Empirical and Theoretical Aspects of the International Arms Trade

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In memory of
my grandfather Dr Fotis Mouzakis, who introduced me to science,
my mother Isidora Mouzaki – Tsigri who introduced me to human sciences,
my grandmother Panagiota Mouzaki – Avgustinou, who introduced me to classical studies.
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Abstract

One way that the Cold War benefited the international environment of security might well be that it allowed more attention to be paid to conflicts of a smaller scale. The persisting large number, however, of hot and cold conflicts scattered around the globe keep fuelling reasonable concerns about security. The defence of a country requires a military sector, the effectiveness of which mainly depends on the quality and quantity of its arsenal. Large countries are usually self-sufficient, producing domestically the majority of their needs, whereas the arms trade is the main source of procurement for the bulk of smaller states. This thesis attempts a combined empirical and theoretical exploration of the international arms market, from a purely quantitative scope. It begins with an examination of the existing literature and the available data and introduces a method to calculate the as yet unavailable prices of imported arms. It tests the validity of the existing theoretical predictions, first in a cross-sectional study and then in a more detailed study of global panel data showing that the existing theory can only partly explain the facts. The data support a standard responsiveness of countries' demand for arms to the military budget and the price of arms, together with an evident need for a deeper theoretical foundation. Following the empirical indications, this thesis first introduces substitution between imported and domestically produced arms in a small-country, partial-equilibrium model of national defence. The applicability of this framework gets further support by an included econometric study. The optional entry of countries to domestic production of arms is finally introduced in a quasi-general equilibrium model of the international arms market, keeping all aspects of previous works. This generalised theory of the contemporary international arms market suggests that policy recommendations that neglect the domestic alternative to imported arms can be misleading.
Contents

1. Introduction ................................................................. 1

2. Literature review
   2.1 Introduction ......................................................... 7
   2.2 Economic theory and the arms trade ......................... 8
   2.3 Empirical literature ............................................... 11
   2.4 Conclusions ......................................................... 13

3. Data, trends and prices of the international arms trade
   3.1 Introduction ......................................................... 16
   3.2 Sources and availability of data ............................... 16
   3.3 General trends ..................................................... 20
   3.4 An implicit price index for imported arms .................. 23
   3.5 Prices of imported arms ......................................... 25
   3.6 Conclusions ......................................................... 26

4. The demand for imported arms: A cross-sectional study
   4.1 Introduction ......................................................... 28
   4.2 Analytical approach
       4.2.1 Preface ........................................................ 30
       4.2.2 Production under fixed factor costs .................... 31
       4.2.3 Domestic production of arms ......................... 33
       4.2.4 Defence planning with internal domestic arms sector 34
       4.2.5 Overview of the modelling framework ............... 35
   4.3 Empirical study of the demand
       4.3.1 A Simple log-linear demand function ................. 36
       4.3.2 Examination of the functional form ................... 39
       4.3.3 A quadratic-logarithmic demand function ........... 43
       4.3.4 Further empirical results ............................... 46
   4.4 The impact of measurement errors
       4.4.1 The problem of latent variables ....................... 49
       4.4.2 One variable measured with error .................... 50
       4.4.3 Two variables measured with error ................... 52
4.5 Conclusions .................................................................................................... 55

5. Regional variations of the demand for arms: a study of panel data

5.1 Introduction .................................................................................................... 57
5.2 The data .......................................................................................................... 58
5.3 Examination for structural heterogeneity ......................................................... 59
5.4 Selection of model and estimation method ...................................................... 64
5.5 Regional estimates of the demand for imported arms ....................................... 67
5.6 Time effects .................................................................................................... 68
5.7 Conclusions ..................................................................................................... 70

6. Differentiated arms and substitution of inputs in defence

6.1 Introduction .................................................................................................. 73
6.2 The role of domestically produced arms in defence ......................................... 74
6.3 The model ....................................................................................................... 77
6.4 Properties of the demand for imported arms .................................................... 81
6.5 Estimation of the demand imported arms ....................................................... 83
6.6 Conclusions .................................................................................................... 87

7. A Generalised Arms Trade Game: The Entry to Domestic Arms Production

7.1 Introduction .................................................................................................... 89
7.2 Strengths and weaknesses of the Levine-Smith approach ................................ 90
7.3 The model
    7.3.1 Industrialised countries .......................................................................... 92
    7.3.2 LDCs involved in conflict ..................................................................... 95
    7.3.3 Arms producing LDCs ........................................................................ .. 96
    7.3.4 Market equilibrium .............................................................................. .. 98
    7.3.5 Examination of model ......................................................................... 100
7.4 Examination of model ................................................................................... 102
7.5 Numerical results .......................................................................................... 104
7.6 Conclusions .................................................................................................. 110

8. Conclusions ........................................................................................................... 111

Appendix A. Appendices of the chapters

A.3 Appendix of chapter 3: the cross-sectional data set ....................................... 117
A.4 Appendix of chapter 4
    A.4.1 Descriptive statistics ........................................................................... 118
    A.4.2 Leverage Values and Residuals ........................................................... 120
A.5 Appendices of chapter 5
    A.5.1 Region Codes ..................................................................................... 121
1. Introduction

At a time when the majority of policies is being intensively analysed by economists, national defence and especially the economic determinants of the international trade in arms appear to have relatively escaped their attention. One of the major activities behind the national provision of defence, the international arms trade is a topic that can be distinguished by its near absence in the economic literature. Still covered by secrecy and political mysticism, the arms trade has always been, and still is, an area of great economic and social significance. After the end of the Cold War the aggregate for the whole world volume of the trade in arms has been steadily falling. According to ACDA [1995], within a decade, from 1985 to 1995, the value of the international arms trade decreased by about three-quarters. Despite this impressive decline, the overall volume of arms transfers still remains at considerably high levels and it could well rise again at any time in the future. The international trade in arms, therefore, needs to be carefully analysed and understood. In other words, the economic determinants of countries' defence activities need to be taken out of the dark ages of secrecy and political despotism and brought under the light of science.

The primarily stated purpose of the procurement of arms is to serve the needs of a country's national defence industry, protecting its citizens from a possible aggressive military offence of another country. Along with this primary and well justified purpose, the procurement of arms for national security is an activity that also implies several negative externalities. In practice, the arms stated to be imported for defensive reasons are often used for purposes of nationalistic aggression and abuse of human rights, inside and outside of the possessing country's national borders. A large number of conflicts are currently active around the world, all of them including at least two sides: usually one more aggressive and one more defensive. It is not always easy, unfortunately, to distinguish between the aggressive and the defensive sides of a conflict and the causes are often hidden a long way back in history. Taking this one step further, it is hard even to reach an agreement about the very definition of aggression per se and the rational basis of popular (and hence political) perceptions of aggressive actions has consistently been very weak. This of course applies in he case of military actions. Borrowing from the views of social psychology 'an act of aggression is a behaviour aimed at causing harm or pain' (Aronson [1980] p. 161). Taking as an example the case of 1999 Balkan
Wars shows that it can be hard to distinguish between defenders and aggressive offenders since all parties involved, including the intervening NATO allies, deliberately took actions that caused *harm and pain*, i.e. death of thousands and economic suffocation of millions of civilians.\(^1\)

The trade in arms differs from any other trade in a number of ways. While the procurement of arms is supposed to create feelings of security for the citizens of a country, the citizens of other countries often perceive it as a potentially offensive capacity. In turn, other countries usually respond to this threat with corresponding increases in their military expenditure and procurement. This feedback process can often lead to an arms race, wasting valuable economic resources and causing a variety of economic and welfare repercussions to the participating countries. Several examples of arms races can be drawn from the recent history, starting from the Cold War, that involved the most powerful states, and cover all the range of sizes down to local conflicts between countries or sub-country groups. The Cold War race of both conventional and arms of mass destruction appears to be in a phase of relaxation, whereas this is not the case for conflicts of a smaller scale. In fact, that large countries have always had the capacity to produce their own arms and the arms trade usually has been the main source of procurement for smaller states, often involved in regional arms races. Well-known examples of arms recipient countries involved in arms races are these between Greece and Turkey, India and Pakistan, Iran and Iraq and North and South Korea.

Along with the motivation that originates from the demand side of the market, the commercial interest and the expansionism of arms-producing firms and their governments can often encourage the arms trade. The fixed-cost requirements for the establishment of an industry of major arms are enormous and only the largest of industrialised countries have been in a position to be near to self-sufficiency in their procurement. This together with the possible existence of positive economies of scale in the production of technologically advanced weapon systems push arms-producing firms to maintain the best possible sales abroad so they can recoup sunk costs. Intensified by the turn down of procurements that followed the end of the Cold War, the need to maintain production levels led arms-producing firms to search for ways to compensate the created demand gap. Furthermore, new supplier countries are at the stage of the realisation of their proclaimed intention to export arms, within politically determined

\(^1\) The 11 week bombarding of Kosovo by NATO allies caused more civilian than military death casualties *(The Observer, 21/6/1991 p.1).*
limits. For example Israel and South Africa are establishing themselves as sellers, having consistently maintained their market shares in the post-Cold War years (ACDA [1995]).

At political level, there is a raising interest in monitor and control of international transfers of arms. Multinational economic agencies such as the IMF, the World Bank and the Development Assistance Committee of the OECD include reductions of military expenditure and procurement in the list of conditionalities related to the provision of economic aid (Levine and Smith [1997]). In 1993 the United Nations established the Arms Transfer Register and the European Union is making significant effort towards the harmonisation of the defence policies of member nations including the reduction of the transfers of arms and dual use technology. Even the government of the US, the leading supplying agent of this market, has recently expressed interest in conversion programs that will reduce the dependency of its arms industry on exports.

Despite the importance of the issue and the need for rational implementation of policies related to the transfers of arms, the economic theory has relatively little to contribute to the understanding of the underlying mechanism behind the international arms market. Recent developments have attempted to reduce this gap in the literature and approach the arms trade from the modern economic perspective of new trade theory (Anderton [1995, 1996]). Adding to this gap, the empirical economic literature is near to non-existing. Most of the existing works are limited to case studies either of a single country or of a small group of countries, with relatively little to contribute in practical terms. An apparent cause, often blamed for the scarcity of thorough empirical studies in this area is the unavailability and the limitations in the quality of the available data (Katrina [1994]).

The unwillingness of many governments to release correct measurements of their defence sector is not the only source of problems in the data on the international arms trade. The literature is rich in references to a range of data problems, regarding issues of measurement and definition. The commonly used offset ways of payment, production under licence and co-production tend to hide the actual value and the net quantity of the international transfers. Furthermore, complementary parts of the bilateral agreements such as technology transfer and counter trade agreements, tied aid, technology transfer, credit arrangements, insurance regarding repayment risks, political concessions etc, contribute in the difficulty of isolating the net value of the arms transfers. The aggregation of the variety of weapon systems into a single homogeneous category may

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2 Among many other works data problems of the arms trade are discussed in Krausse [1992 p. 217-19], Levine, Mouzakis and Smith [1996] and in chapter 3 of this thesis.
blur the measurements, especially with the inclusion of dual use equipment, adding to the large fluctuations at annual basis related to the timing of deliveries. All these problems create uncertainty about the usefulness of the available data series and require special consideration in the development of an empirical study.

It is the twin gap, mainly in the empirical but also in the theoretical literature motivated this study. Based on the hope that there might be as yet unexplained information within the available data, this study first attempted to obtain an important prerequisite in an examination of the demand side: the missing information about prices of imported arms. The provision of this critical input enabled the econometric study of the demand for traded arms, first using a cross-sectional data set with wide country coverage. The outcome of this simple but complete parametric approach of countries defence industries gave encouraging results and provided the motivation for a more detailed examination of panel data. In line with the previous one, this study revealed further empirical facts about the behavioural characteristics of countries demand for imported arms, such as the variability of the demand patterns across the globe and the evolutionary characteristics of defence sector in time. Building up on the empirical evidence the thesis continues with first a small-country partial equilibrium model, and then a more complete quasi-general equilibrium world model of the arms trade.

This thesis essentially binds together five strongly linked but self-contained papers. Each chapter can be read more or less independently, but the natural development of our work follows the order of the chapters. Chapter 2 provides a review of the contemporary theoretical and empirical economic literature on the arms trade. Chapter 3 examines the nature of the data and the calculation of a price index. Chapter 4 presents the preliminary analytical approach together with the results from the cross-sectional study. Chapter 5 provides an econometric examination based on panel data. Chapter 6 reports the attempt to introduce differentiated arm products into a partial equilibrium model of defence and appendix B isolates the related technical discussion about the production function employed. Chapter 7 presents the attempt to introduce the optional entry of countries into domestic production as part of a quasi-general equilibrium model of the international market of arms.

In more detailed description, the material included in this thesis is organised as follows. As mentioned above, chapter 3 isolates the discussion about empirical information directly related to the cross-sectional econometric examination of chapter 4. This separation is made for the purposes of presentation and because of the importance of the data sources as a stand-alone issue. It includes a review of the data sources, a description of the available data, a brief review of the general trends implicit in the data,
the suggested method to calculate a price index for imported arms and examples of calculated prices for a number of cases.

Chapter 4 presents results from an empirical study of the demand for imported arms based on cross-sectional data, including the development of a suitable theoretical model and other technical and data related issues. The main result of this study is the estimation of a well-behaved demand function of arms, which is supported by considerable statistical evidence. The response of the demand for arms to the estimated implicit price index is negative and the response to military expenditure is positive. The responses to military expenditure and to per capita GDP were found to be non-linear. The last part of the chapter examines the impact of measurement errors, and introduces a method that addresses this problem.

Chapter 5 develops an empirical examination of the demand for imported arms, using panel data with near-to-global coverage. It continues the work of the previous chapter introducing the dimension of time into the empirical study of the arms trade with the expansion of the data set from a cross-sectional series to a panel. The data are aggregated in large geographical regions and organizations and cover most of the world. After the preliminary tests for the variability of demand patterns over both regions and time, follows the estimation of a demand function, which allows for structural variation of the demand patterns over both time and region. The results show that demand patterns vary across different regions of the world and there is an evident double-trended effect in time, all well supported by the data.

Chapter 6 introduces a neo-classical small country model for the arms trade, which focuses on the properties of substitution between inputs of the defence industry. In this model countries have the option to produce arms domestically instead of merely relying on imports. This chapter attempts to reconcile economic together with the results of the previously mentioned empirical studies. The model employs an extension of the Stone-Geary function, which allows an input to be either included or completely substituted in the production, with adequate analytical clarity. Finally, the model is estimated with the use of available data, giving a means of comparison of the implied theoretical predictions with the empirical reality. A wide range of production or utility functions can be used to capture a variety of different types of substitution among the inputs of a transformation process. Most of the known utility or production functions, however, would not allow the complete substitution of an input by the others. Appendix B examines an extension of the Stone-Geary production function that provides this characteristic, also allowing for fixed costs in the use of any positive quantity of the substitutable input. Apart from serving the needs of chapter 6, this analysis is applicable
more generally for the purposes of modelling a discontinuous entry of inputs in a
transformation process. In the context of industrial theory, for example, it can provide a
means of analysing the decision of industries for vertical expansion.

The existing economic theory of the arms trade has considered the balance between
importers and suppliers in the international market of arms, but has not yet considered
the option of less developed countries to establish a domestic industry of arms and
become self-sufficient in procurement. According to Levine and Smith [1997, 1998], a
small number of industrialized countries offer arms to the world market, with a large
number of recipient countries on the demand side, all involved in local conflicts. In the
model we develop in chapter 7, however, less developed countries are not limited to be
identical but the size of their economies is allowed to vary, determining their status
either as importers or as producers of arms. Referring to the post-Cold War status of
international security Levine & Smith [1997] concluded that the cooperation of large
arms producing countries could benefit the welfare of both producing and importing
countries. This work shows that the alternative of domestically produced arms implies
limitations on the applicability of this policy and provides a more powerful tool for the
analysis of the international arms trade.

The scope of this study regarding issues of methodology should rather be
characterised as inclined towards formal methods. The continuing debate about the
method in economics includes the very definition of formalism itself. Here we consider
formalism as a systematic and rigorous analytical process, which allows us to identify
what can and what cannot be proved (Dow [1998] pp.1826-27). It implies the use of
deductive mathematical techniques, but only when necessary. In our examination, we
attempt a balanced development of the empirical and modelling aspects, being
convinced that looking at each one of these two sides alone one can easily get
disorientated. According to Krugman [1998], the strong opposition to formalistic
arguments in economics has contributed little to the recent developments. It is important,
however, that independent of the method we use in the development of our ideas they
should finally be expressed simply in words.
2. Literature review

2.1 Introduction

The importance of national defence, which can be successfully indicated by the large amounts of economic resources dedicated to this activity, outlines the need for thorough examination and rational implementation of the related policies, at national and international level. According to ACDA [1995 p.43] the aggregated for the whole world share of military expenditure to the total central government expenditure has decreased from an estimated 19% in 1993 to a non negligible 11.5% in 1993. The existing economic literature, however, appears to focus more on the examination of topics such as the theory of alliances, the determinants of military expenditure or the issue of the arms races, relatively neglecting the economic determinants of the arms trade. There are a number of possible reasons that could be responsible for this absence. These include restrictions in the availability of defence related data and the popular belief that the general means of the economic analysis are inappropriate for defence related issues, supported by the generally authoritative nature of military structures. Another general reason may well be dominance of the Cold War in the arena of international security, during which the large scale of the bipolar conflict of alliances overshadowed the importance of individual countries’ decisions. The generalised threat from weapons of mass destruction and the widespread and strong presence of military superpowers often covered local conflicts, inside and outside national borders, reducing their importance at local or global level. This situation seems to have changed after the end of the Cold War, when the reduction of inter-alliance tensions left an increased national responsibility for regional security. As Anderton [1995] puts it ‘with the end of Cold War there has been an increase in the relative importance of economic causes and consequences of arms transfers.’

This review discusses recent developments of the economic literature, mainly focusing on quantitative aspects of the international arms trade: analytical and empirical. Despite that it is not always easy to separated these two aspects in a particular work the literature is discussed in two separate sections depending on which one of the two is the dominant aspect. The perspective of this thesis, which is thought to follow what is perceived as mainstream methodology used in contemporary economics, has had an
influence on the selection of the works we discuss in this chapter. On these lines the recent advances in (new) trade theory create a fertile ground for developments in the theory of the arms trade. This review does not refer to technical issues, such as those concerning general economic or econometric techniques, which are left for reference inside the relevant chapters. Also, it does not extensively refer to issues related with data and their sources, which are discussed in more detail in chapter 3.

2.2 Economic theory and the arms trade

The approach from an economic perspective seems to characterise only a small minority of the generally extensive literature on defence and military issues. Two concise and well-implemented general readings on defence economics are Hartley and Sandler [1995] and a collection of papers called ‘The Handbook of Defence Economics’, edited by the same authors. Directly related to our topic is chapter 10 ‘The arms trade’ from the former edition and Anderton [1995] from the latter. Both of these chapters provide updated and informative reviews of theoretical and applied developments in the area. Krause [1992] provides an extensive discussion of a variety of economic aspects, the historical and the political nature of international arms trade.

A careful historical approach perhaps provides the safest foundation for the study of a social topic. Extending over three chapters Krause [1992] reviews about two and half millennia of history of the arms trade, beginning from the first recorded arms transfers between Greek city-states. As most essential motive forces behind the arms trade Krause suggests the pursuit of wealth, power and victory in war, providing a well-implemented link of the issue with the general methodology of contemporary economics. Regarding the contemporary dominance of the international arms trade of what he calls 1st tier (largest) producers he argues that arms exports are increasingly maintained by the economic interest of arms producing firms rather than the political willingness of the central decision making instruments. Together with purely strategic advantages of running a domestic arms sector, he suggests economic interest as a central motive in the case of small arms producers, which also generates a strong incentive for exports. He then attributes a generally subordinate role to arms recipient countries in a strongly hierarchical international environment, although he suggests a positive learning curve. Kraus isolates technological innovation as the main driving force of the arms transfers’ life cycle, according to which he divides the world market in three tires of arms producers. He finally outlines the need to link the study of the international arms trade to an understanding of the motives and prospects for arms production.
Anderton [1995] analyses the structure of the international market of arms using ACDA [1995] data. He classifies the existing economic theories of the arms trade into a number of categories, beginning with the supply and demand type of models, represented by Alexander et al [1981]. Another category is this of a neo-classical trade model, based on the homonymous theory of comparative advantage (Ricardo). The models in this category highlight the general equilibrium nature of the arms trade and that it is motivated by countries' differences in terms of tastes and of technology. He continues with a category of models with scale and learning economies. A final category in this classification includes models of imperfect competition and trade, which apply the Krugman and Obstfeld [1994] monopolistic competition model of trade to the international market of arms. In a more recent review, Anderton [1996] links the arms trade with new developments on trade theory. In this extension of his previous classification Anderton adds the category of integrated economy approach of Krugman [1995], which looks into intra- and inter-industry trade and the North-South trade approach of Markusen [1986] for explanations of the heterogeneity in the trade of differentiated types of arms. This review extends the classification of imperfectly competitive models of the arms trade to a number of sub-categories, putting emphasis on models of bilateral oligopoly. Furthermore, Anderton [1996] suggests the product cycle type of model, introduced by Vernon [1966], for linking the arms trade with issues of diffusion of defence technology. Catrina [1994], suggests that theoretical research on the arms trade should 'bring the arms transfers under the national and international control.'

A recent contribution in the theory of the international arms trade, Levine, Sen and Smith [1994], initiated a new line of developments in the area. This model introduces both demand and supply sides of the international market of arms, motivated by countries' need for security from potential military threat from other countries. It then examines the resulting equilibrium, including a number of dynamic issues regarding different types of expectations and of co-operation on the oligopolistic supply side. In a natural continuation of this work Levine and Smith [1995] introduced international intervention of a cartel of large supplier countries, which try to stabilise arms races by appropriate adjustment of the international price of arms. This bring us to the more recent development, Levine and Smith [1997] (LS). This model assumes full depreciation of stocks in each time period, ignores dynamics, solves for the complete quasi-general equilibrium of the world market and improves the formulation of security of the previous models. The latest entrant in this sequence of models on the arms trade is Levine and Smith [1998], a paper that provides a better analytical presentation of the LS model, with the addition of a more realistic organisation of the oligopolistic supply side.
Sections 7.2 and 7.3 of chapter 7 provide a more detailed review of this paper, referring to strengths and weaknesses. In conclusion, LS appears to be an important step towards a quantitative theory of the arms trade with a considerable dose of realism, and is regarded as a benchmark throughout this thesis.

Two areas of defence economics that are closely connected with the arms trade are the determination of military expenditure and the competitive accumulation of arms between nations, often called ‘arms race’. Berhelemy et al [1995] look into domestic growth aspects of allocating resources between defence and domestic growth, evaluating the impact of military expenditure on the economic development of countries. They then calibrate their model using the arms race between India and Pakistan as an example. Levine and Smith [1997b] discuss dynamic issues of regional arms races, using a modelling environment closely linked with that of LS. They examine outcomes of the arms races in time; the stability of non-cooperative equilibria and ways that changes in the price of traded arms affect the behaviour of countries. For the foundations of the modern economic theory on the arms races can be found back in Richardson [1950] and Intriligator [1975]. Among more recent developments Garfingel [1990] examines the allocation of resources in defence and the strategic interaction of a pair of countries involved in an arms race. A static representation of this set-up appears to be the formulation of the demand side of LS model. Van der Ploeg and de Zeeuw [1990] examine a similar environment of dynamic interactions of a pair of conflicting alliances (east-west), rather than countries. Although these works provide useful examinations of particular issues that determine the demand for arms, they do not provide a quantitative theory for the equilibrium of the contemporary international market of arms.

Defence economics and the theory of the arms trade find an attractive application in the implementation of policies that could lead to the establishment of a prospective new superpower: the European Defence Union. Fontanel and Smith [1991] provide an informative and early introduction to this issue, which is later discussed again in LS. Dunne [1995] provides a concise description of the industrial structure of major arms producing and supplying countries and looks into problems arising from the post-Cold War decline of this industry and the need for conversion to civilian applications. Brozka [1994] discusses the issue of financing the arms trade. There is a long history of development of alternative or radical approaches to issues related to the arms trade. Marxist approaches tend to focus on issues of class struggle whereas under-consumptionist approaches look into military expenditure as providing a way out of realisation crises. Dunne [1990] and Smith and Dunne [1994] refer more extensively to alternative approaches.
2.3 Empirical literature

A well-documented contemporary review of empirical work, closely related to our area of interest is provided in Smith [1995], who provides a discussion of theoretical and empirical issues related to the study of the demand for military expenditure, the total amount of resources dedicated to defence industry. Since imported arms are usually one of the largest participants of countries' defence budget the two issues are closely linked, essentially analyse the same issue, the production of defence, from two different perspectives. Furthermore, Smith summarises the strengths and weaknesses of the existing empirical work and makes specific suggestions for further developments, mainly pointing towards a standard parametric approach of the international trade based on appropriate model and data set.

The econometric study of Smith, Humm and Fontanel [1985] has been a cornerstone in the foundation of the empirical part of this thesis. This paper examines the demand for military expenditure using a simple but complete theoretical model and looking in depth into the availability of related data. Particularly useful, especially for the calculation of a price index for imported arms and also for the empirical examinations of chapters 4, 5 and 6 is the review of the available data sources for the arms trade. The authors then conclude that as a result of serious data problems the empirical results should be looked as indications rather than as final measurements and also outline the need for using of a more sophisticated model.

Continuing their previous work, Smith, Humm and Fontanel [1987], take a closer look into the issue of the theoretical implementation of an econometric study of defence industry. They also examine issues of substitution between inputs using a Constant Elasticity of Substitution (CES) production function. They outline the importance of the provision of an appropriate modelling foundation to empirical studies. The applied part of that paper employs a similar model and data to this work, providing interesting empirical results and revealing useful information about the use of the employed theoretical framework and about the general direction of the estimated relationship. On the other hand, the authors conclude that the specification of the particular framework is rather unsatisfactory, given the availability and the quality of data, and outlines a number of specific econometric issues regarding the use of the particular data. This work provides a clear illustrations of the potential of qualitative econometric study of this industry on one hand, but also of the limitations that the restrictions of the data impose on the results.
The estimation based on a flexible functional framework by Okamura [1991] presenting an alternative approach to model specification. Despite the fact that this paper deals with the impact of threat to alliances at Cold War level, it focuses on the importance of conducting econometric studies in defence based on an appropriate theoretical foundation. In order to avoid limitations of previous works, which could well give results subject to specification bias, it employs the Generalised Indirect Translog utility function as a social welfare function. Assuming that the US-Japan alliance perceived a threat from the arms of Soviet Union, Okamura models the demand for procurement, solves for the Logarithmic Linear Expenditure System and finally estimates it. This work provides some interesting results, showing evidence that the allies generally respond to the thread created by the military capability of Soviet Union. In a similar way as Smith et al [1997] he sees benefits from the use of a firm theoretical background in the empirical study of military expenditure. One limitation of this otherwise interesting effort is that it is very demanding in terms of data, which is a serious restriction in this particular topic. Another, directly related with the first, is the small country coverage (limited to Japan and the NATO alliance).

Small country coverage is a common limitation in the majority of recent empirical approaches to the production of military capability. McGuire [1987] used a Linear Expenditure System to estimate the demand for military expenditure in Israel. According to Smith [1995] the most widely applied demand function is the one introduced by Murdoch and Sandler [1984] who used a general functional form in their theoretical work, deriving linear or logarithmic-linear demand functions for military expenditure. Applying this demand system to a small number of industrialised countries they estimated it allowing for cross-equation covariances. Again, this framework assumed a limited formulation of security, making it appropriate for special cases of countries that belong to the same side of an alliance.

Pearson [1989] used econometric analysis to explain arms imports, with some interesting results regarding the use of available explanatory variables. Focusing on developing countries, McKinlay [1989] supported a strong relationship between arms imports and military expenditure. Kinsella [1994] examined the dynamic properties of arms transfers to the Middle East using a Vector Auto-Regression model and Granger causality tests. Anayiotos and Happe [1997] review the recent data and trends in their discussion about financing the arms trade. A recent study, CAAT [1996], provides an updated description of the supply side of the arms trade, well implemented on empirical facts. It argues that the industry of arms generally receives excessive governmental support, which needs to be shifted away from this industry. It also provides estimates for
the subsidies of this industry for the case of the U.K., reported as more than 1 billion worth a year or as a wage subsidy of about £12,500 per worker per year.

Empirical studies on military expenditure are not as scarce as studies on the arms trade. Smith [1994] discusses alternative empirical approaches to the study of countries military expenditure, as time series, cross-sectional or panel data. There are several atheoretical empirical approaches to military expenditure mainly exploring time series with the use of Vector Autoregressive Regressions and Granger causality tests. Examples of the former category could be Chowdhury [1991] who used time series for 55 less developed countries and of the latter Dunne and Smith [1990] and Payne and Ross [1992], who did not find any clear Granger causality patterns in military expenditure. Atheoretical approaches, however, have long now been the target of criticism for the plausibility of their results and their usefulness in the confrontation of theoretical hypothesis. In a parametric approach, with better theoretical properties, Hilton and Vu [1991] estimated a demand of equations for a group of NATO allies.

There seems to be an agreement in the literature that data on defence and especially on the arms trade are limited in quantity and quality. Anderton [1995] and Smith [1995] and Sandler and Hartley [1995] refer to the available data sources. These mainly include SIPRI Yearbooks (e.g. [1992,1993,1995]), ACDA [1995] and the IISS [1991]. All these are annual publications that except the data tables provide analyses and reports on their evolution. Happe and Linn [1994] provide a comparison between SIPRI and ACDA data. SIPRI yearbooks provide reports with extensive coverage of all kinds of issues related to defence. The IISS systemically provide analytical tables with countries’ procurement. This thesis finally used data only from SIPRI and ACDA, which are discussed in more detail in chapter 3.

2.4 Conclusions

The literature on the arms trade could be characterised as well developed in historical and descriptive approaches, developing in theoretical explanations and least developed in empirical implementation. Most of the recent works on the arms trade seem to suggest that there are considerable opportunities for further construction of theoretical and empirical work in the area. Anderton [1995] concludes that ‘almost the whole body of the arms theory remains untapped’. Similar to this is the result of the literature review conducted in the context of this thesis. The recent advances in the theory of the arms trade give incentive for constructive mobility in this literature, as several important aspects of the issue remain as yet uncovered. On the positive side we refer to the
analytical attempts for a positive quantitative theory of the arms trade and, more generally, the determinants of military expenditure. The existing theories, however, only provide a rather simplistic formulation of the demand side of the market ignoring important factors of the demand and the supply sides of the international arms market.

The main strength of recent advances in theoretical modelling appears to be the development of analytical formulations for the demand and the supply side of the arms market and the equilibrium of the world market. The particular way that the characteristics of the two sides have been captured in the LS seems a successful step in the light of the contemporary developments. Despite the importance of this first step in the development of a coherent theory there is an important weakness: The assumption that only few large countries can produce arms and all other countries are net importers, which is apparently contradicted by the reality. In technical terms, the existing theory completely neglects any differentiation between the produced and traded categories of arms, as well as the possibility of substitution between the inputs of defence industry. These important simplifications do not only impose unrealistic simplifications to the description of the demand side of the market but also affect seriously the policy conclusions drawn from the examination of co-operative behaviour of supply side and the results of international regulation of the arms market. In other words, taking account of the alternative of a domestic arms sector may show that the suggested in LS imposition of a tax on imported arms could be ineffective as a means of controlling the arms races. Instead, it could have the opposite result of encouraging the governments to cross from the demand to the supply side of the market. The primary theoretical task, therefore, of this thesis is the introduction of substitution between different categories of arms and labour in the production of military capability and the introduction of a rational decision mechanism for the establishment of domestic arms industries.

Many economists would agree that the quality of the outcome of an empirical study heavily relies on the provision of a coherent and realistic analytical framework and of an appropriate for the needs of the study data set. The message that comes almost unanimously from the literature is that in the case of the international arms trade both these areas seem problematic, with the unavailability or even non-availability of data be the most restrictive in practical terms. While traditionally governments have restricted the release of this information for strategic purposes the recently increasing establishment of international defence databases signals an improvement and opens a new era for the development of empirical studies. The need of more detailed empirical study is also an essential requirement for the development of theoretical work. As Catrina [1994 p.204] puts it 'researchers should show more detachment, devoting their
efforts more to establishing facts than to presenting opinions’. Since arms imports usually absorb the lion’s share from countries’ military budget (see e.g. ACDA 1995) an important obstacle in the development of a parametric empirical approach of defence technology is the unavailability of information about the prices of imported arms. The importance of this critical issue is outlined in the majority of recent studies.

Summarising, we have outlined the problems of the approaches of the international trade in arms found in the economic literature, separated in two different categories: theoretical and empirical. Following the view that the best way to further examine the international arms trade involves a harmonious development of theoretical and empirical work, the rest of this thesis deals with both of these aspects. First we examine the existing data and attempt to calculate a price index of domestic arms, using a relatively simple analytical pattern of for defence industry we proceed to the econometric study of the international arms trade, using cross-sectional and panel data. Based on these results we further develop the analytical approach by introducing differentiated arms and the option of domestic production and re-examine the outcome in an empirical study. Finally we incorporate the experience gained from the combined theoretical empirical attempts in a world market model.
3. Data, trends and prices of the international arms trade

3.1 Introduction

This chapter discusses the stylised facts regarding the arms trade and develops a method for the calculation of a price index for imported arms. Since data on defence are generally known to suffer from qualitative problems, this review looks in detail at the methods employed by the sources. It also makes an effort to extract as much information about the final quality of the data as possible. Series of prices and other data that we discuss in this chapter are finally used in the econometric studies of the following chapters 4 and 5. The following section 3.2 looks carefully into the alternative sources and the availability of data on the arms trade. Section 3.3 proceeds into a preliminary examination of the available data, their general characteristics and underlining trends. Section 3.4 explains a suggested method for the calculation of a price index for imported arms. Section 3.5 gives examples of the evolution of this price index for a number of country aggregates. Appendix A.3 provides the cross-sectional data set used in the empirical study of the following chapter 4.

3.2 Sources and availability of data

A cross-sectional examination of the demand for arms imports requires defence-related information, such as data series on arms imports, prices of arms and military expenditure. It also requires general economic data such as measurements of per-capita income of countries, population, exchange rates, price levels, purchasing power parities (PPP) etc. The selected sources of military information are the Stockholm International Peace Research Institute (SIPRI) and the Arms Control and Disarmament Agency (ACDA). The source of general economic data is version 5.6 of Summers and Heston database known as Penn World Tables (PWT). SIPRI appears as a reliable source, since
it provides clear information about the methodology employed in the preparation of their data series. Although Sweden is a net exporter of arms, Swedish governments have established a reputation for avoiding aggressive related policies and SIPRI has established a reputation as a reliable and independent organisation. This is not exactly the case with ACDA, which is closely linked with the government of the larger exporter of arms USA, a country with a rich background in military activities and manipulation of related information. ACDA, however, has access to information sources such as CIA records, which is not directly available to SIPRI and appears a reliable source.

The numbers of years and countries in our sample, as well as the quality of this information, have largely been determined by the availability of quantitative information. The basic constraint appears to be the availability of data on the arms trade from SIPRI. Data on defence are generally known to suffer from qualitative problems. For that reason, in choosing between the quality and quantity of data, a higher priority is given to quality. Briefly, some of the guidelines in the selection of our data sample are:

- Examine in detail of the quality of data series in use.
- Avoid using data with known problems in their quality.
- Avoid using the most recent observations.
- Avoid putting together data of different quality, ie prefer to loose some observations rather than increase the uncertainty about the final quality of the data set.

As a result of these criteria the final data set includes 38 countries out of about 180.

The calculation of a price index for imported arms, we discuss later in this study, requires observations of arms imports from both SIPRI and ACDA. The coverage of SIPRI tables on the arms trade appears more restricted than the respective of ACDA. According to the conclusions drawn from the evaluation of the data, most appropriate are the two tables that provide annual figures on arms trade, in constant dollars, for 50 countries. The first table originates from SIPRI Yearbook 1992 (SIPRI [1992]) and covers observations from 1987 to 1991 and the second from Yearbook 1995 and covers years from 1990 to 1994 inclusive.

A comparison of the alternative observations for the two years that appear in both of the tables, those of 1990 and 1991, shows considerable discrepancies between the two tables, not only in absolute values, but in relative magnitudes between countries as well. Most likely, these differences originate from revisions made by SIPRI in the later
release. For these reasons, we only use data from Yearbook 1995 (SIPRI [1995]). and, eventually, avoided combination of the two tables. Table 1 gives examples of observations for the same country and year from the two tables. These cases ware selected for their significant changes, illustrating the extent that newly released data can be altered by forthcoming revisions.

Table 1. Revisions in measurements of arms imports

<table>
<thead>
<tr>
<th>1990 Observations</th>
<th>Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Egypt</td>
</tr>
<tr>
<td>Yearbook 1993</td>
<td>1203</td>
</tr>
<tr>
<td>Yearbook 1995</td>
<td>755</td>
</tr>
<tr>
<td>Change %</td>
<td>-37.24%</td>
</tr>
</tbody>
</table>

Note: Annual figures in current million U.S. dollars.

The second series of data on arms imports and series of military forces originate from the annual publication of ACDA ‘World Military Expenditures and Arms Transfers’. Although, ACDA and SIPRI figures on military expenditure appear to follow similar qualitative standards the latter series were eventually chosen for their better performance in practice. SIPRI data on military expenditure are extracted directly from the original database, which was made available to us during the autumn 1995. According to the descriptions of the methods employed by the two sources (ACDA [1995] pp. and SIPRI [1995b]), they attempt to measure amounts that are actually spent in purely military activities, excluding expenditure finally directed to civilian purposes.

After dropping countries and years for which one or some of the tables of military expenditure by SIPRI, arms imports by ACDA, arms imports by SIPRI and per-capita GDP by PWT do not provide observations, the result is series of 38 counties and two years, 1990 and 1991. Given the preference to avoid the use of recently released observations, the years 1990 and 1991 appear as those that best satisfy these two requirements. Another practical reason for the choice of these years is that the currently released (November 1995) Penn World Tables do not provide observations more recent than 1992, and for some countries not even for 1990 and 1991. These countries are dropped from the sample. Since the low number (two) of time observations is inadequate
to consider a time series or panel data study, the average of the two annual observations can be used in a cross-sectional study.

Series of per-capita GDP from PWT are available in a variety of conversions, such as deflated either with Chain or with Laspeyzer indices, corrected for the terms of Trade or for the working force rather than the population etc. Variables that measure values can be transformed in U.S. dollars either with the use of PPP ratios or with exchange rates. Having the possibility to chose between these methods is clearly in the interest of our study. Providing the availability of PPP ratios it is possible to transform the series back to exchange rate equivalents. On the contrary, both SIPRI and ACDA use exchange rates for the transformation of the data (ACDA [1995] p.171). For the deflation of figures on military expenditure SIPRI use Consumers Price Index and for the figures on the arms trade use Producer Price Index for machinery industry of the larger Western exporter countries. A single price index is used for all countries, as there is no evidence that producer prices vary across weapon categories. ACDA deflates figures on the arms trade and military expenditure using implicit GNP deflator for the U.S.

There is a minor modification of one of the observations in the data set. It is the case of Hungary that ACDA gives zero observations of imports for 1990 and 1991. In order to allow the transformation of levels to logarithms the two-year average is interpolated with the average of the neighbouring observations of 1989 and 1991, which are close to zero whatsoever. If SIPRI and ACDA were to be characterised whether as primary or secondary sources for the figures on military expenditure and the arms trade they produce, the former characterisation appears closer to reality. Despite the fact that the governments release their own figures, these organisations collect all available information and produce their own estimates, using the official releases merely as means for comparison. Taking the average of the two annual observations contributes to the reduction of noise in the data, especially by smoothing variations caused by short-term fluctuations or discontinuities of the measured flow. At the same time, the range of averaged years is not too long to blur the contemporaneous relationships of the variables, which is the target of the static nature of a cross-sectional study.

Among the reasons that make years 1990 and '91 attractive for a cross-sectional examination is the generally high volatility of the related variables during these years, apparently a result of the transition to the post-Cold War era. During this period we can expect the stimulated adjustment mechanisms to be reflected in the variations we attempt
to analyse and explain. Furthermore, another positive aspect of this historical time is that
countries have moved towards higher independence in their decisions on defence, having
reduced their commitments to related alliances from previous years. In practice, this
means that countries' imports reflect more clearly their own requirements for security.

3.3 General trends

The first chapter of ACDA [1995], called 'Highlights' provides brief descriptions of
the general trends in the data. The most impressive characteristic of the series is the
sharp decline of international arms trade within the last decade. In the related diagram
ACDA unsuccessfully used a logarithmic scale to plot the aggregate series of the arms
trade giving a misleading impression about the actual scale of the changes. The verbal
description, however, is quite clear: "At the world level the average annual growth rate of
imports was -11.4% for the decade 1983-93 and accelerated to -23.3% during the
second half 1989-93." As ACDA notes, but we can also observe from the diagrams, the
speed of the decline appears to decrease at the end of the covered time period. This slow
down of the decline is rather expected since if 1990-91 rates had continued the
international arms trade would have reached zero levels before 1994.

*Figure 3.1 World aggregates of arms imports by ACDA and SIPRI*
SIPRI measurements have almost systematically been lower than those of ACDA during most of the covered time period, following a decreasing but clearly less steep trend. At about the end of the time period SIPRI imports eventually become higher than ACDA. We can observe the evolution of these two variables at world level of aggregation in Figure 3.1. SIPRI data were spliced, observations 1983-1989 from [1992] Yearbook and observations 1990-1993 from Yearbook [1995].

Figure 3.2 Arms imports of large geographic regions and organisations

Source: ACDA [1995]
At the world level of aggregation, from 1987 the plot shows a relatively stationary and close to linear downward sloping trend, which accelerates in 1991 and finally declines. Since it would be unrealistic to expect zero levels of arms trade, the sharp decline followed by stabilisation appears consistent with dynamic aspects of countries' procurement. Since the arsenal of a country is a stock that increases with the purchase of new arms, either imported or domestically produced and decreases with the use of arms and with their depreciation of the arms in time. Because of the forward-looking component of planning new orders and the delays associated with their delivery, country might face unexpected changes to its desired stock.

In the case that a country attempts to reduce its procurement levels the adjustment of their stocks to the new requirements implies that it has stopped placing new orders but still continues receiving the orders that have already been placed. In time, depreciation of arms reduces the accumulated stock and, when this reaches some critical level the country starts placing new orders again to maintain at the desirable level. It is possible that during the period of de-accumulation a country could consider exporting part of the excessively procured arms. Indeed, from the tables some countries appear to have exported arms only for a single or a couple of years, during the period 1989-91, without having a record for exports before or after this period. Such examples are Ethiopia, Iran, Ireland, Kuwait, Luxembourg, New Zealand, Nigeria. Since it is highly unlikely that these countries produced these arms for such a short time period, it is reasonable to assume that they exported the excessive part of their stocks.

The diagrams of figure 3.2 illustrate the evolution of the arms trade over a decade, aggregated by wide geographical regions and organisations. The data series originate from ACDA [1995]. An examination of the series, aggregated at the level of continents, or large groups of countries, shows that the trends followed by most of the groups are generally in line with the overall trend. Most of the country groups seem to have more or less followed the declining trend, during the covered decade and, in particular, after the withdrawal of the Cold War. NATO countries appear to be an exception to this rule, showing an increasing trend during the first half of the time period, followed by a decline. In the case of Eastern Europe, the turning point from rise to decline (i.e. the peak) precedes three to four years to that of NATO allies.

From the included country groups, those with the earliest turning point are Africa, Middle East, OPEC and South America at about 1984. Central America follows in 1985,
Eastern Europe and Warsaw Pact in 1986 and East Asia in 1997. Last to reduce their levels of imports are the bulk of Western countries, mainly expressed in NATO, OECD, W. Europe and N. America aggregations, having their turning point at about 1988-89. Countries appear to have reduced Cold War levels of procurement following the ranking of economic development. It appears that richer countries preferred to maintain high procurement levels for longer, possibly ensuring higher security, whereas poorer countries seemed to have sought a quicker relief from this burden.

3.4 An implicit price index for imported arms

The price of a good usually plays a critical role in the determination of the demand for that good. Therefore the availability of information on prices is an important prerequisite in an examination of the demand. In practice, we can expect considerable differences in the real prices of imported arms different countries face. A possible cause of this variability is positive or negative externalities of this trade between recipient and supplier countries. When the two parties are involved in an alliance, their procurement implies positive spillover effects to the joint security. Such positive externalities, as well as other possible common national interests of wider nature (eg promotion of bilateral trade), may push prices downwards. A typical case is that a country can face higher prices as a result of negative externalities for security reasons. This is the case of a country that imports arms from a military opponent in bilateral or multilateral terms, for instance during an international ban.

The attempt to acquire a price index for imported arms, at individual country level is based on the comparison of the aggregate figures of arms imports from two different organisations. This calculation is possible because the contents of SIPRI and ACDA measurements are different (Happe and Linn [1994]). The former organisation attempts to measure the aggregate production cost of the imported arms whereas the latter measures the actual value of the transaction. The explanations about the method provided by the organisations support this hypothesis. According to the documentation provided by SIPRI:

"(SIPRI measurement of the arms trade)...is designed as a trend-measuring device to permit measurement of changes in the total flow...(that) reflects in monetary terms both the quantity and the quality of the weapons transferred... the monetary values
chosen do not correspond to the actual prices paid, which vary considerably depending on different pricing methods [Yearbook 1995, p.555]. It must be emphasised that the SIPRI values are not the actual prices of weapons paid in any particular deal [p.5]. The SIPRI values are therefore based on average production costs [Yearbook 1992, p.355]. The results may also be modified according to two other criteria - whether equipment is new, second hand or refurbished in some way; or whether it is transferred direct or produced under licence [SIPRI Fact Sheet 1995 p.4]."

Unlike STRI, ACDA measurements use a standard pricing method by utilising available information on the actual payments of the transaction. In this transformation

"the statistics... are estimates of the value of goods actually derived during the reference year" and there is a warning that "frequently, weapon prices do not reflect production costs," [ACDA 1994 pp.169-70].

The explanations about the employed methodology, provided by two sources suggest that SIPRI measure production cost and ACDA measure values of the aggregate arms imports. The calculation of the production cost takes in account the specific quality of the arms and corrects for depreciation when the arms are previously used. Estimation of the production cost also allows the aggregation of different kinds of arms in a homogeneous volume, with the estimated production cost of each category used as the 'weight' of each weapon system. In this context, SIPRI aggregate figure is a measurement of the total volume of imported arms. On the other hand, ACDA measurements measure the actual amounts paid when the transactions took place, giving the overall value of the imported arms. As value measurements ACDA figures contain information about the actual price of arms, which might differ from the production cost of the imported arms. It should be clear that the suggested divergence between the two methods is relative, rather than absolute, in the sense that each method dominates in the calculation of each organisation in the majority of cases.

The following description ignores possible measurement errors. If $S_i$ is SIPRI measurement of the aggregate volume of arms $M_i$ imported by country $i$ in a year, $q_j$ is the quantity of category $j$ weapon system imported and $c_j$ is the estimated cost of this category we have

(1) \[ S_i = M_i = \sum_j c_j q_j \]
Summarising, SIPRI's volumetric measurement aggregates heterogeneous weapon systems by adding up their production cost, net of depreciation when they are previously used. ACDA measurement $A_i$ adds up the value of each category of arms, where $q_{ij}$ is quantity imported and $p_{ij}$ is the actual price of $j$-th category of arms paid by country $i$. We can then express ACDA value measurement of total imports as

$$A_i = p_i M = \sum_j P_{ij} q_{ij}$$

Based on these assumptions, an implicit price index $p_i$ of the aggregate arms imports of each county is simply given by the ratio of the two measurements

$$\hat{p}_i = \frac{A_i}{S_i} = \frac{\sum_j P_{ij} q_{ij}}{\sum_j c_{ij} q_{ij}} = \sum_j w_{ij} \frac{p_{ij}}{c_{ij}}$$

The weights $w_{ij}$, by which the price-cost ratio of each weapon system contributes to the overall price, are the real cost shares of system $j$ to the total imported volume, i.e.

$$w_{ij} = \frac{c_{ij} q_{ij}}{\sum_j c_{ij} q_{ij}}$$

### 3.5 Prices of imported arms

The diagrams of figure 3.3 display the evolution of the implicit price index for a number of country groups, i.e. organisations such as OPEC and regions. The aggregates included in these diagrams are those for which both ACDA [1995] and SIPRI [1995] provide data, so that the calculation of the price index is possible. From a rough observation, in the majority of cases plotted in the diagrams the price index appears to have followed a general trend associated with that of the volume of the international arms trade: i.e. an increase followed by a decrease. This pattern in the case of prices, however, is not as clear as in the case of the volume or the value of the arms trade.
Figure 3.3 Evolution of the price of arms imports aggregated in large geographic regions and organisations

3.6 Conclusions

According to the evidence provided in this chapter, SIPRI and ACDA appear to measure the volume and the value of the arms trade respectively. At country level, the data on the arms trade from the two sources cover 38 countries and two years. Aggregated in large geographical regions, the available data cover most of the world and eleven annual observations. Using these data, we can calculate the implicit price index of imported arms. The same sources also provide data on military expenditure. As a measurement of countries income, ACDA provides figures of per-capita GNP and also figures of population, so it can be transformed to aggregate level. The widely used Penn World Tables provide series of per-capita GDP, transformed in a variety of ways. Based
on these data, we may consider an econometric examination of the demand for imported arms. Since at country level the available observations are not enough for a study of time series chapter 4 develops a cross-sectional examination of these data. In a separate study, chapter 5 examines the panel of regional aggregated time series.
4. The demand for imported arms: A cross-sectional study

4.1 Introduction

A well-implemented empirical study of the demand for imported arms requires information about the prices that countries face in this market. The unavailability of information about prices possibly has prevented the development of empirical studies, within the context of mainstream quantitative economic techniques. In order to overcome this problem this study uses series of the implicit price index of imported arms, discussed in chapter 3. Using the implicit price index, data for the arms trade, military expenditure and national per-capita income and a simple theoretical model we proceed to a standard cross-sectional econometric examination of countries' demand for imported arms. Along with useful information about the structure of this demand per se, this examination provides a means of evaluating the quality of the implicit price index of imported arms.

Based on the theoretical foundation of the arms trade in this chapter we first specify an analytical model for the needs of our empirical study. Based on that framework we estimate and test alternative functional forms of demand functions, with the use of standard econometric techniques. Finally, we examine the possibility of bias in the estimates, because of measurement errors in the data, and ways to improve our results with respect to this particular problem. The effort to develop an analytical model of the arms trade appropriate for the needs of empirical studies continues in chapter 6. The cross-sectional study of the demand for imported arms in this chapter uses country level data but ignores the dimension of time. The study of a panel of regionally aggregated data in chapter 5 overcomes this problem at the expense of using aggregated data.

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1 Results from this study were presented in the fifth World Peace Science Congress in Amsterdam, in June 1996 and were included in Levine, Mouzakis and Smith [1996] and [1998].
Therefore, the studies of this and the following chapters are complementary, looking at the same issue from different perspectives.

For a theoretical approach of the arms trade two important recent developments are Levine, Sen and Smith [1994] and Levine and Smith [1995]. These two approaches however, do not include in the formulation of the defence industry any internal mechanism of domestic production of arms, as a differentiated input from imported arms. This is also the case of the more recent work Levine & Smith [1997]. Because of the combination of generality, content and analytical tractability, this model is treated as the benchmark for the alternative specifications we examine in the analytical section of this chapter.

Following to the discussion of chapter 2, Smith, Humm. and Fontanel [1987 p.71] provide a valid approach to the applied study of the closely related topic of military expenditure, especially in terms of employed methodology. This paper outlines the importance of a close link between an empirical study and a theoretical framework and uses similar model and data with this work. In an alternative approach, Okamura [1991] used a Translog functional framework. The initial attempt to apply a similar approach in this study was eventually abandoned both because of insufficiency of necessary data and because of low theoretical suitability. Instead, this study uses an explicitly developed analytical set-up.

For general reference on econometric issues this study is mainly based on Greene [1993], Maddala [1977], Kmenta [1990] and Judge et al [1985]. A criticism by Delong [1988] to the results of Baumol [1986] on economic convergence provides an application of the standard analysis of the impact of measurement errors to estimations with one latent explanatory variable. Extending the analysis for the impact of measurement errors from one to two latent variables the analysis by Theil and an addition by Levi [1973] was found particularly useful and applicable. Breuer and Wohar [1996] discuss the impacts of data quality to empirical studies and of the importance of careful selection of quantitative information.

The material in this chapter is organised as follows. Section 4.2 develops an analytical approach of countries' decision mechanism for the allocation of resources in defence, focusing on a theoretical implementation of the empirical study of the demand for imported arms. Section 4.3 presents and discusses econometric results and estimates of countries' demand for arms imports. Last, section 4.4 analyses the impact of
measurement errors to the econometric results, attempting to improve the estimates and reduce the related bias. The appendix of this chapter A.4 provides technical information from the econometric study.

4.2 Analytical approach

4.2.1 Preface

The primary purpose of this attempt to model the demand of countries for imported arms is to provide a demand function for use in a cross-sectional econometric study. A secondary but not less important purpose is an evaluation of the quality of data and the methods involved in their preparation. For this we need to examine the agreement of an appropriate analytical framework with the available empirical information, using appropriate econometric techniques. Using the experience from this examination may further attempt to improve of the analytical model as well as assess the quality of our data. As a main concern, however, the following paragraphs try to keep correspondence between model and empirical findings.

In this examination we assume that one of the inputs of defence, the domestically produced arms, are treated as differentiated from the imported ones. Furthermore, we assume that countries’ domestic industries of arms are natural monopolies, producing arms of lower technology than the imported but more essential for the existence of countries’ defence. In reality, national defence industries are not always but often under public ownership or heavily regulated. A major concern of government when it comes to their domestic industry of arms is security of supply, since in case of conflict the provision of imported arms might be interrupted. On the other hand, the domestic alternative to imported arms is the generally large set-up costs of this production.

For purposes of simplicity, in the following set-up we assume that military budget of countries is pre-fixed by the central government, reflecting public preferences for the security. With the military budget $G$ predefined, our analysis addresses the problem of policy makers to maximise national security by allocating their budget between the available inputs to the production of national defence. Any dynamic aspects are ignored and the static setting is assumed to be structurally stable, at least over some period of
time. We also assume that decision-makers are rational and that countries are not
explicitly aggressive.

4.2.2 Production under fixed factor costs

Assume that the private sector of our typical country produces \( Y \) units of uniform
composite output, which is either used in consumption or as an input on the national
defence sector. The total amount of resources \( G \) allocated to defence industry is spent in
the three inputs. These are quantity \( M \) of imported arms, quantity \( D \) of domestically
produced arms and quantity \( L_K \) of labour. The public sector makes decisions about the
allocation of \( G \) to the alternative inputs. Here we can allow for bureaucratic inefficiency
of the public sector. This means that some part of the budget does not reach the
production but gets wasted in the administrative mechanism. We can write this as

\[
G^* = G^\omega; \quad 0 < \omega \leq 1
\]

where \( G^* \) is the budget finally utilised in the production and \( 1-\omega \) measures the degree of
bureaucratic inefficiency of the military sector.

Assume that labour is available at the cost of wage \( w \), which remains unaffected by
the demand. For a set-up with flexible substitution between inputs assume that the
production of military capability \( K \) follows a CES function

\[
K = \left[ \frac{\kappa_D D^\rho + \kappa_M M^\rho + (1-\kappa_D - \kappa_M) L_K^\rho}{\kappa_D D^{\rho \kappa_D} M^{\rho \kappa_M} L_K^{\rho (1-\kappa_D - \kappa_M)}} \right]^{1/\rho}; \quad \rho \in (-\infty, 0) \cup (0, 1)
\]

The non-negative quantity \( \sigma = (1 - \rho)^{-1} \) is the elasticity of substitution, \( \nu \) is a positive
coefficient that measures the returns to scale and \( \kappa_D \) and \( \kappa_M \) are positive coefficients that
\( \kappa_D + \kappa_M < 1 \). Values of \( \nu > 1 \) and \( \nu < 1 \) indicate increasing and decreasing returns to scale
respectively. Government spending in defence consists of the value of the labour cost (of
the armed forces) and the values of the two stocks of arms. If the price of imported arms
is \( p_M \) and the price of the domestically produced arms is \( p_D \), the budget constraint that
the planner of defence faces is

\[
G^\omega = wL_K + p_M M + p_D D
\]

We also assume that government is a price taker for all three inputs and that the three
factor prices remain unaffected by the demand for the factors. This does not seem to be a
strong assumption for \( p_M \), an international price and not too strong for \( w \), since
employment in the military sector is usually small enough not to affect significantly the
national wage. But it is relatively a stronger assumption for the domestic industry of
arms, which is often heavily regulated and controlled by the state. We relax this
assumption later. The Lagrangian for the maximisation of the objective function (2)
under constraint (3) and the non-negativity constraints for the three inputs is
\[ \mathcal{L} = K(D, M, L_k) - \lambda(wL_k + p_mD + p_DD - G^\alpha) \]
The four first order conditions (FOC) are (3) and
\[ \frac{1}{w} \frac{\partial K}{\partial L_k} = \frac{1}{p_M} \frac{\partial K}{\partial M} = \frac{1}{p_D} \frac{\partial K}{\partial D} = \lambda \]
Solving from the three FOC in (5) the shares of the three inputs are
\[ L_k = l_k M; \quad M = dD; \quad D = gL_k \]
where the three share coefficients are
\[ l_k = \left[ \frac{p_M (1 - \kappa_D - \kappa_M)}{w \kappa_M} \right]^\sigma; \quad d = \left[ \frac{p_M \kappa_M}{p_M \kappa_D} \right]^\sigma; \quad g = \left[ \frac{w \kappa_D}{p_D (1 - \kappa_D - \kappa_M)} \right]^\sigma \]
Under these definitions with the use of the fourth first order condition (3) we can
solve for the demand function of imported arms
\[ M = \frac{G^\alpha}{l_k w + p_M + \lambda^{-1} p_D} \]
as well as for the demands of the two other inputs
\[ l_k = \frac{G^\alpha}{l_k w + l_k^\sigma p_M + \lambda p_D} \]
and
\[ D = \frac{G^\alpha}{\lambda^{-1} w + \lambda p_M + p_D} \]
Notice that the conditions for equilibrium are not independent from the returns to scale
\( \nu \). This is an expected result since CES is a homothetic production function which gives
linear expansion paths for given proportions of the inputs. Then, for any given set of
factor prices the optimum input shares are independent of the level of output.

An important consideration when we analyse either consumer or producer behaviour
is the degree of substitution between the inputs of the transformation. In the case of CES
when \( \sigma > 1 \) inputs are substitutes in production, the isoquant spaces of this
transformation with three inputs are expected to intersect with the axes. In practice, this
means that zero quantities of some of the inputs can be included in the set of optimal solutions. In the case of CES, however, the solutions are not defined (valid) when any of the inputs is zero, since it is not differentiable at zero, so the intersection takes place only asymptotically. In that respect CES family framework fails to capture the complete substitution of an input, a weakness that makes its use less attractive. We return to this issue in chapter 6.

Expanding the demand function for imported arms in (8), using the definitions in (7), it becomes

\[ M = \left( \frac{1 - \kappa_D - \kappa_M}{\kappa_M} \right)^{\sigma \omega^{1-\sigma}} + \left( \frac{\kappa_D}{\kappa_M} \right)^{\sigma \omega^{1-\sigma}} \left( p_D^{1-\sigma} + p_M^{1-\sigma} \right)^{-1} \]

Notice that when price \( p_M \) increases, this demand asymptotically tends to zero. The income elasticity of this demand is

\[ \varepsilon_G = \frac{\partial M}{\partial G} \frac{G}{M} = \omega \]

which shows that this elasticity is independent of the level of \( G \) but it depends on the extend of bureaucratic inefficiency. The two other elasticities take more complicated expressions.

**4.2.3 Domestic production of arms**

In the previous section the planner of defence faces a fixed price of domestic arms, which could be the result of a competitive domestic arms industry. Equilibrium under competition requires that set-up costs are small enough compared to the demand, so that the entry of a relatively large number of firms is possible. If cost and demand factors only allow the entry of a small number of firms, the market structure moves away from that of perfect competition towards some form of oligopoly. In the case of arms industry the competitive structure appears rather unrealistic since this sector is usually either state-owned or heavily regulated. Furthermore, this industry is known to require large set-up costs. In other words the arms industry has an evidently monopolistic character in most of the cases. The majority of arms production takes place in few large states, rather than being evenly distributed around the world. This clearly suggests positive economies of scale, since the defence industry tends to get integrated inside national borders.
According to ACDA [1995] USA held 47% of the world exports in 1993, U.K. 20%, Russia 12% and no other country has more than 5%.

We can combine these requirements in a production function, preserving simplicity in the solutions, if we use a Leontief production function with two inputs, capital $Y_D$ and labour $L_D$. Assume that a part $F$ of $Y_D$ is set up costs and the variable capital $Y_D - F$ is always employed at a constant proportion $b$ with labour, i.e. $Y_D - F = bL_D$ or

$$Y_D = bL_D + F$$

If the units of the domestic product are adjusted to be equal to the average product output (per unit of labour), this function can be written as

$$D = L_D^\delta$$

where $\delta > 0$ is the returns to scale. The overall cost $C_D$ of this production is $C_D = wL_D + Y_D = (w + b)L_D + F$ and, after substituting from (14) it becomes

$$C_D = (w + b)D^{1/\delta} + F$$

Furthermore, assume that the government taxes all profits or subsidises all losses so that the arms industry has zero profits and the price of domestic arms equals the average cost $C_D/D$, i.e.

$$p_D = (w + b)D^{1/\delta} + FD^{-1}$$

If the returns to scale are constant ($\delta = 1$) the decrease of $p_D$ tends to vanish when the $D$ increases but, if $\delta > 1$, the decreasing effect remains. The case of increasing returns to scale is the one we mainly consider here.

### 4.2.4 Defence planning with internal domestic arms sector

Either increasing returns to scale or positive set-up costs result in diminishing unitary cost of the output. In order to maintain simplicity in the solutions we can use the Cobb-Douglas (CD) form of (2) with a unitary elasticity of substitution. Then the budget constraint for the production of defence becomes

$$G^o = wL_K + p_M M + (w + b)D^{1/\delta} + F$$

and the share functions for optimal allocation

$$L_K = \frac{1 - \kappa_D - \kappa_M}{\kappa_M} p_M M; \quad D^{\delta} = \delta \frac{\kappa_D}{1 - \kappa_D - \kappa_M} w + b; \quad D^{\delta} = \delta \frac{\kappa_D}{\kappa_M} \frac{p_M}{w + b} M$$

Using the share functions we can solve from (17) for the demand for imported arms
\[(19)\]
\[M = \frac{\kappa_M}{(\delta - 1)\kappa_D + 1} \frac{G^\omega - F}{p_M}\]

and the demand for military forces

\[(20)\]
\[L_K = \frac{1 - \kappa_D - \kappa_M}{(\delta - 1)\kappa_D + 1} \frac{G^\omega - F}{w}\]

In a similar way, solving for the steady state value of the domestic production of arms \(D\) and substituting from (14) we can solve for the output of the domestic sector

\[(21)\]
\[L_D = D^\delta = \frac{\kappa_D}{(\delta - 1)\kappa_D + 1} \frac{G^\omega - F}{w + b}\]

The system of equations (19), (20) and (21) provides a fairly simple solution with some interesting characteristics. The demand function for imported arms (19) is independent of the labour cost \(w\), inversely related with the price and the returns to scale and has a linear relationship with the effective budget and setup costs. The income, price and cross-price (\(w\) characterises both the prices of the alternative inputs \(L_K\) and \(p_D\)) elasticities of this demand are respectively

\[(22)\]
\[\varepsilon_G = \frac{\partial M}{\partial G} \frac{G^\omega - F}{G^\omega} = \frac{\omega G^\omega}{G^\omega - F}; \quad \varepsilon_{p_D} = \frac{\partial M}{\partial p_D} \frac{p_D}{M} = -1; \quad \varepsilon_w = \frac{\partial M}{\partial w} \frac{w}{M} = 0\]

The unitary price elasticity and zero cross-price elasticity (i.e. the inelastic response of imported arms to wage) are two basic properties of a CD production function with unitary elasticity of substitution. The latter characteristic is a result of symmetric income and substitution effects that cancel each other out. The income elasticity of this demand is independent of the returns to scale \(\delta\) and converges asymptotically to \(\omega\) when \(G\) increases.

**4.2.5 Overview of the modelling framework**

This analytical approach is an attempt to illustrate the structural difference in the behaviour of the countries when the decision for the establishment of a domestic production of arms is internal to the decision mechanism of defence. When a country produces arms, the labour cost has a double effect on the demand for inputs in defence, one through the price of armed forces and another through the price of domestically produced arms. When, for instance, the wage increases the (increasing) substitution effect on the demand for arms imports gets strong enough to offset the decreasing
income effect, giving the result of an inelastic demand. An important limitation of this examination is that the domestic arms industry must always be established, providing that the budget is higher than the set-up costs. Another limitation is that this analysis does not apply when countries cannot afford a domestic sector ($G^o < F$). This limitation originates from the unitary elasticity of substitution of the CD function, which does not allow any of the inputs to be left out from the production.

A production function with higher elasticity of substitution could possibly provide the flexibility of an optional entry of the domestic industry of arms. In the case of CES, the solutions are defined only for positive levels of inputs, excluding zero. When $\sigma_K > 1$ the derived demand functions for the inputs, e.g. in (9), (10) and (11), *asymptotically* converge to zero when their prices rise. The standard bibliography, e.g. Heathfield & Wide [1987], does not seem to provide a straightforward description of the behaviour of CES at the margins of Kuhn-Tucker conditions. Using CES instead of CD as a production function for $K$ the demand function for arms imports becomes a complicated and non-linear inverse function of $M$, which does not take a general algebraic solution and demonstrates the inappropriateness of CES for our specific modelling requirements.

4.3 Empirical study of the demand

4.3.1 A Simple log-linear demand function

Following our theoretical approach in the previous section and the availability of data we may first consider a simple unrestricted log-linear demand function

$\ln M_i = a + \varepsilon_G \ln G_i + \varepsilon_w \ln w_i + \varepsilon_p \ln p_i + u_i$

as a log-linear approximation to (19). Data on military expenditure provide measurements of the military budget $G_i$ and the price of imported arms can be

It can be expressed as

$M + \left[ \left( \frac{w+1}{w} \right)^{\frac{1}{1-\rho}} \left( \frac{K_D}{K_M} \right)^{\frac{1-\rho}{\rho}} \frac{\rho}{\rho+d} P_{M}^{\frac{\rho+d}{\rho}} \right]^{\frac{1}{1-\rho}} = \frac{G^o}{w^{\frac{\rho}{\rho+d}} \left( \frac{1-K_D-K_M}{K_M} \right)^{1-\rho} + P_M^{1-\rho}}$
approximated with the implicit price index, as in chapter 4. Following the conclusions of previous section, the variations of average wage \( w_i \) cause a twin substitution effect in the demand for imported arms, one through the demand for military forces and one through the demand for domestically produced arms. This combined effect is expressed by the associated elasticity of arms imports with average labour costs \( \varepsilon_w \). A reasonable approximation of the average labour cost of the country is a per-capita measurement of GDP. The coefficients \( \varepsilon_G \), and \( \varepsilon_p \) are the elasticities of arms imports to military expenditure and to the price of imported arms respectively and \( u_i \) is a stochastic term.

Along with estimates for the elasticities, this preliminary estimation of an unrestricted demand function can also provide information about the quality of the data used, the validity of the method employed for their calculation and the appropriateness of the adopted modelling framework. For \( w_i \) we use the corrected-for-terms-of-trade series of per-capita GDP by PWT and for \( G_i \) we use the series by SIPRI. Under definitions (1) and (2) of SIPRI and ACDA measurements in the previous chapter, which ignore measurement errors, we can estimate (23) using the cross-sectional data set we described in chapter 3. The estimation of (23) by OLS, which implies the assumption of a well-behaved and normal error term \( u_i \), yields

\[
\ln M_i = 4.65 + 0.44 \ln G_i - 0.285 \ln w_i - 0.51 \ln p_i
\]

\[
(1.32) \quad (0.11) \quad (0.176) \quad (0.17)
\]

\[
[0.001] \quad [0.000] \quad [0.114] \quad [0.005]
\]

\[
R^2 = 0.35 \quad R^2 = 0.29 \quad F = 6.17 \cdot [0.002] \quad FF = [0.614] \quad N = [0.144] \quad H = [0.865]
\]

The standard errors of the estimated coefficients are in brackets and the p-values of the two tail tests for the significance of the coefficients from zero are in square brackets. \( R^2 \) is the coefficient of determination of the regression, \( \bar{R}^2 \) is R-bar squared, \( F \) gives the F-test of the regression and the p-value in square brackets. FF, N and H give p-values from LM tests for the functional form (Ramsey), normality and heteroscedasticity (White) respectively. There is no indication of heteroscedasticity or functional misspecification of the linear equation. The Bera-Jarque test, however, moderately indicates non-normality of the residuals, although the hypothesis of normality fails to be rejected at 5% level of significance.
Despite the moderate goodness of fit of this equation there are several encouraging indications in the results. The signs of the estimates of \( e_p \) and \( e_G \) are theoretically correct and the absolute magnitudes appear reasonable. The price elasticity is estimated at -51\% and the 'income elasticity' is 44\%, both with high statistical significance from zero. If we interpret \( e_G \) as a measure of bureaucratic inefficiency \( a_0 \), the estimate is rather low. Seen as a cross-price elasticity of arms imports to the price of labour \( w \), the estimate of \( e_w \) indicates that the two goods are complements in the production. According to the model in Section 4.2, which is based on a CD production function of military capability, we expect this elasticity to be near zero, but this changes if we consider a production process with a non-unitary elasticity of substitution. Furthermore, this elasticity may also contain substitution effects between imported and domestic arms. For these reasons, the lower statistical significance of this coefficient from zero is not a major nuisance.

The medium level of the coefficient of determination is an expected result, rather than an indication of a poor estimation, mainly for two reasons. One is that the random component of the estimated relationship can be expected to be high, as a result of the specific nature of the arms trade. This may be caused by variations created by the discontinuity of the flow of large deliveries in time, the unpredictability of political factors etc. Apart from purely stochastic variations, the random term of our equation is expected to contain deterministic but unexplained variations of the dependent variable. Examples of such omitted systematic processes are dynamics, simultaneity, endogenous growth effects, the state of domestic production, open economy effects, market structure effects etc. All these systematic but unexplained effects are left to be included in the error term, unavoidably creating expectations of some under-specification bias in the results.

The variables included in the estimated function are those that performed better among the alternatives. The non-nested tests for the use of alternative measurements of the explanatory variables showed that PPP-measurement of GDP performed significantly better than the transformation with exchange rates, a result also reported in Smith et al [1997b]. SIPRI figures of military expenditure performed significantly better than ACDA. This result was similar for all the attempted functional forms and supported with high statistical significance from both sides of the non-nested tests. These tests weakly indicated in favour of the corrected for the terms of trade version instead of other versions of GDP available in PWT. The alternatives we examined included
transformations by Laspeyer and Chain indices and a measurement corrected per equivalent adult (i.e. approximated correction for working force).

The leverage values of the regression, i.e. the elements of the main diagonal of the hat matrix of the regression \( (X'X)^{-1}X' \) show that only two observations can be characterised as influential. For this result we use the criterion of twice the average leverage value (Kmenta [1990] p. 425 or Greene [1993] p. 288). They are the cases of USA and Syria that give values 0.35 and 0.29 respectively, which are well above the critical value \( 2k/n = 0.21 \). Because, however, the values of the respective residuals are small compared to the rest, we concluded that the estimation benefits, rather than gets biased, from the inclusion of these two observations. Thus, the observations are not characterised as outliers and they are retained in the sample, instead of being removed. In practice their removal resulted in an apparent deterioration of the statistics of the estimation.

4.3.2 Examination of the functional form

If ACDA values are used instead of SIPRI as left hand side variable in terms of true (unobserved) magnitudes the equation to estimate becomes

\[
\ln p_M = a + \epsilon_G \ln G_i + \epsilon_w \ln w_i + (\epsilon_p + 1) \ln p_i + u_i
\]

The estimation of this equation, with the use of data, yields identical estimates and almost identical t-ratios with the estimation of (23). The rest of statistical tests, however differ. Interesting is the change of Ramsey LM test \( (\chi^2_1 = 3.998, \text{p-value} = 0.046) \), which indicated in favour of a quadratic functional form at 5% level of significance since there are theoretical reasons to support non-linearity in the equation.

From a closer examination, the estimation of the expanded quadratic form does not provide a clear message about the exact nature of non-linearity. Most significant among the quadratic terms turn out to be the two cross-products rather than squares of the regressors, which directly determine concavity or convexity. Looking for a theoretical explanation of this result, a standard framework that provides a clear connection with a quadratic logarithmic form is the 'translog' model (e.g. Greene [1993] p. 209). An interesting application of this methodology in defence economics can be found in Okamura [1991]. This method, however, is not appropriate for our case since the identification of translog model requires measurements of the output and of all inputs
and their prices. In our case, data on quantities and prices of the inputs are limited and of questionable quality and, most important, the output of defence industry is unobservable.

The establishment of a domestic industry of major arms can be expected to require considerable set-up costs, mainly due to R&D requirements of the related technology. High set-up costs in the domestic production tend to increase the price of domestically produced arms, more when the produced quantity reduces towards zero. This effect is illustrated by the analysis in section 4.2, i.e. equation (16). Countries, however, may choose not to establish a domestic production at all and use only imported arms. Such a discontinuous entry could not possibly be captured by a production function with a unitary elasticity of substitution.

Providing that the set-up costs required for a production of arms are generally high, the size of the production becomes the critical factor for the establishment of a domestic industry. This draws the attention on the association between the size of defence industry and the economic size of the countries. A simple observation of the shares of military expenditure to GDP, e.g. from ACDA [1995], shows that it is near to 2.5% in the majority of the countries. Using the available data, the correlation coefficient between GDP and military expenditure is 90% using SIPRI and 93% using ACDA data, which shows that these two magnitudes tend to move together. The correlation coefficient of GDP with the share of military expenditure to GDP it is -62% and -53% using SIPRI and ACDA data respectively, showing that this share tends to decrease with the size of the economy.

Countries' demand for domestically produced arms depends on the size of the defence industry (economy), which is reflected in the size of the military budget. The decision of countries for the establishment of a costly large-scale domestic industry depends on the size of their military budget and their demand for the product and, possibly, on the level of economic development. Providing that the set-up costs are high, when the military budget is generally small it is unlikely that countries find this establishment beneficial or affordable and they rely upon imports for their procurement. When the demand increases, the impact of set-up costs to the price of the output decreases and the option of entry to the domestic production becomes realistic. If the entry takes place domestic arms substitute for part of imported arms. If the demand becomes very large, with the possible advantage of positive economies of scale and the possibility of benefits from exports, the domestic industry can further substitute
imported arms with domestic. Large and developed economies are likely to enjoy positive externalities in the establishment of a domestic arms sector. Such externalities that essentially reduce production costs may originate from double use of an existing industrial infrastructure (eg nuclear plants) and of investment in R&D, advantages in human capital stocks etc.

**Figure 4.1 Share of arms imports to military expenditure against GDP**

A comparison of the share of the expenditure in imported arms with the economic size of countries provides support to the estimated concave relationship between the demand for imported arms and level of development. Figure 4.1 plots the logarithm of the ratio of arms imports to military expenditure against the logarithm of total GDP, using both SIPRI and ACDA figures. It also includes quadratic trends fitted for the two cases. The two trends are supported by high statistical significance, especially the quadratic component. As a result of the high correlation of GDP with military expenditure we can expect a similar concave relationship between the logarithms of arms imports and GDP. Since the share of arms imports to military expenditure, as it is plotted in figure 4.1, is also in logarithms the non-linear nature of the relationship of this share with the military expenditure is maintained [i.e. $d^2\ln(M/G)/d(\ln G)^2 = d^2\ln M/d\ln G^2$ since $\ln(M/G) = \ln M - \ln G$]. Taking in account the high positive correlation between GDP with military expenditure we could attribute the concavity between $M/G$ and GDP.
to concavity between arms imports and military expenditure. This, however, does not imply that the component of GDP not correlated with military expenditure can not introduce some irrelevant and diversified type of association.

The model in Section 4.2 predicts the demand for arms imports as being inelastic to labour costs. A possible explanation is that the prices of two from the three inputs (of $D$ and $L_K$) depend on labour costs $w$ and the substitution effect of this price is strong enough to fully offset the opposite in direction income effect. This, for instance, happens when we have unitary elasticity of substitution. In the case of a higher elasticity of substitution, which actually appears more realistic, the substitution effect becomes stronger and dominates the income effect. Then, the cross-price elasticity $e_{w}$ can be expected to take positive values compared with the result predicted by (19). This relationship between the elasticity of substitution and the sign of the cross-price elasticity is illustrated in Figure 4.2. In the case of CD production function ($\sigma = 1$) the cross-price income effect $AB$ and the substitution effect $BC$ completely offset each other, whereas in the elastic case ($\sigma > 1$) and in the inelastic case ($\sigma < 1$) substitution and income effects dominate respectively.

**Figure 4.2. Different types of substitution between inputs**

![Figure 4.2](image)

Summarising, we have shown that the final effect of $w$ on arms imports is determined by the combination of a (possibly) increasing effect from substitution with the decreasing effect of the entry of countries to domestic production. If the order of the second effect is higher than that of the first, the result of the combination of these two effects is a relationship of a concave shape, with an increasing part, a maximum and a decreasing part. It is possible that in reality not all these three parts of this pure
concavity will exist, but only the increasing or the decreasing. We come back to this in
the following section.

4.3.3 A quadratic-logarithmic demand function

The hypothesis that the arms imports have concave responses to military expenditure
and the domestic wage can be tested with the estimation of an equation that includes the
squares of these variables.

\[ \ln M_i = a_0 + a_1 \ln G_i + a_2 \ln w_i + a_3 \ln p_i + a_4 \ln^2 G_i + a_5 \ln^2 w_i + u_i \]

where \( a_j, j = 1, \ldots, 5 \) are coefficients. Estimating this equation with OLS and the same data
set we obtain

\[
\begin{align*}
\ln M_i &= -21.64 + 1.52 \ln G_i + 4.74 \ln w_i - 0.56 \ln p_i - 0.06 \ln^2 G_i - 0.29 \ln^2 w_i \\
& \quad (18.44) \quad (0.71) \quad (4.20) \quad (0.17) \quad (0.04) \quad (0.24) \\
& \quad [0.25] \quad [0.04] \quad [0.27] \quad [0.003] \quad [0.16] \quad [0.24] \\
R^2 &= 0.42 \quad R^2 = 0.33 \quad F = 4.66 \quad [0.003] \quad FF = 0.18 \quad N = 0.081 \quad H = 0.49
\end{align*}
\]

These results appear to be in line with the theoretical expectations, as they were
exposed above, including the signs and magnitudes of the estimates. The goodness of fit
has improved and the overall appearance of the t-ratios is satisfactory. The leverage tests
are similar as in the previous estimation except that of the U.S. is about seven times
higher than the average value. Since the corresponding residual is very small the
observation is maintained in the sample. The normality of the residuals appears to have
deteriorated, compared with the previous estimation, but the test still fails to reject
normality at 5% level of significance. The distribution of the residuals appears to be
skewed to the right.

The response of arms imports to changes of the average labour costs is concave and
the slope now depends on \( w_i \). This is in agreement with the theoretical expectations
presented in the previous section. Similarly, the estimated income elasticity of the
demand function now depends on \( G \). The estimates of these two functions are

\[ \varepsilon_w = \frac{\partial \ln M_i}{\partial \ln w_i} = 4.74 - 0.58 \ln w \quad \text{and} \quad \varepsilon_G = \frac{\partial \ln M_i}{\partial \ln G_i} = 1.52 - 0.12 \ln G \]

and evaluated at the sample means of the two variables in logarithms, \( \bar{w} = \sum \ln w_i/n = 8.898 \) and \( \bar{G} = \sum \ln G_i/n = 8.356 \) it is

43
\[ \hat{\varepsilon}_w = -0.47 \quad \text{and} \quad \hat{\varepsilon}_G = 0.57 \]
\[ (0.23) \quad (0.13) \]
\[ [0.049] \quad [0.000] \]

We can test the significance of these point-estimates of the elasticities from zero with the use of t-tests, imposing the linear functions in \( a_2 + 2a_5 = 0 \) and \( a_1 + 2a_4 = 0 \) as restrictions on the estimation of (25). The two estimated standard errors of the restrictions are in brackets and the p-values from the tests of the significance of the elasticities from zero are in square brackets. The estimate of \( \hat{\varepsilon}_G \), evaluated at the mean, has a high significance from zero and a higher value, compared with the estimation of (23). The price elasticity is at the same level but lower in statistical significance. The estimate of \( \varepsilon_w \) has changed more than the other elasticities, increased in value and now passes a 5% test.

The signs of all three estimates of the elasticities are similar as in the estimation of (23) and the estimates are higher in absolute value. The largest increase is that of the estimate of the cross price elasticity \( \hat{\varepsilon}_w \). The estimate of \( \varepsilon_w \) has changed more than the other elasticities and now passes a 5% test of significance but the significance, of price elasticity has somewhat deteriorated. The change in the estimates can be the result of reducing the under-specification bias, compared with the estimation of (25). The coefficients of the two quadratic terms are negative showing that arms imports are expected to have a concave relationship with both \( w \) and \( G \).

Taking in account the actual range of the explanatory variables we may look in more detail into the shape of the estimated responses. The two diagrams in Figure 4.3 present the predicted demand for arms imports as well as the estimated elasticities, as functions of \( G \) and \( w \). The fitted values of \( \ln M \) are calculated at the means of the variables. In the diagram provides the minima \( \ln w_{\min} \) and \( \ln G_{\min} \), maxima \( \ln w_{\max} \) and \( \ln G_{\max} \) and means (bar) from the samples of the two variables, as well as the predicted values of the two variables that maximise these functions \( \ln w^* \) and \( \ln G^* \). To solve for the last two values we can solve from the first order conditions for the maximisation of (27). The estimated maximising values of \( w \) and \( G \) are

\[ \ln w^* = 8.17 \quad \text{and} \quad \ln G^* = 12.66 \]
Following to our results, the response of arms imports to per-capita GDP is purely concave with an increasing and a decreasing section and a maximum in between. This peak is included in the actual range of values in the sample. The two sections are relatively balanced having the peak near to the sample mean. This sample, however, is not representative of the entire population of countries, since it contains more countries with high income than with low and almost excludes countries at the lowest levels of income. This biasedness in the sample happens because economic data are less available for LDCs and if they exist, the quality would probably be poor. This could be the reason why the elasticity evaluated at the mean is on the side of the negative slope. Generally, the estimated relationship appears in agreement with the theoretical pattern presented in the previous section.

According to the analysis of the previous section, development has a twin effect on the arms imports, one through labour costs and another through the capacity to establish the industry. The assumption that, apart from increasing the labour costs, development facilitates the establishment of a domestic industry and tends to push arms imports
downwards seems relatively safe. As section 4.2 illustrated, a unitary elasticity of substitution in the production of defence implies inelastic responses of arms imports to labour costs. If this were the case, because of the negative impact of development to the arms imports through set-up costs, the slope of this function should be negative. Only with a higher than unit elasticity of substitution the positive substitution effect of \( w \) to \( M \) can dominate the negative income effect and yield a positive elasticity. Since \( e_w \) is estimated to be first positive and then negative when \( w \) increases, this could be an indication of a higher than unit elasticity of substitution in the production of \( K \).

4.3.4 Further empirical results

The previous section examines a specification mainly selected for its clear theoretical interpretation, rather than for having the best fit. Looking for further empirical information about the functional form of the underlying demand function, we might also examine alternative functional forms mainly selected for their acceptance by the data. Applying a standard specification method the quadratic form with improved fit is

\[
\ln M_i = b_0 + b_1 \ln G_i + b_2 \ln w_i + b_3 \ln p_i + b_4 \ln^3 w_i + \nonumber \\
+ b_5 \ln G_i \ln w_i + b_6 \ln w_i \ln p_i + b_7 \ln G_i \ln p_i + u_i
\]

where \( b_i \)'s are coefficients. The estimation of (26) yielded

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>( b_0 )</th>
<th>( b_1 )</th>
<th>( b_2 )</th>
<th>( b_3 )</th>
<th>( b_4 )</th>
<th>( b_5 )</th>
<th>( b_6 )</th>
<th>( b_7 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>-37.08</td>
<td>2.37</td>
<td>7.39</td>
<td>-3.81</td>
<td>-0.35</td>
<td>-0.19</td>
<td>0.59</td>
<td>-0.24</td>
</tr>
<tr>
<td>Standard error</td>
<td>21.0</td>
<td>1.34</td>
<td>4.3</td>
<td>2.3</td>
<td>0.24</td>
<td>0.15</td>
<td>0.31</td>
<td>0.12</td>
</tr>
<tr>
<td>p-value</td>
<td>0.09</td>
<td>0.09</td>
<td>0.096</td>
<td>0.11</td>
<td>0.16</td>
<td>0.19</td>
<td>0.06</td>
<td>0.05</td>
</tr>
</tbody>
</table>

\( R^2 = 0.50, \quad R^2_* = 0.38, \quad F = 4.3 [0.002], \quad FF = 0.50, \quad N = 0.062, \quad H = 0.43 \)

The overall appearance of the statistics from the estimation of this form has improved, compared with the estimation of (25). The exception is the normality of the residuals, the distribution of which now appears to be skewed to the right. While the normality of the error term does not affect the properties of OLS estimates it mainly affects the statistical tests and confidence intervals, which rely upon the assumption of a normal error term. We may overcome this problem by examining empirical distributions of the estimates using Bootstrap methods. The leverage tests indicated the same influential observations as in the previous estimation, again with small
corresponding residuals that indicate the inclusion of two observations in the sample
does not bias but improves the estimation. In a similar way as in the estimation of the
previous section the three elasticities now become

\[
\begin{align*}
\hat{\varepsilon}_\omega &= \frac{\partial \ln M}{\partial \ln w} = 7.39 - 0.7 \ln w - 0.19 \ln G + 0.59 \ln p \\
\hat{\varepsilon}_\sigma &= \frac{\partial \ln M}{\partial \ln G} = 2.37 - 0.19 \ln w - 0.24 \ln p \\
\hat{\varepsilon}_\rho &= \frac{\partial \ln M}{\partial \ln G} = -3.81 + 0.59 \ln w - 0.24 \ln G
\end{align*}
\]

and the elasticities evaluated at the sample means

\[
\hat{\varepsilon}_\omega = -0.49, \quad \hat{\varepsilon}_\sigma = 0.52 \quad \text{and} \quad \hat{\varepsilon}_\rho = -0.57
\]

The estimate of \( \varepsilon_\sigma \) from (26) is little changed from the estimation of (23), whereas the
two other elasticities are higher in absolute value. From the statistics of this estimation
some coefficients have low statistical significance and the deterioration of \( R^2 \) indicates
an over-specified equation.

An alternative form we examined in the contents of our search for additional
information included countries’ population in levels along with a quadratic logarithmic
specification. After eliminating the terms with insignificant statistical tests we arrived at
the hybrid model

\[
(27) \quad \ln M_i = b_0 + b_1 \ln G_i + b_2 \ln w_i + b_3 \ln p_i + b_4 \ln ^2 G_i + \\
+ b_5 \ln G_i \ln w_i + b_6 \ln w_i \ln p_i + b_7 \ln G_i \ln p_i + b_8 \pi_i + u_{i}''
\]

where \( b_i \)'s are coefficients. The estimation of (29) yielded

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>( b_0 )</th>
<th>( b_1 )</th>
<th>( b_2 )</th>
<th>( b_3 )</th>
<th>( b_4 )</th>
<th>( b_5 )</th>
<th>( b_6 )</th>
<th>( b_7 )</th>
<th>( b_8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>-30.22</td>
<td>3.84</td>
<td>4.61</td>
<td>-3.71</td>
<td>-0.15</td>
<td>-0.64</td>
<td>0.69</td>
<td>-0.36</td>
<td>-0.002</td>
</tr>
<tr>
<td>Standard Error</td>
<td>12.69</td>
<td>1.37</td>
<td>1.80</td>
<td>2.11</td>
<td>0.08</td>
<td>0.22</td>
<td>0.30</td>
<td>0.15</td>
<td>0.00075</td>
</tr>
<tr>
<td>p-value</td>
<td>0.024</td>
<td>0.009</td>
<td>0.016</td>
<td>0.089</td>
<td>0.074</td>
<td>0.008</td>
<td>0.029</td>
<td>0.021</td>
<td>0.013</td>
</tr>
</tbody>
</table>

\( R^2 = 0.58 \quad R^2 = 0.47 \quad F = 5.07 \quad [0.001] \quad FF = 0.20 \quad N = 0.001 \quad H = 0.97 \)

The goodness-of-fit \( R^2 \) of this specification is high, just one percent lower from that
of the an unrestricted quadratic form, where \( R^2 \) takes the highest value from all the
specifications tested. The normality of the residuals safely fails the tests, the distribution
of which is clearly skewed to the right. The tests for heteroscedasticity and functional
form give satisfactory results. The overall appearance of the t-ratios of the coefficients
seems satisfactory with all the coefficients passing a 10% level of significance test and
only two of them fail a 5% test. There are seven observations indicated as influential from the leverage tests, but all of them with clearly small corresponding residuals, so they were all retained in the sample.

The three elasticities now become

\[
\hat{\varepsilon}_w = \frac{\partial \ln M}{\partial \ln w} = 4.61 - 0.64 \ln G + 0.69 \ln p \\
\hat{\varepsilon}_G = \frac{\partial \ln M}{\partial \ln G} = 3.84 + 0.31 \ln G - 0.64 \ln w - 0.36 \ln p \\
\hat{\varepsilon}_p = \frac{\partial \ln M}{\partial \ln p} = -3.71 + 0.69 \ln w - 0.36 \ln G
\]

and evaluated at the sample means of logarithmic variables \( \bar{w} = 8.898 \) and \( \bar{G} = 8.356 \)

\[
\hat{\varepsilon}_w = -1.15 \quad \hat{\varepsilon}_G = 0.78 \quad \hat{\varepsilon}_p = -0.61
\]

\[
(0.31) \quad (0.14) \quad (0.15)
\]

\[
[0.001] \quad [0.000] \quad [0.000]
\]

All three estimates of the elasticities evaluated at the sample means and their significance from zero rose in absolute value compared to the previous estimations. In a similar way as before we can solve for the maximum of the only non-linear response between \( M \) and \( G \), using the FOC from (27). This is

\[
G^* = -b_1/b_4 - (b_5/b_4) \ln w - (b_7/b_4) \ln p = 25.6 - 4.27 \ln w - 2.4 \ln p
\]

The second order condition confirms that this is a maximum. According to this set-up the response of \( M \) to the other two variables is linear. This form appears to suggest that the higher the size of countries’ military industry (extent of militarisation) the less they tend to rely on imports. According to this functional form, population and per-capita GDP provide separately the two ingredients of GDP. military expenditure includes information about the product of these two variables (total GDP), as it is highly correlated with it. Furthermore, it contains information orthogonal to GDP (the non-correlated part), which is the intensity of militarisation of the country. It appears reasonable that all these three factors, the militarisation, the size and the level of development of the country affect the decision for an establishment of a domestic industry of arms. Then, the character of the countries tends to change from being mainly importers of arms towards higher self-sufficiency of procurement, and towards the exports of arms.
4.4 The impact of measurement errors

4.4.1 The problem of latent variables

This section examines the relaxation of one of the working assumptions, that there are no measurement errors in the data used in the estimations. On the contrary, we expect considerable errors in defence related data. The impact of these errors would be possible bias in the estimation of the coefficients and the calculation of statistical tests, which may influence the course of development of this analysis and the correctness of the conclusions. Among general textbooks on econometrics a helpful reference to this issue is Maddala [1977]. Greene [1993] discusses the problem of identification of the unobserved magnitudes, for the case of a two-variable regression and mentions the existence of implications for the multivariable case, but does not cover the case of two explanatory variables measured with error. A more extensive presentation of this issue can be found in Judge et al [1985].

The aim of this examination is to obtain information about the direction and the magnitude of bias in the estimates. The rather good overall appearance of the estimated demand functions and the high statistical significance of the estimate of price elasticity can be used as a confirmation that the adopted method to calculate prices has been successful. This conclusion gets further support if the direction of bias in the estimated elasticities is downwards in absolute value. This implies evidence for even more responsive relationships than those estimated. Furthermore, the indication of downwards bias implies stronger statistical significance of the coefficients from zero. And vice-versa: the closer the true elasticities are to zero the weaker the statistical support for the estimated relationship gets.

In a similar way as in Baumol [1986] and De Long [1988], any measurement error in SIPRI values would appear in both sides of the estimated function making the error term correlated with this regressor. As Levi has shown and Maddala illustrates [1977 pp. 292-4], in the case of two variable regression the estimation bias can be bounded by estimating the inverse function. The direction of bias is always downwards in absolute terms. Similarly, when the regressors are more than one, and only one variable is measured with error, the bias of this variable's estimated coefficient is downwards and a boundary can be calculated. The bias to the other error-free variables, nevertheless, can be either upwards or downwards, and it can be estimated. The same method does not
directly apply when more than one explanatory variables are measured with error. For the case of two variables, however, Theil has proposed an approximate calculation of the bias. In the following sections the impact of measurement errors on the estimations will be examined using alternative approaches.

4.4.2 One variable measured with error

4.4.2.1 The inverse equation approach

Starting this examination from a simple case the definition of SIPRI and ACDA measurements, shown in forms (1) and (2) of chapter 3, we can include multiplicative lognormal error terms with unit means and constant variances. If $e_{Si}$ and $e_{Ai}$ are the logarithms of these terms, the definition of SIPRI measurement can be written as

\begin{equation}
\ln S_i = \ln M_i + e_{Si}, \quad e_{Si} \sim N(0, \sigma^2_S)
\end{equation}

and of ACDA

\begin{equation}
\ln A_i = \ln p_i + \ln M_i + e_{Ai}, \quad e_{Ai} \sim N(0, \sigma^2_A)
\end{equation}

Then, the estimated price index becomes

\begin{equation}
p_i = \ln A_i - \ln S_i = \ln p_i + e_{pi}
\end{equation}

where

\begin{equation}
e_{pi} = e_{Ai} - e_{Si}
\end{equation}

Under this definition of $p_i$ we can re-write (25) as

\begin{equation}
\ln S_i = a + e_{G} \ln G_i + \epsilon_w \ln w_i + e_p \ln p_i + v_i
\end{equation}

where the composite, but still normal, error term $v_i$ is

\begin{equation}
v_i = u_i - e_{Ai} + (1 - e_p) e_{Si}
\end{equation}

In the case that the error terms $e_{Si}$, $e_{Ai}$ and $u_i$ distribute independently from each other and from $p_i$ and $M_i$ we have

\begin{equation}
cov(\ln p_i, v_i) = (e_p - 1) \sigma^2_S
\end{equation}

Always assuming only $\hat{p}_i$ is measured with error, the bias in the estimation of $e_p$ can be bounded by estimating the function that the LHS and the explanatory variable with the error has been inverted. It is interesting that when $e_p = 1$ the bias does not exist, or it becomes negligible, since in this case $cov(\ln \hat{p}_i, v_i) = 0$ and the error terms is uncorrelated with $p$. Indeed, the estimates of $e_p$ are not far from zero, especially in the estimations of the extensive forms, i.e. (26) and (27).
Although the method of inverse function is exact only in the case of two variable regression, an approximate estimation can be obtained from

\[
\ln p_i = a_0 + a_1 \ln G_i + a_2 \ln w_i + (1/\varepsilon_p') \ln S_i + \nu_i
\]

where \(a\)'s are coefficients and \(\varepsilon_p\)' the other estimated boundary. The interval in which the real price elasticity is expected to lie can be defined by the estimations of (23) and (36), after we solve for \(\varepsilon_p\). This is

\[-0.51 > \varepsilon_p > -2.24\]

This is a rather wide bound but it can be used to support the argument of elastic response of the arms imports to prices and the good performance of the estimated price index. The statistics of this estimation are similar to those from the estimation of (23) including the statistical significance of the estimate (0.5% two tail). If this method is applied to the more extensive form (25), the obtained bounded region is similar and, also, lies within this interval.

### 4.4.2.2 One unobservable regressor in multiple regression

The method of inverse function can provide indications about the direction and the magnitude of bias on the estimated coefficient of the variable with the error. The application of this asymptotic method is exact only in the case of two variable regression, in which the bias is always downwards in absolute value. Levi [1973] has examined the impacts in the case of one (only) regressor measured with error in a multiple regression. Levi shows that the direction of the bias of the coefficient of the variable with the error is similar as in the two variable case: *With only one variable measured with error, in multiple regression, the coefficient of the variable is underestimated in absolute terms.*

Levi's analysis, however, shows clearly that there could be bias in the estimates of the coefficients of the other regressors, which are assumed to be correctly measured. The direction of this bias for a regressor depends on the direction of correlation between the regressor with the unobserved variables. If a regressor is positively correlated with the unobserved regressor, the direction of bias is downwards in absolute value. If it is negatively correlated the bias is upwards. This result becomes particularly interesting since the predictions it provide can be totally inverted, if a second variable is assumed to be measured with error. In the present case, the estimated correlation coefficient of the
price index $p_i$ with measurements of military expenditure $G_i$ is negative, with a statistically significant sign (see sub-section 4.4.3). Thus, if $G_i$ (and always $w_i$) are correctly measured $\hat{\delta}_p$ is expected to be upwards biased because of the errors in $p_i$. In the analysis that follows we also relax the assumption that $G_i$ is correctly measured. The outcome outlines the risk from a simplistic statistical approach since the relaxation of an additional assumption inverts the direction (sign) of the estimated bias of $\varepsilon_G$.

### 4.4.3 Two variables measured with error

Moving one step further from the assumption that only one explanatory variable is measured with error we now assume two such variables. A good candidate is SIPRI measurement of military expenditure, first because of the high significance of the estimated coefficient and secondly the expectation that measurement errors can be correlated with this of $p_i$. Using the usual assumptions this can be written as

$$\ln \hat{G}_i = \ln G_i + \varepsilon_{Gi}, \quad e_{Gi} \sim \text{NN}(0, \sigma_G^2)$$

Any possible errors in PWT data on per-capita income are ignored.

For the case of two explanatory variable regression Theil has calculated approximate expressions for the biases when both of them are measured with errors. A possible way to overcome this difficulty could be to examine the case of removing one of the explanatory variables in (23) and obtain estimates of the bias for the remaining variables, hoping that the removal of this variable will not affect largely the results of Theil's analysis. An estimate of the bias from the omission of a regressor in the estimation of the restricted form can be observed and the variance-covariance matrix of the regression is not affected [Stuart p.64]. Then, we may apply results of Theil's analysis to the original estimates. Indeed, with the use of the statistics of the estimation of (23) one could drop the per capita income from the regression, even using a 10% level of significance.

The restricted version of (23) then is

$$\ln Si = a + \varepsilon_G'\ln G_i + \varepsilon_p'\ln p_i + u_i$$

and OLS yields

$$\ln Q = 2.93 + 0.34 \ln G_i - 0.47 \ln p_i$$

(0.094)  (0.17)

$[0.001]$  $[0.010]$  $R^2 = 0.30$  $F = 7.6$  $FF = 0.8$  $N = 0.228$  $H = 0.947$
The change of the statistics is minor and the bias from the omission of the variable is small and towards zero for both the estimates of the elasticities. Taking this as an encouraging indication one can proceed to Theil’s analysis assuming that the bias from under-specification does not affect significantly the calculation of bias from measurement errors for the remaining regressors. A closer examination of this methodology has not been detected in the literature, and might be worthy of a closer examination. If this assumption holds the two results can be combined.

Theil’s asymptotic approximation for a correction of bias from measurement errors are

\[ \hat{\theta}_G - \theta_G = -(1 - \rho^2)^{1/2} (\theta_G \theta_G - \rho \theta_G \theta_p) \]

\[ \hat{\theta}_p - \theta_p = -(1 - \rho^2)^{1/2} (\theta_p \theta_p - \rho \theta_G \theta_p) \]

where \( \rho = \text{cov}(\ln G_i, \ln p_j)[\text{var}(\ln G_i)\text{var}(\ln p_j)]^{-1/2} \) is the correlation coefficient between the true values of the two variables and \( \theta_G, \theta_p \) are the ratios of the error variances to the variances of the respective true values. Since the estimated signs of the two elasticities, i.e. \( \epsilon_G > 0 \) and \( \epsilon_p < 0 \), are strongly supported both by the obtained empirical evidence and the underlying theory the direction of bias solely depends on \( \rho \). Whatever the value of the unobserved but always positive ratios \( \theta_G \) and \( \theta_p \) and given the necessary for OLS assumption that \( |\rho| = 1 \), the only condition that the bias is downwards in absolute value for both estimates is

\[ \rho > \max \left( \frac{\theta_G \epsilon_G}{\theta_G \epsilon_p}, \frac{\theta_p \epsilon_p}{\theta_G \epsilon_G} \right) \]

In this case (39) is negatively and (40) positively valued. Since both quantities in the RHS of (41) are non-positive one can notice that if \( \rho \) is positive the direction of bias is always towards zero for both variables. The power of this desirable for the support of the explanatory power of the estimated equation result, has a feedback effect, since the indications about the bias adds to the statistic significance of the elasticities from zero and so on.

The step that naturally follows is to obtain information about the value, or at least the sign of \( \rho \). From the data, the estimated value is \( \hat{\rho} = 0.398 \) and, if possible errors are ignored, a hypothesis that \( \rho \leq 0 \) can safely be rejected at 1% level of significance using a t-test (one tail p-value is 0.7%). Furthermore, if measurement errors are taken in
account, \( \rho \) is most likely underestimated, along with the significance of the test. This becomes apparent if we expand the numerator and denominator of the calculation of \( \rho \), using covariance and variance rules. In terms of sample measurements (denoted with an s) it is

\[
\hat{\rho} = \frac{s\text{cov}(\ln G_i, \ln p_i) + s\text{cov}(\ln G_i, e_{p_i}) + s\text{cov}(e_{G_i}, \ln p_i) + s\text{cov}(e_{G_i}, e_{p_i})}{\sqrt{s\text{var}(\ln G_i) + s\text{var}(e_{p_i}) + 2s\text{cov}(\ln G_i, e_{p_i}) + s\text{var}(\ln p_i) + 2s\text{cov}(\ln p_i, e_{p_i})}}
\]

Looking into the large sample properties of \( \rho \) they depend on the assumptions about the errors. If the two errors distribute independently from each other and from the true magnitudes \( G_i \) and \( p_i \), it is \( \text{plim} \text{cov}(e_{G_i}, e_{p_i}) = \text{cov}(e_{G_i}, e_{p_i}) = 0 \), \( \text{plim} \text{cov}(\ln G_i, e_{p_i}) = 0 \) and \( \text{plim} \text{cov}(e_{G_i}, \ln p_i) = 0 \). Then we can write

\[
\text{plim} \hat{\rho} = \frac{\text{cov}(\ln G_i, \ln p_i)}{\sqrt{\text{var}(\ln G_i) + 2\sigma^2_{e_G} \sqrt{\text{var}(\ln p_i) + 2\sigma^2_p}}}
\]

If the two error terms are not independent this result does not necessarily hold. In this case a non-zero term \( \text{cov}(e_{G_i}, e_{p_i}) \) appears in the numerator of (43) reducing the underestimation of \( \rho \). From an algebraic observation of (42) we may notice that only with very weakly correlated true magnitudes (small \( \rho \) near zero) and strongly correlated errors the increase in the numerator could possibly offset the increase of the denominator. A similar result holds if the error terms are also correlated with the true magnitudes where all the terms in (42) remain, after taking probability limits. In general, over-estimation of \( \rho \) is quite unlikely and might only take place under very strong state of correlation between the variables. Putting this result together with the relatively high value of the estimate \( \hat{\rho} \) and its high statistical significance from zero we may conclude that a negative value of \( \rho \) and, therefore, overestimation of the two elasticities (in absolute value) is most unlikely.

For a calculation of the bias of the two estimates, except a value for \( \rho \), we need estimates for coefficients \( \theta_G \) and \( \theta_p \). This information is not directly obtainable, since neither the sources provide it nor a method of calculation is directly apparent. Having examined the direction of the bias and calculated a bound for it we could examine the sensitivity of bias dependent on various values of the parameters. The diagrams in Figure 4.5 present corrected estimates of price elasticity, noted \( \tilde{e}_{p_i} \), for different values of \( \rho, \theta_G \) and \( \theta_p \). Similar corrections in absolute value apply to the estimate of \( e_G \).
The calculation of corrected-for-bias point estimates of the two elasticities requires specific values for all the parameters involved ($\rho$, $\theta_c$ and $\theta_p$). The available data are not adequate for an estimation of these parameters. In other words we have clear indications for the direction of bias but not for its absolute value for both parameters. In the general case that the three coefficients take intermediate values the corrected estimates for both elasticities are close to unity. This result is consistent with the calculations of the inverse equation approach, which bounded $e_p$ between $-0.51$ and $-2.24$. Thus, assuming moderate values $\rho = 0.5$ and $\theta_c = \theta_p = 0.35$ and the initial estimates of the two elasticities from (25) the two corrected estimates take values near unity, i.e.

$$\tilde{e}_p = -0.95 \text{ and } \tilde{e}_G = 0.97$$

4.5 Conclusions

The econometric results of this chapter appear to provide some considerable evidence that the demand of countries for internationally traded arms generally follows patterns predicted by the theory. It is interesting that the analysis of measurement errors enforces the argument of a responsive relationship of the demand to the price and the military budget, instead of blurring the clarity of the results. This responsiveness on its own turn provides support to the underling theoretical assumptions about the production technology of military capability, with substitution between imported, domestic arms and armed forces and the near-to-rational behaviour of the governments.
The good overall appearance of the estimated demand functions and in particular the strong statistical support for the inclusion of the calculated price index, in all estimated forms, indicates for the usefulness of the index per se. In other words, the assumption about the two pricing methods SIPRI and ACDA used to produce aggregate figures of arms imports, seems to be confirmed since the implied price index has been well accepted by the data. This information can be utilised in other quantitative studies on arms trade, possibly more sophisticated than this one, which has been designed as a general first approach. In this study, the selection of data is carefully based on the quality and the exact identity of the included information and contributed to the quality of the estimates. Thus, an interesting qualitative conclusion regarding the data can be drawn ex-post from the performance or the combined econometric and modelling work that was based on them. In other words, the rather encouraging econometric results also signals information about the quality of data.

There are several ways that this study can continue. One is to estimate a demand system of equations for the three inputs to defence based on the presented model and providing that information on domestic production becomes available. Another is to introduce time, if adequate observations are available and proceed with panel data estimations on a static framework or an investigation the dynamic properties of the arms trade, and a possible combination of the two concepts. The examination of the impact of measurement errors can be developed further; first by examining the implications of under-specification to the calculation of bias and second by developing bootstrap analysis. From a modelling point of view, the issue of the entry of domestic producers needs more attention. This issue is further discussed in chapters 6 and 7.
5. Regional variations of the demand for arms: a study of panel data

5.1 Introduction

In chapter 4 we examine the demand for imported arms using a cross-sectional data set of a relatively wide sample of 38 countries. This cross-sectional study, however, ignores possible evolution of the demand patterns over time and limits the structure of the demand to be the same for all countries. These restrictions open the way for heterogeneity bias in the results. Furthermore, the sample of countries in the cross-section data set does not represent countries from different areas of the world evenly, but includes relatively more developed than developing countries. This uneven representation of countries makes possible the existence of sampling bias in the results. This chapter attempts to overcome these limitations by time and longitudinal dimensions in the examination. In terms of theoretical background, this study follows closely the theoretical implementation of the previous chapter.

Since the necessary data for a world-wide examination at country level are not available, this study is based on data aggregated into large geographic regions, at continental or sub-continental level. A practical advantage of this aggregation is that the groups generally include countries with several similarities regarding the important characteristics of national defence, such as regional threat levels, economic development, political status etc. Thus, the outcome of this study refers to the representative or average country of each group, ignoring any differences of the countries within the groups.

The rest of this chapter is organised as follows. Section 5.2 looks at the specifications and the nature of the data set, in addition to the general examination of chapter 3. Section 5.3 examines for heterogeneity of the demand structure across the two dimensions of the panel and reports selected results from statistical tests. Section 5.4 raises issues about model selection and the method of estimation. Section 5.5 presents estimates of the

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1 Early results of this work have been presented in the conference 'Globalisation of European Military Industry and the Arms Trade' at Middlesex University in September 1997 by the author and in ASSA meetings, Chicago, January 1998 by Professor Ron Smith in a review. Later these results were released in more detail in Mouzakis [1999].
demand and its regional variation. Finally, section 5.6 focuses on the characteristics of the evolution of the arms trade over time and presents the estimated time trends. The appendix of this chapter A.5 provides statistical results, tables with the data and other related technical information.

5.2 The data

The data set we use in this study consists of aggregate measurements of arms imports from both SIPRI and ACDA and measurements of total and per capita GNP and Military Expenditure by ACDA. The panel covers 11 annual observations, from 1983 to 1993 and 8 regions. For purposes of comparison we also collected data for 3 international organisations as well as the world aggregate. The 8 regions are Africa, North America, South America, Central America, Asia (aggregate of Southern and Eastern), M. East, Europe and Oceania. The three organisations are NATO, OECD and OPEC. Series from ACDA originate from World Military Expenditures and Arms Transfers (WMEAT) (ACDA [1995]). Series from SIPRI on arms imports originate from SIPRI Yearbooks: years 1981-1984 from Yearbook 1992 (SIPRI [1992]) and years 1985-1993 from Yearbook 1995 (SIPRI [1995]). Per capita GNP is measured in constant 1990 thousands US dollars, where all other values are expressed in million US dollars.

From a careful examination of the origin of the data it comes up that the measurements of per-capita GNP and population series provided by ACDA originate from the World Bank tables. Measurements of arms imports by ACDA and SIPRI are calculated by the two organisations using various sources of information, in an attempt to provide the best possible qualitative improvements. The two organisations, moreover, use different methods to calculate the aggregate figure of imported arms they report. It is this divergence of contents that allowed the calculation of a price index for imported arms, discussed in chapter 3, which provides a more detailed discussion of the definition of the variables. Further discussion of this issue can be found in Levine, Mouzakis & Smith [1996], in Levine, Mouzakis & Smith [1998] and, of course, in the technical appendices of ACDA and SIPRI publications (especially in SIPRI [1995b]).

The grouping of counties in regions and organisations we use is this of ACDA. With the two organisations using similar methods to measure military expenditure, it is more convenient to use ACDA measurement since the coverage corresponds with the observations on arms imports. For similar reasons we used GNP transformed to US
dollars with IMF exchange rates instead of Purchasing Power Parities measurements used in chapter 4. This series is provided by ACDA, in the same tables. According to chapter 4 the alternative use of the two variables made little difference in the estimations. Since the use of SIPRI measurements of the arms trade is unavoidable, however, it is fortunate that both organisations use similar country groups. The only exception that the contents of a group slightly differ is the case of ‘Asia’ that ACDA provides the sub-aggregates ‘East Asia’ and ‘South Asia,’ which we added up to match the aggregate by SIPRI in the best possible way. The unavailable observation by SIPRI for Central America for 1992 has been replaced by linear interpolation of years 1991 and 1993. Last, constant values of ACDA of 1993 were transformed to base year 1990 using the IMF deflator of US dollar (=1.091).

The diagrams in figure 3.2 in chapter 3 present the evolution of the value of imported arms for a number of aggregates, using data from the panel we examine in this chapter. From a simple observation of the data series the volume of the arms trade and military expenditure generally follow a double trended pattern, with an evident declining tendency after the end of the Cold War. Figure 3.3 presents the evolution of the estimated prices. A general pattern of increases before the end of Cold War, and decreases after, appears to dominate the evolutions of all three magnitudes.

5.3 Examination for structural heterogeneity

A standard way to begin a study of panel data is to examine for structural homogeneity of a theoretically suggested model across the two dimensions of the panel, i.e. across time and longitudinal dimensions. In the case that the tests indicate against structural homogeneity the model is finally estimated on pooled data, otherwise the estimation must take account of the type of variation suggested by the data. Structural variation can either be captured by a variable intercept or by a more general variable coefficient model. Heterogeneity of any of these two types may occur across time or longitudinal dimensions or both. About the kind of structural variation we need to choose between fixed and random kinds of variation. All these combinations of different types of heterogeneity add up to several different cases and require a careful selection of the method of estimation, based on the results of appropriate statistical tests.

In the case of total homogeneity over both time and regions the standard log-linear demand function for imported arms has the form
\( q_{it} = a + b_1 m_{it} + b_2 g_{it} + b_3 p_{it} + e_{it} \)

where \( q_{it} \) is the logarithm of SIPRI volume of imported arms of region \( i \) for year \( t \), \( m_{it} \) is the logarithm of ACDA military expenditure, \( p_{it} \) is the logarithm of the implicit price index for imported arms and \( g_{it} \) is the logarithm of per-capita GNP by ACDA. Coefficient \( a \) is an intercept term, \( b \)'s are slope coefficients and \( e \) is an error component. The implicit price index is the ratio of SIPRI to ACDA measurement of arms imports for every observation. The preliminary examination for variation of the model structure over regions and time includes examination for variability of either intercept term, or all coefficients (variability of slopes with invariable intercept is rarely a meaningful assumption). If there is evidence for variability, the question that naturally follows is about the kind of variability, i.e. if the nature of the variation is fixed or random.

We may begin testing for variability of the intercept term. First, we assume homogeneity over time and test the hypothesis of variable intercepts over regions \( H_2 \)

\( q_{it} = a + a_i + b_1 m_{it} + b_2 g_{it} + b_3 p_{it} + e_{it} \) where \( \sum_i a_i = 0 \)

against the hypothesis of overall homogeneity \( H_1 \), using a fixed effects model estimated by OLS. Continuing, we can examine for variability of the intercept over time \( H_3 \)

\( q_{it} = a + \gamma_i + b_1 m_{it} + b_2 g_{it} + b_3 p_{it} + e_{it} \) where \( \sum_i \gamma_i = 0 \)

Finally, we may test for the hypothesis of both variations in a two-factor fixed effects model \( H_4 \)

\( q_{it} = a + a_i + \gamma_i + b_1 m_{it} + b_2 g_{it} + b_3 p_{it} + e_{it} \) where \( \sum_i a_i = \sum_i \gamma_i = 0 \)

If the statistical tests indicate for variation we may then investigate the kind of variation by testing for fixed against random effects models. Notice that the previous models are nested into each other in succession, i.e. \( H_1 \) is nested into both \( H_2 \) and \( H_3 \), all these are nested into \( H_4 \) whereas \( H_2 \) and \( H_3 \) are not nested into each other.

Similarly, we may test for variability of all coefficients, either over regions as in \( H_5 \)

\( q_{it} = a_i + b_{1i} m_{it} + b_{2i} g_{it} + b_{3i} p_{it} + e_{it} \)

or over time as in \( H_6 \)

\( q_{it} = a_i + b_{1t} m_{it} + b_{2t} g_{it} + b_{3t} p_{it} + e_{it} \)

Next, we may consider a model that allows for variation of all coefficients over region and, also, of intercept only over time \( H_7 \)

\( q_{it} = a_i + \gamma_i + b_{1i} m_{it} + b_{2i} g_{it} + b_{3i} p_{it} + e_{it} \) where \( \sum_i \gamma_i = 0 \)

Last in the queue is an unrestricted model that allows for variations of all coefficients over both time and region \( H_8 \)

\( q_{it} = a_{it} + b_{1it} m_{it} + b_{2it} g_{it} + b_{3it} p_{it} + e_{it} \)
H₈ is the most general (unrestricted) model in the rank, with all other models nested in it. Apart from H₈, model H₂ is nested into H₅ and H₇ but not into H₆ and model H₅ is nested into H₇. Model H₃ is nested into H₆ and H₇ but not into H₂. Models H₅ and H₆ are not nested. Since the results from standard (nested) F-tests between nested models provide adequate evidence for model selection, there is no further need for non-nested tests.

**Table 5.1 Results from estimated Fixed Effects models**

<table>
<thead>
<tr>
<th>Model</th>
<th>Type of coefficient variation</th>
<th>RSS</th>
<th>DoF</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₁</td>
<td>Overall homogeneity (pooled estimation)</td>
<td>62.36</td>
<td>84</td>
<td>0.68</td>
</tr>
<tr>
<td>H₂</td>
<td>Intercept over regions (one factor)</td>
<td>20.44</td>
<td>77</td>
<td>0.88</td>
</tr>
<tr>
<td>H₃</td>
<td>Intercept over time (one factor)</td>
<td>41.71</td>
<td>74</td>
<td>0.75</td>
</tr>
<tr>
<td>H₄</td>
<td>Intercept over both regions and time (two factors)</td>
<td>10.12</td>
<td>67</td>
<td>0.93</td>
</tr>
<tr>
<td>H₅</td>
<td>All coefficients over regions (variable coefficients)</td>
<td>4.18</td>
<td>56</td>
<td>0.97</td>
</tr>
<tr>
<td>H₆</td>
<td>All coefficients over time (variable coefficients)</td>
<td>31.96</td>
<td>44</td>
<td>0.68</td>
</tr>
<tr>
<td>H₇</td>
<td>All coefficients over regions &amp; intercept only over time</td>
<td>1.66</td>
<td>46</td>
<td>0.98</td>
</tr>
<tr>
<td>H₈</td>
<td>All coefficients over both regions and time</td>
<td>0.40</td>
<td>16</td>
<td>0.99</td>
</tr>
</tbody>
</table>

**Table 5.2 Nested tests between alternative Fixed Effects models**

<table>
<thead>
<tr>
<th>Test</th>
<th>F-value</th>
<th>D.o.F. Numerator</th>
<th>D.o.F. Denomin.</th>
<th>p-value</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₂</td>
<td>22.56</td>
<td>7</td>
<td>77</td>
<td>0.00%</td>
<td>Reject H₁</td>
</tr>
<tr>
<td>H₃</td>
<td>3.66</td>
<td>10</td>
<td>74</td>
<td>0.05%</td>
<td>Reject H₁</td>
</tr>
<tr>
<td>H₄</td>
<td>6.83</td>
<td>10</td>
<td>67</td>
<td>0.00%</td>
<td>Reject H₂</td>
</tr>
<tr>
<td>H₅</td>
<td>10.37</td>
<td>21</td>
<td>56</td>
<td>0.00%</td>
<td>Reject H₂</td>
</tr>
<tr>
<td>H₆</td>
<td>0.45</td>
<td>30</td>
<td>44</td>
<td>98.82%</td>
<td>Fail to reject H₃</td>
</tr>
<tr>
<td>H₇</td>
<td>6.98</td>
<td>10</td>
<td>46</td>
<td>0.00%</td>
<td>Reject H₃</td>
</tr>
<tr>
<td>H₈</td>
<td>1.68</td>
<td>30</td>
<td>16</td>
<td>13.75%</td>
<td>Fail to reject H₇</td>
</tr>
</tbody>
</table>

Table 1 presents the results from the estimation of the models by OLS and table 2 presents the statistical tests between combinations of these models. The results of the F-tests from the attempted OLS regressions indicate clearly in favour of variable coefficients over regions and of variable intercepts (only) over time, with considerable statistical confidence. Variation of coefficients over region gets strongly supported by the tests of H₃ against H₂ and the variation of intercepts over time gets supported by the tests
of $H_3$ against $H_1$ and of $H_7$ against $H_5$. The tests of $H_6$ versus $H_3$ and of $H_8$ versus $H_6$ lead conveniently to the conclusion that there are not significant variations of the slope coefficients over time. In other words, the tests unanimously promote model $H_7$ as the one that captures more successfully the variations across the two dimensions of the panel.

As an alternative to fixed effects we may consider random effects models. The time invariant random effects model $H_9$ can be written as

$$q_{it} = \alpha + b_1m_{it} + b_2g_{it} + b_3p_{it} + u_i + e_{it}$$

and the respective region invariant model $H_{10}$ as

$$q_{it} = \alpha + b_1m_{it} + b_2g_{it} + b_3p_{it} + w_t + e_{it}$$

where $u_i$ and $w_t$ are the random effects of the two cases, also assumed to follow the standard assumptions. Combining the two effects, we have the Two Factor Random Effects model $H_{11}$

$$q_{it} = \alpha + b_1m_{it} + b_2g_{it} + b_3p_{it} + u_i + w_t + e_{it}$$

Estimating random effects models $H_9$, $H_{10}$ and $H_{11}$ by Feasible Generalised Least Squares we may test against fixed effects models $H_2$, $H_3$ and $H_4$ respectively using Hausman tests.

**Table 5.3 Hausman tests between Fixed and Random Effects models**

<table>
<thead>
<tr>
<th>Test</th>
<th>$\chi^2$ value</th>
<th>DoF</th>
<th>$p$ - value</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_9$ against $H_2$</td>
<td>36.92</td>
<td>3</td>
<td>0.0000%</td>
<td>Reject $H_9$</td>
</tr>
<tr>
<td>$H_{10}$ against $H_3$</td>
<td>7.84</td>
<td>3</td>
<td>4.9329%</td>
<td>Reject $H_{10}$ at 5%</td>
</tr>
<tr>
<td>$H_{11}$ against $H_4$</td>
<td>15.49</td>
<td>3</td>
<td>0.1441%</td>
<td>Reject $H_{11}$</td>
</tr>
</tbody>
</table>

Table 5.3 presents the results of Hausman tests. Both hypotheses of random effects over regions and time get clearly rejected: individually from the tests of $H_9$ versus $H_2$ and of $H_{10}$ versus $H_3$ respectively and jointly from the more general test of $H_{11}$ against $H_4$. Note that Hausman test tests a null hypothesis of identity of OLS and Generalised Least Squares (GLS) against the alternative of biased GLS estimation. Since both estimations are used in the calculation of the test statistic this test has undesirable large sample properties under the alternative hypothesis (GLS is inconsistent, see e.g. Greene [1993 p.479]). The strong statistical confidence of the tests of table 5.3, which apparently fall into this case, can be used to compensate any doubts about their validity adding confidence to the result of fixed effects variations.

Along with the statistical evidence, the fixed type of variations appears more suitable to the theoretical expectations. Our examination of large regional aggregates implies the assumption that regional groups consist of countries with relatively homogeneous
characteristics. Indeed, the large majority of countries in most of the regional groups appear to have similar levels of development, security problems, political regime profiles etc. On the other hand, these parameters generally differ from one region to another, which implies deviations for defence policies of countries that belong to different regions. Therefore, any divergence in the behaviour of each group from the others is not a result of a random sampling process but an expected event.

Choosing between fixed and random effects, Hsiao [1986 p. 43] concludes the following: ‘When inferences are going to be confined to the effects in the model, the effects are more appropriately considered fixed. When inferences will be made about a population of effects from which those in the data are considered to be a random sample, then the effects should be considered random.’ In our case the regions in the data cover almost the entire population, which is the whole world. In that sense, the case of random sampling is unsuitable, since we do expect significant divergences in the pattern of demand across regions. Thus, both theoretical and empirical evidence apparently point in the direction of fixed rather than random effects. It is noteworthy, however, that the process of grouping countries into regional aggregates may possibly create some randomness in the structural variation across the regions.

In the previous paragraphs we based the selection between fixed and random effects models on tests from a variable intercept model. If we consider the more general case of variable coefficient model statistical testing for the kind of variations becomes even more complicated and statistical inferences more unclear. Hsiao [1996, p.136] repeats for this case that the choice between fixed and random coefficients depends on whether we are making inferences conditional on the individual sample or unconditional on the population characteristics. Here, we make conditional inferences from regions, which (nearly) cover the entire population rather than a sample and are a-priori expected by the theory to differ. Providing that the data clearly indicate for the variation of coefficients the question is whether we use a fixed coefficients model or a Swamy type Random Coefficients Model. According to Judge et al [1985] ‘...the important consideration is likely to be whether the variable coefficients are correlated with the explanatory variables. If they are... the fixed coefficients model can be used [p.544].’ If coefficients are known to be correlated with the regressors fixed variations are more appropriate, otherwise we can gain in efficiency using the Random Coefficients Model.

In our case, any regional effects of the coefficients are indeed expected to correlate with the regressors, a fact that gets eventually supported by the results. For instance, more industrialised countries (high per capita income) enjoy advantages in the
establishment of a domestic industries of arms. This alternative to the complete reliance on imports enhances substitution between inputs in the production of defence and, finally, is likely to affect the estimated elasticities. Also, poorer countries happen to be more often involved in internal or external conflicts, possibly making their responses to income and prices more inelastic. Once again, the choice of fixed type of variations appears as more suitable, but we return to this issue in the following section.

5.4 Selection of model and estimation method

One of the results of the cross-sectional study of chapter 4 is that the aggregate measurement of GDP has a better fit to the data than the per-capita measurement. According to the proposed explanation an aggregate measurement of the economic size of countries indicates their ability to establish a domestic arms industry. The per-capita measurement measures the level of development of the country, factor that possibly affects the establishment of a domestic industry of arms. The difference between the two measurements is population, a basic indicator of countries’ size. Thus, per-capita measurement purely measures development but ignores size whereas the aggregate measurement contains a mix both effects.

There are three mainly reasons for choosing to use per-capita, instead of total GNP in this study. First, the differences between the estimations with the two variables are rather small, both in terms of estimates and statistical tests. Second, per-head income provides an approximation of the average labour cost of the country and the estimated coefficient reflects the impact of development on the demand for arms. Since armed forces and imported arms are the two main inputs in the production of military capability this coefficient partly measures the cross-price elasticity of the demand for arms to the price of labour. Furthermore, this coefficient may include other effects of development on the demand for the arms trade, such as development related changes of public preferences for security, technological or human capital factors, the ability to produce domestic arms etc. Third and most important, estimating over aggregated groups of countries the total GNP does not provide the required information about the economic size of countries but only about the size of the regional group. In other words, it is not possible to distinguish if a large figure shows many small countries or few large. For these reasons the use of per-capita GNP appears more appropriate for this study.
About the estimation method of a fixed effects variable coefficients model Judge et al [1985 p.539] clearly suggest that ‘when the response coefficients... are fixed parameters... (the system) can be viewed as the Seemingly Unrelated Regression (SUR) model.’ A similar suggestion is made by Hsiao [1996 p.134]. The appropriate method to estimate SUR model depends on the state of correlation between the error terms of different regions. If the disturbances of the equations of different regions are correlated, a GLS estimation of SUR gives more efficient estimates from the separate OLS estimates of each region (Greene [1993 p.489]). If the errors between different regions are uncorrelated, i.e. the variance-covariance (V-C) matrix is diagonal, SUR reduces to the case of ‘Group Heteroskedastic Model’ and GLS estimation has no efficiency gains over separate OLS regressions (Greene [1993 p.488]).

Separate OLS estimates for each region have good large sample properties and provide consistent estimates of the V-C matrix (Greene [1993 p.453]), independent of the state of correlation between the errors of different regions. A SUR model can either be estimated by a two step Feasible Generalised Least Squares estimator (FGLS) or by a Maximum Likelihood estimator (ML). Using these estimates we can then test for the significance of the covariances, in order to evaluate the benefits from estimating as a system, rather than as independent equations. In our case, the number of available observations is small and favours an estimation method with good small sample properties.

Prior to the selection of an appropriate estimation method the it is important to chose the final functional form of the estimated model. The tests of the previous section make clear the need to allow for variation of all coefficients across regions and, also, for variation of intercept term (only) over time. The selected model H₆, however, allows for fixed effects over time but restricts them to be equal for all regions, since the number of observations is not adequate for the estimation of separate fixed effects for each region. Looking closer at the estimated fixed effects, which are presented later in Section 5.5, we can easily distinguish the familiar double trended pattern: first increasing and then decreasing, approximately following the developments of the Cold War.

The estimation of a SUR model by FGLS requires the inclusion of global dummy variables for the time effects, as in model H₇, is practically difficult and not directly available in standard econometric packages. An alternative method to account for time effects, within the limits posed by the sample size, is to include independent quadratic trend for each region, as an approximation of the estimated double trended pattern. This
solution restricts the form of the estimated trend to a parabola but relaxes the restriction of equal effects between regions. Using a quadratic time trend, the equation to estimate is

\[ q_{it} = a_i + b_{1i} m_{it} + b_{2i} g_{it} + b_{3i} p_{it} + b_{4i} t + b_{5i} t^2 + u_{it} \]

A standard test for the correlation between error terms of different regions is the likelihood ratio statistic \( l_{LR} = T(\sum_i \ln s_i^2 - \ln |S|) \), \( i = 1, \ldots, n \), where \( n = 8 \) is the number of regions, \( s_i \) is the ML estimator (=RSS/n) of the OLS standard error of the regression of region \( i \) and \( |S| \) is the determinant of the estimated V-C matrix from the iterative ML estimation (Greene [1993 p. 492]). Under the null hypothesis of a diagonal V-C matrix \( l_{LR} \) distributes \( \chi^2 \) with \( n(n - 1)/2 = 28 \) degrees of freedom (i.e. the number of covariances restricted to be zero). From our estimations we calculate \( \sum_i \ln s_i^2 = -36.90 \), \( \ln |S| = -64.85 \), so \( l_{LR} = 307.45 \), with a p-value less than \( 10^{-45} \). The significance of this test suggests that the covariances jointly are different from zero and, therefore, SUR is the appropriate model to estimate.

From the literature, neither FGLS nor ML methods are known to have a general advantage in the estimation of SUR, and the answer seems to depend on the specific nature of the data set (Greene [1993 p.493]). A more detailed implementation of this issue requires Monte Carlo experiments, raising statistical issues beyond the scope of this study. According to Judge et al [1985 p.493], however, in the case of small sample size with a large number of equations simulation results have indicated that FGLS model could provide better finite sample properties. To allow for actual comparison of the results, in this study we estimated (11) with all three methods.

In practice, the iterative ML estimation converged after a single iteration and the differences between FGLS and ML estimates appeared rather unimportant. The ML estimates, however, had considerably smaller standard errors and stronger tests, result not unexpected since the former method uses maximum likelihood whereas the latter uses unbiased estimators of V-C matrix. With 11 annual observations and estimating 6 coefficients (including intercept and trends) the estimation has 5 degrees of freedom, so the two estimates of V-C differ by a factor of 2.2 (=11/5). FGLS is expected to provide better asymptotic properties than ML method, especially in our case of a small sample. Note that the latter method gave stronger statistical results with smaller estimated errors, apparently as a result of small sample bias in the estimation of V-C.
5.5 Regional estimates of the demand for imported arms

Table 5.4 presents the estimated elasticities and statistics from the SUR estimation of (12) by a two-step FGLS method, using with an unbiased estimator of variance-covariance matrix for the first step. The estimates of the effects of the time trends are not reported in this section but in Table 6 of Section 5 that follows. The general look of these results appears encouraging and on line with those from the cross-sectional estimations of LMS. Most of the regions have positive budget (military expenditure) and negative price elasticities, with strong statistical support. All five statistically significant price elasticities are negative. The only significant negative budget elasticity is this of Middle East. Except this coefficient, the two remaining income and two price elasticities with incorrect signs, are insignificant. All remaining regressions have at least one significant slope coefficient and the first 6 regressions have significant F-tests.

Table 5.4 FGLS estimates of a model with a quadratic trend

<table>
<thead>
<tr>
<th>Region</th>
<th>Intercept</th>
<th>Mil. Exp.</th>
<th>P-C GNP</th>
<th>Price</th>
<th>R²</th>
<th>DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Africa</td>
<td>-7.79 (2.46)</td>
<td>2.12 (1.24)</td>
<td>-0.66 (0.17)</td>
<td>-0.19 (0.03)</td>
<td>0.997</td>
<td>3.32</td>
</tr>
<tr>
<td>2. N. America</td>
<td>27.8 (23.3)</td>
<td>0.30 (0.92)</td>
<td>-2.49 (2.58)</td>
<td>-1.01 (0.13)</td>
<td>0.873</td>
<td>2.26</td>
</tr>
<tr>
<td>3. C. America</td>
<td>31.6 (15.4)</td>
<td>2.30 (1.12)</td>
<td>-5.74 (2.01)</td>
<td>-0.68 (0.31)</td>
<td>0.893</td>
<td>2.45</td>
</tr>
<tr>
<td>4. S. America</td>
<td>-23.1 (5.78)</td>
<td>2.22 (0.20)</td>
<td>1.73 (0.78)</td>
<td>-0.88 (0.03)</td>
<td>0.996</td>
<td>3.04</td>
</tr>
<tr>
<td>5. Asia</td>
<td>35.4 (21.6)</td>
<td>1.28 (0.57)</td>
<td>-5.63 (3.60)</td>
<td>0.15 (0.28)</td>
<td>0.876</td>
<td>2.39</td>
</tr>
<tr>
<td>6. Europe</td>
<td>12.8 (14.4)</td>
<td>-0.03 (0.21)</td>
<td>-0.26 (1.79)</td>
<td>-0.96 (0.28)</td>
<td>0.940</td>
<td>2.95</td>
</tr>
<tr>
<td>7. Mid. East</td>
<td>-11.1 (8.08)</td>
<td>-2.85 (1.27)</td>
<td>6.20 (2.40)</td>
<td>0.14 (0.18)</td>
<td>0.695</td>
<td>1.20</td>
</tr>
<tr>
<td>8. Oceania</td>
<td>-180 (27.2)</td>
<td>-0.039 (0.34)</td>
<td>-20.4 (3.05)</td>
<td>-0.65 (0.11)</td>
<td>0.811</td>
<td>3.17</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses. Estimates significant at 5% are in **bold-italics**; significant at 1% are also **underlined**. The significance of $R^2$ is tested from the F-test of the regression. DW stands for Durbin-Watson statistic.

According to the statistical tests, Africa has the strongest estimates, with all coefficients significant at 1%. Next follow South and Central Americas with all their coefficients significant at either 5% or 1%. Europe and North America also have strong equations, both showing elastic responses to military expenditure and inelastic to price. Furthermore, most regions show either elastic responses to military expenditure and inelastic to price or vice versa. Exceptions are South and Central Americas, which appear
relatively elastic to both of these effects. Middle East diverts from the rule, showing the lowest fit and inverted signs of income and price elasticities and is the only region that Durbin-Watson statistic indicates for positive serial correlation of the error term. Reasons behind this divergence could be the special nature of security in this region, the occurrence of Gulf War during the examined historical period and possibly omitted dynamics.

More developed areas as N. America, Europe and Oceania tend to appear less elastic to military expenditure and more elastic to price than other regions. Less developed areas as Africa and Asia have these effects inverted, whereas areas of intermediate level of development, as Central and South Americas, appear relatively more elastic in both effects. Middle East shows a clearly negative response to military expenditure. If this effect is interpreted as an income elasticity the negative sign implies that in this case imported arms are an inferior input. Taking in account the indication for positive autocorrelation, the low level of this elasticity could be an indication for downsizing of stocks, apparently an adjustment of their arsenals to the new desirable levels. Indeed, this region presents far the highest positive response to per-capita GNP. If this coefficient is interpreted as a cross-price elasticity between arms imports and armed forces the estimate indicates for high substitution between these two inputs. Except Middle East, South America is the only other region with a positive elasticity to per-capita GNP. All other regions have this elasticity negative, with sizes that vary from close to zero in the case of Europe up to -20 in the case of Oceania.

5.6 Time effects

Although the tests for variation in section 5.3 clearly indicate in favour of variable intercept fixed effects over time the available degrees of freedom do not allow the estimation of independent time effects for each region. As a second best alternative we may estimate fixed effects over time in the context of a variable coefficient model by restricting them to be equal across all regions. Figure 5.1 presents the estimated by OLS fixed effects over time from H7, relative to year 1983, and their standard errors as the radius of the bubbles. The diameter of the bubbles defines an about 68% confidence interval on the vertical axis.

The sequence of the estimated time effects appears to form a relatively smooth and well-shaped double-trended pattern. It is an average trend for the whole world, increasing
until 1987 and decreasing from 1988 onwards. A comparison of this plot with the similar but raw appearance of the arms trade in the figures of chapter 3 reveals that the undertaken parametric examination of the data allowed the isolation of a smooth time trend, which consistently follows the changes in intensity of the Cold War. From a careful observation of the declining part of the estimated trend appears slightly convex, corresponding to the logical expectation that this drastic decline cannot be permanent.

Figure 5.1 Estimated fixed effects over time from the two-way model

![Fixed Effects Over Time](image)

Note: Fixed effects relative to 1993. Width of bubbles give estimated standard errors

The alternative estimation of the underlined time effects of section 5.4 relaxes the restriction of imposing the same fixed effects to all regions by allowing for a different time trend for each region in exchange for being restricted to a parabolic shape. This alternation of restrictions is dictated by the limited degrees of freedom. Table 5.5 reports the remaining results from the FGLS estimation of the previous section. It presents the estimates of the parabolic time trend for the eight regions, including the estimated coefficients of $t$ and $P^2$, their standard errors and the year of the predicted turning point. The turning point of the inverted U shaped double trended patterns can be calculated by the first order condition for maximum from (12) setting $\frac{\partial Q}{\partial t} = 0$ which gives $t^*_{\text{max}} = -\frac{b_4}{2b_5}$. The value of $t$ increases from unit in 1983 to 11 in 1993.

All regions have negative quadratic coefficient, which corresponds to a time trend of a concave shape. Not surprisingly, the estimated peak appears at about the time that Cold War ended for most of the regions. From the 7 regions Africa, North America, South America, Asia and Middle East show estimated trends with high statistical significance. These regions have their turning points estimated between 1986 and 1989 except Asia that has a delayed turning point in 1992. Central America and Europe have their estimates insignificant, both having relatively early turning points and a purely declining time trend for the estimated time period. Oceania has a significant, declining and almost linear trend.
The OLS estimate of the whole world gave coefficients 0.0573 and -0.113 for the linear and quadratic terms respectively, with a turning point predicted in 1986, both with low significance from zero.

**Table 5.5 Estimated quadratic time trend for the eight regions**

<table>
<thead>
<tr>
<th>Region</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coeff. of $t$</td>
<td>.158</td>
<td>.253</td>
<td>-.064</td>
<td>.217</td>
<td>.511</td>
<td>-.045</td>
<td>.136</td>
<td>-.222</td>
</tr>
<tr>
<td>Standard Error</td>
<td>.029</td>
<td>.110</td>
<td>.148</td>
<td>.073</td>
<td>.160</td>
<td>.253</td>
<td>.072</td>
<td>.112</td>
</tr>
<tr>
<td>Coeff. of $t^2$</td>
<td>-.030</td>
<td>-.021</td>
<td>-.0046</td>
<td>-.019</td>
<td>-.027</td>
<td>-.0067</td>
<td>-.027</td>
<td>-.0049</td>
</tr>
<tr>
<td>Standard Error</td>
<td>.0025</td>
<td>.0076</td>
<td>.0123</td>
<td>.0073</td>
<td>.0060</td>
<td>.0173</td>
<td>.0092</td>
<td>.0086</td>
</tr>
</tbody>
</table>

Notes: Code numbers of regions are available in Table 5. Estimates significant at 5% are in bold-italics; significant at 1% are also underlined. Coefficient of $t$ for region 7 has a p-value 0.057. Maximads are rounded to the nearest year.

**5.7 Conclusions**

It is an encouraging indication when the result of technically complicated work comes out clearly and free of internal contradictions. Panel data techniques are technically demanding, produce large volume of statistical output and it is often difficult to distinguish the underlying message and support it with clarity. In this study, the results came up in a relatively clear way, without confusing contradictions. In line with the results of chapter 4, this examination showed that the demand for arms imports generally follows standard patterns with positive responses to the budget and negative responses to the price, both of reasonable strength. The demand for imported arms was found to vary across different regions of the world. Over time, the demand appears to follow a double trended pattern, first increasing and then decreasing. The responses to the explanatory variables (slope coefficients), and therefore the structure of the model, were found to be constant over time. The variations of the structure over regions appear to be of the fixed rather than the random, possibly indicating that the variations are endogenous in the process.

Despite the limited sample size, the estimated regional demand functions enjoy considerable statistical support, quite strong in some of the cases. Most regions have elastic response to military expenditure and negative price elasticities with estimated
values near unity. Since this result is in line with those of chapter 4, the results of the two studies contribute in support of each other. Middle East appears to diverge from this rule, having the signs of the two elasticities inverted and indications of possible underlying dynamics. Relatively less developed areas like Africa, Asia and the Middle East showed more elastic responses to military expenditure and more inelastic responses to price. More developed areas, as North America, Europe and Oceania have elastic responses to prices and inelastic to military expenditure. Central and South Americas had both of these effects relatively more elastic. The estimated effect of per-capita GNP on the demand for arms imports varied, taking a positive value in the case of Middle East and South America and negative in all other cases.

Over time, the demand for imported arms seems to have increased until 1987, succeeded by a relatively steep and steady decline until the end of the examined time period. The two underlining trends have smooth and approximately linear shape. The declining part shows a weak tendency for stabilisation (convexity) in the most recent years. When the time effect is estimated independently for each region, with the use of a quadratic trend, all regions turn out to follow the twin pattern of increases followed by decreases. Most regions had a turning point between roughly after the end of the Cold War, i.e. between 1986 and 1989. All these results are supported by considerable statistical evidence. The estimated independence of the slope coefficients from the strong trend-related variations of the imported arms is quite an interesting and possibly a meaningful result. Providing that the limited sample size does not allow for any confident conclusions, it is possible that this independence indicates that there is a separation between the arms imports related to the Cold War and those required for the national security of the country, at regional level. On the other hand, this could simply be the result of particular characteristics of the production process. For a further examination of this issue we need a more realistic and flexible analytical background.

A weakness of this study is the relatively simplistic form of the estimated demand equation, a result of limitations in the availability of data and the sample size. Another one is the simplistic form of the model used in this examination. These limitations indicate possible directions that this research may continue. A more detailed empirical examination requires more data. Using the latest ACDA and SIPRI releases we may extend our data set across time. An extension of this examination from aggregate-regional to country level needs to overcome the difficulty of several zero observations, due to missing information. This could be possible with the use of techniques appropriate for this type of measurement errors, as some type of Tobit model with measurement
errors. Moreover, this examination requires a more powerful analytical model, the development of which is attempted in the following chapter 6.
6. Differentiated types of arms and substitution of inputs in defence

6.1 Introduction

In the context of national defence policy, governments consider the possibility of having a domestic industry of arms. If such an industry does not exist and they decide that producing arms domestically is beneficial for national defence and the general public welfare of their country, they proceed with the installation. Otherwise, they avoid this generally costly establishment and rely for their national defence entirely on imported arms and labour. This chapter develops a small country model to examine the optimal allocation of resources in defence, allowing for the option to include a domestic arms industry or not. The three inputs in the production of military capability - imported, domestic arms and labour - are considered as imperfect substitutes in this production. Some minimal quantities of the first two inputs are completely necessary for the production, whereas domestically produced arms are not.

Based on empirical evidence, chapter 4 we have seen that the demand for arms imports follows standard patterns instead of the opposite, which has often been asserted in the literature. Based on cross-sectional estimations of demand functions and a data set that covers 38 countries, our examination came to the conclusion that the demand for imports follows negative responses to the price of arms and positive responses to the size of the military budget. Furthermore, there is evidence for non-linear logarithmic components in the demand function, implying responses of a concave shape to military expenditure and to per-capita income of the countries.

For an explanation of the hump-shaped estimates of the demand for arms chapter 4 suggested that at low levels of military expenditure, countries do not consider the establishment of a domestic industry. Instead, they mainly rely on imported arms, so that

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1 Early results of this work have been presented in the meeting on 10th of July 1997 of the group working on arms trade. I am grateful to the participants of this meeting for helpful comments and criticism.
the demand for them appears elastic to the size of the budget. At high levels of military turnover, although, countries tend to produce their own arms, partly substituting imported arms and possibly exporting too. Concluding, chapter 4 outlined the need for deeper theoretical explanation of the empirical findings. The study of panel data in chapter 5 added further evidence and increased the confidence in these results. It verified the systematic changes of the demand pattern with the size and the development of countries. It also provided evidence that despite the large variations of the arms trade the Cold War the underlining demand pattern does not change in time.

In this chapter we develop a small country model that focuses on the entry of countries to domestic production of arms as well as on the demand for the three main factors used in defence: imported, domestic arms and armed forces. It is a partial equilibrium model because it ignores the impact of allocations in defence on national factor prices, assuming that defence sector is small enough so that these effects are negligible. It is a small country model in the sense that it ignores the impact of each country’s demand for arms imports on the world market price, which is assumed fixed. As a payoff for these simplifying assumptions the model provides a relatively simple and clear analytical tool, suitable for the needs of econometric studies.

The material included in the chapter is organised as follows. Section 6.2 looks into empirical evidence and outlines the requirements from an analytical set-up. Section 6.3 presents the model. Section 6.4 examines the properties of the new model and compares them with previous empirical results. For a further assessment of the applicability of the developed theory section 6.5 presents results of an econometric estimation of this model. Finally, appendix A.6 provides statistical results from the econometric application.

6.2 The role of domestically produced arms in defence

Observation of available data shows that the vast majority of countries import arms (ACDA [1995]), but only few large countries have a well-developed domestic arms industry. From the 166 countries included in ACDA tables, there are only eight countries and the ex-Soviet republics with zero figures of imports for the covered historical period (Bhutan, Central African Republic, Fiji, Malta, Swaziland and Trinidad and Tobago). All eight countries are very small both in terms of economic size and, with the exception of Malta, in terms of economic development. There are reasons that suggest, however,
of Switzerland to 0.12 in the case of labour intensive Morocco (1990-1991 averages of SIPRI volumetric arms imports in constant 1990 million dollars divided by ACDA military forces in thousand persons – see section A.6.1). Note that we may not simply attribute the large difference in the composition of inputs between these two cases to the availability of domestic arms, since Switzerland is an established producer of arms, whereas Morocco is not. The large variability of this index across different countries indicates that in reality defence technology allows for a variety of different mixes of the three inputs under discussion. In other words, the empirical evidence appears to support the assumption of considerable substitution among inputs in defence.

For analytical purposes, we may simplify the variety of arms into two aggregated and homogenous categories: imported and domestically produced. There are three inputs altogether in the production of military capability, the two categories of arms and labour. The two categories of arms (domestic and imported) are assumed to be imperfect substitutes in the production of military capability, since countries can defend themselves using imported arms, without having to establish a domestic industry. Imported arms and armed forces, moreover, differ from domestic arms by having the characteristics of 'necessity,' whereas domestic arms rather have the characteristics of a 'luxury'. This distinction is based on the fact that the smaller defence budget is the more defence tends to rely on the former two inputs, whereas when military expenditure grows larger countries tend to use domestic arms.

Searching the literature, it has not been possible to detect an analytical framework that captures the required characteristics, together with relative analytical simplicity. It appears, moreover, that the desired features can be provided by an extension of Stone-Geary production function, discussed in detail in appendix B. In its standard form Stone-Geary function allows for some minimal (positive) quantities of the inputs, called 'subsistence quantities,' to be absolutely necessary for any output to exist. Appendix B examines an extension of Stone-Geary function in which the subsistence quantity of one only input can be negative, representing a minimum amount of the input always available as a free good. Furthermore, Appendix B introduces fixed costs to the use of this input, which can be avoided if the input is not used. The result of this alteration apparently provided the desirable characteristics, including the entry of domestic production, when the budget rises enough.

A simplifying assumption in this work is to consider the budget of defence as prefixed, reflecting government's perception of country's needs for security. Moving
first, the central government defines the level of budget. Then the military sector allocates resources maximising output. This implies that central government is not informed about the returns to scale of the defence industry, neglecting any disproportional effects of the budget to the produced output. The assumption that governments face restrictions in the availability of information about the returns of their own defence industries appears more realistic if we take into account the general state of secrecy that usually covers the internal organisation of national defence sectors. Central governments often face limitations of information needed to assess the performance of their own defence industry. Such an assessment requires a detailed comparative study of the characteristics of the production with those of a wide sample of other countries, which is generally not available. As a consequence, governments' assessments mainly rely on the information they receive from their own defence industry. But even the information that originates from this main source might face limitations in quality, for reasons related to the bureaucratic character of military structure. Thus, without thorough implementation from available comparative studies, governments first roughly assess the cost curve of security, and decide about the desirable level of defence expenditure. This fixed amount of resources is then made available to the management of defence sector, which is expected to allocate resources efficiently.

6.3 The model

Assume that the only output of countries' military sector is military capability $K$, which finally creates national security. Also, assume that the government of our representative country has fixed preferences in terms of allocating resources between national defence and other expenditures. According to this assumption, central government moves first and defines the budget of defence, which is then fixed. With this budget fixed, the planner of defence allocates it to the factors of production with the purpose of maximising the output of military capability. Inputs to the production are domestic arms, imported arms and armed forces (labour). We assume a static setting where inputs and output are subject to full depreciation in each time period.

The planner of defence may consider the establishment of a domestic arms industry. The set-up costs for the establishment of such an industry are generally large and they mainly consist of expenditure in R&D. There are mainly two reasons why we
differentiate domestically produced from imported arms. One is that domestically produced arms provide higher security of supply, especially in the case of a war, when they are most needed. The other reason originates from the abstraction process of aggregating numerous different kinds of arms in a single homogenous good. Both categories of imported and domestically produced arms are essentially bundles of a variety of different kinds of arms. If the domestic sector is established, the quality and composition of the bundle of domestic arms could be adapted to the specific needs of the country, better than those imported. In this context, it seems reasonable that the availability of a domestic industry of arms can raise the returns to scale of the defence sector.

In terms of substitution between inputs we assume that the two categories of arms are imperfect substitutes in this production. Imported arms and labour are imperfect complements and a minimum quantity of them, the subsistence quantity, is necessary for any production to exist. Beyond this quantity they are imperfect substitutes. Domestic arms are gross substitutes in the production with imported arms and labour; and they may fully be substituted by the other two inputs. In this set-up, the option to export domestically produced arms is not taken into consideration.

Let the production function for military capability be

\[
K = (D - \gamma_D)^{\kappa_D} (M - \gamma_M)^{\kappa_M} (L - \gamma_L)^{\kappa_L}; \quad \kappa_D + \kappa_M + \kappa_L = 1
\]

where \( K \) is military capability, \( \kappa_D, \kappa_L, \) and \( \kappa_M \) are coefficients that \( \kappa_D, \kappa_M, \kappa_L \in (0, 1) \). Variable \( M > \gamma_M \geq 0 \) stands for imported arms and \( \gamma_M \) is the subsistence quantity of them. Variable \( L > \gamma_L \geq 0 \) is military forces and \( \gamma_L \) is their subsistence quantity. \( L \) excludes labour employed in domestic arms industry. Domestically produced arms \( D \geq 0 \) have a negative subsistence quantity \( \gamma_D < 0 \). The price of labour is \( w \) and of imported arms is \( p_M \), both assumed fixed. When the domestic production is not established \( D \) is simply set to zero, \( \gamma_D^{\kappa_D} \) becomes a fixed coefficient in (1) and, as A2 shows, returns to scale are lower.

For the domestic industry of arms we assume a Leontief production function with constant returns to scale. If we adjust the measurement units of \( D \) to be equal to the average product of labour we may write the production function as

\[
D = L_D
\]
where \( L_D \) is the labour employed in this industry. The production of \( D \) requires total capital \( Y_D \), which consists of fixed capital \( F \geq 0 \) and variable capital \( bL_D \) in a constant proportion \( b \) with labour

\[
Y_D = bL_D + F
\]

The price of labour is \( w \) and the price of capital is normalised to unity. Then, the total cost function of \( D \) is \( C_D = wL_D + Y_D = (w + b)D + F \). Dividing this expression through by \( D \) we obtain

\[
p_D = (w + b) + \frac{F}{D}
\]

where \( p_D \) is the average and \( (w + b) \) is the marginal cost of domestically produced arms. Also, \( p_D \) is the break-even price at which the profits of this industry vanish. According to (4), when the set-up costs are not zero, the average cost diminishes with output. We also assume that factor prices are fixed, remaining unaffected by the demand for capital and labour.

In the analysis that follows we examine two cases: when the domestic arms industry is established and when it is not. Following to A2, we may use the binary (dummy) variable \( E \) to denote the two cases, that is set \( E = 1 \) when the industry is established set \( E = 0 \) when it is not. In terms of notation, we distinguish between the two cases using a tilde (\( \tilde{\cdot} \)) to distinguish the variables that refer to the case without domestic production. If \( G \) is the total defence budget \( G - EF \) is the disposable budget, net of set-up costs for the domestic industry. Without domestic sector \( E = EF = 0 \) and the disposable budget is \( G \). When domestic industry is established \( EF = F \) and the disposable budget is \( G - F \). Under these assumptions, the budget constraint for the production of military capability is

\[
G \geq (w + b)ED + p_M M + wL + EF
\]

Of course, \( E = 0 \) implies that \( D = 0 \) too in which case (5) reduces to \( G \geq p_M M + wL \).

If the planner of national defence has the objective to maximise (1) subject to the constraint (5) we may solve this problem using the Langrangian method. Solving this problem, when the domestic industry is established the demand for imported arms is

\[
M = \gamma_M + \frac{\kappa_M}{p_M} \left[ G - F - (w + b)\gamma_D - p_M \gamma_M - w\gamma_L \right]
\]

the demand for armed forces is

\[
L = \gamma_L + \frac{\kappa_L}{w} \left[ G - F - (w + b)\gamma_D - p_M \gamma_M - w\gamma_L \right]
\]

and the demand for domestically produced arms is
When the domestic industry is not established \( D = 0 \) and the demands for the two remaining inputs are

\[
\tilde{M} = \gamma_M + \frac{\kappa_M}{1 - \kappa_D} \frac{G - p_M \gamma_M - w \gamma_L}{p_M}
\]

and

\[
\tilde{L} = \gamma_D + \frac{\kappa_L}{1 - \kappa_D} \frac{G - p_M \gamma_M - w \gamma_L}{w}
\]

The two systems of demand equations have different characteristics dependent on the establishment of the domestic industry. Apparently, an advantage of this set-up is the algebraic simplicity of the solutions.

A standard requirement of Stone-Geary function is that the budget should necessarily cover the cost of subsistence quantities of the inputs, i.e. \( G > \tilde{G}_L \) where

\[
\tilde{G}_L = p_M \gamma_M + w \gamma_L
\]

When the domestic industry is established the non-negativity constraint of input \( D \) requires from (8) that \( G > G_L \) where

\[
G_L = F + p_M \gamma_M + w \gamma_L - \frac{1 - \kappa_D}{\kappa_D} (w + b) \gamma_D
\]

Otherwise, if \( G < \tilde{G}_L \) domestic production is never established. When the budget exceeds this critical level the planner chooses the kind of production that gives higher output. A2 has shown that in the range \([G_L, \infty)\) there is a critical level of the budget \( G^* \) that the two cases give equal output. When the budget is lower than this critical level producing \( K \) without a domestic sector of arms gives higher output than producing it with an established domestic sector. When \( G > G^* \) output with the use of domestic arms dominates.

Setting the indirect utility functions for the two cases equal, with some algebra we obtain the necessary and sufficient condition for indifference to the establishment of domestic industry of arms

\[
G - \frac{( - p_D \gamma_D)^{\kappa_D}}{\kappa_D^{1 - \kappa_D} (1 - \kappa_D)^{1 - \kappa_D}} (G - p_M \gamma_M - w \gamma_L)^{1 + \kappa_D} = [F + (w + b) \gamma_D + p_M \gamma_M + w \gamma_L] = 0
\]
Appendix A2 proves that $G^*$ always exists and it is unique, providing that prices and parameters take values from their valid ranges. As a real polynomial of $G$, expression (13) does not take a general algebraic solution. Since a single real solution does exist, however, we may overcome this problem by solving for $G^*$ with the use of numerical methods.

Summarising, we have determined three critical levels of the budget, following order of magnitude

$$GL \leq G \leq G^* \leq G_L$$

Comparison of $G$ with these values determines the establishment of the domestic industry

$$E = \begin{cases} 
0 & ; \ G < G^* \\
1 & ; \ G > G^* \\
\text{undefined} & ; \ G = G^* 
\end{cases}$$

When $G = G^*$ either cases give equal output, the planner is indifferent and the solution undefined. This is the reason why we may not write condition (15) in a necessary and sufficient form. In inverted form the necessary conditions for the establishment are

$$\begin{align*}
E = 0 & \Rightarrow G \leq G^* \\
E = 1 & \Rightarrow G \geq G^*
\end{align*}$$

6.4 Properties of the demand for imported arms

One of the targets set out for this study is to provide some theoretical explanation to the results of the cross-sectional study of chapter 4, i.e. the estimated equation

$$\ln M = -21.64 - 0.56 \ln p_M + 1.52 \ln G - 0.06 (\ln G)^2 + 4.74 \ln w - 0.29 (\ln w)^2$$

Only the price elasticity is estimated as constant, independent of the level of $p_M$. Income elasticity $\varepsilon_G$ decreases when $G$ grows but stays always positive, nearing zero in the case of the largest observation, this of the U.S. The elasticity of imports to per-capita income $\varepsilon_w$ changes from positive to negative when $w$ grows and so do the estimated responses of the demand, with a turning point just below the sample mean of $w$. Analysis for the impact of measurement errors indicated possible underestimation of $\varepsilon_G$ and $\varepsilon_w$.

From the demand functions for imported arms (6) and (9) we may calculate the own price elasticities for the two cases.
Similarly, the income elasticities are

\begin{equation}
\varepsilon_G = \kappa_M \frac{G}{p_M M} > 0 \quad \text{and} \quad \tilde{\varepsilon}_G = \frac{\kappa_M}{1 - \kappa_D} \frac{G}{p_M M} > \varepsilon_G > 0
\end{equation}

and the elasticities of imports with respect to the average national wage

\begin{equation}
\varepsilon_w = -\kappa_M \frac{w(y_D + y_L)}{p_M M} \quad \text{and} \quad \tilde{\varepsilon}_w = \frac{\kappa_M}{1 - \kappa_D} \frac{w y_L}{p_M M} w < 0; \quad \tilde{\varepsilon}_w > \varepsilon_w
\end{equation}

Since \( y_D < 0 \) and \( y_M < 0 \) the elasticity of armed imports to labour costs with the domestic sector established \( \varepsilon_w \) can either be negative or positive, dependent on the relative magnitude of the two subsistence quantities. When the domestic sector is not established this elasticity is limited to be negative. This might well be the case in the cross-sectional study of chapter 4 where this elasticity was estimated to change from negative to positive when \( w \) rises from lower to higher levels.

In either cases, own price elasticity is negative and might either increase, decrease with or remain unaffected from the establishment of the domestic sector, depending on parameter values. Notice that if \( F > -(w + b)y_D \) the demand for \( M \) is always more elastic without the domestic sector established. This encouraging result appears in line with the constant price elasticity estimated in chapter 4 or, at least, not contradictory. Income elasticity \( \varepsilon_G \) is positive, depends on \( G \) and decreases when the domestic sector is established. Since the establishment of domestic sector depends on \( G \) as well, when \( G \) increases beyond \( G^* \) the domestic sector gets established and imports \( M \) and elasticity \( \varepsilon_G \) jump discontinuously to lower levels. Without domestic sector, the elasticity of imported arms to labour costs is negative. When the domestic sector is established elasticity \( \varepsilon_w \) has a higher value than when it is not and it is positive iff \( -y_D > y_L \).

The two diagrams of Figure 6.1 plot the reaction of countries \( \hat{M} \) (dashed curve), as it was estimated in the cross-sectional study of chapter 4, as well as examples of reactions predicted by the model \( \bar{M} \) and \( M \). Diagram A presents the demand for arms as a function of the budget \( G \) and diagram B as a function of labour costs (\( w \)). A comparison of the predicted by the theory with the estimated reactions shows that the two are in line or, at least, they are not contradictory. Diagram B gives an example of the predicted response to changes of economic development of the country, which mainly include the impact of
labour costs. Again, the set of two linear reactions provides a possible explanation to the estimated parabolic response.

Figure 6.1 The demand for imported arms: Model prediction and empirical estimates

6.5 Estimation of the demand imported arms

The model we presented above can provide analytical means for an empirical examination of the demand for AI, providing the availability of data. Following the approach of chapter 4, we may consider quantities G, w, M, \( p_m \) and L as observable, i.e. military expenditure, average national wage, price of imported arms and armed forces respectively. Data series for the domestic production of arms \( D \) are not directly available in an appropriate form for our requirement form. Nevertheless, some sources do provide some information on domestic production (e.g. SIPRI, IISS) and construction of such a series could be a possibility. Any attempt, however, to approximate a series of domestic production, which would unavoidsably raise questions about the quality of the constructed data series. Alternatively, as the following analysis demonstrates, we may avoid the quantitative measurement of countries' domestic production of arms and
estimate the model only with the use of information about variable $E$, i.e. if countries have a domestic industry of arms or not.

Separating observable from unobservable quantities, we can rearrange the twin demand function for arms (6) and (9) as

\[(20) \quad p_M M = -\kappa_M (b \gamma_D + F) + \kappa_M G + (1 - \kappa_M) \gamma_M p_M - \kappa_M (\gamma_D + \gamma_L) w, \quad E = 1\]

\[(21) \quad p_M M = \frac{\kappa_M}{1 - \kappa_D} G + \frac{\kappa_L}{1 - \kappa_D} \gamma_M p_M - \frac{\kappa_M}{1 - \kappa_D} \gamma_L w, \quad E = 0\]

respectively, where $\kappa_L + \kappa_M + \kappa_D = 1$. Let $E_i$ be the dummy variable for the establishment of a domestic arms sector for country $i$, which equals 1 when a domestic industry is established and equals 0 when it is not. Also, following chapter 3, let $A_i (= p_{Mi} M_i)$ be the ACDA measurement of the value of arms imports and $p_{Mi}$ the implicit price index of imported arms. Then, equations (20) and (21) can be written jointly with the use of intercept and slope dummy variables as

\[(22) \quad A_i = m_0 E_i + m_1 G_i + m_2 p_{Mi} + m_3 w_i + m_4 (E_i G_i) + m_5 (E_i p_{Mi}) + m_6 (E_i w_i) + u_{Mi}\]

where $m_j$ are coefficients and $u_{Mi}$ is a random term.

Having estimated form (22) we may solve for the coefficients of (20) and (21) as follows. Directly from the definition of dummy variables we obtain

\[(23) \quad \kappa_M = m_1 + m_4\]

and from the coefficients of $G$ from (20) and (21) we obtain

\[(24) \quad \kappa_D = \frac{-m_4}{m_1}\]

and

\[(25) \quad \kappa_L = 1 - \kappa_D - \kappa_M\]

Continuing, we may isolate $\gamma_M$ from the coefficient of $p_M$ in (21), which after simplifications takes the form

\[(26) \quad \gamma_M = \frac{m_2}{1 - m_1}\]

From (21) we have

\[(27) \quad \gamma_L = \frac{-m_1}{m_1}\]

and from the coefficient of $w$ in (20) we may isolate
The coefficients $b$ and $F$ are not identified. For a calculation of them requires additional information to that provided by form (22).

For an estimation of (22) we may use the same cross-sectional data used in chapter 4, which consist of 1990-1991 average annual figures of arms imports and military expenditure by SIPRI, arms imports by ACDA and per-capita GDP from Penn World Tables (available in appendix A.4). Arms imports and military expenditure are in constant 1990 million U.S. dollars and per-capita GDP is in constant 1990 U.S. dollars. The former three variables are transformed to U.S. dollars with the use of average annual exchange rates whereas per-capita GDP only is transformed with the use of Purchasing Power Parities. The implicit price index for imported arms is calculated by dividing the ACDA measurement of the value with SIPRI volumetric estimate of imported arms. Unlike chapter 4, however, the estimated equation is not logarithmic but the variables are in levels.

Systematic measurements of countries' production of arms are not generally available, with adequate coverage and quality for the needs of our study. As an approximation we may use available figures on largest arms producing firms and, also, tables on arms exports, assuming that a permanently high level of arms exports indicates for a well developed domestic industry in the country. This information is available in the two tables by SIPRI Yearbook 1993 '100 Larger Arms Producing Companies' [pp.470-4] and '25 Leading Suppliers of Major Conventional Weapons' from SIPRI Yearbook 1995 [p.493]. From all countries in the sample 10 countries have entries in both tables. These are Canada, France, Germany, Israel, Italy, Netherlands, Spain, Switzerland, UK and USA. Only in the table of 100 larger companies were India and Japan and only in the table of 25 larger exporters were Brazil, China, South Korea, Norway, Pakistan and Poland. Thus, these 18 countries were characterised as having an established domestic sector leaving the remaining 21 counties characterised as net importers.

Table 6.1 presents the results from the estimation of (22) with OLS and, also, the solutions for the coefficients of the model from (23), (24), (25), (26), (27) and (28) including the standard errors and the p-values of the two-tailed tests for the significance from zero. The $R^2$ of the regression is 0.655 and the standard tests of the residuals take relaxing values. In particular, White's F-test for heteroscedasticity gives a p-value 0.908.
to the hypothesis of homoscedasticity, Ramsey's F-test for functional miss-specification gives a p-value 0.209 to the hypothesis of correct linear specification and the $\chi^2$ test for the hypothesis of normal distribution of the residuals has a p-value of 0.209.

Table 6.1 Estimated coefficients

<table>
<thead>
<tr>
<th></th>
<th>$m_0$</th>
<th>$m_1$</th>
<th>$m_2$</th>
<th>$m_3$</th>
<th>$m_4$</th>
<th>$m_5$</th>
<th>$m_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coeff.</td>
<td>524.74</td>
<td>0.1347</td>
<td>78.027</td>
<td>-0.1747</td>
<td>-0.1292</td>
<td>-110.00</td>
<td>0.0117</td>
</tr>
<tr>
<td>St. Err.</td>
<td>179.70</td>
<td>0.0250</td>
<td>31.754</td>
<td>0.0102</td>
<td>0.0251</td>
<td>118.53</td>
<td>0.0153</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\kappa_M$</th>
<th>$\kappa_D$</th>
<th>$\Gamma_L$</th>
<th>$\gamma_M$</th>
<th>$\gamma_L$</th>
<th>$\gamma_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coeff.</td>
<td>0.0056</td>
<td>0.9585</td>
<td>0.0359</td>
<td>90.391</td>
<td>0.1296</td>
<td>0.8946</td>
</tr>
<tr>
<td>St. Err.</td>
<td>0.0022</td>
<td>0.0178</td>
<td>0.0159</td>
<td>35.775</td>
<td>0.06371</td>
<td>1.9655</td>
</tr>
<tr>
<td>p-value</td>
<td>0.014</td>
<td>0.000</td>
<td>0.031</td>
<td>0.016</td>
<td>0.050</td>
<td>0.652</td>
</tr>
</tbody>
</table>

The two coefficients $\kappa_M$, $\kappa_D$ and $\kappa_L$ have values within the acceptable range (0, 1) and their signs are well supported by the t-tests. Assuming that $\kappa_L = 1 - \kappa_M - \kappa_D$, testing for the significance of this coefficient from zero is equivalent to the test for significance of the sum of the two other coefficients from unity. Subsistence quantities $\gamma_M$ and $\gamma_L$ are also positive with signs significant from zero at 5% level. Note that as a result of the normalisation of the units of labour equal to the average output of the arms industry, coefficient $\gamma_L$ is measured as a proportion of the domestic procurement of the country. The last estimated parameter subsistence quantity $\gamma_D$ instead of the expected negative has a positive estimated value, which implies a contradiction with one of the fundamental assumptions of the model. Comparing the low statistical significance of $\gamma_D$, however, with the high significance of the sign of all other coefficients we may well conclude that the hypothesis of a negative $\gamma_D$ is not contradicted by the data.

The empirical results so far appear either in line or at least not contradictory to the hypotheses of the model. It should be clear that the scope of this econometric examination is not an in depth empirical study of the arms trade, but merely a test for the applicability of the theory. The weak aspects of these results include the over-identification of the estimated system, measurement errors in the variables, limited information about domestic procurement. Another reason might be that our model limits the establishment of arms industry to have the same fixed cost for all countries.
Apparently, this typical simplifying assumption (e.g. used in LS) contributes to the divergence of this theory from the empirical reality, the examination of which probably requires a more flexible specification. This is partly relaxed in the analysis of the following chapter 7, which assumes different fixed costs for large and smaller producing countries.

6.6 Conclusions

Setting up a model for the international arms trade one can hardly fail to realise that any single analytical attempt would be far from explaining all the underlining mechanisms of this field of activity. I could be useful to clarify, therefore, that the suggested analysis is not intended to provide a stand-alone theory of the arms trade, but it is merely the result of an attempt to capture particular aspects of the issue that appear to be neglected by other work. The final form of the model developed in this chapter has been largely based on the technical assumption of modifying Stone-Geary function in a way that it provides the desired properties of substitution. As a newly introduced method little tested yet, however, it should be looked from a critical perspective. From the results, however, it appears that the performance of the developed framework corresponds to the initial requirements.

In line with empirical evidence, the analysis in this chapter suggests that the establishment of a domestic industry of arms may have a considerable effect on the demand of countries for internationally traded arms. With the establishment of a domestic sector domestic arms substitute for imported arms and the demand for the latter category becomes less responsive to the military budget of the country. Also, the effect of economic development on this demand changes from positive to negative. The effect of the price of arms imports, however, appears to remain always negative and unaffected by the existence of a domestic industry. These results appear to provide suitable answers to the questions brought up by the cross-sectional examination of chapter 4.

The econometric estimation of the developed framework with available data provided useful evidence for the assessment of the performance and the applicability of this analysis in empirical studies. The high estimated value of the weight on domestically produced arms indicates that the establishment of a domestic industry and the availability of domestic along with imported arms has a positive effect on the returns to
scale of defence industry. There is also evidence for considerable subsistence quantities of imported arms and labour, but not enough evidence to support the negative sign of the subsistence quantity of domestic arms. All other coefficients had strong statistical support of their signs and small standard errors. One possible way that this study could continue is the examination of a wider theoretical framework, introducing a welfare function of the central government. This would allow for a more realistic distribution of resources between defence and other sectors and the relaxation of the assumption of a prefixed defence budget. Another more complete approach would be to use the developed formulation of the demand side in a world market model. In terms of econometrics, the availability of data series of arms production would enable the estimation of a complete version of the model.
7. A Generalized Arms Trade Game: The Establishment of Domestic Arms Industries

7.1 Introduction

Economic reality is usually far too complicated to be adequately explained by any single analytical set-up. Theoretical models require abstraction, so that they can isolate specific mechanisms and analyse a few at a time. The development of some new and more powerful theory usually stimulates a critical attitude and an appetite for a more realistic approach that eliminates some of the weaknesses of previous theory. The contents of this chapter are the result of an attempt to upgrade the existing economic analysis of the international world market by introducing a more powerful analytical tool that captures includes a domestic sector of arms as an optional input in defence industry. More specifically, this chapter generalises the Levine & Smith [1997] (LS) model of the international arms trade, by introducing a decision mechanism for entry of countries to domestic production.

There are two main reasons that make LS theory an appropriate platform for further analytical developments. First, it examines the equilibrium of the international market of arms, taking account of the special nature of both demand and supply sides. Second, the formulation of national security it employs for the supply side is adapted to the post-Cold War era. Instead of the previously established bipolar antagonism of alliances, large arms producing countries are not involved in arms races but instead they collaborate for the purposes of regional security. On the empirical side, the econometric examination of the demand for imported arms in chapters 4 and 5 revealed some new aspects of empirical reality, requiring theoretical explanation.

In LS the world is divided into two groups of identical countries: One group consists of a small number of supplying industrialised countries (ICs), which produce arms for the needs of domestic procurement and for exports. Producer countries are self-sufficient in the sense that domestically produced arms are the only arms these countries use to produce military capability. These countries are not involved in arms races and they are concerned with regional peacekeeping. For that purpose these countries prepare for military intervention, in case that one or more pairs of countries
in the other group get engaged in conflict. The other group consists of a larger number of arms-buying countries. The economies of these countries are of considerably smaller size and of lower level of development and their defence relies on imported arms (LDCs).

The objective of this work is the relaxation of two basic simplifying assumptions of LS theory: the assumption that all of these countries have the same economic size and the assumption that LDCs do not have the option to produce arms domestically. A direct result of the first extension is that the income, consumption, military capability, procurement and demand for arms of recipient countries varies rather than being identical for all the countries in the group. A result of the second extension is that LDCs consider the establishment of a domestic industry of arms, dependent on cost factors, their economic size and the price of international traded arms. The combination of these two extensions to LS model determines the number of LDCs' that enter to domestic production of arms, leaving the remaining countries of this group as net importers. The result is a generalisation of LS model, which includes an endogenous solution of the number of countries on the demand and supply side of the market.

The presentation of this chapter is organised in four sections. Section 7.2 looks in more detail into LS theory, providing a brief review of its characteristics and an outline of its strengths and weaknesses. Section 7.3 presents analytically the developed model. First it presents the LS set-up of supply side followed by the description of the demand side and a description of the overall equilibrium. Section 7.4 examines in more detail the analytical properties of the developed model, attempts an analytical comparison with LS results and prepares for the numerical examination that follows. Finally, section 7.5 discusses the performance and the predictions of the model based on the results of numerical simulations.

7.2 Strengths and weaknesses of the Levine-Smith approach

The LS model assumes that the world is separated into two groups – a small number of large, industrialised and identical countries on the supply side and a large number of small less developed and also identical countries on the demand side. Arms production of supplier countries is regulated and subsidised by their governments. The barrier that inhibits new entrants into the market is the generally large R&D costs of this production. In the post-Cold War era, arms producing countries do not directly perceive military threat from each other and so they can be seen as not involved in
arms races. They have concerns about regional security, however, which depends (inversely) on the amount of weapons held by the LDCs, and they are jointly preparing for intervention in case of one or more conflicts between them (i.e. they anticipate a role for international policing). Supplier countries increase their security when either their own military capability or the regional security increase, or both. Security and private consumption are the two types of expenditure that absorb industrialised countries’ national income, which is exogenous and remains unaffected, and are the two inputs to national welfare. The final allocation of resources is such that the opportunity cost of the two inputs is equal.

The demand side consists of a large number of net arms importing countries, which are all involved in conflict in pairs. They are all identical having the same income, which is predefined but is considerably lower than that of ICs. These countries are involved in arms races in the sense that the security of each recipient country depends on their on procurement and inversely on their opponents. As for ICs they optimise their public welfare controlling expenditure in defence. Putting together the demand and supply side, LS solve for the equilibrium of the market for the case that the ICs co-operate in their defence policies and for the case they do not.

Several characteristics of the LS model received criticism, which helpfully was reported in the publication following the paper [pp. 362-367]. Of course, the referees of the paper and other critics had access to a more complete version of the paper in terms of analysis than was finally released. An expected target of critics was that the model is static not distinguishing between stocks and flows of arms, suggesting the need to introduce dynamics. Another suggestion, typical for this topic, is that since recipient countries are not aggressive (i.e. they do not derive utility from attacking neighbours) a number of minor alterations to assumptions result in an equilibrium with zero level of armament. David Begg replied to this criticism by arguing that the equilibrium with no procurement is unstable and does not derive feelings of security to the members of public. The need for defence emerges from the uncertainty of possible aggressive behaviour of a neighbour country, a concern that is historically well justified.

Directly related with the work of this paper are the comments LS paper received about the option of countries to establish their own production of arms if arms produced cost less than the imported arms. Lucrezia Reichlin [p. 363], Patrick Rey [p. 365] and Ron Smith (in his reply [p.367]) point out the need to consider this alternative to imports and endogenise this entry. The decision to establish of a domestic industry was suggested to be dependent on the cost factors of this
production and the level of international price of traded arms. This is the main objective of this study. Furthermore, chapter 4 estimated that the reaction of countries' demand for imported arms to the military budget has a positive slope but convex shape. It is elastic at low levels of military expenditure (and income) but tends to become completely inelastic when we reach the largest expenditure levels (that of US). The same convex relationship appears between the demand for imported arms and exports. According to LMS this effect is primarily created by substitution of imported with domestic arms, the unitary price of which falls when output level rises. The two diagrams in figure 4.3 of help demonstrating this relationship.

7.3 The model

Any attempt to generalise an existing theory implies that the new and wider structure encompasses the preceding one as well as including other additional components. In this sense the model we build in this section includes the entire LS model, as well as a number of new components. There are few changes of notational conventions, from those of LS. In LS the two exogenously defined country groups are called 'suppliers' and 'buyers,' noted by indices 's' and 'b.' In our model Less Developed Countries (LDCs) are not necessarily importers or 'buyers' and always producing 'Industrialised Countries' (ICs) may not be the only exporters. Keeping the same notation for the two groups, indices s and b now denote ICs and LDCs respectively. Only when we need to distinguish between importing and producing LDCs we use indices 'm' and 'p' respectively.

7.3.1 Industrialised countries

All n_s countries in the group of suppliers are assumed to be identical and the following description applies to any of them. As a result of this we use indices to denote a specific supplier country only when necessary. If g_s is national procurement and x_s is exports of each supplier country y = g_s + x_s is the total production of arms. For purposes of simplicity we may assume a linear cost function for the production of arms identical for each producer

\[ C_s(y) = D_s + c_s y + d_s y^2 \]

where \( D_s \) is the fixed cost, required for the establishment of this industry and \( c_s \) is the marginal cost of this production.
Let $P$ be the price of internationally traded arms and $p_s$ be the procurement price that government buys domestically produced arms. The profits of supplier country's arms industry per period are

$$\Pi_s = p_s g_s + P x_s - C_s(y) = \pi_s(p_s, P) - D_s$$

where $\pi_s(p_s, P)$ denotes the operating profits, dependent on the two prices. The participation constraint for this industry requires $\Pi_s \geq 0$, whereas by controlling the procurement price $p_s$ government makes sure that the industry breaks even, taxing all profits or subsidising all losses. Thus, $\Pi_s = 0$ and applying (2) in (1) we obtain

$$p_s = \frac{C_s(y) - P x_s}{g_s} = p + \frac{D_s - \pi_s(P, P)}{g_s}$$

where $w_s$ the national wage.

The production of military capability of supplier countries uses two inputs, assumed perfect complements and employed in a fixed proportion. If $L_s$ is labour employed in national defence, with the use of Leontief production function and appropriate adjustment of measurement units $L_s = g_s$. As a result of that, the expenditure of defence sector becomes $w_s L_s + p_s g_s = (w_s + p) g_s$. Let $B_m = (w_s + p) m$ be the budget of defence of IC. National governments of supplier countries make decisions for the distribution of national income $Y_i$ between defence expenditure $B_m$ and other spending, say consumption $C_s$, facing a resource constraint

$$Y_i = C_s + (w_s + p) g_s$$

Assuming an adequately small military sector we may ignore any effects of resource allocations in defence to national income and factor prices. Thus, we define $Y_i$ and $w_s$ as fixed quantities, exogenous to the model.

The social welfare of ICs follows a Cobb-Douglas welfare function

$$U_s = C_s^{\alpha_s} S_s^{1-\alpha_s}, \alpha_s \in (0,1)$$

where $C_s$ is consumption and $S_s$ is security and $\alpha_s$ is a coefficient. Supplier countries have a security function with two components

$$S_s = (\alpha_s Y_i + \beta_s g_s)^{1-\mu} S_s^{\mu}, \mu \in [0,1]$$

The first term (in parenthesis) is the domestic component of security, where $\alpha_s Y_i$ and $\beta_s g_s$ capture the fixed and variable benefits of defence respectively and $\mu$ is a coefficient. This domestic security function does not contain an arms race component implying that supplier countries do not perceive a direct military threat.

The second component of (6) is regional security and, as in LS, it is defined as

$$S_r = G_s - r \beta_s \frac{K_r}{n_b}$$
Where $G_s = \sum_{i=1}^{n} g_{ni}$ is the aggregate procurement of all ICs. Similarly to LS, parameter $r$ captures the number of regional conflicts that producer countries can successfully intervene at a time and $\beta$ is a coefficient similar to $\beta$. $K_r$ represents the aggregate military capability of all LDCs and $K_{r/n_b}$ the average. In the simpler case of LS, all recipient countries are identical and the average military capability of recipients depends only on the arms exported by ICs. In our case, the armaments of LDCs include their own output, which varies from one country to another and can either be retained as procurement or exported to other LDCs. Thus, we may define regional security as

$$K_r = M + G_b$$

where $M$ is the aggregate imports and $G_b$ is the aggregate procurement of importing and self sufficient LDCs respectively.

The governments of supplier countries control $g_i$ and $x_i$ with an objective to maximise social welfare. Let $g_{ni}, x_{ni}$ and $p_{ni}$ be procurement, exports and procurement price of supplier country $i$. The first order conditions for the maximisation of (5) are

$$\frac{\partial U}{\partial x_i} = \frac{-\omega_i}{Y_s - (w_s + p_s)g_{ni}} \frac{\partial p_{ni}}{\partial x_i} + \mu(1 - \omega_s) \frac{\partial S_r}{\partial x_i} = 0$$

and

$$\frac{\partial U_{ni}}{\partial g_{ni}} = -\omega_i \frac{w_s + p_s + g_{ni}}{Y_s - (w_s + p_s)g_{ni}} + (1 - \omega_s) \left[ \frac{(1 - \mu)\beta_s}{\alpha_s Y_s + \beta_s g_{ni}} + \mu \frac{\partial S_r}{\partial g_{ni}} \right] = 0$$

Let $X_s = \sum_{i=1}^{n} x_{ni}$ be the aggregate exports of the whole group of ICs. If $X_s$ are the total arms exports of LDCs let $X = X_s + X_b$ be the total world exports. In a non-cooperative Nash settlement $\partial X/\partial x_{ni} = \partial(x_{ni} + \Sigma_{j\neq i} x_{ji} + X_b)/\partial x_{ni} = 1$ and $\partial G_s/\partial g_{ni} = \partial(g_{ni} + \Sigma_{j\neq i} g_{ji})/\partial g_{ni} = 1$. If producer countries cooperate in a common defence policy, taking an equal share of the world exports market they set $x_{ni} = x_s$ and $g_{ni} = g_s$. Then they perceive $X_s = n_s x_s$ and $G_s = n_s g_s$, so $\partial X/\partial x_{si} = n_s$ and $\partial G/\partial g_{si} = n_s$. Following the neat notation of LS we may define $\Phi$ as

$$\Phi = \frac{\partial X}{\partial x_s} = \frac{\partial G_s}{\partial g_s}$$

that is $\Phi = n_s$ and $\Phi = 1$ in the cases with and without coordination respectively. From (3) we obtain
\[
\frac{\partial p}{\partial x_{si}} = \frac{c + 2d(g_n + x_n) - P - x_n}{g_n}
\]
and
\[
\frac{\partial p}{\partial g_{si}} = \frac{c + 2d(g_n + x_n) - P_n}{g_{si}}
\]
and with the use of (11)
\[
\frac{\partial P}{\partial x_{si}} = \frac{\partial P}{\partial X} \frac{\partial X}{\partial x_{si}} = P'(X)\Phi = \frac{\Phi}{X'}
\]
where \(X = \partial X/\partial P\). The solution for the allocations of ICs will be completed at the end of this section, after the discussion about LDCs.

### 7.3.2 LDCs involved in conflict

The general profile of recipient countries in this model has two mainly differences from LS. First, still organised in pairs of identical countries, the national income of countries across different pairs is not limited to be equal but is allowed to vary. Second, the countries in this group face the option to establish their own production and fully substitute imported arms, providing they can face the set-up cost of this industry. In total there are \(n_b\) pairs of recipient countries. Each country in the \(i\)-th pair has income \(Y_{bi}\), which takes a value in the range \([Y_L, Y_H]\). For the purposes of simplicity in this paper we will assume that \(Y_{bi}\) follows a uniform distribution. Let \((Y_L + Y_H)/2\) be the mean of the distribution and \(\Delta = (Y_H - Y_L)/2\) be the deviation of the extremes from the mean (implying that \(Y_L = Y_b - \Delta\) and \(Y_H = Y_b + \Delta\)). Furthermore, assume that countries are ordered according to their income, i.e. that \(i > j \Leftrightarrow Y_{bi} > Y_{bj}\).

Government's objective is the maximisation of social welfare
\[
U_{bi} = C_{bi} S_{bi}^{1 - \phi_b},
\]
where \(C_{bi}\) is consumption and \(S_{bi}\) is security. As in LS, the security function of each country has the form
\[
S_{bi} = \alpha_b Y_{bi} + \beta_b m_i - m_i^*,
\]
where \(m_i^*\) is the imports of the opponent country \(\alpha_b > 0\) and \(\beta_b \geq 1\) are coefficients similar to \(\alpha\) and \(\beta\), capturing the fixed and variable benefits of defence respectively. Each country in the pair imports quantity \(m_i\) of arms. The same expression holds for both rivals. All recipients are assumed to face the same price \(P\) of internationally traded arms. Using the same Leontief production function of military capability as for
producer countries, each unit of arms requires a unit of labour. Let $B_m = (w_s + p_b)m$ be the defence budget of importing LDCs. The government of each country in pair i allocates income between consumption $C_{bi}$ and military spending $B_m$ facing the national resource constraint

$$Y_{bi} = C_{bi} + (w_{bi} + P)m_i$$

In a symmetric Nash equilibrium countries maximise (15) with respect to $m_i$ under the constraint (17) and, as in LS, from the first order condition we can solve for the demand for imports

$$m_i = m(Y_{bi}) = AY_{bi}$$

where

$$A = A(P) = \frac{(1 - \omega_b) \beta_b - \omega b \alpha_b (w_b + P)}{(\beta_b - \omega_b) (w_b + P)}$$

and

$$A' = \frac{\partial A(P)}{\partial P} = \frac{-(1 - \omega_b) \beta_b}{(\beta_b - \omega_b) (w_b + P)^2}$$

Factor A and its derivative of $A'$ have been determined for purposes of simplicity as they are often used in the analysis that follows.

### 7.3.3 Arms producing LDCs

Coming to the essence of the extension of this work beyond the LS model, an LDC has the option to establish a domestic industry of arms and produce for the needs of domestic procurement, as well as export. Following the assumptions of supply side, we assume linear technology with a cost function

$$C_b(q_i) = D_b + c_b q_i$$

where $C_b(q_i)$ is the total cost for the first country of pair i, $D_b$ is the fixed costs for the establishment of this industry and $c_b$ is marginal cost. Notice that the existence of setup cost causes diminishing average cost in this production.

Domestic and imported arms are assumed as perfect substitutes in countries' procurement and the production of military capability. In order to make a decision about the establishment of a domestic arms industry LDCs compare the total cost of the output with that of importing the same quantity of arms. If the unitary cost of domestic arms exceeds the price of imports, i.e. $Pm_i < C_b(m_i)$, the government cancel the establishment of domestic production and the country remains a net recipient of arms. Otherwise, if $Pm_i > C_b(m_i)$ instead of importing arms they produce them domestically and possibly export as well. In this setting, however, we rule out the option of LDCs to export for reasons we discuss later. When $C_b(q_i) = Pm_i$, countries are indifferent between the two options. Setting $q_i = m_i$, with the use of (20) we may
solve for the critical level of imports \( m^* = D(P - c_b) \). Furthermore, with the use of (18) and (19) we may solve for the critical for the establishment level of income

\[
y^* = \frac{D_b}{A(P - c_b)}
\]

Since (18) defines a monotonic transformation of \( Y_b \) to \( m \), \( Y^* \) takes a unique solution from (21). According to this setting, if this critical level falls within the range of income of LDCs, i.e. if \( Y^* \in [Y_L, Y_H] \), it separates their population in two groups: 2\( n_m \) countries with income less than \( Y^* \) that merely import arms and 2\((n_b - n_m) \) countries that produce and export. If \( Y^* < Y_L \) all LDCs are recipients and if \( Y^* > Y_H \) they are all producers. Generally, country \( i \) produces arms iff \( Y_i > Y^* \). Thus, under the assumption that \( Y_i \) distributes uniformly in \([Y_L, Y_H]\) the number of LDC importers is given by 2\( n_m \) where

\[
n_m = \begin{cases} 
\frac{n_b}{Y_H - Y_L} ; & Y^* \geq Y_H \\
\frac{n_b}{Y_H - Y_L} \left( \frac{D_b}{A(P - c_b)} - Y_L \right) ; & Y_L < Y^* < Y_H \\
0 ; & Y^* \leq Y_L 
\end{cases}
\]

providing that that \( 0 \leq Y_L \leq Y_H \). Notice that the necessary condition for the intermediate case implies that \( Y_H - Y_L > 0 \), i.e. a non-trivial distribution of \( Y_{bi} \).

In a similar way as for industrialised supplier countries, instead of (16) the security function of producer LDCs becomes

\[
S_{bi} = \alpha_b Y_{bi} + \beta_b g_{bi} - g_{bi}^* 
\]

where \( g_{bi} \) and \( g_{bi}^* \) are domestic procurements of the country and its opponent respectively. Thus, instead of (17), the resource constraint of producing LDCs becomes

\[
Y_{bi} = C_{bi} + (w_{bi} + p_{bi})g_{bi}
\]

In this examination we consider only ICs having access to the international market whereas LDCs do not face this option but produce only for the needs of their domestic procurement. The reason for this discrimination is that if we allow LDCs to freely compete with the ICs, the model predicts that they take over the largest part of the market. This contradicts the empirical evidence, apparently because the world market is not price competitive. This interesting result is further discussed later in this paper. The working assumption in this set-up is \( x_b = X_b = 0 \) and the profits of LDC’s arms industry are

\[
\Pi_{bi} = p_{bi} g_{bi} - C_{bi} (q_i) = \pi_b (p_{bi}, P) - D_b
\]
where $\pi_b(p_b, P) = (p_{bi} - c_i)g_{bi}$ are the operating profits and $q_i = g_{bi}$. The profits of the industry vanish if the government offers to the firm the procurement price

\[ p_{bi} = \frac{C_b(q_i)}{g_{bi}} = P + \frac{D_b - \pi_b(P, P)}{g_{bi}} = c_b + \frac{D_b}{g_{bi}} \]

For a welfare maximising government, the FOC for optimal allocation of procurement is

\[ \frac{\partial U_{bi}}{\partial g_{bi}} = \frac{w_b + p_{bi} + g_{bi}}{Y_b} - \frac{w_b + c_b}{Y_b} + (1 - \omega_b)\beta_b \alpha_b Y_{bi} + \beta_b g_{bi} - g_{bi} = 0 \]

From (26) we solve for $\partial p_{bi}/\partial g_{bi} = -(p_{bi} - c_b)/g_{bi}$. Substituting for $p_{bi}$ in (27) from (26) and applying symmetry ($g_{bi} = g_{bi}$), after simplifications LDCs procurement takes the form

\[ g_{bi} = g_{bi}(Y_{bi}) = \Lambda_y Y_{bi} + \Lambda_y'(w_b + c_b)D_b ; n_m < n_b \]

where

\[ \Lambda_y = \Lambda(c_b) = \frac{(1 - \omega_b)\beta_b - \omega_b \alpha_b (w_b + c_b)}{(\beta_b - \omega_b)\lambda} \quad \text{and} \quad \Lambda_y' = \frac{\partial \Lambda(c_b)}{\partial c_b} = \frac{- (1 - \omega_b)\beta_b}{(\beta_b - \omega_b)(w_b + c_b)^2} \]

Finally, providing the solution for $g_{bi}$, we may eliminate it from (26) and complete solution for procurement price (see summary of the model at the end of this section).

### 7.3.4 Market equilibrium

Having set out the profiles of the agents involved in our arms trade game, in this section we put demand and supply together and determine the complete form of our model. First, we examine the aggregates of demand side under the assumption that income of importing LDCs follows a uniform distribution. In the case that $Y^* < Y_L$ the group of importers has no members. If $Y_L < Y^* \leq Y_H$ we may express the average national income of importing LDCs as $Y_m = (Y_L + Y^*)/2$. Using the wider definition of $n_m$ we may substitute for $Y^*$ after solving from it from (21) and express $Y_m$ as

\[ Y_m = \frac{1}{n_m} \sum_{i=1}^{n_m} Y_{bi} = Y_b - \frac{n_b - n_m \Delta}{n_b} ; n_m > 0 \]

If $M = 2 \sum_{i=1}^{n_m} m_i$ is the aggregate imports of LDCs with the use of (18) and (31) the average imports of LDCs $m = M/2n_m$ take the form

\[ m = m(Y_m) = \Lambda Y_m \]

The producing group of LDCs has average income
\[
Y_p = \frac{1}{n_b - n_m} \sum_{i=n_m+1}^{n_b} Y_{bi} = Y_b + \frac{n_m}{n_b} \Delta \quad ; n_m < n_b
\]

Total procurement \(G_b = 2 \sum_{i=n_m+1}^{n_b} g_{bi}\) and average procurement

\[
g_b = g_b(Y_b) = \Lambda_Y Y_p + \Lambda'_Y (w_b + c_b) D_b \quad ; n_m < n_b
\]

Summing up the aggregate exports of ICs and under the assumption that producing LDCs do not export we can put the demand supply together and define equilibrium in this market

\[
X = n_x x_x = M = 2n_m m
\]

Assuming that both groups of producing countries are fully informed about the market structure, the exports of exporting countries influence the world price and the demand in two ways. One is directly through the responsiveness of the demand of the existing importing countries and the other is indirectly by influencing the number of producing LDCs and importing LDCs. If the group of producing LDCs is empty \((n_m = n)\) the latter effect vanishes, reducing to the LS case. Since \(X = M\), substituting for \(n_m\) from (22) and (21) and differentiating the RHS of (34) with respect to \(P\) the relative responsiveness of the aggregate demand to price takes the form

\[
\frac{X'}{X} = \frac{\partial M/\partial P}{M} = \frac{\partial n_m/\partial P}{n_m} + \frac{\partial m/\partial P}{m} = \left(\frac{2}{P - c_b} + \frac{\Lambda'_Y}{\Lambda}\right) \quad ; 0 < n_m < n_b
\]

Returning to the discussion about regional security we may now look closely at the LDCs' regional military capability. From the definitions of \(S_r\) and \(K_r\) it is clear that procurement of supplier countries has no direct effect on military capability of LDCs. Thus, differentiating (7) with the use of (11) we obtain

\[
\frac{\partial S_r}{\partial G_s} = \frac{\partial S_r}{\partial G_s} = \Phi
\]

Likewise procurement, exports of supplier countries also have no direct effect on procurement and exports of LDCs, so \(\partial K_r/\partial x_{st} = \partial X/\partial x_{st} = \Phi\) and

\[
\frac{\partial S_r}{\partial x_{st}} = -\frac{r}{n_b} \beta_x \Phi
\]

This completes the solution of all the components of this model.

The entrance of LDCs to domestic production depends on the average cost of the demanded quantity of arms and, finally, on the economic size of each country. If it exceeds the critical level \(Y^*\) countries produce and vice versa. In other words, if the critical level for entrance lies within the income range of LDCs, i.e. \(Y^* \in (Y_L, Y_H)\), it
separates it in two groups: the group of producers with higher income and that of importers with lower income. This happens if, and only if, the condition

\[ Y_L < \frac{D_b}{\Lambda(P - c_b)} < Y_H \]

is satisfied. If the RHS of inequality (38) does not hold, there are no producing LDCs. In this case the model reduces to a case similar to LS with the additional characteristic that LDCs are no more limited to be identical. If LHS of (38) does not hold, all countries enter into domestic production, which is a rather unrealistic case.

7.3.5 Summary of the model

Putting together the various pieces of this model we may summarise it as follows.

i. Number of importing LDCs

\[ n_m = \begin{cases} 0 & ; D_b / \Lambda(P - c_b) \leq Y_b - \Delta \\ \frac{D_b}{2\Delta} \left( \frac{1}{\Lambda(P - c_b)} - \frac{1}{Y_b - \Delta} \right) & ; Y_b - \Delta < D_b / \Lambda(P - c_b) < Y_b + \Delta \\ \frac{D_b}{n_b} & ; D_b / \Lambda(P - c_b) \geq Y_b + \Delta \end{cases} \]

ii. ICs’ procurement price from (21) and (22)

\[ P_s = P + \frac{D_s - (P - c_s)(g_s + x_s) + d(g_s + x_s)^2}{g_s} \]

iii. ICs’ regulated exports from (7), (9), (11), (12), (14), (37) and the definition of \( \rho \)

\[ \begin{cases} \frac{c_s + 2d(g_s + x_s) - P}{Y_s - (w_s + P_s)g_s} = \frac{(1 - \omega_s)\mu}{\beta_s} & ; n_m > 0 \\ x_s = 0 & ; n_m = 0 \end{cases} \]

iv. ICs’ procurement from (7), (10), (11), (13) and (36)

\[ \frac{\omega_s}{Y_s - (w_s + P_s)g_s} [w_s + c_s + 2d(g_s + x_s)] = \frac{(1 - \omega_s)(1 - \mu)\beta_s}{\alpha_s Y_s + \beta_s g_s} + \frac{(1 - \omega_s)\mu \Phi}{n_s g_s - r \beta_s K_r} \]

v. Market equilibrium using (31) and (22)

\[ X = n_s x_s = M = 2n_m m \]

Complementary definitions include LDCs’ average demand for imports

\[ m = \Lambda \left( Y_b - \frac{n_b - n_m}{n_b} \Delta \right) \]

the responsiveness of the demand to price
\[
\frac{X}{X'} = \begin{cases} 
\frac{2}{P - c_b} + \frac{A'}{A} \quad ; 0 < n_m < n_b \\
A/A' \quad ; n_m = n_b
\end{cases}
\]

the regional military capability

\[(8') \quad K_r = 2n_mm + 2(n_b - n_m)g_b\]

the average procurement of producing LDCs

\[(33) \quad g_b = \begin{cases} 
A_p \left( Y_b + \frac{n_m}{n_b} \Delta \right) + A'_p (w_b + c_b)D_b \quad ; n_m < n_b \\
0 \quad ; n_m = n_b
\end{cases}\]

and the definitions of \(A, A', A_p\) and \(A'_p\)

\[A = \frac{(1 - \omega_b) \beta_b - \omega_b \alpha_b (w_b + P)}{(\beta_b - \alpha_b)(w_b + P)} \quad \text{and} \quad A' = \frac{-(1 - \omega_b) \beta_b}{(\beta_b - \omega_b)(w_b + P)^2}\]

\[A_p = \frac{(1 - \omega_b) \beta_b - \omega_b \alpha_b (w_b + c_b)}{(\beta_b - \omega_b)(w_b + c_b)} \quad \text{and} \quad A'_p = \frac{-(1 - \omega_b) \beta_b}{(\beta_b - \omega_b)(w_b + c_b)^2}\]

The above system of 5 equations solves for 5 of the endogenous variables: \(n_m, P, p, g_b\) and \(x_i\). It is a non-linear system, which does not appear to take a complete algebraic solution. Instead, it can be solved with the use of numerical methods.

Having calculated a solution for the price of arms \(P\), we may then derive the individual solutions for each LDC. For the completeness of this summary these are

vi. Importing LDCs' demand

\[(18) \quad m_i = AY_i\]

vii. Producing LDCs' procurement

\[(28') \quad g_{bi} = \begin{cases} 
A_p Y_{bi} + A'_p (w_b + c_b)D_b \quad ; n_m < n_b \\
0 \quad ; n_m = n_b
\end{cases}\]

viii. Producing LDCs' procurement price

\[(26) \quad P_{bi} = c_b + \frac{D_b}{g_{bi}} \quad ; n_m < n_b\]

These four additional equations provide vectors of solutions for the remaining variables \(\{m_i\}, \{p_{bi}\}\) and \(\{g_{bi}\}\).

In a summary, the parameters of the model are \(D_s, D_b, c_s, c_b, d, w_s, w_b, \omega_s, \omega_b, \alpha_s, \alpha_b, \beta_s, \beta_b, \beta_r, \mu, r, \Phi, n_s, n_b, Y_s, Y_b\) and \(\Delta\) (or equivalently \(Y_L\) and \(Y_H\)). It is noteworthy that in (22) the necessary for \(0 < n_m < n_b\) condition \(Y_H > Y^* > Y_L\) implies that \(Y_H > Y_L\), i.e. that the distribution of LDCs income is not trivial. This relaxes any possible
worries for division by zero in expressions (22), (35) and (9'). If \( Y_H = Y_L n_m \) either equals 0 or \( n_b \) and providing that \( n_m \) is non-zero the model reduces to the LS case.

7.4 Examination of model

The summary of the previous section displays the set-up of our model but there is still need to examine its internal consistency and capacity to produce results with desirable properties. Since this model incorporates the complete LS model this examination builds upon the existing experience of LS’s examination. It focuses on the performance of the model in the presence of the newly introduced components and assesses the capacity of the model to reproduce the observed stylised facts. The examination of the model includes two stages: an analytical followed by a numerical.

Imposing in this examination that a number of LDCs enter the domestic arms production, let \( s = n_n/n_b \) be the ratio of importing LDCs, which implies that \( 1 - s = (n_b - n_m)/n_b \) is the ratio of the remaining arms-producing LDCs.

In our model (as in LS) the world is divided in two parts ICs and LDCs, let

\[
\theta = \frac{2n_b Y_b}{n_r Y_s}
\]

be the ratio of the total income of LDCs to that of ICs. To examine the behaviour of the model, we may avoid the implications arising from the utility function, by ignoring regional security concerns and setting \( \mu = 0 \). Solving from (9') we obtain

\[
MC_s = c_s + 2d(g_s + x_s) = P + \frac{X \Phi}{X' n_s} = MR_s(P)
\]

This condition implies that under Laissez Faire (\( \mu = 0 \)) industries export the quantity of arms that make marginal revenue equal to marginal cost. If the elasticity of demand is \( \alpha(P) = -(\partial M/\partial P)P/M = -PX/X \) we can express the marginal revenue of arms industry as \( MR_s(P) = P [1 - 1/n_s \alpha(P)] \).

Substituting for \( MR(P) \) in (10') we can solve for ICs’ procurement level.

\[
g_s = \frac{1 - \omega_s \alpha_s}{\beta_s \omega_s [w_s + MR_s(P)]}
\]

Using the definitions of defence budgets \( B_s \) and \( B_m \) the income share of ICs expenditure in defence takes the form

\[
r_s = \frac{B_s}{Y_s} = (w_s + p_s) \frac{g_s}{Y_s}
\]

for ICs and
\[ r_m = \frac{B_m}{Y_b} = \frac{(w_b + P)Y}{Y_b} \]

for the average importing LDCs. Without any loss of generality we may further assume that \( \Delta = Y_b \) so that the uniform distribution of LDCs' income varies from 0 to \( 2Y_b \). Under this simplifying assumption using (31') and (43) we arrive at

\[ m/Y_b = r_m/(w_b + P) = s\Lambda \]

The procurement price that ICs' governments purchase domestic arms includes either a subsidy of a tax, dependent if it is higher or lower than the international price \( P \). Let the subsidy (or tax if negative) of ICs be \( \tau_s = p_s - P \). Given \( \tau_s \), from (42) and (43) the income share of ICs procurement takes the form

\[ g_s = \frac{r_s}{w_s + P + \tau_s} \]

If \( r_x = P x_r/B_s = P x_s/(w_s + p_s)g_s \) is the share of exports to defence budget of the same group, solving with the use of (42) for the income share of exports we obtain

\[ \frac{x_s}{Y_s} = \frac{r_x r_s}{P} \]

In the context of this examination we may also simplify the model assuming that ICs produce under constant marginal cost, i.e. that \( d = 0 \).

Using available information we can roughly observe quantities \( r_s, r_{xs}, r_m, \) subsidy \( \tau_s \) and ratio \( s \) and adjust our model to meet these requirements. From these ratios we can calculate values for \( x_s/Y_s \) and \( m/Y_b \), using (44) and (46). The market cleaning condition (34), then, takes the form \( x_s/Y_s = s\alpha(m/Y_b) \). Based on the specified quantities we may further calculate the corresponding level of LDCs to ICs income ratio

\[ \theta = \frac{1}{s} \frac{x_s/Y_s}{m/Y_b} \]

Moving towards a complete re-parameterisation of the model, so that it can be brought in line with the observed stylised facts, we can determine the corresponding sizes of the remaining parameters. Relationships (41) and (44) hold simultaneously, involving a number of common parameters. If we also consider labour costs \( w_s \) and \( w_b \) and the variable benefits of defence \( \beta_s, \beta_b, \) and \( \beta_r \) as observable coefficients, limiting \( \alpha_s = \alpha_b \) and \( \omega_r = \omega_s \), we may then define appropriate values for these remaining parameters. Since (41) and (44) together with the definition of \( \Lambda \) in (19) consist a non-linear system of equations a possible solution of it requires the use of iterative numerical methods. The only as yet undefined coefficient of this system is LDCs marginal cost \( c_b \), which we examine later. For any given value of \( c_b \), however, we
may calculate the responsiveness of the demand side to prices $X/X$ and, thus, the corresponding value of $c_s$, using the $MC = MR$ condition (40) for ICs. This gives

$$c_s = P + \frac{X}{X'} \frac{1}{n_s}$$

The last two parameters remaining for definition are the set-up costs for the two groups. Applying $\Delta = Y_b$ to (22) we can determine the level of LDCs' set-up costs that corresponds to the desirable level of ratio $s$.

$$D_b/Y_b = 2s(P - c_b)\Delta$$

Last, given that $d = 0$ from the participation constraint (3) we can derive the corresponding cost of ICs industry as

$$\frac{D_s}{Y_s} = \left[ (1 + \tau_s) - c_s \right] \frac{\bar{g}_s}{Y_s} + \frac{P - c_s}{Y_s}$$

This completes the construction of the model in terms of observable and unobservable quantities and prepares the background for the calibration and numerical examination that follow.

7.5 Numerical results

We now turn to numerical results based on a baseline calibration. Since the system of equations, which solves for the endogenous variables of the model, is not linear we must calculate solutions using Gauss-Newton iterative method, providing that solutions exist. In order to allow for comparison of these results with those of LS we use the same values of parameters, in most of the cases. The parameters have different values only when the differences of the two models make it necessary. The rearrangement of the model in the previous section essentially prepares the baseline calibration of the simulations and also follows the LS method when possible.

In our baseline calibration, where possible we used the same values with LS. This applies to shares $r_s$, $r_{xs}$, $r_m$ and $r$, coefficients $\beta_s$, $\beta_b$ and $\beta_r$, labour costs $w_s$ and $w_b$, number of interventions $r$, numbers of ICs and couples of LDCs $n_s$ and $n_b$, coefficient of regional concern $\mu$, coefficient $d$ and the type of game $\Phi$. A newly introduced share is the ratio of importing LDCs $s$ and a new parameter is the spread of income distribution of LDCs $\Delta$. With these values and the method set out in the previous section we derive parameters $\omega_s$, $\omega_b$, $\alpha_s$, $\alpha_b$, $D_s$, $D_b$, $c_s$ and the relative income level $\theta$. The only parameter as yet without a value is critical for the entry of LDCs to production, the marginal cost $c_b$. Figure 7.1 displays the change of derived parameters
as a result of changing \( c_b \). Note that predefined shares are used only in the baseline calibration for the calculation of derived parameters, whereas these values of all coefficients (but not these shares) are used in the simulations.

**Figure 7.1 Calibrated coefficients as a function of \( c_b \)**

Figure 7.1 contains four diagrams showing the values of some calibrated magnitudes for a range of values of \( c_b \). The top left diagram gives ICs marginal cost \( c_t \) and 45° line, which shows the difference from \( c_b \). Notice that in order to keep the desirable shares imply that the marginal cost of ICs' has to stay higher than that of LDCs, even when \( c_b \) takes a large value near this of ICs. Left bottom diagram plots the calibrated parameters \( \omega_s, \omega_b, \alpha_t \) and \( \alpha_b \), which stay close to the LS levels with a smooth change. The inverted responsiveness of the demand \( XIX' \), at the right bottom plot, changes rather drastically compared to LS case, apparently as a result of LDCs option of autonomy. The top right diagram gives the derived set-up costs for the two groups, expressed as shares of each group's average income. Table I gives the values of all old and new, predefined and derived baseline values of parameters for a chosen value 0.58 of \( c_b \).

The LS paper provides evidence in support of the specific values of predefined parameters and shares. Namely, the shares correspond on average to the figures of
standard empirical databases, as SIPRI and ACDA, and beta coefficients correspond to the folk theorem that an attacker has to achieve an at least 3:1 superiority in military capability. A unitary deviation to average income ratio $\Delta/Y_b$ implies that LDCs’ uniform distribution of income spreads from zero up to level $Y_b$, a pattern which is not far from reality. The lower marginal cost of LDCs arms industry may be mainly attributed to lower labour costs and possibly cheaper raw materials.

### Table 7.1 Values of predefined and derived parameters

<table>
<thead>
<tr>
<th>Shares</th>
<th>$r_s$</th>
<th>$r_{xx}$</th>
<th>$r_m$</th>
<th>$r_s$</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.03</td>
<td>0.12</td>
<td>0.05</td>
<td>0.3</td>
<td>23/24</td>
</tr>
<tr>
<td>Predefined Parameters</td>
<td>$\beta_r$, $\beta_b$, $\beta_r$</td>
<td>$w_s$</td>
<td>$w_b$</td>
<td>$r/n_b$</td>
<td>$\mu$</td>
</tr>
<tr>
<td>Value</td>
<td>3</td>
<td>1.3</td>
<td>1</td>
<td>1/6</td>
<td>0.05</td>
</tr>
<tr>
<td>Predefined Parameters</td>
<td>$\Phi$</td>
<td>$n_s$</td>
<td>$n_b$</td>
<td>$c_b$</td>
<td>$\Delta/Y_b$</td>
</tr>
<tr>
<td>Value</td>
<td>1</td>
<td>4</td>
<td>12</td>
<td>0.58</td>
<td>1</td>
</tr>
<tr>
<td>Derived Parameters</td>
<td>$\omega_s$, $\omega_b$</td>
<td>$\alpha_s$, $\alpha_b$</td>
<td>$c_s$</td>
<td>$D_s/Y_s$</td>
<td>$D_b/Y_b$</td>
</tr>
<tr>
<td>Value</td>
<td>0.8586</td>
<td>0.182</td>
<td>0.913</td>
<td>0.0048</td>
<td>0.021</td>
</tr>
</tbody>
</table>

Set-up costs mainly consist of investment in R&D, which is required for the technical innovations that make produced arms useful, in each time period. Note that the two set-up costs, which measure values, are represented as income shares of each group’s average income. In order to compare the absolute levels of set-up costs of the two groups we need to express them as proportions of the same amount, say of $Y$, [i.e. $D_b/Y_s = D_b/Y_b(\theta n_s/2n_b)$]. After this transformation when $c_b$ takes the baseline value 0.58, $D_b/Y_s$ takes the value 0.00053, which is about nine times lower than the respective ratio of ICs. The assumption that LDCs face lower R&D costs is based on the possibility of imitating IC’s innovations from R&D. Imitation of ICs’ technology can take place in reality, providing that the length of the single time period we examine is long enough for copying process. Notice that the derived relationship of set-up costs becomes more extreme when $c_b$ rises beyond its baseline value. Thus, keeping the LS calibration, the set-up cost relationship is more realistic for relatively lower values of $c_b$.

Solving the model for a range of different values of a specific parameter we can examine the impact to the endogenous magnitudes and the overall equilibrium of the market. The simulation illustrated in figure 7.2 uses the baseline parameter values of table 7.1, for a range of values of LDCs’ marginal cost. Of course, the shares of table 7.1 do not necessarily hold any more but they vary with $c_b$. 

106
Figure 7.2 Trade, procurement, prices, security and welfare as a function of LDCs' marginal cost

Diagram A plots the procurement and exports of ICs. Diagram B plots the average imports and, when LDCs are producing, the average domestic procurement of LDCs. Diagram C plots the now endogenous number of producing LDCs. Diagram D plots the international price of traded arms in equilibrium and the two prices of domestic
procurement of the two groups of producing countries. Diagram E plots the security for either type of LDCs, diagram F plots welfare in logarithms for the three groups of countries and diagrams G and H plot regional and total security for ICs respectively. All magnitudes that refer to any of LDCs’ groups are group averages. As diagram C shows when \( c_b \) is higher than a critical level of about 0.6, the number of arms producing LDCs reaches zero and LDCs do not produce but merely import arms. Passing from this critical level some of the endogenous variables of the model change discontinuously. When \( c_b \) is equal to this critical level LDCs are indifferent to entering and the solution is undefined.

From a closer observation of these results, \( c_b \) has a negative effect on the equilibrium price of imported arms \( (P) \), which keeps falling when \( c_b \) rises. This happens because the higher the \( c_b \) the lower the number of producing LDCs the higher the number of importing. More aggregate imports imply more exports and therefore less unitary cost. This happens until \( c_b \) reaches the level that LDCs are not producing arms any more, when the demand for traded arms suddenly becomes more inelastic and the price discontinuously jumps to a higher level. The exports of ICs and the imports of the increasing number of LDCs increase with \( c_b \) but they fall when LDCs move out of production, apparently as a result of the price increase. With the implied reduction of LDCs procurement it is an expected result that regional security rises together with \( c_b \), following to the decrease of LDCs total procurement. The reduction of ICs welfare is a result of the opportunity cost of the increased arms exports. Higher \( c_b \) causes both increases in the unitary subsidy and in the amount of exports, which result in a stronger rise of the total subsidy. This together with the decrease of domestic procurement offset the benefits in regional security with a final result the slight reduction of national security.

Figure 7.3 illustrates the results of a simulation that solves the model for changing values of the weight of regional security in the ICs assessment of security, i.e. \( \mu \). As diagram C indirectly shows, when \( \mu \) increases the number of arms producing countries first increases and then decreases. When \( \mu \) exceeds some critical level, shown in the diagrams, there no producing LDCs in equilibrium but they are all net importers. Whilst \( \mu \) increases from an initially low level, ICs increase their procurement so that they can have increase capability of intervention in possible regional conflicts. Using procurement prices as an instrument to control arms exports they lower it, reducing the subsidy they provide to their domestic industry. As a result of this policy the market price of arms increases with the resulting reduced impact on LDCs’ imports. This reduction is accelerated from the implied relaxation of the
intensity of arms races. The increase of the price of arms first encourages the entry of
the largest LDCs to domestic production. When the price increases more, however,
the further decrease of the demand for military capability discourages the entry of
LDCs into production. This is a direct result of the assumptions of this model that
LDCs establish their domestic arms industry when the total cost of their imports is
equal or higher with that of producing the same quantity domestically. When $\mu$
increases enough and LDCs do not produce arms the price of arms and the trade
(imports - exports) jump discontinuously at higher and lower levels respectively.

**Figure 7.3** Trade, procurement, prices and number of producers as a
function of ICs' weight on regional security

The case illustrated in figure 7.3 allows the comparison of the predictions of this
model with those of LS. LS have shown that cooperation between large arms
producing countries may cause Pareto improvements, increasing the welfare of the
two country groups. In each model the benefits of ICs originate from the elimination
of market failures that originate from the oligopolistic nature of the market of arms.
Furthermore, the non co-operative behaviour of ICs implies additional negative
externalities. One of them is the free rider problem between ICs, caused by the public
good nature of regional security. Another is the lack of collaboration over R&D cost
of each country’s arms industry. In LS the welfare improvements of LDCs are a result of the reduction of arms races due to the increase of the price of imported arms. In our model, however we have seen that the entry of countries to domestic industry can either be encouraged or discouraged by an increase of the price.

7.6 Conclusions

An analytical model becomes useful in practice when it helps to understand phenomena as yet unexplained, uncovering their generating mechanisms so that this knowledge can be used to make predictions. Another purpose of theoretical structures, independent of predictability, is to provide a rational basis for the implementation of policy decisions. In this context, the main target of the type of models we discuss in this chapter is to penetrate under the noisy surface of the arms trade figures and reach the essential principals that motivate the agents involved in their actions.

The model, as it appears in this paper, took its final form following the realisation that the assumption of price competitiveness in the world market of arms did not allow a realistic calibration of the model. This version excludes the exports of arms producing LDCs from the supply side of the market. In fact, under competitive entry of producers to the supply side of the world market and using the calibration lines followed by both Levine-Smith and this work, LDCs dominate the world market. Since the model reproduces the remaining observable stylised facts only when the marginal cost of LDCs is considerably smaller than that of industrialised countries, if they were to compete in price with ICs they would earn a vast share of exports leaving ICs with exports near zero. This possibly indicates that the small market share of LDCs on the supply side is not a result of competition in price with ICs.

This chapter has mainly covered an introductory analytical examination of the developed theory, leaving space for further, more in depth examination of the predictions of this analysis, though new numerical experiments. These could examine the impact of changing various parameters of the model as well as the type of game (common defence policy of ICs). These results must carefully selected so that they can be brought in line and compared with the results of LS model. Having understood in more depth the predictions of this theory we need to provide links with reality and specific policy recommendations. The strength of this study is that it provides a generalised theory of the arms trade. Research on these directions continues.
8. Conclusions

It is a fortunate event when some encouraging results come out from an early stage of a study. The results of the cross-sectional examination stimulated the interest, motivated for a deeper study of the subject and raised a number of challenging questions. According to the results of chapter 4, the demand for imported arms generally follows patterns that are in line with the theoretical expectations. These results show that the price of imported arms has a negative effect and the level of countries' expenditure in defence has a positive effect on demand. The effect of military expenditure, however, seems to be stronger in the case of smaller countries, with respectively smaller budgets, but gets weaker when military expenditure is relatively large. In a similar way, when the level of economic development rises the demand of countries appears to get weaker, possibly because of the substitution of imported by domestically produced arms.

Supporting that countries' demand for imported arms generally follows standard patterns, directly implies that policies for the arms trade may consider using tools commonly used in trade policies. Our results appear to suggest that the price of internationally traded arms can be an effective policy tool for the control of this market. This contradicts the previous belief that the demand for arms mostly depends in non-price factors and that information about the actual prices that countries pay for the arms they import is unobtainable. In chapter 4, however, the estimated demand pattern is not directly associated with the theory but there is a loose connection between the two. Also, the theoretical examination developed in this chapter leaves several issues open making it inappropriate for the needs of empirical work. Therefore, we needed to develop both the empirical and theoretical aspects of this particular approach, attempting to bridge the gap between the two. This was the target of chapters 6 and 7, which developed theoretical structures that better accord with the empirical facts.

There are some other useful conclusions we may draw from the outcome of chapters 3 and 4. The theoretically acceptable properties and the statistical confidence of the responses to the calculated price index give useful feedback about the whole idea of using this index. In particular, it shows that SIPRI and ACDA series of arms imports in
fact indicate volume and value respectively. It is interesting that the analysis of measurement errors enforces the argument of responsive relationships and the correspondence of the estimations with the theory, instead of blurring the clarity of our results. Another useful indication came from our preference to be very selective about the quality of data, monitoring closely their contents. Being selective about qualitative properties of data has greatly contributed to the satisfactory results. Putting this vice versa, we can draw an interesting qualitative conclusion regarding the data, ex post from their satisfactory performance. It should be clear, though, that this does not imply that data on the arms trade are free of problems but merely that they are not bad enough to be useless (e.g. see Sandler & Hartley [1995], p. 254).

The study of regional panel data gave some relatively clear results, free of important contradictions. From the outcome of this study we can isolate the following four results. First, the results are in line with those from the cross-sectional one (chapter 4): the essence of these is that the demand for imported arms generally follows standard patterns with positive responses to the budget and negative responses to the price, both of reasonable strength. A second selected result is that the structure of the demand is found to vary across different regions of the world. The form of this variation appears to be of the fixed rather than the random type, in both cases. This is an indication of the endogeneity of the variations to the process. Third, over time demand appears to follow a double trend, first increasing and then decreasing. Fourth, the structure of demand appears to be constant over time. In the following paragraphs we refer to these results more analytically.

Despite the limited sample size, the pattern of the estimated regional demands is generally well supported by the data. In some cases this support is quite strong. Most regions have a positive response to military expenditure and a negative response to price with elasticities estimated near unity. The Middle East appears to divert from this rule, having the direction of these two responses inverted and indications of possible underlying dynamics. Relatively less developed areas like Africa, Asia and Middle East showed stronger responses to military expenditure and weaker responses to price. On the contrary, more developed areas, such as North America, Europe and Oceania generally showed elastic responses to prices and inelastic responses to military expenditure. Central and South Americas had both of these effects relatively more elastic. The effect of per-capita GNP on the demand for arms imports varied, taking a positive value in the
case of Middle East and South America and negative in all other cases. Providing that
the structure of countries' demand for arms varies across different regions of the world,
we can expect the outcome of price related policy measures to vary accordingly. This
means that the effectiveness of demand-oriented policies will depend on what degree
these variations are understood and taken into account.

Over time, the demand for imported arms seems to increase until 1987, succeeded by
a relatively steep and steady decline until the end of the examined time period. Each part
of this twin trend has a smooth and approximately linear shape. The declining part
shows a tendency to stabilise (convexity), in the most recent years. From our results,
however, this trend appears to be the only variation of the demand over time, whereas
we have significant evidence that the underlying structure of the generating mechanism
of this demand remains unchanged. Since the change in the level (trend) follows closely
the evolution of the intensity of the Cold War we may well attribute the trend component
of the demand. If this is true, we have isolated the impact of the Cold War from the
endogenous processes of this demand and we have demonstrated that the two are
separable. When the time effect is estimated independently for each region, with the use
of a quadratic trend, all regions turn out to follow the same twin pattern, with a small
variation of their turning points.

An important determinant of the quality of empirical work is the close
correspondence of the applied methodology with an underlining theoretical framework.
This points towards one of the main weaknesses of our empirical studies in chapters 4
and 5, the use of an ad hoc simple analytical framework rather than an explicitly
specified one. The theory discussed in the context of the cross-sectional estimation is
linked but not directly connected with the estimated demand functions. There is also no
direct link between the results of the panel data study and some particular model of the
arms trade. Having obtained a considerable amount of empirical information, the natural
progress is to aim at theoretical developments that will bridge this gap and develop a
close link between theory and practice. The analysis in chapter 6, therefore, is the result
of an attempt to introduce a number of characteristics into the preliminary analytical
approach of chapters 4 and 5 that brings it closer to the actual empirical results.

A critical for our analysis characteristic of the suggested production function is the
concavity of the response of imported arms to both military expenditure and per-capita
income of countries. In practice, this implies that when countries are either larger or
richer (more developed – higher per capita GDP) they tend to produce their own arms so they are less dependent on imports. In other words, we need to introduce into our models an endogenous process for the entry of countries into domestic production as well as differentiated categories of arms in the analysis of the arms trade, characteristics not provided by other works. Such a differentiation could also be used to resolve the commonly mentioned need to distinguish between major and minor categories of arms. Following to these requirements, the final form of our model in chapter 6 has been based largely on the use of a modified form of the Stone-Geary function, which is further discussed in appendix B and provides the desired properties of substitution. It should be mentioned that it has not been possible to detect an analysis of the particular properties of this framework in the literature. From the results of our examination, however, it appears that the performance of the framework employed corresponds to the expectations.

According to the predictions of the model developed in chapter 6 the establishment of a domestic industry of arms has a considerable effect on the demand of countries for internationally traded arms. When the domestic industry becomes established, domestic arms directly substitute for part of the initially imported arms and the demand for the latter category becomes less responsive to the military budget of the country. Also, the effect of economic development on this demand decreases, possibly changing from positive to negative. The effect of the price of arms imports, however, appears to remain always negative and unaffected by the state of establishment, in line with the empirical results of chapter 4. The attempt to apply this theory in an empirical also yielded some interesting results. The estimated parameters have significant and theoretically correct signs. These results, however, do not provide estimates for all the parameters of the model, a complete evaluation of which requires additional data.

The material included in chapter 7 is the result of an effort to incorporate some of the findings of the previous work into the contemporary international arms trade, which considers both the demand and supply side of this market. It is an attempt to generalise the Levine-Smith quasi-general equilibrium, world market model by introducing the optional entry of countries to domestic production and bring the theory of the arms trade nearer to the empirical facts. An initial attempt to incorporate both the mechanism of entry and the differentiated categories of arms showed that this could possible but requires an examination the scale of which exceeds the limits this thesis. Chapter 7, thus,
attempts to endogenise the entry of countries to domestic production, always assuming a single, non-differentiated category of arms that can either be imported or domestically produced.

The examination of the model developed in chapter 7 predicts that an increase of the price of traded arms encourages countries to produce their own arms, passing from the reliance on imports to self-sufficiency. In an interesting way, it shows that the opposite may also happen. Under appropriate conditions the raising of price might prevent arms races and encourage countries to avoid arms production and spend more on peaceful activities, rather than on security. It gives encouraging evidence that if large arms-producing industrialised countries become adequately concerned about regional security, other countries might prefer to keep out of major arms producing activities. Another interesting result from this study is that it is impossible to allow for price competitiveness in the world market of arms and keep the market shares as we observe them in the real world. In fact, under competitive entry of producers to the supply side of the world market and using the calibration lines followed by both Levine-Smith this model predicts that LDCs would dominate the world market. The contradiction of this result with the observed reality possibly indicates that an assumption of price competitiveness in the international market of arms is unrealistic. It should be clear that chapter 7 is a very technical study, which requires further examination.

There are several ways that the research developed in the context of this thesis may continue. For a further development of the empirical research we need to extend our data set, both in terms of quality and quantity. Regarding the expansion of our data set using the latest releases of ACDA and SIPRI, we may increase the number of observations of our set over time. An extension of this examination from aggregate-regional to country level needs to overcome the difficulty of several zero observations, due to missing information. Such an attempt could be possible with the use of econometric techniques appropriate for this type of measurement errors, such as a form of Tobit type model.

A possible way to continue the study of the small-country model could be an examination of a wider theoretical framework, introducing a welfare function of the central government and move towards general equilibrium models. This would allow for a more realistic distribution of resources between defence and other sectors of the economy. The empirical part of chapter 6 is a preliminary examination rather than a complete estimation of the model. A complete identification of the model requires
additional information, such as data on countries’ arms production. We may attempt to put together data series on countries’ domestic production either in a direct or indirect way. Alternatively, we may employ the assumption of the panel data study that the countries of each geographical region have equal income shares of military expenditure and attempt to bring the two studies together, identifying a complete and more powerful model.

An attractive way to further develop the model of chapter 6 would be to incorporate its useful characteristics in a much more powerful model, such as that of chapter 7. Although chapter 7 reproduces the effect of entry to domestic production of arms, the formulation of the decision of this entry is more simplistic than in the small country model. It also allows for only a single category of arms, without differentiation between domestic and imported ones. If the production framework of chapter 6 were incorporated into the world model of chapter 7 the result would possibly avoid the limitations of each. Research in these directions continues, always motivated by the hope that a clearer understanding of international security processes will contribute towards less abuse of military power.
A. Appendices of the chapters

A.3 Appendix of chapter 3: the cross-sectional data set

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<th>Arms Imports</th>
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Note: All series are averages of the years 1990 and 1991 and are measured in constant 1990 million U.S. dollars. Exception is ACDA imports for Hungary which is average of the years 1989 and 1992.

A.4 Appendices of chapter 4

A.4.1 Descriptive statistics

List of Variables and their Descriptions

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<th>Variable(s)</th>
<th>Description</th>
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Statistics

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Coef of Variation: .15315 .17542 .18776 .15992 6.9732 .094893 .094893

Estimated Correlation Matrix of Variables

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List of Variables and their Descriptions

- LSHSIIMME: LOG of AI to ME share from SIPRI
- LSHACIMME: LOG of AI to ME share from ACDA
- LTOTGDPT: LOG of total GDP from PWT (corrected for the terms of trade) in million constant (1990) U.S. dollars.
- LTOTGDPT2 := LTOTGDPT²

Statistics

Sample period: 1 to 38
Variable(s): LSHSIIMME LSACIMME LTOTGDPT
Maximum: 0.17552 -0.87086 15.4976
Minimum: -6.8095 -5.5344 10.6104
Mean: -2.5027 -2.7847 12.4331
Std. Deviation: 1.3864 1.1910 1.2170

Estimated Correlation Matrix of Variables

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### A.4.2 Leverage Values and Residuals

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Mean (=k/n; n = 38): 0.105 (k = 4) 0.158 (k = 6) 0.210 (k=8)  
Mean × 2: 0.210 0.316 0.421

Note: Observations with leverage values higher than double of the mean are characterised as influential and their figures are in italics and underlined.
A.5 Appendices of chapter 5

A.5.1 Region Codes

The 11 aggregates are coded as follows:

- 0: World aggregate
- 1: Africa
- 2: North America
- 3: Central America
- 4: South America
- 5: Asia
- 6: Europe
- 7: Middle East
- 8: Oceania
- 9: NATO
- 10: OECD
- 11: OPEC

A.5.2 The data

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<td>0.81148</td>
<td>7.007600614</td>
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<td>96</td>
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<td>12.73064388</td>
<td>15.5811362</td>
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<td></td>
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</tbody>
</table>
Sources: 1) ACDA 'WMEAT 1994' and 2) SIPRI Yearbook 1992 (years 1983-84) and Yearbook 1995 (years 1985-93).

Notes:

1. All figures are in logarithms. Observations 1983-84 of SIPRI imports originate from SIPRI yearbook 1992 and observations 1985-93 from Yearbook 1995. All other variables originate from ACDA. The implicit price index ARPI is calculated by dividing SIPRI imports by ACDA. Therefore in logarithms ACDA observation of imported arms can be calculated by adding up the SIPRI observation with the implicit price index.

2. Per-capita GNP is measured in US $ and other values in constant (1990) million U.S.

3. The unavailable SIPRI observation for C. America 1992 has been replaced by linear interpolation of neighbouring observations '91 and 92.

4. The base year of ACDA values is changed from 1993 constant US dollars to 1990, using the IMS deflator (=1.091).

A.6 Appendix of chapter 6: Statistical results

Estimation of form (25)

<table>
<thead>
<tr>
<th>Regressor</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>T-Ratio[Prob]</th>
</tr>
</thead>
<tbody>
<tr>
<td>DOM</td>
<td>524.7431</td>
<td>179.6998</td>
<td>2.9201(.006)</td>
</tr>
<tr>
<td>SIME</td>
<td>.13479</td>
<td>.025019</td>
<td>5.3076(.000)</td>
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<tr>
<td>ARPI</td>
<td>78.2073</td>
<td>31.7539</td>
<td>2.4629(.020)</td>
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</table>

Ordinary Least Squares Estimation

38 observations used for estimation from 1 to 38
GDPC  .017471  .010237  -1.7067 [.098]
SIMEDOM -1.12920  .025112  -5.1450 [.000]
ARPIDOM -1.10.0035  118.5281  -.92808 [.361]
GDPCDOM .011749  .015285  .76866 [.448]

R-Squared .65461  R-Bar-Squared .58776
S.E. of Regression 294.7857  F-stat. F(6,31) 9.7922 [.000]
Mean of Dependent Variable 487.3606  S.D. of Dependent Variable 459.1245
Residual Sum of Squares 26938S7  Equation Log-likelihood -266.1287
Akaike Info. Criterion -273.1287  Schwarz Bayesian Criterion -278.8603
DW-statistic 2.2607

Diagnostic Tests

<table>
<thead>
<tr>
<th>Test Statistics</th>
<th>LM Version</th>
<th>F Version</th>
</tr>
</thead>
<tbody>
<tr>
<td>B: Functional Form</td>
<td>*CHSQ(1)= .24619 [.620]</td>
<td>*F(1,30)= .19562 [.661]</td>
</tr>
<tr>
<td>C: Normality</td>
<td>*CHSQ(2)= 3.1270 [.209]</td>
<td>Not applicable</td>
</tr>
<tr>
<td>D: Heteroscedasticity</td>
<td>*CHSQ(1)= .013244 [.908]</td>
<td>*F(1,36)= .012551 [.911]</td>
</tr>
</tbody>
</table>

B: Ramsey's RESET test using the square of the fitted values
C: Based on a test of skewness and kurtosis of residuals
D: Based on the regression of squared residuals on squared fitted values

Analysis of Function(s) of Parameter(s)

Based on OLS regression of ACIM on:

<table>
<thead>
<tr>
<th>DOM</th>
<th>SIME</th>
<th>ARPI</th>
<th>GDPC</th>
<th>SIMEDOM</th>
</tr>
</thead>
<tbody>
<tr>
<td>38 observations used for estimation from 1 to 38</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Coefficients A1 to A7 are assigned to the above regressors respectively.
List of specified functional relationship(s):
kappaM=a2+a5 ; kappaD=-a5/a2 ; kappaL=1-(a2+a5)+a5/a2 ; gammaM=a3/(1-a2) ; gammaL=-a4/a2 ; gammaD=a4/a2-(a4+a7)/(a2+a5) ;

<table>
<thead>
<tr>
<th>Function</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>T-Ratio [Prob]</th>
</tr>
</thead>
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<tr>
<td>kappaM</td>
<td>.0055870</td>
<td>.0021627</td>
<td>2.5833 [.015]</td>
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<tr>
<td>kappaD</td>
<td>.95855</td>
<td>.017794</td>
<td>53.8679 [.000]</td>
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<tr>
<td>kappaL</td>
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<td>.015872</td>
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<td>gammaM</td>
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<td>35.7746</td>
<td>2.5267 [.017]</td>
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<tr>
<td>gammaL</td>
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<td>.063713</td>
<td>2.0345 [.050]</td>
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<tr>
<td>gammaD</td>
<td>.89463</td>
<td>1.9655</td>
<td>.45517 [.652]</td>
</tr>
</tbody>
</table>
Appendix B. The emergence of a new input: An extension of Stone-Geary function

B.1 Introduction

An important consideration regarding the use of a utility or a production function is that it provides has to do with the type and degree of substitution between inputs. Ranking from one extreme of perfect substitutes to the other extreme of perfect complements the extent of substitutability among the inputs of a transformation process is a critical concern in the selection of the appropriate analytical tool. Whilst the contemporary literature on transformation functions seems to focus on the development of less restrictive and more powerful analytical frameworks, it is little informative on the issue of the complete substitution of an input by the others. This paper examines an extension of Stone-Geary function, which appears to provide the desirable properties. It also examines the case that the substitutable input requires a fixed cost, if it is used.

This work has been motivated from an attempt to provide theoretical explanation to the results of chapter 4. Using cross-sectional data chapter 4 estimated the responses of the demand for internationally traded arms to the military budget of the countries and to per-capita GDP having concave shapes. The explanation proposed for the hump-shaped response of the demand for imported arms to the military budget has as follows: When the budget increases countries tend to substitute imported arms with an alternative input, domestically produced arms, for two mainly reasons. First, the two aggregated inputs are differentiated but substitutes in the production of military capability. Second, the production of domestic arms requires considerable set up costs, so the opportunity price of this input falls when the demand for it rises. A similar explanation was suggested for the concave response to the level of development, measured by per-capita GDP.

In order to examine the previously stated theoretical hypotheses, we need an analytical tool capable to describe the state of transition from the type of production that includes to the type that excludes the substitutable input. In other words, we examine the decision
for vertical expansion of the production process. The element that requires special
treatment, from analytical point of view, is the binary nature of a 'yes' or 'no' to the
vertical expansion. In technical terms, we need to incorporate a binary and not continuous
variable to standard neo-classical analysis of transformation (which implies continuity).

In terms of geometry, we focus on the property of production functions that iso-
product curves intersect the axes when the inputs are substitutes. The popular Constant
Elasticity of Substitution function (CES), with flexible properties of substitution, was
found inappropriate for our requirements, revealing some peculiar properties. Even
though iso-output curves of CES appear to intersect the axes when the inputs are
substitutes, this intersection is merely asymptotic since CES function is not defined at
zero quantities of its inputs. Using Chung [1994] for a contemporary review on
transformation functions, the examination of generalised forms of CES showed a similar
weakness to capture the complete substitution of an input.

The need for this study rose from the poor results of our literature review in detecting
a transformation function (either titled as production or utility functions) with the
desirable properties. Therefore, this paper discusses a modification of Stone-Geary (SG)
function, which appears to provide the requested properties with the additional benefit of
relative analytical simplicity. The modification applied to SG essentially extents the
standard range of the parameters, allowing one - and only one - of the subsistence
quantities to be negative. This alteration provides the desirable properties of substitution
between a particular input and the others, and allows for production with and without
that input.

The following pages present the set up of the suggested functional framework, an
examination of its properties and an assessment of its validity. In its general form, this
framework allows for fixed costs in the use of the input in question. The presentation
includes three sections. First, we discuss the properties of SG and other functional forms.
Section two stresses out the new framework and the last section examines the properties,
assesses the performance and looks to the suitability of this analysis.
B.2 The choice of production function

As a starting point we may briefly look at ways that elasticity of substitution affects the geometry of production functions. In the case of a function with unitary elasticity of substitution (CD is a typical example), all iso-quants (iso-product curves, surfaces, volumes or hyper-planes in the cases of two, three, four or more inputs respectively) have the axes of the inputs as asymptotes (e.g. see Heathfield & Wibe [1987], pp. 90-104). A more general function, with parental relationship to CD, is the CES function. When inputs are complements the asymptote of every isoquant is located at some positive quantity of the input, denoting the lowest possible quantity required for any level of output, called *subsistence level*. When the elasticity of substitution is higher than unit the isoquants are expected to intersect the axes of the inputs, allowing for complete substitution of one input by the others. Of course, the range of the function is set accordingly to rule out negative values of the inputs.

A closer examination of CES, however, shows that this function is not defined for zero levels of any of the inputs and the iso-output curves have asymptotical intersects with the axes of output map. While the use of CES, however, tends to produce complicated algebra it carries the unnecessary for this study characteristic of restricting to an equal elasticity of substitution between all inputs. This limitation could be avoided by the less restricted Generalised CES which, likewise CES, rules out zero levels of inputs and it is even more complicated (e.g. see Chung [1994] pp. 126-30).

Stone-Geary function (e.g. Chung [1994] pp. 23-31) has mainly been used as a utility function in the well-known Linear Expenditure System and, like CES, can be considered as a generalisation of CD. In logarithmic form, we can define SG function as follows

\[
\ln U = \sum_{i=1}^{n} \kappa_i \ln(q_i - \gamma_i) 
\]

where \(0 < \kappa_i < 1\), \(\Sigma \kappa_i = 1\), \(U\) is output (utility or product), \(q_i\) are quantities of the inputs, \(\gamma_i\) are subsistence quantities so that \(q_i > \gamma_i > 0\). It is non-homothetic but strictly monotonic and convex, thus well defined as a utility or production function. Assuming constant prices the budget constraint is \(G = \sum p_i q_i\), where \(p_i\) is the price of input \(i\). Solving for the standard consumer's (or producer's) optimisation problem the demand for the inputs of this process is given by
Looking closer into this function the own price elasticities of demands are

\[ \varepsilon_u = \frac{\partial q_i}{\partial p_i} \frac{p_i}{q_i} = -\kappa_i \frac{G - \sum_j p_j \gamma_j}{p_i q_i} < 0 \]

Examination of (3) easily reveals that using SG implies that demand is always inelastic (|\(\varepsilon_u\)| < 1). The cross-price elasticities are

\[ \varepsilon_{ij} = \frac{\partial q_i}{\partial p_j} \frac{p_j}{q_i} = -\kappa_i \frac{p_j \gamma_j}{p_i q_i} < 0 \quad (i \neq j) \]

which limit inputs to be gross complements. The income elasticity is

\[ \varepsilon_{G_i} = \frac{\partial q_i}{\partial G} \frac{G}{q_i} = \kappa_i \frac{G}{p_i q_i} > 0 \]

which restricts inferior goods. The Allen-Uzawa cross-partial elasticities of substitution are

\[ \sigma_{ij} = \frac{\prod_h p_{ih} \gamma_{ih}}{p_i p_j q_i q_j} < 1 \quad (i \neq j; \; i, j \in h) \]

limiting the inputs to gross complements, as the cross-price elasticities also show. From the previous description SG function does not seem to accomplish the requirements of this study, since it would not allow the elasticity of substitution to be higher than unit between any pair of inputs.

**B.3 Complete substitution in transformation**

The properties of SG in the standard form are designed to model consumer behaviour, where the name *subsistence levels* of \( \gamma \) coefficients has its origin. In its standard form inputs are imperfect complements and subsistence quantities are restricted to be positive. There is a modification in the assumptions, however, that enables SG to provide the desirable properties. This is to allow subsistence quantity of one only input to be negative. The second modification of this setting, from standard SG set-up, is to allow for fixed costs in the use of the input in question. The impact of this alteration can either be seen as causing a diminishing price of the input or as reducing the budget by a fixed
amount (of set-up cost) but leaving the price of that input fixed, equal to marginal cost. This convenient trick prevents algebraic implications and allows for a neat solution.

Assume that all assumptions hold as in usual SG set-up except one: the subsistence level of only one input, say input $f$, is negative. The subsistence quantities of all other inputs remains positive as usual, or

\[ (7) \quad \ln U = \sum_{i=1}^{n} \kappa_i \ln (q_i - \gamma_i) \]

where $0 < \kappa_i < 1$, $\sum_i \kappa_i = 1$, $q_{i\neq f} > \gamma_{i\neq f} > 0$ and $\gamma_f < 0 \leq q_f$. The negative subsistence level $\gamma_f$ allows input $f$ to be a gross substitute instead of being limited to a net complement with all other inputs. Now, input $f$ can be fully substituted by the other inputs, in which case (7) takes the form

\[ (8) \quad \ln \tilde{U} = \kappa_f \ln \left( -\gamma_f \right) + \sum_{i\neq f} \kappa_i \ln (q_i - \gamma_i) \]

Note that (8) is just a special case of (7) where $q_f$ is set to zero. As a convention, we distinguish the type of transformation that excludes input $f$ from the case that includes it by noting the related variables with a tilde ($\tilde{\cdot}$).

The second alteration we introduce to SG is that only when input $q_f$ is excluded the disposable budget is $G$, whereas when $f$ is included it reduces to $G - F$, because of fixed costs $F$ in the use of $f$. The fixed amount of set-up costs $F$ has to be spent when any $q_f > 0$ is used in production. As a result of the restriction $q_i < \gamma_i$, in either types of transformation the necessary condition for any output to exist ($U > 0$) is that the budget can cover at least the expenditure for subsistence quantities of all other than $f$ inputs

\[ (9) \quad G > \tilde{G}_L = \sum_{i\neq f} P_i \gamma_i \]

When $q_f$ is included in the transformation, the budget in addition to $\tilde{G}_L$ has to cover fixed costs $F$. Thus, a necessary but not sufficient condition for using $q_f$ is

\[ (10) \quad G > G_L = \tilde{G}_L + F \]

Including input $f$ in the process, the demand functions for inputs are essentially as in the general cases (2) with the budget reduced by the fixed costs

\[ (11) \quad q_i = \gamma_i + \kappa_i \frac{G - F - \sum_{i\neq f} P_i \gamma_i}{p_i} \quad (i \neq j), \quad q_f > 0 \]

Under (8), however, the returns to scale are decreasing compared to (7) and the demand equations for inputs take the form
Solving the problem of output maximisation, we may first look into restrictions (9) and (10). When $G_L > G > \tilde{G}_L$ the use of input $f$ in the production is impossible and we can directly detect the demand for other inputs using (12), without any implication. To determine the optimal allocation when $G > G_L$, where either types of production are possible, we must ensure that both output is maximised and also that the non-negativity of inputs is satisfied. While the non-negativity of other than $f$ inputs is not problematic, following the standard SG case, we must ensure that the demand for $q_f$ is positive. Thus, applying the restriction $q_f > 0$, with the use of (11) we obtain

$$q_f > 0 \Rightarrow G > G_H$$

where $G_H = G_L - \frac{1 - \kappa_f}{\kappa_f} p_f \gamma_f$. Notice that under the assumptions that $\gamma_f < 0$ and $F \geq 0$ we always have $G_H > G_L \geq \tilde{G}_L$. Thus, condition (13) fully replaces condition (10).

Having detected the necessary conditions and solutions for the allocation of resources in each of the two types of production, we need a sufficient condition that defines the optimum type of production in the general case. We can do this using the indirect utility functions for each case. Substituting from the demands for inputs (11) and (12) in the production functions for the two cases, after simplifications the two indirect utility functions become

$$u = (G - F - \sum_i p_i \gamma_i) \prod_{i=1}^{n} \left( \frac{\kappa_i}{\beta_i} \right)^{\gamma_i}; \quad q_f > 0$$

and

$$\bar{u} = \frac{(-\gamma_f)\kappa_f}{(1 - \kappa_f)^{-\gamma_f}} \left( G - \sum_{i \neq f} p_i \gamma_i \right)^{-\kappa_f} \prod_{i \neq f} \left( \frac{\kappa_i}{\beta_i} \right)^{\gamma_i}; \quad q_f = 0$$

From the definition of maximality, the condition for the inclusion of $g$ in the production is

$$q_f > 0 \iff u > \bar{u}$$

Substituting from (14) and (15) to the right hand side of (16) and rearranging we obtain

$$q_f > 0 \iff z > 0$$

where
Variable $z$ is proportional to the output surplus from the inclusion of input $f$. Note that it is impossible to obtain a general solution for the inverse function of $z(G)$. Therefore, we can only solve for values of $G$ that correspond to given values of $z$ with the use of numerical methods.

Differentiating $z$ with respect to $G$ once we get $\frac{dz}{dG} = 1 - a\left(\frac{\tilde{G} - \tilde{G_L}}{\kappa_f}\right)^{\kappa_f}$ and twice we get $\frac{d^2z}{dG^2} = a\kappa_f\left(1 - \frac{\tilde{G} - \tilde{G_L}}{\kappa_f}\right)\left(\frac{\tilde{G} - \tilde{G_L}}{\kappa_f}\right)^{\kappa_f - 1} > 0$, showing convexity of $z$ to $G$.

Solving from the first order condition $\frac{dz}{dG} = 0$ for the global minimum of $z$ we obtain the neat result that $z$ gets its minimum at $G_H$. Thus, $z(G)$ is strictly increasing in its range $[G_H, \infty)$. The domain of $z$ is bounded downwards, by the global minimum $z(G_H)$ but not upwards, since $\lim_{G \to \infty} z = \infty$. We can also show that $z(G_H)$ is always negative. Substituting for $G_H$ in (18) we obtain $z(G_H) = -a\left[F - \left(1 - \kappa_f\right)\frac{p_f \gamma_f}{\kappa_f}\right]^{\kappa_f} - \frac{p_f \gamma_f}{\kappa_f}$ and after substitution for $a$ and some algebraic manipulation we obtain $z(G_H) < 0 \iff \frac{F}{1 - \kappa_f}\left(\frac{-p_f \gamma_f}{\kappa_f}\right)^{\kappa_f} > 0$, QED. Thus, he have shown the initial hypothesis that $z$ is never higher than zero.

Summarising, we have shown that in its range $z(G)$ is continuous, strictly increasing and quasi-convex, with a negative lowest bound and upwards unbounded. Since 0 is always included in the domain of $z$ using the mean value theorem we have shown that there is always a value of $G$, say $G^*$, that satisfies $z(G^*) = 0$. Thus, $z$ has at least one root in its range. Furthermore, since $z(G)$ is a strictly increasing function, using the fixed point theorem we have shown that this root is unique. This proof can be expressed in a more formal way as follows.

**Theorem**

Let function $U = \prod_{i=1}^n \left(q_i - \gamma_i\right)^{\kappa_i}$ map a transformation of $(q_1, q_2, ..., q_n)$ into $R$ where $n \in \mathbb{N}$, $0 < \kappa_i < 1$, $\sum \kappa_i = 1$, $q_{i\neq f} > \gamma_{i\neq f} > 0$, $q_f \geq 0$, $\gamma_f < 0$ and $p_f$, $\kappa$, and $\gamma_i$ are fixed. If
\[ G = \begin{cases} \sum_{i} p_i q_i + F; & q_f > 0 \\ \sum_{i \neq f} p_i q_i; & q_f = 0 \end{cases} \text{ where } F \geq 0, \text{ then there exists one and only one} \]

\[ G^* \in (G_H, \infty), \text{ where } G_H = \tilde{G}_L + F - \frac{1 - \kappa_f}{\kappa_f} p_f \gamma \text{ and } \tilde{G}_L = \sum_{i \neq f} p_i \gamma_i, \text{ such that the} \]

constrained maximisation of \( U \) under \( G \) implies that:

\[ \tilde{G}_L < G < G^* \Rightarrow \begin{cases} q_i = \gamma_i + \frac{\kappa_i}{1 - \kappa_f} \frac{G - \sum_j p_j \gamma_j}{p_i}; & (j \neq f, i \in j) \text{ and} \\ q_f = 0 \end{cases} \]

\[ G > G^* \Rightarrow q_i = \gamma_i + \kappa_i \frac{G - F - \sum_j p_j \gamma_j}{p_i}; & (i \in j) \]

Note that by definition, when \( G = G^* \) either previous solutions give equal output.

An interpretation of the previous result can be as follows. At the lowest boundary of the budget that both types of transformation are possible (i.e. when \( G = G_H \)) transformation with input \( f \) excluded always gives equal or higher output than with \( f \) included. Thus, at \( G_H \) the output of transformation without input \( f \) weakly dominates the output of transformation with input \( f \), whereas if \( F > 0 \) the later type strictly dominates the former. When \( G \) rises, excluding \( f \) keeps dominating until \( G \) reaches the critical level \( G^* > G_H \). When \( G \) exceeds \( G^* \) production with \( f \) included dominates in terms of output. When \( G = G^* \) the two types of transformation give equal output and the dominant type is undefined. The critical level of transition \( G^* \) is unique.

### B.4 Examination of the new system

We may illustrate the performance of the presented modification of SG function using a simple example with two inputs: \( f \) with a negative subsistence level \( \gamma_f \) and \( q \) with a usual positive subsistence level \( \gamma_q \). Input \( q \) can represent a combination of a number of other inputs. Figure B.1 presents a case without fixed costs in the use of \( q_f \); i.e. \( F = 0 \), and a non zero subsistence quantity \( \gamma_f \). Diagram A of Figure B.1 plots the iso-output map and the solution of the constraint maximisation problem. All iso-quant curves have two asymptotes, one in each axis, determined by quantities \( \gamma_f \) and \( \gamma_q \).
When the budget is $G_1$ the producer maximises output at $u_1$ using $q_1 = G_1/p_q$ quantity of input $q$ and excludes input $f (=0)$. Notice that the budget constraint has a smaller slope than the slope of $u_1$ isoquant at the intersection with the axis of $q$. When the budget is $G_2 = G_H$ input $f$ is still marginally excluded from the transformation, although an infinitesimal increment of $G$ implies the entry of $f$ in the process. If $G_3 > G_H$ maximisation of output requires quantities $q_3$ and $f_3$ of the two inputs. The double diagram B of Figure B.1 plots the demand for the two inputs (Engel's curves) and output ($u$), all as functions of $G$. Curve $u$ gives the output with input $f$ included and curve $\tilde{u}$ (dashed line) gives the output with $f$ excluded. Maximum output is given by $\tilde{u}$ when $G$ lies between $G_L$ and $G_H$ and switches to $u$ when $G > G_H$.

**Figure B.1 Equilibrium without fixed costs**

Figure B.2 illustrates the case that $F > 0$ and Diagram A plots the indifference map of this case. From technical point of view, the specific distinction of this case from the previous is that the budget constraint is non-continuous at the intersect with the axis of $q$. When $f > 0$ the budget constraint is represented by the usual straight line with slope $-p_f/p_q$ but at $f = 0$ (i.e. at the intersection with the axis of $q$) it does not include the usual
endpoint at $g = G/p_q$. Instead, it includes a point which is detached from the rest of the curve, shifted upwards by amount $F/p_q$. This shift represents the release of resources from fixed costs $F$ when input $f$ is not used, measured in units of $q$. If, for example, the budget is $G_1$ and input $f$ is included in the process maximum output is $u_1$. When input $f$ is excluded, though, output reaches level $\bar{u}_1 > u_1$ using quantity $\bar{q}_1$ of input $q$. When the budget is $G_2 = G^*$ allocation $(q_2, 0)$ and allocation $(\tilde{q}_2, \tilde{f}_2)$ yield exactly the same output $u_2 = \bar{u}_2$, which is a case of indifference. If the budget grows higher than $G^*$, as in the case of $G_3$, output level $u_3$ that requires $q_3$ and $f_3$ quantities of the inputs dominates output $\bar{u}_3$.

In a similar way as in the previous example, Diagrams B present the output and demands for inputs as a function of $G$, when $F > 0$. The difference between the previous and this case is that the demand for input $q$ is non continuous any more but consists of two line fragments detached at $G^*$. Output is still a continuous function of $G$ but not differentiable, changing discontinuously slope at $G^*$.

Figure B.2 Equilibrium when input $f$ requires fixed costs

The transition from the type of transformation that includes $f$ to the type that excludes it, and vice versa, depends on the relative prices of inputs, the budget and the other
parameters of the production. Under the new definition, simple observation of the elasticities from (3), (4), (5) and (6) reveals that the properties of SG are affected in a desirable way. The own price elasticity is not any more limited below unity, for all inputs except the input substitute $f$. The cross price elasticities and cross partial elasticities of substitution now reflect the substitutability of this input by not being limited below zero and unity respectively. In particular, (6) shows that input $f$ is a gross substitute with any other input since for any $i \in n$, $\sigma_f = \sigma_i > 1$.

The type of transformation that excludes $f$ differs from the case that includes it, in that neither the technological coefficient $(-\gamma_f)^{\kappa_f}$ is unitary nor is the sum of $\kappa$ coefficients, i.e. $\sum_{i \neq f} \kappa_i < 1$. Another difference between the properties of the two types is that transformation type that excludes input $f$ has lower returns to scale to the type that includes it. Since SG is not homothetic the returns to scale are not globally equal but depend on the inputs. In order to compare the returns to scale of the two alternative methods of production we can examine the change of the ratio of the two outputs, when all inputs except $f$ are equal and change by a factor $l$, for any $q_f > 0$.

$$
\frac{U}{\tilde{U}} = \frac{(lq_f - \gamma_f)^{\kappa_f} \prod_{i \neq f} (lq_i - \gamma_i)^{\kappa_i}}{(-\gamma_f)^{\kappa_f} \prod_{i \neq f} (lq_i - \gamma_i)^{\kappa_i}} = \left( \frac{lq_f - \gamma_f}{-\gamma_f} \right)^{\kappa_f} > 1
$$

The fact that $U$ grows faster than $\tilde{U}$ when inputs grow at the same proportion indicates for higher returns to scale of the former function.

For an establishment of the link between the presented analytical framework and reality we may return to the original motivation of this work. It all started from an attempt to provide a coherent theoretical explanation to the empirical findings of chapter 4 on the demand for Arms Trade. The quadratic logarithmic function specified in chapter 4 included concave responses of the demand for one input (imported arms) to the budget. The working assumption of chapter 4 is that after some critical size of the budget a new input enters in the production: the domestically produced arms. Imported arms are differentiated from imported arms but they also are imperfect substitutes, so with their entrance they replace part of the imported and decrease the reliance on them. This has the result that the demand both shifts and bends downwards, giving to the curves the humped shape.
Perhaps the application of our framework to firm theory is the most apparent, the presented analysis, although, might find other applications in the areas of either firm or consumer theory. In the context of consumer theory, for instance, SG function allows for positive subsistence quantities of inputs that are absolutely necessary for the transformation. A negative subsistence quantity of a good in consumer's utility transformation may be the result that this good is free up to some given quantity. Further increase of the quantity of this good may require an considerable set-up cost and additional variable costs dependent on the quantity. An example could be homemade food. The majority of consumers can enjoy the quality of some home made food with a small effort and a low cost. If the consumer has strong preferences for home-made food would probably need to hire personnel for that purpose, set-up the appropriate installations, pay wages etc.

A useful tool that helps to secure that the performance of an analytical model is indeed the one it appears to be and that the underline logic is not flawed is the numerical simulation with the help of computer. This becomes more essential especially when part of the analytic structure of the model cannot take a complete algebraic solution, as in our case. Therefore, building numerical examples of this model and comparing its predictions with the observed facts is not only an assessment of its usefulness but, also, an illustration of its performance in practice. The results of various simulations of this framework appear to confirm the discussed theoretical results. The examples drawn in Figures A.1 and A.2 are not freehand drawings but numerically simulated applications of the model. Appendix B.A provides the output of such a simulated example.

B.5 Conclusions

The alteration of Stone-Geary function, presented in this paper, has initially been motivated by the needs of a particular empirical study. Our attempt to develop an analytical framework that provides the required properties followed the poor results of the literature review to detect an available alternative. The performance of the framework we develop here is generally satisfactory regarding the initial requirements. Using a single production function, technically as simple as SG, our framework allows an input either to be included or excluded from a transformation process, providing solutions for the
demand of all inputs in either cases. We also examine the case that the substitutable input requires a fixed cost.

According to the standard assumptions of Stone-Geary function and those that originate from alterations we introduced, we can relate the entry of the input in question to the transformation with the size of the resource constraint. In a brief description this has as follows. There is a minimal required level of the fixed resource (the budget) for any output to exist. This is the standard in Stone-Geary framework amount that only examines positive only subsistence quantities of the inputs. When the budget exceeds the necessary for the subsistence quantities level the production first takes place without the substitutable input, providing that the negative subsistence quantity of it is smaller than zero. When the budget grows beyond some critical level that output using the substitutable input equals to output without it (indifference) the input in question is the included in the transformation. In this work we prove that this critical level always exists and it is unique, and we may calculate a value for it with the use of iterative methods.

For a confirmation of the proclaimed theoretical results the developed framework has been examined with the use of analytical and tested with the use of numerical methods, without any contradictory results. This does not imply, however, that further investigation and further examination for possible affinity with some existing work are not needed. Chapter 6 further examines of the applicability of this framework in the context of applied econometric work. A single empirical attempt, however, might not be adequate to assess the applicability of this framework. Such an evaluation might require a number of applications, both in the context of firm and consumer theory.

B.A Appendix: A numerical simulation

Figure B.3 displays the outcome of a numerical simulation of an example with three inputs $q$, $y$ and $f$ using Matlab. The critical for the transition level of income $G^*$ is calculated by iteration process (Gauss-Newton method), since it is not possible to inverse the real expression in (18). As the output of the simulation shows, the level of budget varies from 20 to 60, the fixed cost of using input $f$ is 15 and the prices of the three inputs are set to unit. The subsistence levels of $q$ and $y$ are 3 units and the subsistence level of $f$ is $-3$ and the $\kappa$-coefficients 0.3, 0.3 and 0.4 respectively. $G_c$ is 21 ($=15+2\times3$) and the low
bound for using $f$ is $G_H = 25.5$. The level of transition $G^*$ is calculated at about 45.69. We conclude that the performance of the numerical simulation is in line with our expectations. A similar result came from all similar attempts.

Figure B.2 Numerical results: The discontinuous entry of the input

GFsim: $F=15$, $kq=ky=.3$, gamma$q$=gamma$y$=3, gamma$f$=-3, $pq=py=pf=1$


Campaign Against the Arms Trade (CAAT), 1996, Killing Jobs, based on work by Dunne P, Smith R and Cooper N.


Levine P and Smith R, 1998, The arms trade game: From Laissez Faire to a common defence policy, mimeo, Department of Economics, University of Surrey.


