fluid by discrete fluid elements, the most basic of which being molecules. The same differential equations as those used to describe transport of the continuum fluids are applied to each fluid element in order
to resolve its effective transport properties at a microscopic level. In contrast to continuum models, discrete models enable fundamental study of physical and chemical interaction at microscopic level. Furthermore, they are more appropriate to study transport processes in porous media at the pore level.

However, discrete models are computationally very demanding. This restricts their application to the study of very small volumes - of the order of a few centimetres - where a realistic discrete treatment is desired. This restriction can, to some degree, be improved by simplifying inter-particle interactions or particle dynamics and/or clumping fluid particles together to form mesoscopic particle (Biggs and Humby, 1998). One of the discrete methods that adopts the latter approach is the lattice gas automata (LGA) model. These models are a class of cellular automata (Neumann, 1966) used for the simulation of fluid dynamics. LGA models were used by Humby (1998) to model fluid and colloid transport in porous media.

A brief description of LGA

Lattice gas automata consist of a regular lattice on which each site may take a finite number of states. The lattice evolves in discrete time steps, according to a set of rules governing the interaction of sites with one another. Broadwell (1964) was probably - the first researcher who developed an automata type model for fluid simulations. In his model velocity was taken as the only discrete variable while space and time were taken as continuous.

In the early 70's, Hardy et al. developed the first lattice gas model which had a completely discrete phase space and time and could be viewed as made up of "Boolean molecules". However, because of the anisotropy associated with the model, it did not recover the Navier-stokes equation when density and momentum varied slowly in space and time. Then Frisch et al. (1986) replaced the underlying square lattice of HPP with a hexagonal one (fig 6.1) which overruled the discrepancies associated with HPP and mimic Navier-stokes equations at low Mach numbers (Frisch, et al. 1986).
As stated above, FHP model and its extensions are the most popular models which, are most commonly used. The original FHP (two-dimensional, six bit) lattice gas model consists of identical particles on a hexagonal lattice unit. All particles have the same mass and they reside only on the sites of a hexagonal lattice. There are six different particle momentum states at each lattice site associated with the direction $c_i = [\cos (2\pi i/6), \sin (2\pi i/6)]$, $i=1...6$. An exclusion rule is imposed so that no more than one particle exists at a given site. There are two microscopic updating processes at each discrete time step, propagation and collision. In the propagation process, a particle moves from its present site to the nearest neighbour site in the directions, $c_i$; all particles have the same speed ($=1$) and the same kinetic energy. In the collision process, particles at each site are redistributed according to some collision rules among the six momentum states so that mass and momentum are conserved. Some of these collision rules are given in Fig. 6.2.

![Collision rules for particle movement on a triangular lattice.](image)

In order to decrease viscosity and increase the Reynolds number, one can add up to three rest particles to each node. This gives rise to the other versions of the FHP model, known as, FHP-II and FHP-III.
The addition of rest particles can increase the Reynolds coefficient, the ratio of Reynolds and Mach number, hence allowing Reynolds numbers three to six times greater than those achieved by the original model, FHP-I (Biggs and Humby, 1998).

The four dimensional face-centered-hypercubic lattice (FCHC) was proposed by d’Humieres et al. (1986) to simulate three-dimensional problems. The FCHC lattice is the set of all points on the integral lattice for which the sum of the coordinates is even. Each lattice site has 24 nearest neighbors that are a distance $\sqrt{2}$ away. Particle collisions in each site can involve up to 24 particles and conserve mass and momentum. The velocity of a particle site $x$ can be defined as $v_i$, $i=1,2, 24$. A periodic condition for the fourth dimension is used, leading to the pseudo-FCHC mode. The FCHC model forms the basis of all three-dimensional lattices.

Since the development of FHP model, a great deal of work has been done on the simulation of many physical systems. At the present time, lattice gas methods can solve, in addition to Navier-stokes equations, many other partial differential equations including; Burgers equation, the Poisson equation, the wave equation, and the diffusion equation. They can also model many physical phenomena, such as; single phase and two phase fluid flow through porous media, turbulent flow, phase transitions, multi phase flows, chemical-reacting flows, magnetohydrodynamics, liquid crystals, thermodynamics, and semiconductors (Chen et al. 1991).

**Application of LGA to porous media**

Lattice gas automata are currently being applied to problems related to porous media. Recent advances in cellular automata, CA, and computer sciences, however, suggest that accurate calculations of microscopic flow are predictable in arbitrary complex pore-space geometries. The relevant advances in fluid mechanics are the advent of the discrete lattice gas, a cellular automaton fluid. Although CA fluids may be implemented on any computer, new, massively parallel machines perform these flow computations considerably more efficiently than conventional computers.

The low Reynolds numbers associated with porous media and their complex geometries makes them attractive for LGA simulations, LGA has been used by many investigators to study porous media. When LGA is used to simulate porous media,
the Navier-stokes equations are solved without the usual simplifications (Alder et al., 1990). Hagen-Poiseuille flow within a network of one-dimensional pores has been simulated using LGA (Bryant et al., 1993).

The majority of the work has been done to develop LGA models as a tool to predict the permeability of porous media. These studies have been stimulated by the fact the experimental determination of permeability is time consuming and limited to relatively small samples compared to the real-world applications (van Genabeek and Rothman, 1996).

Balasubramanian et al (1987) were the first to examine the potential application of LGA to model flow through porous media. They obtained Darcy's law from flow in two-dimensional channel in the presence of randomly distributed point scatters which produced high porosities which limited their studies. Rothmans (1988) was the first to confirm Darcy's law at a more realistic voidage and his simulation showed excellent quantitative flow characteristics. Following Rothmans findings several workers focussed their attention on obtaining permeability using LGA models (Humby, 1998).

The first three-dimensional study has been carried out by Chen et al. (1991). They simulated the flow through randomly generated porous media using an FCHC model. They reported only qualitative findings that both the pressure and velocity distributions within the media were typical of real systems.

To a lesser extent, LGA has been used for the simulation of two-phase flow in porous media. Jeulin (1992) studied the problem of dispersion in porous media. The simulation achieved by labeling and tracking particles as they move through a system. Chen, Doolen and Mattaeus, (1991); McCarthy, (1994); Kohring, (1991); Rothman, (1988); Rothman and Keller, (1998); Rothman and Zaleski, (1989); Somers and Rems, (1989 & 1991); used CA models to study the flow of two immiscible fluids in two dimensional porous media. Rem and Somers (1989) also studied the same problem in three dimensions.
Conclusions and future work

It is recommended that LGA, which inherently allows the fundamental study of systems to be made and offers accurate, efficient and simple method of simulating transports phenomena through complex geometries such as porous media, to be employed as a simulating tool to investigate the following.

As it has been discussed in the previous sections, there are many correlations available to predict the transport coefficients, permeability and effective thermal conductivity, in porous media. The discrepancy in the prediction of these coefficients could be attributed to the limitations associated with continuum models. Therefore a fundamental study of the transport processes could be made. The followings are suggestions for future work in this area.

Single-phase thermal LGA

Previous studies on thermal LGA mainly concentrate on conduction in solids (a fluid at rest). Therefore the existing collision rules are designed for systems with conduction as a means of heat transfer mechanism. Heat transfer in porous media includes the conduction in solid particles, convection and conduction in the liquid phase and also the conduction-convection at the solid-liquid interface. Therefore in addition to collision rules which have been presented for the conduction in solids (a fluid at rest) by Chopard et al. (1988) and Chen et al. (1995), collision rules are to be devised for solid phase as well as the liquid phase which are straightforward. A set of collision rules has to be constructed for the solid-liquid interface so that they transfer energy at the interface and prevent the passage of the fluid particles into solid, or, visa versa at the same time. These rules must satisfy the conservation law. The simulation can first be done for a moving and a stationary fluid and then be extended to solid-fluid system (i.e. porous media).

Two-phase LGA simulation

Fluid flow

To date, a number of studies have been made concerning two-phase immiscible flow. The same principal can be applied to two-phase flow in porous media. Each fluid is
represented by a color so that they can be tracked down during the simulation stages. The collision rules within the bulk of each fluid are straightforward. Collision rules at the boundaries of the two fluids must be so that they divert each colored particle towards the area with high concentration of particles with the same color and hence form the interfaces. In addition to collision rules for fluid-fluid interfaces, no-slip collision rules are to be implemented to take account of fluid-solid interfaces. However, for flow involving solid surfaces, wettability of the different fluids is simply imposed by coloring wall sites proportional to the wettability of the fluids.

**Heat transfer**

Once the fluid flow algorithm for the simulation of fluid flow has been formulated, the work can be extended to develop an algorithm for two-phase thermal LGA. Collision rules at the interfaces, in addition to providing areas of different color concentration representing each fluid, must allow the transfer of energy while preventing elements of each fluid from passing the boundaries.

The above can be extended to gas-liquid systems. The respective model can use simple ‘long range’ interactions to induce a single species LGA fluid to separate into regions of differing density, one of low density akin to that of a gas, and the other of high density which may be viewed as a liquid. Once the two regions are simulated then suitable collision rules similar to the above can be devised to simulate fluid flow and heat transfer for the same system.
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Appendix A-1 Nomenclature for equations 2.95-2.97

\[ \rho = \rho_1 \varepsilon_1 + \rho_g (1 - \varepsilon_1) \]  
\[ (A-1) \]

\[ \bar{u}_1 = -K \frac{K_{nl}}{\mu_1 (\varepsilon_1)} (\nabla p_1 - \rho_1 g) \]  
\[ (A-2) \]

\[ \bar{u}_g = -K \frac{K_{rg}}{\mu_{gl} (\varepsilon_1)} (\nabla p_g - \rho_g g) \]  
\[ (A-3) \]

\[ \rho \bar{u} = \rho_1 \bar{u}_1 + \rho_g \bar{u}_g \]  
\[ (A-4) \]

\[ \nu = \left( \frac{K_{nl} (\varepsilon_1)}{v_1} \frac{K_{rg} (\varepsilon_1)}{v_g} \right)^{-1} \]  
\[ (A-5) \]

\[ h_1 = c_{pg} T \]  
\[ (A-6) \]

\[ h_g = c_{pg} T + \left[ c_{p,l} + c_{p,g} \right] T_{\text{sat}} + h_{fg} \]  
\[ (A-7) \]

\[ \lambda_c = \lambda_s + \sqrt{\varepsilon_1 \left( \lambda_1 (\varepsilon_1) + \lambda_g (\varepsilon_g) \right)} \]  
\[ (A-8) \]
Photograph B-1 Test rig
Photograph B-2 Peristaltic pump

Photograph B-3 Perspex test section

Photograph B-4 Gasometer
Photograph B-5 Test section features

Photograph B-6 Holding tank features
Photograph B-7 Spiral grooves for heating wire, longitudinal grooves for wall thermocouples and bores for bed thermocouple insertion.

Photograph B-8 Spiral grooves for heating wire, longitudinal grooves for wall thermocouples and bores for bed thermocouple insertion.
Appendix B-2 Calibration of thermocouples

The detailed locations of the thermocouples are illustrated in Fig. B-1.

Fig. B-1 Detail of thermocouple locations at two axial positions along the test section.

The heat conducted through the radial distance is given by

$$q = \frac{2\pi \lambda l \cdot (T_{TC} - T_s)}{\ln(r_o/r_i)} \quad (B-1)$$

Multiplying equation (A.1) by

$$\frac{(r_o - r_i)}{(r_o - r_i)} \quad (B-2)$$

and taking

$$\frac{(r_o - r_i)}{\ln(r_o/r_i)} \quad (B-3)$$

as the mean radius, $r_m$. Then the heat transfer rate per unit area is

$$\frac{q}{A} = \frac{\lambda (T_{TC} - T_s)}{x} \quad (B-4)$$

By performing a steady state energy balance the wall, bed and surface temperatures can be related.
\[ \frac{q}{A} = U \cdot (T_{TC} - T_b) = \alpha \cdot (T_s - T_b) = \frac{\lambda}{x} \cdot (T_{TC} - T_s) \]  

Equation (A-5) can be rearranged to give:

\[ \frac{1}{U} = \frac{1}{\alpha} + \frac{x}{\lambda} \]  

(B-6)

If \( \alpha \) can be related to some measurable quantity such as velocity, then an estimation for \( x/\lambda \), using equation (A-6), can be found. The following equations, which are well documented in any standard heat transfer textbook, are assumed to be valid.

\[ \alpha = F(Re, Pr) \]  

(B-7)

\[ \alpha = a \cdot Re^m \cdot Pr^n \]  

(B-8)

If all properties remain constant then

\[ \alpha \propto u^m \]  

(B-9)

Sieder and Tate, 1936 give the numerical value of the exponent \( m \), as 0.333 for laminar flow. Noting this and grouping all the constants of proportionality into a single constant, \( a \), one obtains:

\[ \frac{1}{U} = \frac{a}{u^{0.333}} + \frac{x}{\lambda} \]  

(B-10)

Equation (A-10) shows that a plot of \( 1/U \) versus \( 1/u^{0.333} \), for each thermocouple, gives \( x/\lambda \) as the intercept and \( a \) as the slope. Therefore, the values of \( \lambda/x \) can be calculated.

Table A-1 illustrates the values of \( \lambda/x \) for different thermocouples.

<table>
<thead>
<tr>
<th>TC</th>
<th>TC11</th>
<th>TC12</th>
<th>TC13</th>
<th>TC14</th>
<th>TC21</th>
<th>TC22</th>
<th>TC23</th>
<th>TC24</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda/x )</td>
<td>4673</td>
<td>4710</td>
<td>5935</td>
<td>7035</td>
<td>5086</td>
<td>5668</td>
<td>6050</td>
<td>5412</td>
</tr>
</tbody>
</table>

Table B-1 The values of \( \lambda/x \) for different thermocouples.
Fig. B-2 \( \frac{1}{U} \) versus \( \frac{1}{u^{0.333}} \) used to estimate \( \lambda_{ss}/x \)
Appendix B-3 Investigation of the uniformity of the heating provided by the electrical heating wire wound around the test section.

Electric heating elements are used for the easy implementation of constant heat fluxes in small heating applications. However, a few studies have carried out to evaluate the uniformity of their heating when they are mounted on the outside surface area of cylinders. The following quantifies the non-uniformity in heat flux arises from the uneven heating issued by considering the axial and radial temperature distribution in the test section wall.

<table>
<thead>
<tr>
<th>$r_o$</th>
<th>21 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_i$</td>
<td>16 mm</td>
</tr>
<tr>
<td>$H$</td>
<td>8 mm</td>
</tr>
</tbody>
</table>

Fig. B-3 Schematic diagram of the test section together with the heating wire

Poisson and Quack (1972) have studied the effect of uneven heating by solving the two dimensional conduction partial differential equation for the configuration shown in Fig. B-3 and presented their results in graphical form.

Considering the dimensions of the test section used in the present study the temperature difference between the hottest and the coldest points along the inner wall of the test section is about 0.7% of the temperature difference between the inner surface of the test section and the bulk temperature of the fluid flowing inside which can be considered to be negligible. Therefore the assumption of constant heat flux is valid with accepted accuracy.
Appendix C-1 Program code for the numerical solution of equation (4.14)

Main driver for finding the velocity profile in a porous media

parameter (maxn=1000)
real*8  f,fr(maxn),u(maxn),ubig(maxn),delta_R
real*8  rt(2*maxn),ut(2*maxn)
real*8  r(maxn),r0,kk,del,neu,ud
real*8  a(maxn),b(maxn),c(maxn),d(maxn)

open (unit= 10, file='out.1')

r0=16
kk=4.62d-11
delta=0.3616
ud=50.d0/48000.d0

f=kk/(r0**2*delta)

write(*,*) Enter the number of internal nodes
read(*,*)nodes

nsect=nodes+1

if(maxn.lt.nsect+1)then
   write(*,*)'halt due to maxn<nsect+1 ; increase maxn'
endif

write(10,*) R0-R u(R0-R)
write(10,*) ---- -------

u(1)=0.d0 / * BC1 */
ut(1)=0.d0 / * BC1 */
ut(2*nsect+1)=0.d0 / * BC1 */

delta_R=r0/nsect

do i=1,nsect+1
   r(i)=1.d0*(i-1)/nsect
endoi

do i=1,nsect+1
rt(i)=-r(nsect+2-i)
rt(2*nsect+2-i)=-rt(i)
enddo

do i=1,nsect-1
  a(i+1)=(f/delta_R**2)-(f/(2.0*r(i+1)*delta_r))
  b(i)=-(1.0+2.0*f/delta_r**2)
  if(i.ne.nsect-1)c(i)=f/delta_r**2+f/(2.0*r(i+1)*delta_r)
  d(i)=-1.0
endo
call tridag(a,b,c,d,ubig,nsect-1)
do i=1,nsect-1
  u(i+1)=ubig(i)*ud
  ut(i+1)=u(i+1)
  ut(2*nsect+1-i)=ut(i)
endo

u(nsect+1)=u(nsect) /* BC2 */
ut(nsect+1)=ut(nsect) /* BC2 */
do i=1,2*nsect+1
  write(10,*)rt(i),ut(i)
endo
end

SUBROUTINE tridag(a,b,c,r,u,n)
INTEGER n,NMAX
REAL a(n),b(n),c(n),r(n),u(n)
PARAMETER (NMAX=1000)
INTEGER j
REAL bet,gam(NMAX)
if(b(1).eq.0.)pause 'tridag: rewrite equations'
bet=b(1)
u(1)=r(1)/bet
do 11 j=2,n
  gam(j)=c(j-1)/bet
11 continue
end
bet=b(j)-a(j)*gam(j)
if(bet.eq.0.)pause 'tridag failed'
u(j)=(r(j)-a(j)*u(j-1))/bet
continue
do 12 j=n-1,1,-1
   u(j)=u(j)-gam(j+1)*u(j+1)
12 continue
return
END
Start

Input

Read no. of nodes in both direction

Apply first boundary condition for axial profile, \( T(r, 0) = T_i(r) \)

Compute the elements of the tri-diagonal design matrix

Solve set of linear equation to find axial and radial temperature profile

Apply second boundary condition \( \frac{\partial T_{(r,1)}}{\partial x} = 0 \) for axial profile and both boundary conditions for radial profile \( \frac{\partial T_{(0,x)}}{\partial r} = 0 \), \( \lambda \frac{\partial T_{(R,x)}}{\partial r} = q^* \)

Report the results

Stop
Appendix C-2 Solution of the energy equation for single-phase flow.

For most flow conditions, an energy equation, which regards the porous medium as a homogeneous medium can be derived to satisfactorily represent the transport through the solid-fluid system. To derive such an equation, first the energy equation is volume averaged Carbonell and Whitaker (1984) and Whitaker (1986).

The steady, volume-averaged equation for the solid phase is

\[ 0 = \nabla \cdot \lambda_s \langle \nabla T_s \rangle + \frac{1}{\Omega_s} \int q_s^e \cdot n_{sf} dA \]  

where \( \lambda_s \) is the solid-phase conductivity, \( q_s \) the interfacial solid-phase heat flux, and \( n_{sf} \) is a unit normal to the interfacial area \( A_{sf} \). The equation for the fluid contains both conduction and convection,

\[ \rho_f c_p f \left( u_p \cdot \nabla \langle T_f \rangle + \nabla \langle u_p \cdot T^* \rangle \right) = \nabla \cdot \lambda_f \langle \nabla T_f \rangle + \frac{1}{\Omega_f} \int q_f^e \cdot n_{fs} dA \]  

where \( u_p \) is the pore velocity and all quantities in the above equation correspond to the fluid phase. The last term corresponds to the interfacial heat flux between the fluid and the solid. The second term on the left-hand side is the dispersion term, resulting from local variations in velocity and temperature, Whitaker (1969).

By defining the spatially averaged temperature

\[ \langle T \rangle = \delta \langle T_f \rangle + (1 - \delta) \langle T_s \rangle \]  

and the effective stagnant conductivity (based on no flow condition),

\[ \lambda_o \nabla \langle T \rangle = \delta \lambda_f \langle \nabla T_f \rangle + (1 - \delta) \lambda_s \langle \nabla T_s \rangle \]  

and the dispersion conductivity,
The fluid and solid equations are combined to yield the homogeneous energy equation as follow;

\[ \rho_f c_{p,f} \langle \mathbf{u} \cdot T^* \rangle = \nabla \cdot (\lambda_e \nabla \langle T \rangle) \]  

\[ \text{(C-6)} \]

where \( \lambda_e \) is the effective conductivity, \( \lambda_e = \lambda_o + \lambda_D \). The model assumes the existence of local thermal equilibrium between the two phases, \( (T_s - T_f = 0) \), which is valid under most flow conditions (DeWasch and Forment, (1972), Whitaker (1986)), or equivalent local temperature gradients \( (\nabla T_s = \nabla T_f) \). Under these conditions, a homogeneous model accurately represents the thermal transport and existing empirical relations can be used to evaluate the stagnant and dispersion conductivities. However, if the particle size is large or the flow rate is high, or if the ratio of the solid-to-fluid conductivities is not close to unity, the homogeneous model may no longer be valid.

The volume-averaged, steady energy equation in a cylindrical porous media assuming Darcian flow and neglecting the third dimension (axi-symmetric flow) is therefore;

\[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} = \frac{u_D \partial T}{A_e \partial x} - \frac{\partial^2 T}{\partial x^2} \]  

\[ \text{(C-7)} \]

where \( A_e \) is the effective thermal diffusivity and defined as follow,

\[ A_e = \frac{\lambda_e}{\rho_f c_{p,f}} \]  

\[ \text{(C-8)} \]

The last term on the right hand side of the equation (C-7) takes care of the axial conduction. Since its contribution to the convection heat transfer is not significant, therefore upon omitting the axial conduction term, the energy equation reduces to:

\[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} = 0 \]  

\[ \text{(C-9)} \]
The initial and boundary conditions applicable to the solution of the partial differential equation (C-10) with respect to the test bed under consideration are as follows:

Initial condition: \[ T(r,0) = T_0 \] (C-12)

Boundary condition (1) \[ \frac{\partial T}{\partial r} \bigg|_{r=0} = 0 \] (C-13)

Boundary condition (2) \[ \frac{\partial T}{\partial r} \bigg|_{r=R_e} = \frac{q''}{\lambda_e} \] (C-14)

where \( q'' \) is the heat flux. Taking
\[
\theta_{(r,x)} = T_{(r,x)} - T_0
\]  
(C-15)

then

\[
\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} - \beta \frac{\partial \theta}{\partial x} = 0
\]  
(C-16)

The initial and boundary conditions are

Initial condition: \[\theta_{(r,0)} = 0\] (C-17)

Boundary condition (1) \[\frac{\partial \theta}{\partial r} \bigg|_{r=0} = 0\] (C-18)

Boundary condition (2) \[\frac{\partial \theta}{\partial r} \bigg|_{r=R_o} = \frac{q''}{\lambda_e}\] (C-19)

Hence the general solution will be of the following form

\[
\theta_{(r,x)} = \psi_{(r,x)} + \phi_{(r)} + \phi_{(x)}
\]  
(C-20)

Upon differentiation equation (C-20) yields

\[
\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} - \beta \frac{\partial \psi}{\partial x} + \frac{d^2 \phi}{dr^2} + \frac{1}{r} \frac{d \phi}{dr} - \beta \frac{d \phi}{dx} = 0
\]  
(C-21)

Applying initial boundary conditions (1) and (2) respectively to equation (C-20) yields

Initial condition: \[\theta_{(r,0)} = \psi_{(r,0)} + \phi_{(r)} + \phi_{(0)} = 0\] (C-22)

Boundary condition (1) \[\frac{\partial \theta}{\partial r} \bigg|_{r=0} = \frac{\partial \phi}{\partial r} \bigg|_{r=0} + \frac{d \phi}{dr} \bigg|_{r=0} = 0\] (C-23)

Boundary condition (2) \[\frac{\partial \theta}{\partial r} \bigg|_{r=R_o} = \frac{\partial \phi}{\partial r} \bigg|_{r=R_o} + \frac{d \phi}{dr} \bigg|_{r=R_o} = \frac{q''}{\lambda_e}\] (C-24)
To convert the initial and boundary conditions into more familiar and homogeneous terms, the following mathematical transformations are done

\[ \Phi(0) = 0 \Rightarrow \psi(r,0) = -\phi(r) \]  
\[ \frac{d\phi}{dr} \bigg|_{r=0} = 0 \Rightarrow \frac{\partial \psi}{\partial r} \bigg|_{r=0} = 0 \]  
\[ \frac{d\phi}{dr} \bigg|_{r=R_e} = q'' \Rightarrow \frac{\partial \psi}{\partial r} \bigg|_{r=R_e} = 0 \]

Using equations (C-25), (C-26) and (C-27), equation (C-21) is separated to yield

\[ \frac{d^2\phi}{dr^2} + \frac{1}{r} \frac{d\phi}{dr} - \beta \frac{d\phi}{dx} = 0 \]  
and

\[ \frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi}{\partial r} - \beta \frac{\partial \varphi}{\partial x} = 0 \]

Since \( \phi \) and \( \varphi \) in equation (C-28) are independent of each other therefore

\[ \frac{d^2\phi}{dr^2} + \frac{1}{r} \frac{d\phi}{dr} = \beta \frac{d\phi}{dx} = C \equiv \text{Cons tan t} \]  

Therefore

\[ \frac{d^2\phi}{dr^2} + \frac{1}{r} \frac{d\phi}{dr} = C \]  
\[ \beta \frac{d\phi}{dx} = C \]

Equation (C-31) can be integrated to give
\[ \phi(r) = \frac{C}{2} r^2 + C_2 \]  

where \( C_2 \) is a constant. The value of \( C \) can be found using the boundary condition given by equation (C-27). Substitution of \( C \) in equation (C-33) yields

\[ \phi(r) = \frac{q''}{2R_o \lambda_e} r^2 \]  

(C-34)

The integration of equation (C-32) yields

\[ \varphi(x) = \frac{2q''}{R_o \lambda_e \beta} x + C_1 \]  

(C-35)

The value of constant \( C_1 \) can be found using boundary condition (B-25), hence

\[ \varphi(x) = \frac{2q''}{R_o \lambda_e \beta} x \]  

(C-36)

To solve equation (C-29), the separation of variables method is used. The solution to the partial equation is assumed to take a product form

\[ \psi(r, x) = R(r) \cdot X(x) \]  

(C-37)

Substituting equation (C-37) into equation (C-29) and dividing the sides by \( R \cdot X \) yields

\[ \frac{1}{R} \left( \frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} \right) = \frac{\beta}{X} \frac{dX}{dx} \]  

(C-38)

It can be observed that each side of equation (C-38) is independent of the other because \( x \) and \( r \) are independent variables. This requires that each side be equal to some constant. We may thus obtain two ordinary differential equations in terms of this constant,

\[ \frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} + \Lambda^2 R(r) = 0 \]  

(C-39)
\[ \beta \frac{dX}{dx} = -\Lambda^2 X(x) \]  

(C-40)

The solution of equation (C-39) yields

\[ R(r) = a_1 J_0(\Lambda r) + a_2 Y_0(\Lambda r) \]  

(C-41)

Using equations (C-26) and (C-27), \( a_2 = 0 \) and the solution to the equation (C-41) is

\[ R_n(r) = a_n J_0(\Lambda_n r) \]  

(C-42)

Where

\[ \Lambda_n = \frac{\text{The nth zero function of } J_1}{R_0} \quad \text{and } n = 0, 1, 2, 3, \ldots \]  

(C-43)

The general solution to equation (C-40) is

\[ X_n(x) = C_n^* \exp \left( -\frac{\Lambda_n^2 x}{\beta} \right) \]  

(C-44)

Substituting equations (C-42) and (C-44) into equation (C-37) and taking the product of \( a_n \) and \( C_n^* \) as \( C_n \) we obtain the general solution to equation (C-29) as

\[ \varphi(r, x) = \sum_{n=0}^{\infty} C_n \exp \left( -\frac{\Lambda_n^2 x}{\beta} \right) J_0(\Lambda_n r) \]  

(C-45)

Since \( J_1(0) = 0 \), then \( \Lambda_0 = 0 \) and because \( J_0(0) = 1 \), therefore

\[ \varphi(r, x) = C_0 + \sum_{n=1}^{\infty} C_n \exp \left( -\frac{\Lambda_n^2 x}{\beta} \right) J_0(\Lambda_n r) \]  

(C-46)

Substituting equations (C-34)-(C-46) into equation (C-20), we obtain the general solution for equation (C-16) as follow
\[ \theta(r, x) = C_0 + \sum_{n=1}^{\infty} C_n \exp \left( -\frac{\Lambda_n^2 x}{\beta} \right) J_0(\Lambda_n r) + \frac{q''}{2R_o \lambda_e} r^2 + \frac{2q''}{R_o \lambda_e \beta} x \]  (C-47)

The expressions for \( C_0 \) and \( C_n \) with reference to Fourier-Bessel series are as follows

\[ C_0 = \frac{-q'' R_0}{4 \lambda_e} \]  (C-48)

\[ C_n = \frac{2q''}{\lambda_e R_0 \Lambda_n^2} \frac{J_2(\Lambda_n R_0)}{J_0^2(\Lambda_n R_0)} \]  (C-49)

Therefore the temperatures at the wall, \( T_w \) and at the centre, \( T_c \) are given in the following

\[ T(R_o, x) = T_0 + \sum_{n=1}^{\infty} C_n \exp \left( -\frac{\Lambda_n^2 \Delta_e x}{u_D} \right) J_0(\Lambda_n r) + \frac{2q'' \Delta_e}{R_o \lambda_e u_D} x \]  (C-50)

\[ T(0, x) = T_0 - \sum_{n=1}^{\infty} C_n \exp \left( -\frac{\Lambda_n^2 \Delta_e x}{u_D} \right) x + \frac{2q'' \Delta_e}{R_o \lambda_e u_D} x \]  (C-51)

And therefore the heat transfer coefficient becomes

\[ \alpha = \frac{1}{R_o/4 \lambda_e + \sum_{n=1}^{\infty} C_n \exp \left( -\frac{\Lambda_n^2 \Delta_e}{u_D} x \right) J_0(\Lambda_n R_o)} \]  (C-52)

Finally the functional dependence of the Nusselt number can be expressed as

\[ \text{Nu} = \frac{1}{R_o/4 + \sum_{n=1}^{\infty} \lambda_e C_n \exp \left( -\frac{\Lambda_n^2 \Delta_e}{u_D} x \right) J_0(\Lambda_n R_o)} \]  (C-53)

Where \( C_n' = C_n / q'' \)
Appendix C-3 Discretization procedure and the resulting nodal equations to solve equation (4.46).

The volume-averaged, steady energy equation for a cylindrical porous medium in which the axial conduction is taken into account is given as follows:

\[ \frac{\partial^2 \theta}{\partial t^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \beta_s \frac{\partial \theta}{\partial x} = \frac{\partial^2 \theta}{\partial x^2} \]  

where

\[ \theta(r) = T(r) - T_o(r) \quad \text{and} \quad \beta_s = u_D / A_{es} \]  

Taking the number of increments in radial and axial direction as \( m \) and \( n \) respectively, then

\[ \Delta r = \frac{R_o}{m} \quad \text{and} \quad \Delta x = \frac{L}{n} \]  

The following boundary conditions applicable to solve equation (C-54) are

\[ \frac{\partial \theta}{\partial r} \bigg|_{r=0} = 0 \]  

\[ \frac{\partial \theta}{\partial r} \bigg|_{r=R_s} = \frac{q''}{\lambda_{es}} \]  

\[ \theta(r,0) = 0 \]

Equation (C-54) is discretized using: a central difference for the first and second terms on the left hand side and also for the last term on the right hand side and an upward difference for the first term on the right hand side of the equation. The scheme also uses forward and backward differences for the first and second boundary conditions, respectively. Since the domain is parabolic, a three point backward difference is used for the axial conduction term. The algebraic equations are

(A) boundary conditions:

C-3-1
\[ \theta_{2,j} = \theta_{1,j} \quad \text{(C-60)} \]

\[ \theta_{m+1,j} = \theta_{m,j} + \frac{q^* \Delta r}{\lambda_{es}} \quad \text{(C-61)} \]

\[ \theta_{i,1} = 0 \quad \text{(C-62)} \]

(B) central points:

\[
\left(1 - \frac{\Delta r}{2r_{i,j}}\right) \theta_{i-1,j} + \left(-2 - \frac{u_D(\Delta r)^2}{A_e \Delta x} - 2 \left(\frac{\Delta r}{\Delta x}\right)^2\right) \theta_{i,j} + \left(1 + \frac{\Delta r}{2r_{i,j}}\right) \theta_{i+1,j} = \\
- \left(\frac{\Delta r}{\Delta x}\right)^2 \theta_{i,j+1} - \left(\frac{u_D(\Delta r)^2}{A_e \Delta x} + \frac{\Delta r^2}{\Delta x^2}\right) \theta_{i,j-1} \quad \text{(C-63)}
\]

(C) left hand face:

\[
\left(1 + \frac{\Delta r}{2r_{i,j}}\right) \theta_{i+1,j} + \left(-\frac{\Delta r}{2r_{i,j}} - 1 - \frac{u_D(\Delta r)^2}{A_e \Delta x} - 2 \left(\frac{\Delta r}{\Delta x}\right)^2\right) \theta_{i,j} = \\
- \left(\frac{\Delta r}{\Delta x}\right)^2 \theta_{i,j+1} - \left(\frac{u_D(\Delta r)^2}{A_e \Delta x} + \frac{\Delta r^2}{\Delta x^2}\right) \theta_{i,j-1} \quad \text{(C-64)}
\]

(D) right hand face:

\[
\left(1 - \frac{\Delta r}{2r_{i,j}}\right) \theta_{i-1,j} + \left(\frac{\Delta r}{2r_{i,j}} - 1 - \frac{u_D(\Delta r)^2}{A_e \Delta x} - 2 \left(\frac{\Delta r}{\Delta x}\right)^2\right) \theta_{i,j} = \\
- \left(\frac{\Delta r}{\Delta x}\right)^2 \theta_{i,j+1} - \left(\frac{u_D(\Delta r)^2}{A_e \Delta x} + \frac{\Delta r^2}{\Delta x^2}\right) \theta_{i,j-1} - \frac{q^* \Delta r}{\lambda_{es}} - \frac{q^* \Delta r^2}{2\lambda_{es} r_{i,j}} \quad \text{(C-65)}
\]

(E) lower face

\[
\left(1 - \frac{\Delta r}{2r_{i,j}}\right) \theta_{i-1,j} + \left(-2 - \frac{u_D(\Delta r)^2}{A_e \Delta x} - 2 \left(\frac{\Delta r}{\Delta x}\right)^2\right) \theta_{i,j} + \left(1 + \frac{\Delta r}{2r_{i,j}}\right) \theta_{i+1,j} = \\
- \left(\frac{\Delta r}{\Delta x}\right)^2 \theta_{i,j+1} \quad \text{(C-66)}
\]
(F) upper face:

\[
\begin{align*}
&\left(1 - \frac{\Delta r}{2r_{i,j}}\right)\theta_{i-1,j} + \left(-2 - \frac{u_D(\Delta r)^2}{A_e\Delta x} + \frac{(\Delta r)^2}{\Delta x}\right)\theta_{i,j} + \left(1 + \frac{\Delta r}{2r_{i,j}}\right)\theta_{i+1,j} = \\
&-\left(\frac{\Delta r}{\Delta x}\right)^2\theta_{i,j-2} + \left(-\frac{u_D(\Delta r)^2}{A_e\Delta x} + 2\frac{\Delta r^2}{\Delta x^2}\right)\theta_{i,j-1} \\
&= \left(\frac{\Delta r}{\Delta x}\right)^2\theta_{i,j+1} \quad \text{(C-67)}
\end{align*}
\]

(G) lower left hand corner:

\[
\begin{align*}
&\left(1 + \frac{\Delta r}{2r_{i,j}}\right)\theta_{i+1,j} + \left(-\frac{\Delta r}{2r_{i,j}} - 1 - \frac{u_D(\Delta r)^2}{A_e\Delta x} - 2\frac{(\Delta r)^2}{\Delta x}\right)\theta_{i,j} = \left(\frac{\Delta r}{\Delta x}\right)^2\theta_{i,j+1} \\
&= \frac{q^*(\Delta r)^2}{\lambda_{es}} \frac{2}{2\lambda_{es} r_{i,j}} \quad \text{(C-68)}
\end{align*}
\]

(H) lower right hand corner:

\[
\begin{align*}
&\left(1 - \frac{\Delta r}{2r_{i,j}}\right)\theta_{i-1,j} + \left(\frac{\Delta r}{2r_{i,j}} - 1 - \frac{u_D(\Delta r)^2}{A_e\Delta x} - 2\frac{(\Delta r)^2}{\Delta x}\right)\theta_{i,j} = \left(\frac{\Delta r}{\Delta x}\right)^2\theta_{i,j+1} \\
&-\left(\frac{\Delta r}{\Delta x}\right)^2\theta_{i,j-2} - \left(-\frac{u_D(\Delta r)^2}{A_e\Delta x} + 2\frac{\Delta r^2}{\Delta x^2}\right)\theta_{i,j-1} \quad \text{(C-69)}
\end{align*}
\]

(I) upper left hand corner:

\[
\begin{align*}
&\left(1 + \frac{\Delta r}{2r_{i,j}}\right)\theta_{i+1,j} + \left(-\frac{\Delta r}{2r_{i,j}} - 1 - \frac{u_D(\Delta r)^2}{A_e\Delta x} + \frac{(\Delta r)^2}{\Delta x}\right)\theta_{i,j} = \\
&-\left(\frac{\Delta r}{\Delta x}\right)^2\theta_{i,j-2} - \left(-\frac{u_D(\Delta r)^2}{A_e\Delta x} + 2\frac{\Delta r^2}{\Delta x^2}\right)\theta_{i,j-1} \quad \text{(C-70)}
\end{align*}
\]

(J) upper right hand corner:

\[
\begin{align*}
&\left(1 - \frac{\Delta r}{2r_{i,j}}\right)\theta_{i-1,j} + \left(\frac{\Delta r}{2r_{i,j}} - 1 - \frac{u_D(\Delta r)^2}{A_e\Delta x} + \frac{(\Delta r)^2}{\Delta x}\right)\theta_{i,j} + \left(1 + \frac{\Delta r}{2r_{i,j}}\right)\theta_{i+1,j} = \\
&-\left(\frac{\Delta r}{\Delta x}\right)^2\theta_{i,j-2} - \left(-\frac{u_D(\Delta r)^2}{A_e\Delta x} + 2\frac{\Delta r^2}{\Delta x^2}\right)\theta_{i,j-1} - \frac{q^*\Delta r}{\lambda_{es}} - \frac{q^*\Delta r^2}{2\lambda_{es} r_{i,j}} \quad \text{(C-71)}
\end{align*}
\]
The following flow chart applies to a code for a program for the solution of the algebraic equations.

1. Start
2. Input
3. Read no. of nodes in both direction
4. Apply first boundary condition for axial profile, \( T(r,0) = T_i(r) \)
5. Compute the elements of the tri-diagonal design matrix
6. Solve set of linear equation to find axial and radia temperature profile
7. Apply second boundary condition \( \frac{\partial T_{(r,1)}}{\partial x} = 0 \) for axial profile and both boundary conditions for radial profile \( \frac{\partial T_{(0,x)}}{\partial r} = 0 \), \( \lambda_e \frac{\partial T_{(R,x)}}{\partial r} = q^* \)
8. Report the results
9. Stop
The program code
CLS
CLEAR
TROFF

This program solves the algebraic equations resulted from the discretization of the differential equation of temperature distribution in porous media including the axial conduction effect.

Input variables

INPUT "Radius, m ="; R
INPUT "Length, m ="; L
INPUT "FLOW RATE, 50, 75, 100, 125, 150, 175, 200, CC/MIN. ="; FLO
INPUT "Inlet temperature, C ="; TI
INPUT "Fluid thermal conductivity, W/m C ="; KF
INPUT "Solid thermal conductivity, W/m C ="; KS
INPUT "Fluid density, kg/m^3 ="; RO
INPUT "Fluid specific heat, j/kg C ="; CP
INPUT "Heat flux, W/m^2 ="; HF
INPUT "porosity, % ="; POR
INPUT "% ERROR, .1-1E-3"; DELTA

The following are the notations which are used in this program.

TOLD = TEMPERATURE OF THE PREVIOUS STEP
TC = CENTRE TEMPERATURE, THE SUB. INDICATES THE CALCULATION POINT
TNEW = TEMPERATURE OF THE PRESENT STEP
HC = HEAT TRANSFER COEFFICIENT AT THE CENTRE, THE SUBSCRIPT INDICATES THE POINT AT WHICH IT IS BEING CALCULATED
HB = BULK HEAT TRANSFER COEFFICIENT, THE POINT AT WHICH IT IS BEING CALCULATED
TW = WALL TEMPERATURE THE SUBSCRIPT INDICATES THE POINT AT WHICH IT IS BEING CALCULATED
Q1 = MC(Tout - Tin)
Q3 = Q(P1*D*L)
M = NO. OF INCEAMENTS IN R DIRECTION
N = NO. OF INCREAMENTS IN X DIRECTION

M = 120; N = 38
UD = (FLO / 6E+07) / (3.1415 * R^2)
KE = KF ^ (POR / 100) * KS ^ (1 - (POR / 100))

C-3-5
ALFA = KE / (RO * CP)
DR = R / M: DL = L / N: K1 = (DR / DL) ^ 2
K2 = UD * DR ^ 2 / (ALFA * DL): K3 = -HF * DR / KE
K4 = -HF * DR ^ 2 / (2 * KE): AREA = 3.1415 * R ^ 2
MASS = RO * UD * AREA: A2 = -2 - 2 * K1: A4 = K1: A5 = K2 + K1
B3 = K1: B4 = K2 + K1: C3 = K1: C4 = K2 + K1: D2 = -2 - 2 * K1
D4 = K1: F3 = K1: G3 = K1: N2 = -2 - K2 + K1: N4 = -K2 + 2 * K1
N5 = -K1: M3 = -K2 + 2 * K1: M4 = -K1: P5 = -K1
P4 = 2 * K1 - K2

DIM TOLD(M + 1, N + 1), TNEW(M + 1, N + 1), EL1(300), EL2(300), EL3(300)
DIM CON(300), NU1(300), NU2(300), TB(300)
TOLD
FOR I = 1 TO M + 1
FOR J = 1 TO N + 1
TOLD(I, J) = 5: TNEW(I, J) = 5
NEXT J
NEXT I
1 CONT = CONT + 1
REM VERTICAL SWEEPING
FOR I = 2 TO M
FOR J = 2 TO N + 1
TOLD(I, J) = TNEW(I, J)
NEXT J
NEXT I
REM LOWER FACE
REM LEFT CORNER
IF J = N + 1 THEN 5
IF J > 2 THEN 6
IF I = 2 THEN
EL2(I) = -1 - DR / (2 * R1) - 2 * K1 - K2
EL1(I) = 1 + DR / (2 * R1)
CON(I) = -F3 * TOLD(I, J + 1)
REM RIGHT CORNER
ELSEIF I = M THEN
EL2(I) = -1 + DR / (2 * R1) - 2 * K1 - K2
EL3(I) = 1 - DR / (2 * R1)
CON(I) = -G3 * TOLD(I, J + 1) + K3 + K4 / R1

C-3-6
REM BETWEEN RIGHT AND LEFT CORNERS

ELSE

EL3(I) = 1 - DR / (2 * (I - 1) * DR)
EL2(I) = D2 - K2
EL1(I) = 1 + DR / (2 * (I - 1) * DR)
CON(I) = -D4 * TOLD(I, J + 1)
END IF
GOTO 10

REM UPPER FACE

REM LEFT CORNER
5 IF I = 2 THEN

EL2(I) = -1 - DR / (2 * R1) - K2 + K1
EL1(I) = 1 + DR / (2 * R1)
CON(I) = M3 * TNEW(I, J - 1) + M4 * TNEW(I, J - 2)

REM RIGHT CORNER
ELSEIF I = M THEN

EL2(I) = -1 + DR / (2 * (I - 1) * DR) - K2 + K1
EL3(I) = 1 - DR / (2 * (I - 1) * DR)
CON(I) = K3 + K4 / R1 + P4 * TNEW(I, J - 1) + P5 * TNEW(I, J - 2)

REM BETWEEN RIGHT AND LEFT CORNER
ELSE

EL3(I) = 1 - DR / (2 * (I - 1) * DR)
EL2(I) = N2
EL1(I) = 1 + DR / (2 * (I - 1) * DR)
CON(I) = N4 * TNEW(I, J - 1) + N5 * TNEW(I, J - 2)
END IF
GOTO 10

REM INTERNAL NODES

REM LEFT NODES
6 IF I = 2 THEN

EL2(I) = -1 - 2 * K1 - DR / (2 * R1) - K2
EL1(I) = 1 + DR / (2 * (I - 1) * DR)
CON(I) = -B3 * TOLD(I, J + 1) - B4 * TNEW(I, J - 1)
ELSEIF I = M THEN

REM RIGHT NODES
EL2(I) = -1 - 2 * K1 + DR / (2 * R1) - K2
EL3(I) = 1 - DR / (2 * (I - 1) * DR)
CON(I) = -C3 * TOLD(I, J + 1) - C4 * TNEW(I, J - 1) + K3 + K4 / R1
ELSE

REM CENTRAL NODES
EL3(I) = 1 - DR / (2 * (I - 1) * DR)
EL2(I) = A2 - K2
EL1(I) = 1 + DR / (2 * (I - 1) * DR)
CON(I) = -A4 * TOLD(I, J + 1) - A5 * TNEW(I, J - 1)
END IF
10 NEXT I

K22 = 2: K23 = M
GOSUB 2
FOR I = 2 TO M
  TNEW(I, J) = CON(I)
NEXT I
NEXT J
FOR I = 2 TO M
  FOR J = 2 TO N + 1
    TOLD(I, J) = TNEW(I, J)
  NEXT J
NEXT I

FOR J = 2 TO N + 1
  TNEW(M + 1, J) = TNEW(M, J) - K3
  TNEW(1, J) = TNEW(2, J)
NEXT J

REM HORIZONTAL SWEEPING
FOR I = 2 TO M
  FOR J = 2 TO N + 1
    RL = (I - 1) * DR
    REM LEFT FACE
    REM BOTTOM CORNER
    IF I = M THEN 7
    IF I > 2 THEN 8
    IF J = 2 THEN
      EL2(J) = -1 - DR / (2 * RL) - 2 * K1 - K2
      EL1(J) = F3
      CON(J) = -(1 + DR / (2 * RL)) * TOLD(I + 1, J)
      REM UPPER CORNER
      ELSEIF J = N + 1 THEN
      EL3(J) = M3
      EL2(J) = 1 + DR / (2 * RL) + K2 - K1
      CON(J) = (1 + DR / (2 * RL)) * TOLD(I + 1, J) - M4 * TOLD(I, J - 2)
      ELSE
      REM BETWEEN UPPER AND LOWER CORNERS

C-3-8
$\text{EL3}(J) = \text{B4}$
$\text{EL2}(J) = -\text{DR} / (2 \ast \text{R1}) - 2 \ast \text{K1} - 1 - \text{K2}$
$\text{EL1}(J) = \text{B3}$
$\text{CON}(J) = -(1 + \text{DR} / (2 \ast \text{R1})) \ast \text{TOLD}(I + 1, J)$

END IF
GOTO 20

REM RIGHT FACE
REM BOTTOM CORNER
7 IF J = 2 THEN
  $\text{EL2}(J) = -1 - 2 \ast \text{K1} + \text{DR} / (2 \ast \text{R1}) - \text{K2}$
  $\text{EL1}(J) = \text{G3}$
  $\text{CON}(J) = -(1 - \text{DR} / (2 \ast \text{R1})) \ast \text{TOLD}(I + 1, J) - 1 - \text{K2}$
ELSEIF J = N + 1 THEN
  $\text{EL2}(J) = 1 + \text{DR} / (2 \ast (I - 1) \ast \text{DR}) - \text{K2} + \text{K1}$
  $\text{EL3}(J) = \text{P4}$
  $\text{CON}(J) = (1 - \text{DR} / (2 \ast \text{R1})) \ast \text{TNEW}(I - 1, J) - \text{K3} - \text{K4} / \text{R1} - \text{P5} \ast \text{TOLD}(I, J - 2)$
ELSE

REM BETWEEN UPPER AND LOWER CORNERS
  $\text{EL3}(J) = \text{C4}$
  $\text{EL2}(J) = \text{DR} / (2 \ast \text{R1}) - 2 \ast \text{K1} - 1 - \text{K2}$
  $\text{EL1}(J) = \text{C3}$
  $\text{CON}(J) = (1 - \text{DR} / (2 \ast \text{R1})) \ast \text{TNEW}(I - 1, J) - \text{K3} - \text{K4} / \text{R1}$
END IF
GOTO 20

REM INTERNAL NODES
REM LOWER NODES
8 IF J = 2 THEN
  $\text{EL2}(J) = \text{D2} - \text{K2}$
  $\text{EL1}(J) = \text{D4}$
  $\text{CON}(J) = -(1 - \text{DR} / (2 \ast \text{R1})) \ast \text{TNEW}(I - 1, J) - (1 + \text{DR} / (2 \ast \text{R1})) \ast \text{TOLD}(I + 1, J)$
ELSEIF J = N + 1 THEN
REM UPPER NODES
  $\text{EL2}(J) = -N2$
  $\text{EL3}(J) = N4$
  $\text{CON}(J) = (1 - \text{DR} / (2 \ast \text{R1})) \ast \text{TNEW}(I - 1, J) + (1 + \text{DR} / (2 \ast \text{R1})) \ast \text{TOLD}(I + 1, J) - N5 \ast \text{TOLD}(I, J - 2)$
ELSE
REM CENTRAL NODES
  $\text{EL3}(J) = \text{A5}$

C-3-9
$\text{EL2}(j) = A2 - K2$

$\text{EL1}(j) = A4$

$\text{CON}(j) = -(1 - \text{DR} / (2 \times R1)) \times \text{TNEW}(i - 1, j) - (1 + \text{DR} / (2 \times R1)) \times \text{TOLD}(i + 1, j)$

END IF

20 NEXT J

K22 = 2: K23 = N + 1

GOSUB 2

FOR J = 2 TO N + 1

$\text{TNEW}(i, j) = \text{CON}(j)$

NEXT J

NEXT I

FOR J = 2 TO N + 1

$\text{TNEW}(m + 1, j) = \text{TNEW}(m, j) - K3$

$\text{TNEW}(1, j) = \text{TNEW}(2, j)$

NEXT J

NEXT I

TB(J) = -R * TNEW(M + 1, J)

FOR I = 2 TO M STEP 2

R1 = (I - 1) * DR

TB(J) = TB(J) + 4 * R1 * TNEW(I, J) + 2 * I * DR * TNEW(I + 1, J)

NEXT I

RB = TB(J) - R * TNEW(M + 1, J)

FOR J = 12 TO M STEP 2

R1 = (I - 1) * DR

TB(J) = TB(J) + 4 * R1 * TNEW(I, J) + 2 * I * DR * TNEW(I + 1, J)

NEXT I

TB(J) = TB(J) * DR / 3 * 2 / R^2

NEXT J

FOR J = 2 TO N + 1

NU1(J) = 2 * R / (TNEW(M + 1, J) - TNEW(2, J)) * (3 * TNEW(M + 1, J) - 4 * TNEW(M, J) + TNEW(M - 1, J)) / (2 * DR)

NU2(J) = 2 * R / (TNEW(M + 1, J) - TB(J)) * (3 * TNEW(M + 1, J) - 4 * TNEW(M, J) + TNEW(M - 1, J)) / (2 * DR)

NEXT J

Q1 = MASS * CP * TB(N + 1)

Q3 = HF * 2 * 3.1415 * R * L

PRINT "ITERATION="; CONT

PRINT Q1; Q3

REM INTERPOLATION

HC31 = HF / (TNEW(M + 1, 15) - TNEW(2, 15))

HC32 = HF / (TNEW(M + 1, 16) - TNEW(2, 16))

HC3 = .25 * HC31 + .75 * HC32

HC41 = HF / (TNEW(M + 1, 24) - TNEW(2, 24))

HC42 = HF / (TNEW(M + 1, 25) - TNEW(2, 25))

HC4 = .25 * HC41 + .75 * HC42

C-3-10
HC21 = HF / (TNEW(M + 1, 7) - TNEW(2, 7))
HC22 = HF / (TNEW(M + 1, 8) - TNEW(2, 8))
HC2 = 0.25 * HC21 + 0.75 * HC22
HC51 = HF / (TNEW(M + 1, 32) - TNEW(2, 32))
HC52 = HF / (TNEW(M + 1, 33) - TNEW(2, 33))
HC5 = 0.25 * HC51 + 0.75 * HC52
TW3 = 0.25 * TNEW(M + 1, 15) + 0.75 * TNEW(M + 1, 16)
TW4 = 0.25 * TNEW(M + 1, 24) + 0.75 * TNEW(M + 1, 25)
TB3 = 0.25 * TB(15) + 0.75 * TB(16)
TB4 = 0.25 * TB(24) + 0.75 * TB(25)
HB3 = HF / (TW3 - TB3)
HB4 = HF / (TW4 - TB4)
TC2 = 0.25 * TNEW(2, 7) + 0.75 * TNEW(2, 8)
TC3 = 0.25 * TNEW(2, 15) + 0.75 * TNEW(2, 16)
TC4 = 0.25 * TNEW(2, 24) + 0.75 * TNEW(2, 25)
TC5 = 0.25 * TNEW(2, 32) + 0.75 * TNEW(2, 33)
PRINT "TW3="; TW3
PRINT "TW4="; TW4
PRINT "TB4="; TB4
PRINT "TB3="; TB3
PRINT "TC2="; TC2
PRINT "TC3="; TC3
PRINT "TC4="; TC4
PRINT "TC5="; TC5
PRINT "HC2="; HC2
PRINT "HC3="; HC3
PRINT "HC4="; HC4
PRINT "HC5="; HC5
PRINT "HB3="; HB3
PRINT "HB4="; HB4
PRINT
PRINT "THE PROGRAM IS RUNNING"
FOR I = 2 TO M
FOR J = 2 TO N
    IF ABS((TNEW(I, J) - TOLD(I, J)) / TNEW(I, J)) > DELTA THEN 1
NEXT J
NEXT I
FOR J = 2 TO N
    TNEW(M + 1, J) = TNEW(M, J) - K3
    TNEW(1, J) = TNEW(2, J)
OPEN "OUT.DAT" FOR OUTPUT AS #3
PRINT #3, USING "***. *** "; TB3
CLOSE #3
PRINT "END"
END

******************************************************************************
***
SUBROUTINE 2(This subroutine solves algebraic equations using Thomas
Algorithm)
******************************************************************************
***
2 K24 = K22 + 1
FOR L1 = K24 TO K23
    K25 = EL3(L1) / EL2(L1 - 1)
    EL2(L1) = EL2(L1) - K25 * EL1(L1 - 1)
    CON(L1) = CON(L1) - K25 * CON(L1 - 1)
NEXT L1
CON(K23) = CON(K23) / EL2(K23)
FOR L1 = K24 TO K23
    L2 = K23 - L1 + K22
    CON(L2) = (CON(L2) - EL1(L2) * CON(L2 + 1)) / EL2(L2)
NEXT L1
RETURN
Appendix C-4 Two-phase flow regimes

Photograph C-1, ascending flow
L = 150 cm$^3$/min., G = 500 cm$^3$/min.

Photograph C-2, ascending flow
L = 150 cm$^3$/min., G = 1600 cm$^3$/min.

Photograph C-3, ascending flow
L = 150 cm$^3$/min., G = 2800 cm$^3$/min.

Photograph C-4, ascending flow
L = 150 cm$^3$/min., G = 7800 cm$^3$/min.
Appendix C-5 Identification of the thermally fully developed region

In order to identify the thermally fully developed region when the flow of fluid through the porous medium is initiated under constant heat flux, it is useful to consider the analogous situation for an unpacked tube with uniform wall heat flux. For such a case, it is well known that the bulk temperature, $T_b$, rises linearly downstream and that in the fully developed thermal region, the wall temperature, $T_w$, also rises linearly and at the same rate as $T_b$ (Holman, 1989). Similarly, for flow through a packed tube (the present investigation) under constant heat flux, the thermally fully developed region can be identified by observing how far downstream the measured values of $T_w$ and $T_b$ plot as straight lines having the same slopes.

Fig. C-2 shows a typical plot of the axial variation of measured values of $T_w$ and calculated values of $T_b$ obtained in the present study.

![Fig. C-2 Axial variation of wall and bulk temperatures.](image)

The difference $(T_b - T_o)$ is everywhere very nearly proportional to the downstream distance, $x$, thus
\[ T_b - T_o = a \cdot x \quad \text{C-72} \]

where \( T_o \) is the inlet temperature and \( a \) is the slope of the line that can be obtained by linear regression. For the thermally fully developed region, \( (T_w - T_o) \) may be presented as follows:

\[ T_w - T_o = a \cdot x + b \quad \text{C-73} \]

where \( a \) is as determined by equation (C-72) and \( b \) is the intercept. For all forced convection data obtained in this study, it was observed that \( (T_w - T_o) \) was nearly constant for \( x/d_c \geq 8 \).