ENVIRONMENTAL SYSTEM ANALYSIS:  
THE APPLICATION OF LINEAR PROGRAMMING 
TO LIFE CYCLE ASSESSMENT 

Volume I 

A dissertation submitted towards 
the Degree of Doctor of Philosophy 

by 

Adisa Azapagić 

Centre for Environmental Strategy 
University of Surrey 

December 1996
This dissertation is presented in two volumes. Volume I includes Chapters 1-8, Appendices 1-4 and a part of Appendix 6. The rest of Appendix 6 and Appendices 5 and 7 are presented in Volume II. Each appendix is related to a chapter with the corresponding number; however, there is no appendix to Chapter 3.

Volume II contains company specific data which are confidential and it is available with the prior permission in writing of Borax-Europe Ltd.

©Adisa Azapagić 1996
SUMMARY

The objective of this work is to contribute to the methodological development of Life Cycle Assessment (LCA). Its intention is also to demonstrate more systematic use of LCA as a tool in environmental system analysis and management.

One of the unresolved problems in LCA is allocation of environmental burdens and impacts in the Inventory and Impact Assessment stages. Allocation refers to the problem of associating environmental burdens and impacts to each functional input or output of a multiple-function system. It is argued here that allocation is an artifact of applying LCA to individual products rather than to the whole productive system. To solve this problem, a new marginal allocation approach is proposed, based on whole system modelling. The marginal approach is applicable where the LCA study is concerned with incremental changes to a particular system. This work proposes the use of LP modelling as a valuable tool in representing marginal allocation in general. The allocation coefficients are then equivalent to the marginal values calculated at the solution of the LP model.

One of the main potential uses of LCA in environmental decision-making lies in providing a quantitative basis for achieving improvements in environmental performance of a system throughout the life cycle. However, this is associated with another problem in LCA: identifying the optimum solutions and choosing the best possible alternative in a system with multiple objectives. This work proposes the use of multiobjective optimisation whereby the system is simultaneously optimised on a number of environmental objective functions, defined by resource usages and emissions to air, water, and solid wastes. This approach provides a range of environmental optima which define the Pareto or non-inferior surface. The multidimensional Pareto surface then offers a number of possibilities for improving environmental performance of the system. Multiobjective Linear Programming (MOLP) is used in this work as a particular tool for identifying and evaluating the best possible options for environmental management of the product system.

Since system improvements cannot be carried out on the basis of environmental LCA only, it is also shown in this work that the compromise between environmental and economic performance can be found thus enabling the choice of Best Practicable Environmental Option (BPEO) not entailing excessive cost. The value of this approach in environmental system analysis lies in providing a set of alternative options for system improvements rather than a single prescriptive solution, which may be optimal but not appropriate for a particular situation.
These theoretical developments are tested, supported and demonstrated by application to a case study of an existing mineral-processing system producing five boron products. It is shown that LCA can successfully be combined with MOLP to satisfy both economic and environmental criteria for better performance of the whole system.
ACKNOWLEDGEMENTS

My thanks are due to a number of people:

Professor Roland Clift for his advice, guidance and above all his friendship which helped me carry on;

Mrs Sheila Sutherland for giving generously of her time and advice and helping invaluably in many ways;

ClifMar Associates Ltd. and the Committee of Vice Chancellors and Principals of the UK who made this research possible with financial support;

Friends and colleagues in the Centre for Environmental Strategy for the fun I had working with them;

US Borax Inc. and Borax Europe Ltd. who provided data for the case study and so helped in completing this project;

Mr Jonathan Rainer who enabled the contact with Borax and helped in various ways throughout;

Mr Gerald Pepper, Mr Art Beckerman, Dr Wayne Cooper, Ms Dara English, Mr Mike Kirby, Mr Joe Siefke, Mr Donald Kennedy, Mr Joe Newman, Mr David Parker, Mr Larry Voronyak and many others in the US Borax who assisted in data collection and made my stay at Boron very pleasant indeed;

Professor Hans Müller-Steinhagen for his support and understanding;

Dr Norman Kirkby for reading Chapters 2 and 3 and giving helpful suggestions for improvements;

Dr Alan Millington who kindly offered to teach Chemical Reaction Engineering to help me finish the dissertation;

And finally, but by no means the least, to my husband Slobodan, for his love without which little else would make much sense.
# CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUMMARY</td>
<td>iii</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>v</td>
</tr>
<tr>
<td>ABBREVIATIONS</td>
<td>ix</td>
</tr>
<tr>
<td>NOMENCLATURE</td>
<td>x</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xiii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xv</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>CHAPTER 1 LIFE CYCLE ASSESSMENT AND WHOLE SYSTEM MODELLING</td>
<td>4</td>
</tr>
<tr>
<td>1.1 Background</td>
<td>4</td>
</tr>
<tr>
<td>1.2 Methodological Framework for Life Cycle Assessment</td>
<td>5</td>
</tr>
<tr>
<td>1.3 Whole System Modelling and Life Cycle Assessment</td>
<td>12</td>
</tr>
<tr>
<td>1.4 Concluding Remarks</td>
<td>15</td>
</tr>
<tr>
<td>CHAPTER 2 THE CONCEPT OF LINEAR PROGRAMMING</td>
<td>16</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>16</td>
</tr>
<tr>
<td>2.2 Linear Programming Defined</td>
<td>17</td>
</tr>
<tr>
<td>2.3 Linear Programming Model and Solution - An Example</td>
<td>18</td>
</tr>
<tr>
<td>2.4 Dual Values</td>
<td>21</td>
</tr>
<tr>
<td>2.5 Sensitivity Analysis</td>
<td>24</td>
</tr>
<tr>
<td>2.6 Multiobjective Linear Programming</td>
<td>25</td>
</tr>
<tr>
<td>2.7 Concluding Remarks</td>
<td>29</td>
</tr>
</tbody>
</table>
CHAPTER 3  ALLOCATION OF ENVIRONMENTAL BURDENS IN MULTIPLE-FUNCTION SYSTEMS

3.1 Introduction 30
3.2 Procedures for Allocation in Multiple-function Systems 34
3.3 Marginal Allocation in Multiple-function Systems 41
3.4 Concluding Remarks 59

CHAPTER 4  MULTIOBJECTIVE DECISION-MAKING AND LIFE CYCLE ASSESSMENT

4.1 Classification of Multiobjective Programming Methods 61
4.2 Pareto Analysis - A History of Welfare Economics 65
4.3 Multiobjective Linear Programming and Life Cycle Assessment 68
4.4 Concluding Remarks 78

CHAPTER 5  LIFE CYCLE ASSESSMENT OF A MULTI-OUTPUT PRODUCT SYSTEM - A CASE STUDY

5.1 Goal Definition and Scoping 80
5.2 Inventory Analysis 84
5.3 Impact Assessment 109
5.4 Concluding Remarks 112

CHAPTER 6  ALLOCATION OF ENVIRONMENTAL BURDENS IN THE BORON PRODUCTS SYSTEM

6.1 Allocation in the Boron Products System 113
6.2 Linear Programming Model of the Boron Products System 117
6.3 Linear Programming and the Marginal Allocation Revisited 118
6.4 Results of Marginal Allocation in the Boron Products System 121
6.5 Allocation in the Cogeneration Plant 139
6.6 Concluding Remarks 144

CHAPTER 7  MULTIOBJECTIVE OPTIMISATION OF THE BORON PRODUCTS SYSTEM

7.1 Optimisation on Environmental Performance 147
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.2</td>
<td>Optimisation on Economic and Environmental Performance</td>
<td>156</td>
</tr>
<tr>
<td>7.3</td>
<td>Concluding Remarks</td>
<td>164</td>
</tr>
<tr>
<td>CHAPTER 8</td>
<td>CONCLUSIONS</td>
<td>166</td>
</tr>
<tr>
<td>REFERENCES</td>
<td></td>
<td>R-1</td>
</tr>
<tr>
<td>APPENDIX 1</td>
<td></td>
<td>A1-1</td>
</tr>
<tr>
<td>APPENDIX 2</td>
<td></td>
<td>A2-1</td>
</tr>
<tr>
<td>APPENDIX 4(4a and 4b)</td>
<td></td>
<td>A4-1</td>
</tr>
<tr>
<td>APPENDIX 5</td>
<td></td>
<td>A5-1</td>
</tr>
<tr>
<td>APPENDIX 6</td>
<td></td>
<td>A6-1</td>
</tr>
<tr>
<td>APPENDIX 7</td>
<td></td>
<td>A7-1</td>
</tr>
</tbody>
</table>
# ABBREVIATIONS

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>10Mol</td>
<td>Ten mol borate</td>
</tr>
<tr>
<td>5Mol</td>
<td>Five mol borate</td>
</tr>
<tr>
<td>AB</td>
<td>Anhydrous borate</td>
</tr>
<tr>
<td>ABA</td>
<td>Anhydrous Boric Acid</td>
</tr>
<tr>
<td>BA</td>
<td>Boric Acid</td>
</tr>
<tr>
<td>K/B</td>
<td>Kernite to Borax ratio</td>
</tr>
<tr>
<td>BOD</td>
<td>Biological Oxygen Demand</td>
</tr>
<tr>
<td>BPEO</td>
<td>Best Practicable Environmental Option</td>
</tr>
<tr>
<td>CBA</td>
<td>Cost-Benefit Analysis</td>
</tr>
<tr>
<td>COD</td>
<td>Chemical Oxygen Demand</td>
</tr>
<tr>
<td>EA</td>
<td>Environmental Audit</td>
</tr>
<tr>
<td>EIA</td>
<td>Environmental Impact Assessment</td>
</tr>
<tr>
<td>EPS</td>
<td>Expanded Polystyrene</td>
</tr>
<tr>
<td>FB</td>
<td>Fluid Bed (dryer)</td>
</tr>
<tr>
<td>GWP</td>
<td>Global Warming Potential</td>
</tr>
<tr>
<td>GWPI</td>
<td>Global Warming Potential Indirect</td>
</tr>
<tr>
<td>HC</td>
<td>Hydro-Carbons</td>
</tr>
<tr>
<td>HDPE</td>
<td>High Density Polyethylene</td>
</tr>
<tr>
<td>HIPS</td>
<td>High Intensity Polystyrene</td>
</tr>
<tr>
<td>ISO</td>
<td>International Organisation for Standardisation</td>
</tr>
<tr>
<td>LCA</td>
<td>Life Cycle Assessment</td>
</tr>
<tr>
<td>LCI</td>
<td>Life Cycle Inventory</td>
</tr>
<tr>
<td>LDPE</td>
<td>Low Density Polyethylene</td>
</tr>
<tr>
<td>LP</td>
<td>Linear Programming</td>
</tr>
<tr>
<td>MOLP</td>
<td>Multiobjective Linear Programming</td>
</tr>
<tr>
<td>OD</td>
<td>Ozone Depletion</td>
</tr>
<tr>
<td>PM</td>
<td>Particulate Matter</td>
</tr>
<tr>
<td>POCP</td>
<td>Photochemical Oxidants Creation Potential</td>
</tr>
<tr>
<td>PP</td>
<td>Polypropylene, Primary Process</td>
</tr>
<tr>
<td>SETAC</td>
<td>Society for Environmental Toxicology and Chemistry</td>
</tr>
<tr>
<td>TDS</td>
<td>Total Dissolved Solids</td>
</tr>
<tr>
<td>TSP</td>
<td>Total Suspended Particulates</td>
</tr>
<tr>
<td>TSS</td>
<td>Total Suspended Solids</td>
</tr>
<tr>
<td>US EPA</td>
<td>United States Environmental Protection Agency</td>
</tr>
</tbody>
</table>
**NOMENCLATURE**

- $a_i$: Input/output coefficients of a process or activity (t/t)
- $b$: Emissions of dioxin (t/t)
- $b_{\text{max}}$: Maximum emission of dioxins (t/t)
- $B$: Total emissions of dioxin (t)
- $b_{c_{j,i}}$: Environmental burden coefficients (t/t)
- $b_H$: Marginal change of dioxin emissions with $H$, for constant $M$, $L$, and $T$ (t/t)
- $b_{j,l}$: Marginal allocated product-related burden (t/t)
- $b_{j,p}$: Marginal allocated process-related burden (t/t)
- $b_{j,m}$: Marginal burden allocated to the material availability (t/t)
- $b_{j,i}$: Marginal burden allocated to the capacity (t/t)
- $b_{j,z}$: Marginal burden allocated to the heat requirement (t/t)
- $B_j$: Environmental burden (t)
- $b_M$: Marginal change of dioxin emissions with $M$, for constant $H$, $L$, and $T$ (t/t)
- $b_L$: Marginal change of dioxin emissions with $L$, for constant $M$, $H$, and $T$ (t/t)
- $b_T$: Marginal change of dioxin emissions with $T$, for constant $M$, $H$, and $L$ (t/t)
- $C$: Cost objective function ($\)\$
- $C^*$: Optimum value of the cost function obtained by single-objective optimisation ($\)\$
- $c_i$: Capacity of a process or an operation unit (t)
- $c_i$: Coefficients in the objective function; cost coefficients in the cost objective function ($$/t)
- $CVA$: Volume of air in the Critical volume approach (m$^3$)
- $CVW$: Volume of water in the Critical volume approach (m$^3$)
- $D_j$: Demand on the output of the products (t)
- $e_{c_{\text{GWP},j}}$: GWP coefficient for burden $B_j$ (t/t)
- $e_{c_{k,j}}$: Environmental impact coefficients (t/t)
- $e_{k,m}$: Marginal impact allocated to the material availability (t/t)
- $e_{k,i}$: Marginal impact allocated to the capacity (t/t)
- $e_{k,l}$: Marginal allocated product-related impact (t/t)
- $e_{k,z}$: Marginal impact allocated to the heat requirement (t/t)
- $e_j$: Right hand side coefficients or parameters of the constraints (t/t)
- $E_k$: Environmental impact (t); (m$^3$)
- $F$: Economic objective function ($\)\$, (£)
- $f_j = \sum_{i=1}^{l} a_{ji}x_i$
- $GWP$: Global warming potential objective function (t)
GWP* Optimum value of the global warming potential objective function obtained in the single-objective optimisation

h Chlorine fraction in waste
H Total chlorine content in the waste
H₂ Heat requirement in the system
M Total mass of waste processed in the waste incinerator
l Specific calorific value of the waste
L Total calorific value of the waste
P Total production objective function
P* Optimum value of the P objective function obtained in the single-objective optimisation
P₁ Output of the product 1
PR Profit objective function
pr₁ Coefficients in the profit objective function
Qₜ Heat availability
Rₘ Raw material availability
Rₚ Process-related parameter
Sₘ Supply of primary material m
T Combustion temperature in the incinerator
uₙ Material-(or product-) related parameter
Uₙ Material-(or product-) related partial derivative
vₘ Process-related parameter
Vₘ Process-related partial derivative
wₗ Multiplier in eqn. (A2.16)
wₜ Dual value of the active constrained objective in multiobjective optimisation
wₗ Multiplier in eqns. (A2.15) and (A2.16)
Xᵢ Output from a process or activity (operation level)

Greek letters

λⱼ Dual or marginal values; Kuhn-Tucker multipliers
εₗ Right hand side of the constrained objective in multiobjective optimisation
εₗₗ Lower bound on εₗ
εₗₗₗ Upper bound on εₗ
r Number of different values of εₗ
r Number of multiobjective optimisations
Vectors

\[ \mathbf{x} \]  \quad \text{I-dimensional vector of decision variables}

\[ \mathbf{X} \]  \quad \text{Feasible decision region}

\[ \mathbf{x}^q \]  \quad \text{Optimal solution of the } q \text{th objective}
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 1.1</td>
<td>The state of development of LCA methodology</td>
<td>11</td>
</tr>
<tr>
<td>Table 2.1</td>
<td>Optimal solution for the LP example</td>
<td>21</td>
</tr>
<tr>
<td>Table 2.2</td>
<td>Dual values as a result of change of the state of the system</td>
<td>23</td>
</tr>
<tr>
<td>Table 3.1</td>
<td>Possible approaches to representing technologies in foreground and background systems</td>
<td>35</td>
</tr>
<tr>
<td>Table 3.2</td>
<td>Marginal allocation in the example of the co-product system</td>
<td>54</td>
</tr>
<tr>
<td>Table 3.3</td>
<td>Change of marginal values with change in the state of the system</td>
<td>55</td>
</tr>
<tr>
<td>Table 3.4</td>
<td>Marginal allocation in the open-loop recycling example</td>
<td>58</td>
</tr>
<tr>
<td>Table 4.1</td>
<td>Pay-off table for the two-objective optimisation of the polymers system</td>
<td>73</td>
</tr>
<tr>
<td>Table 5.1</td>
<td>Functional units in relation to the intended use of the study</td>
<td>84</td>
</tr>
<tr>
<td>Table 6.1</td>
<td>Product categories with respect to a different type of packaging</td>
<td>123</td>
</tr>
<tr>
<td>Table 6.2</td>
<td>Changes in the allocated CO₂ emissions with changes in the state of the system</td>
<td>136</td>
</tr>
<tr>
<td>Table 6.3</td>
<td>Allocation methods considered for the steam cogeneration</td>
<td>144</td>
</tr>
<tr>
<td>Table A2.1</td>
<td>Pay-off table for a multiobjective problem</td>
<td>A2-5</td>
</tr>
<tr>
<td>Table A4.1</td>
<td>Pay-off table for the two-person zero-sum game</td>
<td>A4-9</td>
</tr>
<tr>
<td>Table A4.2</td>
<td>LP matrix for the example of the polymers system</td>
<td>A4-12</td>
</tr>
<tr>
<td>Table A5.1</td>
<td>Environmental burdens for Blasting and transport</td>
<td>A5-35</td>
</tr>
<tr>
<td>Table A5.2</td>
<td>Environmental burdens for Primary crusher</td>
<td>A5-38</td>
</tr>
<tr>
<td>Table A5.3</td>
<td>Environmental burdens for Mining</td>
<td>A5-40</td>
</tr>
<tr>
<td>Table A5.4</td>
<td>Environmental burdens for Secondary crusher</td>
<td>A5-42</td>
</tr>
<tr>
<td>Table A5.5</td>
<td>Environmental burdens for Primary process</td>
<td>A5-44</td>
</tr>
<tr>
<td>Table A5.6</td>
<td>Environmental burdens for Boric acid</td>
<td>A5-47</td>
</tr>
<tr>
<td>Table A5.7</td>
<td>Environmental burdens for Anhydrous boric acid</td>
<td>A5-49</td>
</tr>
<tr>
<td>Table A5.8</td>
<td>Environmental burdens for Anhydrous borax</td>
<td>A5-51</td>
</tr>
<tr>
<td>Table A5.9</td>
<td>Environmental burdens for Packaging and shipping</td>
<td>A5-53</td>
</tr>
<tr>
<td>Table A5.10</td>
<td>Environmental burdens for the Steam plant</td>
<td>A5-56</td>
</tr>
</tbody>
</table>
Table A5.11 Environmental burdens for the Cogeneration
Table A5.12 Total environmental burdens from the boron system
Table A5.13 Total environmental impacts from the boron system
Table A5.14 Environmental impacts from the boron system normalised to the world annual impacts
Table A5.15 Classification factors used in the Impact Assessment stage
Table A6.1 Environmental burdens and cost coefficients in the boron LP model
Table A6.2 Environmental burdens allocated to the products
Table A6.3 Environmental impacts allocated to the products
Table A7.1 Results of single-objective optimisations on the environmental burdens
Table A7.2 Results of single-objective optimisations on the environmental impacts
Table A7.3 Pay-off table for single-objective optimisations on the environmental burdens
Table A7.4 Pay-off table for single-objective optimisations on the environmental impacts
Table A7.5a Selected noninferior solutions optimisations on two environmental burdens (Gas and NOx)
Table A7.5b Selected noninferior solutions optimisations on two environmental impacts (GWPI and POCP)
Table A7.6 Pay-off table for single-objective optimisations on the environmental impacts, total production and costs
Table A7.7 Selected noninferior solutions optimisations on Global warming potential (GWP), Production (P) and Costs (C)
Table A7.8 Selected noninferior solutions optimisations on Global warming potential (GWP), Production (P) and Costs (C)
Table A7.9 Selected noninferior solutions optimisations on Global warming potential (GWP), Ozone depletion (OD), Production (P) and Costs (C)
Table A7.10 Selected noninferior solutions optimisations on Global warming potential (GWP), Ozone depletion (OD), Production (P) and Costs (C)
**LIST OF FIGURES**

<table>
<thead>
<tr>
<th>Fig.</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Stages in the life cycle of a product</td>
<td>5</td>
</tr>
<tr>
<td>1.2</td>
<td>Technical framework for Life Cycle Assessment</td>
<td>7</td>
</tr>
<tr>
<td>1.3</td>
<td>Interactions between LCA stages</td>
<td>10</td>
</tr>
<tr>
<td>1.4</td>
<td>Environmental system analysis</td>
<td>13</td>
</tr>
<tr>
<td>1.5</td>
<td>Life Cycle Assessment and Linear Programming</td>
<td>14</td>
</tr>
<tr>
<td>2.1</td>
<td>Graphical presentation of solution for the LP example</td>
<td>20</td>
</tr>
<tr>
<td>2.2</td>
<td>Graphical interpretation of noninferiority</td>
<td>26</td>
</tr>
<tr>
<td>2.3</td>
<td>Feasible region and noninferior set in objective space for the LP example</td>
<td>27</td>
</tr>
<tr>
<td>3.1</td>
<td>Multiple-function systems where the problem of allocation occurs</td>
<td>31</td>
</tr>
<tr>
<td>3.2</td>
<td>Foreground and background system</td>
<td>33</td>
</tr>
<tr>
<td>3.3</td>
<td>Avoiding allocation by system enlargement</td>
<td>36</td>
</tr>
<tr>
<td>3.4</td>
<td>Variation with waste composition of dioxin emitted per tonne waste</td>
<td>48</td>
</tr>
<tr>
<td>3.5</td>
<td>Variation with waste properties of total dioxin emitted</td>
<td>49</td>
</tr>
<tr>
<td>3.6</td>
<td>Simplified LCA flow diagram for the co-product example</td>
<td>51</td>
</tr>
<tr>
<td>3.7</td>
<td>Allocation by marginal values for the co-product example</td>
<td>52</td>
</tr>
<tr>
<td>3.8</td>
<td>Simplified LCA flow diagram for the open-loop recycling example</td>
<td>57</td>
</tr>
<tr>
<td>4.1</td>
<td>Relationship among multiobjective methods</td>
<td>62</td>
</tr>
<tr>
<td>4.2</td>
<td>A welfare frontier for a two-individual society</td>
<td>66</td>
</tr>
<tr>
<td>4.3</td>
<td>LCA flow diagram of the polymers system</td>
<td>72</td>
</tr>
<tr>
<td>4.4</td>
<td>Noninferior curve for environmental optimisation of the polymers system</td>
<td>74</td>
</tr>
<tr>
<td>4.5</td>
<td>Polymer production for selected noninferior solutions</td>
<td>75</td>
</tr>
<tr>
<td>4.6</td>
<td>Noninferior curve for economic and environmental optimisation of the polymers system</td>
<td>76</td>
</tr>
<tr>
<td>4.7</td>
<td>Polymer production for selected noninferior solutions</td>
<td>77</td>
</tr>
<tr>
<td>4.8</td>
<td>Choosing the compromise solution</td>
<td>78</td>
</tr>
<tr>
<td>5.1</td>
<td>Schematic representation of the background and foreground subsystems</td>
<td>83</td>
</tr>
<tr>
<td>5.2</td>
<td>Air view of the foreground system</td>
<td>85</td>
</tr>
</tbody>
</table>
Fig. 5.3 Flow diagram of the foreground system 87
Fig. 5.4 Blasting and transport: Comparison of resource requirements 92
Fig. 5.5 Blasting and transport: Comparison of emissions to air, water and land 92
Fig. 5.6 Primary crusher: Comparison of resource requirements 93
Fig. 5.7 Primary crusher: Comparison of emissions to air, water and land 93
Fig. 5.8 Total mining: Comparison of resource requirements 94
Fig. 5.9 Total mining: Comparison of emissions to air, water and land 94
Fig. 5.10 Secondary crusher: Comparison of resource requirements 95
Fig. 5.11 Secondary crusher: Comparison of emissions to air, water and land 95
Fig. 5.12 Primary process: Comparison of resource requirements 97
Fig. 5.13 Primary process: Comparison of emissions to air, water and land 97
Fig. 5.14 Boric acid: Comparison of resource requirements 99
Fig. 5.15 Boric acid: Comparison of emissions to air, water and land 99
Fig. 5.16 Anhydrous boric acid: Comparison of resource requirements 100
Fig. 5.17 Anhydrous boric acid: Comparison of emissions to air, water and land 100
Fig. 5.18 Anhydrous borax: Comparison of resource requirements 101
Fig. 5.19 Anhydrous borax: Comparison of emissions to air, water and land 101
Fig. 5.20 Refinery packing and shipping: Comparison of resource requirements 103
Fig. 5.21 Refinery packing and shipping: Comparison of emissions to air, water and land 103
Fig. 5.22 Boric acid packing and shipping: Comparison of resource requirements 104
Fig. 5.23 Boric acid packing and shipping: Comparison of emissions to air, water and land 104
Fig. 5.24 Steam plant: Comparison of resource requirements 105
Fig. 5.25 Steam plant: Comparison of emissions to air, water and land 105
Fig. 5.26 Cogeneration: Comparison of resource requirements 107
Fig. 5.27 Cogeneration: Comparison of emissions to air, water and land 107
Fig. 5.28 Total environmental burdens: Comparison of resource requirements 108
Fig. 5.29 Total environmental burdens: Comparison of emissions to air, water and land 108
Fig. 5.30 Total environmental impacts of the boron products system 111
Fig. 5.31 Environmental impacts normalised to the world annual impacts 111
Fig. 6.1 Avoiding allocation by system enlargement 114
Fig. 6.2 Avoiding allocation by the "avoided burdens" approach 115
Fig. 6.3 Environmental burdens in the boron products system allocated by the marginal approach 122
Fig. 6.4 Environmental impacts in the boron products system allocated by the marginal approach 122
Fig. 6.5 CO₂ emissions allocated by the marginal approach 124
Fig. 6.6 Comparison of different allocation methods for CO₂ emissions 125
Fig. 6.7 SO₂ emissions allocated by the marginal approach 126
Fig. 6.8 Comparison of different allocation methods for SO₂ emissions 127
Fig. 6.9 Solid waste allocated by the marginal approach 128
Fig. 6.10 Comparison of different allocation methods for solid waste 129
Fig. 6.11 Abiotic reserve depletion allocated by the marginal approach 130
Fig. 6.12 Comparison of the allocation methods for abiotic reserve depletion 131
Fig. 6.13 Changes in the allocated CO₂ emissions with change of the state of the system 133
Fig. 6.14 Changes in the allocated fuel consumption with change of the state of the system 134
Fig. 6.15 Process- and product-related burdens in the boron system 137
Fig. 6.16 Avoiding allocation in the cogeneration system by system enlargement 140
Fig. 6.17 Avoiding allocation in the cogeneration system by the "avoided burdens" approach 141
Fig. 6.18 "Avoided burdens" approach and allocation on the heat content basis 142
Fig. 6.19 Comparison of different allocation methods in the steam cogeneration system 143
Fig. 6.20 Sensitivity analysis: Change of the total burdens compared to the "avoided burdens" approach 145

Fig. 7.1 Improvements in the boron system: environmental burdens 151
Fig. 7.2 Improvements on the boron system: environmental impacts 151
Fig. 7.3 Noninferior curve for multiobjective optimisation on Gas and NOx objective functions 153
Fig. 7.4 Selected noninferior solutions for multiobjective optimisation on Gas and NOx objective functions 154
Fig. 7.5 Noninferior curve for multiobjective optimisation on GWPI and POCP objective functions 155
Fig. 7.6 Selected noninferior solutions of multiobjective optimisation on GWPI and POCP objective functions 156
Fig. 7.7 Noninferior surface for optimisation on GWP, P and C objectives 158
Fig. 7.8 Noninferior subsets for optimisation on GWP, P and C, for constant GWP 159
Fig. 7.9 Noninferior subsets for optimisation on GWP, P and C, for constant P 159
Fig. 7.10a Noninferior surface for optimisation on OD, GWP, P and C, for P/P*=0.982 162
Fig. 7.10b Noninferior surface for optimisation on OD, GWP, P and C, for P/P*=0.987 162
Fig. 7.10c Noninferior surface for optimisation on OD, GWP, P and C, for P/P*=0.992 163
Fig. 7.10d Noninferior surface for optimisation on OD, GWP, P and C, for P/P*=1.000 163
Fig. A5.1 Flow diagram of the Mining subsystem A5-6
Fig. A5.2 Mine: Flow diagram of the Blasting and transport subsystem A5-7
Fig. A5.3 Mine: Flow diagram of the Primary crusher area A5-8
Fig. A5.4 Flow diagram of the Secondary crusher A5-9
Fig. A5.5 Flow diagram of the Primary process A5-10
Fig. A5.6 Primary process: Flow diagram of the Dissolvers area A5-11
Fig. A5.7 Primary process: Flow diagram of the Thickeners area A5-12
Fig. A5.8 Primary process: Flow diagram of the 5Mol crystallisers area A5-13
Fig. A5.9 Primary process: Flow diagram of the 10Mol crystallisers area A5-14
Fig. A5.10 Primary process: Flow diagram of the Cold 10Mol crystallisers area A5-15
Fig. A5.11 Primary process: Flow diagram of the Product dryers area A5-16
Fig. A5.12 Flow diagram of the Boric acid plant A5-17
Fig. A5.13 Boric Acid: Flow diagram of the Reactor area A5-18
Fig. A5.14 Boric Acid: Flow diagram of the Thickeners area A5-19
Fig. A5.15 Boric Acid: Flow diagram of the Crystallisers area A5-20
Fig. A5.16 Boric Acid: Flow diagram of the First stage vacuum filter area A5-21
Fig. A5.17 Boric Acid: Flow diagram of the Second stage vacuum filter area A5-22
Fig. A5.18  Boric Acid: Flow diagram of the Spent wash system area  A5-23
Fig. A5.19  Boric Acid: Flow diagram of the Settlers, Centrifuges, and Dryers area  A5-24
Fig. A5.20  Flow diagram of the Anhydrous boric acid plant  A5-25
Fig. A5.21  Flow diagram of the Anhydrous borax plant  A5-26
Fig. A5.22  Flow diagram of the Refinery packing and shipping subsystem  A5-27
Fig. A5.23  Flow diagram of the Boric acid packing and shipping subsystem  A5-28
Fig. A5.24  Flow diagram of the Steam plant  A5-29
Fig. A5.25  Flow diagram of the Cogeneration plant  A5-30
Fig. A5.26  Flow diagram for natural gas  A5-31
Fig. A5.27  Flow diagram for diesel fuel  A5-31
Fig. A5.28  Flow diagram for ammonium nitrate  A5-32
Fig. A5.29  Flow diagram for soda ash  A5-32
Fig. A5.30  Flow diagram for sulphuric acid  A5-32
Fig. A5.31  Flow diagram for the paper bag packaging  A5-33
Fig. A5.32  Flow diagram for the polypropylene packaging  A5-33
Fig. A5.33  Packaging: Comparison of resource requirements  A5-34
Fig. A5.34  Packaging: Comparison of emissions to air, water and land  A5-34

Fig. A6.1  Nuclear electricity allocated by the marginal approach  A6-40
Fig. A6.2  Hydro-electricity allocated by the marginal approach  A6-40
Fig. A6.3  Coal reserves allocated by the marginal approach  A6-40
Fig. A6.4  Oil reserves allocated by the marginal approach  A6-41
Fig. A6.5  Gas reserves allocated by the marginal approach  A6-41
Fig. A6.6  Other non-renewable resources allocated by the marginal approach  A6-41
Fig. A6.7  Renewable resources allocated by the marginal approach  A6-42
Fig. A6.8  CO emissions allocated by the marginal approach  A6-42
Fig. A6.9  NOx emissions allocated by the marginal approach  A6-42
Fig. A6.10  VOC emissions allocated by the marginal approach  A6-43
Fig. A6.11  Emissions of metals to air allocated by the marginal approach  A6-43
Fig. A6.12  Dust emissions allocated by the marginal approach  A6-43
Fig. A6.13  Halide emissions allocated by the marginal approach  A6-44
Fig. A6.14  Waste water discharge allocated by the marginal approach  A6-44
Fig. A6.15  Emissions of metals to water allocated by the marginal approach  A6-44
Fig. A6.16  Total dissolved solids (TDS) allocated by the marginal approach  A6-45
Fig. A6.17  Total suspended solids (TSS) allocated by the marginal approach  A6-45
Fig. A6.18  Emissions of oil and greases allocated by the marginal approach  A6-45
<table>
<thead>
<tr>
<th>Fig. A6.19</th>
<th>Chemical oxygen demand (COD) allocated by the marginal approach</th>
<th>A6-46</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. A6.20</td>
<td>Biological oxygen demand (BOD) allocated by the marginal approach</td>
<td>A6-46</td>
</tr>
<tr>
<td>Fig. A6.21</td>
<td>Solid waste (landfill) allocated by the marginal approach</td>
<td>A6-46</td>
</tr>
<tr>
<td>Fig. A6.22</td>
<td>Direct global warming potential (GWP) allocated by the marginal approach</td>
<td>A6-47</td>
</tr>
<tr>
<td>Fig. A6.23</td>
<td>Indirect global warming potential (GWPI) allocated by the marginal approach</td>
<td>A6-47</td>
</tr>
<tr>
<td>Fig. A6.24</td>
<td>Ozone depletion potential (OD) allocated by the marginal approach</td>
<td>A6-47</td>
</tr>
<tr>
<td>Fig. A6.25</td>
<td>Acidification potential allocated by the marginal approach</td>
<td>A6-48</td>
</tr>
<tr>
<td>Fig. A6.26</td>
<td>Nutrification potential allocated by the marginal approach</td>
<td>A6-48</td>
</tr>
<tr>
<td>Fig. A6.27</td>
<td>Photochemical oxidants creation potential (POCP) allocated by the marginal approach</td>
<td>A6-48</td>
</tr>
<tr>
<td>Fig. A6.28</td>
<td>Human toxicity allocated by the marginal approach</td>
<td>A6-49</td>
</tr>
<tr>
<td>Fig. A6.29</td>
<td>Landfill volume (excluding overburden) allocated by the marginal approach</td>
<td>A6-49</td>
</tr>
</tbody>
</table>
INTRODUCTION

The impact of human activities on the environment is not a new phenomenon. Since the earliest societies, human activities such as use of fire, agriculture and management of grazing animals have influenced the natural world and processes in it. However, it is only with the development of industrial technology and use of fossil fuels that human actions have had more significant detrimental effects on the environment. Industrialisation did not bring the advancement of the world only, it also brought about a number of environmental problems which now pose potential threats not only to humans themselves but to other organisms in the biosphere. Resource depletion, air, water and land pollution, the extinction of species, and the disappearance of wild life are some of the examples of the environmental problems which have emerged as a result of intensified human interventions into the environment.

Today, ever increasing technological power enables humans to transform the environment in different ways, changing radically the nature and extent of environmental impacts of human activities. One of the main resulting problems is that many of these activities may not have an immediate effect and some may have a more global impact on the environment. This is becoming apparent with the increasing awareness of the cumulative and synergistic effects of some of the environmental impacts over space and time. For instance, emissions of greenhouse gases can occur locally, but the resulting greenhouse effect will have a global character.

Assigning responsibilities to those who have caused the environmentally damaging effects on such a global scale is not a simple matter. Due to the interrelated human activities and the collective and possibly synergistic nature of their interventions in the environment, it is no longer possible to put blame for environmental degradation on an individual, company, industrial concern or state. In a global context, where the concept of the environment has broadened to include the whole world, the responsibility for preserving the environment and finding ways to reduce environmentally harmful impacts of human activities falls on all of us.

It is in this changing understanding of the environment and its broadened boundaries that the concept of Life Cycle Assessment (LCA) started to emerge in the early seventies. Although now present as a concept for about twenty years, LCA has received wider attention and methodological development only with the beginning of the nineties when its relevance as an environmental management aid in both corporate and public decision-
making became more evident. For many, LCA represented a paradigm shift and it irrevocably changed the way the environmental problems were seen. It clearly pointed out that, if sustainable solutions to environmental problems are to be found, then they must be sought on a more global level.

Today, the methodology is still developing and a number of unresolved issues remain to be addressed. This dissertation represents an attempt to assist in solving some of the methodological problems in LCA. Its intention is also to demonstrate more systematic use of LCA as a tool in environmental system analysis and management.

As an introduction to the subject, Chapter 1 focuses on some of the methodological problems and, after outlining the history of LCA and its present methodological framework, it sets out to explain the importance of whole system modelling and optimisation in the context of LCA. A specific mathematical modelling and optimisation tool used in this work is Linear Programming (LP), which is introduced and discussed in Chapter 2. In the view of possible applications of LP in LCA, the emphasis there is placed on interpreting the LP solutions, rather than on the theory of LP. Chapter 2 concludes with an overview of Multiobjective LP, a decision-making tool proposed in this work for use in the context of LCA.

One of the unresolved methodological problems in LCA is allocation of environmental burdens and impacts in multiple-function systems. This is the subject of Chapter 3. There, the allocation problem is introduced and illustrated on specific examples of the co-product, waste treatment and recycling systems. The approach to solving allocation by using LP and marginal values developed in this work is then explained and discussed for these three examples.

Chapter 4 represents a review of multiobjective-decision making in the context of LCA. A number of the techniques and tools, developed to help in multiobjective decision-making processes, are discussed in this chapter. They are compared and contrasted with Multiobjective Linear Programming (MOLP), a specific decision-making tool used in this work. This chapter, therefore, focuses on a possible application of MOLP in the decision making process, with particular emphasis on its use in the LCA context.

To demonstrate the application of LP in LCA, as well as the potential of LCA as a tool in environmental system management, a case study of the co-production of boron products has been carried out and these results are presented in Chapter 5. In Chapter 6, LP and the marginal approach are used to allocate the burdens and impacts among
Introduction

different outputs in the boron system as part of the Inventory and Impact Assessment stages.

The system is then optimised on environmental performance to identify a range of optimum solutions and possibilities for improvements. In addition, the economic performance is also optimised, to illustrate that both environmental and economic criteria can be satisfied in a system with conflicting objectives. It is argued that this method provides a more effective approach to environmental management of product system by offering a range of alternative optimal solutions and enabling decision-makers to choose the best practicable environmental option not entailing excessive cost. These considerations are part of the Improvement Assessment stage and are presented in Chapter 7.

The main findings and the conclusions of this thesis are summarised in Chapter 8. The recommendations for future work are also given in this chapter.
CHAPTER 1

LIFE CYCLE ASSESSMENT AND
WHOLE SYSTEM MODELLING

Life Cycle Assessment is becoming an increasingly important decision-making aid in environmental system management. Its potential is being recognised not only by industry but also by policy makers and planners, educators and others. The LCA methodology is still developing and a number of unresolved issues remain to be addressed. This chapter introduces some of these problems and, after outlining the history of LCA and its present methodological framework, it explains the need for a whole system modelling approach in the context of LCA.

1.1 Background

Life Cycle Assessment is a technique for assessing the environmental performance of a product, process or activity from "cradle to grave", i.e. from extraction of raw materials to final disposal. Today's LCA originates from "net energy analysis" studies, which were first published in the 1970's (e.g. Smith, 1969; Bousted, 1972; Hannon, 1972; Sundstrom, 1973) and considered only energy consumption over a life cycle of a product or a process. Some later studies included wastes and emissions (Hunt and Franklin, 1974; Barber et al., 1977; Ayres, 1978; Lundholm and Sundstrom, 1985; Bousted, 1989), but none of them went further than just quantifying materials and energy use. At this point it was clear that a more sophisticated approach to complex environmental issues was needed.

As a result, in 1990, the Society for Environmental Toxicology and Chemistry (SETAC) initiated activities to define LCA and develop a general methodology for conducting the LCA studies. Soon afterwards, the International Organisation for Standardisation (ISO) started similar work on developing principles and guidelines on the LCA methodology. Although SETAC and ISO worked independently of each other, they reached a general consensus on the methodological framework. It is expected that both bodies will produce the final documents on the internationally standardised LCA methodology by the Spring 1997. This methodology is described in the following section. Because of the involvement of the author of this thesis in the work of SETAC from an early stage of development of the LCA methodology, the further discussion is related to that work.
However, since the differences between SETAC and ISO methodology are only in a matter of detail, the discussion presented here also applies to the work of ISO.

### 1.2 Methodological Framework for Life Cycle Assessment

Life Cycle Assessment, as defined by SETAC, is "a process to evaluate the environmental burdens associated with a product, process, or activity by identifying and quantifying energy and materials used and wastes released to the environment; to assess the impact of those energy and material uses and releases to the environment; and to identify and evaluate opportunities to effect environmental improvements" (Consoli, 1993). It follows the life cycle of a product, process or activity from extraction of raw materials to final disposal, including manufacturing, transport, use, re-use, maintenance and recycling (Fig. 1.1).

![Fig. 1.1 Stages in the life cycle of a product](image-url)
As illustrated in Fig. 1.2, the methodological framework for conducting LCA comprises four main stages (Fava et al., 1993; Consoli, 1993):

1. Goal Definition and Scoping
2. Inventory Analysis
3. Impact Assessment
4. Improvement Assessment

1.2.1 Goal Definition and Scoping

The first and probably most critical phase of an LCA study is Goal Definition and Scoping. This component includes defining the purpose of the study and its intended use, i.e. whether the study is going to be used internally by a company for improving the performance of the system or externally, e.g. for influencing public policy. Scoping explains what assumptions have been made and why, and defines the limitations of the study and the system boundaries, including its spatial and temporal limits. One of the most important parts of an LCA study - the functional unit - is also defined in this stage. The functional unit is a quantitative measure of the output of products or services which the system delivers. In comparative studies it is crucial that the systems are compared on the basis of equivalent function, i.e. functional unit. Some examples of the functional unit are "the quantity of packaging used to deliver a given volume of beverage" or "the amount of detergent necessary for an average household wash". This stage also includes an assessment of the data quality and establishing the specific data quality goals. Goal Definition and Scoping are constantly reviewed and refined during the process of carrying out an LCA, as additional information on the system becomes available.

1.2.2 Inventory Analysis

Life Cycle Inventory (LCI) analysis is the second stage in undertaking an LCA study. It is the most objective of all LCA stages and represents a quantitative description of the system through material and energy balances. Inventory Analysis includes:

- further definition of the system and its boundaries;
- representing the system in the form of flow diagrams;
- data collection;
- allocation of environmental burdens;
- calculation and reporting of the results; and
- sensitivity analysis.

On the basis of the system definition in the Goal Definition and Scoping stage, the system is further defined and characterised in the LCI in order to clearly identify the data needs. The system is disaggregated into a number of inter-linked subsystems and their interconnectedness is shown by flow diagrams. Depending on how detailed data are available, the subsystems can represent the operation units or a group of the units. Each subsystem is described in detail by flows of materials and energy, as well as emissions to air and water and solid wastes. All inputs and outputs of the subsystems are balanced in this stage and data are normalised with respect to the unit output from each subsystem.

On the basis of the data collected for a period statistically relevant for the study, the environmental burdens, i.e. resource depletion and emissions to air and solid wastes, are calculated for the whole system and the results are listed in the inventory tables and may be represented graphically. Environmental burdens include, for instance, fossil fuel consumption, emissions of sulphur dioxide, emissions of metals to water, volume of the solid waste, etc.

Fig. 1.2 Methodological framework for Life Cycle Assessment
(Consoli, 1993)
Since the data quality varies in all LCIs, it is also necessary to perform a sensitivity analysis in order to identify the effects that data variability, uncertainties and data gaps have on the final results of the study. The Inventory Analysis stage also includes allocation of environmental burdens in multiple-function systems, such as co-product systems, waste treatment and recycling. The allocation problem is addressed in detail in Chapter 3.

1.2.3 Impact Assessment

The effects of the environmental burdens identified and quantified in the Inventory Analysis stage are assessed and characterised in the Impact Assessment stage. This part of LCA is based on both quantitative and qualitative procedures to characterise and assess the environmental impacts of a system. It consists of three steps: Classification, Characterisation and Valuation.

Classification is a qualitative step of the Impact Assessment stage, in which the burdens are aggregated into a smaller number of impact categories which may lead to impacts on human and ecological health and to depletion of resources. The aggregation is carried out over all potential impacts of the burdens, so that one burden can be associated with a number of impacts. For instance, VOCs contribute to both global warming and ozone layer depletion. Among others, two methods most commonly used for classification of the impacts are problem-oriented and medium-oriented approach. In the problem-oriented approach, the burdens are aggregated according to the relative contributions to specific potential environmental effects. The effects most commonly considered in LCA are: Resource depletion, Global warming potential (direct and indirect), Ozone depletion, Acidification, Nutrification, Photochemical oxidant formation potential, Human toxicity and Ecotoxicity. The medium-oriented approach aggregates the burdens according to the medium into which they are released, i.e. air, water and soil.

Characterisation is the process for estimating the potential impacts of the aggregated burdens. This is a quantitative stage of LCA and should be based on the scientific findings on the relevant environmental impacts. In the problem-oriented approach, the impacts are calculated relative to a reference substance. For instance, CO₂ is a reference gas for determining the Global warming potential of other related gases, such as CH₄ and other VOCs. The calculation procedure for different impact categories in the problem-oriented approach developed by Heijungs et al. (1992) is the most widely used so far and its details are given in Appendix 1. On the other hand, the medium-oriented or critical
volume approach uses regulatory standards to calculate volumes of air and water that would be necessary to dilute the emissions to such an extent that the maximum allowable concentration is not exceeded. This method, therefore, includes critical air and water volumes; in addition energy consumption and volume of the solid waste are also calculated (see Appendix 1). Lack of regulatory standards for a number of the burdens (e.g. CO$_2$, CH$_4$, etc.) is one of the limiting factors for the use of this approach in the Impact Assessment stage. Further considerations in this work will, therefore, focus on the problem-oriented approach.

The impacts identified and quantified in the problem-oriented approach can also be normalised on the total emissions or extractions in a certain area over a given period of time (Guinée, 1993). It is argued that, since LCA is global in its character, total world annual impacts should be used as the basis for Normalisation. Total emissions of global warming gases and world resource depletion can be calculated relatively easily; however, other impacts, such as acidification or human toxicity, are more difficult to determine on the global level so that normalisation is still not a reliable method for comparing different environmental impacts from a system.

The final and the most subjective step of the Impact Assessment stage is Valuation, in which the relative significance of different impacts is weighted so that they can be compared among themselves. As a result, different environmental impacts are reduced to a single environmental impact function, as a measure of environmental performance. A number of techniques have been suggested for use in Valuation. They are mainly based on expressing preferences either by decision-makers, "experts" or by the public. Some of these methods include Multiattribute Utility Theory, Analytic Hierarchy Process, Impact Analysis Matrix, Cost-Benefit Analysis and Contingent Valuation (Fava et al., 1993). However, because of a number of problems associated with using these techniques in the context of LCA, there is no consensus at present on how to aggregate the environmental impacts into a single environmental impact function. Some of these methods are further defined and discussed in Chapter 4 and Appendix 4.

1.2.4 Improvement Assessment

On the basis of the results of the Inventory Analysis and Impact Assessment stages, Improvement Assessment identifies and evaluates the options for improving the environmental performance of the system. This stage can be carried out before an LCA study is completed because the opportunities for improvements can be detected at an
early stage of carrying out the study. The redesign of the product or a process, as a result of the Improvement stage, is not part of the LCA - it is one of its applications. Similar to the Valuation stage, the methodology for this phase of LCA has not been agreed upon yet. Indeed, ISO has decided to combine Valuation and Improvement Assessment into a phase called Interpretation, and not to attempt to develop a standard at least at this stage.

LCA is an iterative process in which all stages are closely interrelated to each other and, depending on the results of subsequent stages, they can be changed and refined. This is illustrated schematically in Fig. 1.3.

As already mentioned, the LCA methodology is still being developed and, as summarised in Table 1.1, several issues remain to be addressed. Life Cycle Inventory (LCI) is the best developed and documented stage in LCA; however there are still unresolved problems that require further research. One of these problems is allocation of environmental burdens in multiple-function systems (Consoli, 1993). Within it, development of modelling techniques and procedures for co-product systems, waste treatment and recycling, with appropriate allocation of burdens to sources, is particularly important.

Fig. 1.3 Interactions between LCA stages
(Fava et al., 1993)
Impact Assessment is in an early stage of development; it has been defined but requires a lot of further work. A model for adequate allocation of the environmental impacts in multiple-function systems, similar to allocation in the Inventory stage, is also required. Furthermore, it is necessary to develop an appropriate procedure for the decision-making process as a part of the Valuation and Improvement Assessment stages. Moreover, Improvement Assessment needs to be properly defined and an appropriate methodological framework developed, including possibilities for improvement, selection of options and assessment of feasibility (Consoli, 1993). In order to apply system analysis in the context of LCA, SETAC also acknowledges a need for development of computer models and software which are realistic and universally acceptable (Consoli, 1993).

As set out in the Introduction, this thesis represents an attempt to solve some of these problems in LCA methodology. In particular, it focuses on the allocation problem in the Inventory and Impact Assessment stages. It also addresses the Improvement stage, to develop a procedure that can be applied as a tool in the decision-making process within LCA. It is argued in this work that, by applying this methodology, the Valuation stage can be avoided. In this way, Valuation and Improvement Assessment are in effect combined, as in the current ISO approach. This novel and powerful approach to solving the methodological problems in LCA identified above, is based on whole system modelling by Linear Programming and is introduced in the following section. A detailed discussion of the proposed methodology is presented in the rest of this thesis.
1.3 Whole System Modelling and LCA

LCA is generally accepted as an application of system analysis whose prime objective is to provide a picture of the interactions of an activity with the environment, thus serving as a tool for environmental management. Its main advantage over other, site-specific, methods for environmental analysis, such as Environmental Impact Assessment (EIA) or Environmental Audit (EA), lies in broadening the system boundaries to include all burdens and impacts in the life cycle of a product or a process, and not focusing on the emissions and wastes generated by the plant or manufacturing site only.

As an environmental management tool, LCA has two main objectives. The first is to quantify and evaluate the environmental performance of a product or a process and so help decision-makers choose between alternative products and processes. Another objective of LCA is to provide a basis for identifying and assessing potential improvements in the environmental performance of a system. This can be of particular importance to process industries and engineers, because it can advise them on how to modify a system to decrease its overall environmental impacts.

As reviewed in Pedersen and Christiansen (1992), LCA has been, so far, most commonly applied to individual products or activities. The essence of the approach proposed in this work is analysis of a complete production system by representing it as a Linear Programming (LP) model (Azapagic and Clift, 1994). It is based on the kind of thermodynamic and system analysis which are central to process engineering. Therefore, the first step in any analysis must be definition of the system under study. In Life Cycle Assessment, this is done in the Goal Definition and Scoping stage. The environment is then interpreted in the thermodynamic sense as "that which surrounds the system", i.e. the whole Universe except the system under study. Thus for these purposes, "the environment" is defined along with the system, by exclusion. On this basis, Figure 1.4 shows schematically the general problem of environmental system analysis. The system of interest produces goods and services, which are treated together as outputs. To generate these outputs, inputs of energy and materials are required. The system will inevitably also produce environmental burdens: in general, resource depletion, emissions to air and water, and solid waste.

In a site-specific environmental analysis, such as EIA or EA, the system is the plant or manufacturing site and the inputs are related to the inputs of material and energy to that plant. On the other hand, in the LCA context, the system boundaries are drawn from "cradle to grave" to include all burdens and impacts in the life cycle of a product or a
process, so that the inputs into the system become primary resources. This approach also applies to multiple-function systems, i.e. co-product, recycling or waste treatment systems, provided that the system boundary is drawn correctly. For instance, for re-use and recycling, the output must be the service, so that the material used to provide the service (e.g. the packaging or container) is treated as part of the system. It then follows that all alternative ways to provide the same service, for example using primary or secondary materials, must be considered as part of the whole system.

The use of LP for whole system modelling in the context of LCA is appropriate for several reasons. Firstly, LCA is based on linear models of human economic activities and the environment, which means that environmental burdens are assumed to be directly proportional to the number of functional units produced (Heijungs et al., 1992; Huppes and Schneider, 1994). Hence there is no benefit at the present stage of development of LCA methodology in introducing non-linearities into the environmental system analysis.

Secondly, LP is able to relate the burdens and impacts identified in LCA to the functional units on the basis of physical and technical relationships. These include relationships as fundamental as material and thermal balances as well as descriptions of the technical performance of the units and operations in the system. In addition, they also include the burdens associated with each operation and the constraints on the operational level of each unit.
Thirdly, modelling a system through LP is able to take into account complex interactions between different parts of a system and so describe the behaviour of the system. LP can also accommodate changes in the state of the system and so show changes in the environmental burdens which result from changes in activities. As discussed in Chapter 3, this property of LP is particularly useful for solving the problem of allocation of environmental burdens among different inputs or outputs from multiple-function systems through its marginal or dual values.

One of the objectives of LCA is to identify the possibilities for improving the environmental performance of a system in the Improvement stage. In many cases the possibilities will be many and optimum ways of improving the performance will not always be obvious. It is argued in this work that Multiobjective Linear Programming (MOLP) can be used in the Improvement Assessment stage, to identify a range of optimum options for the environmental improvements in the system. However, system improvements cannot be carried out on the basis of environmental LCA only; other factors, such as economic and social criteria, have to be considered as well. Since LP can also incorporate these aspects of the system analysis, as shown schematically in Fig. 1.5, this provides the additional motivation for using LP in the LCA context.

Fig. 1.5 Life Cycle Assessment and Linear Programming
1.4 Concluding Remarks

Life Cycle Assessment is becoming an increasingly important tool in environmental system analysis and management. The methodology is still developing and a number of issues remain to be resolved. This chapter has introduced some of the problems that LCA practitioners and decision-makers are facing. To help resolve some of them, whole system modelling by LP is proposed. This is the subject of the rest of this thesis.

Prior to explaining the idea behind this approach, the basics of system modelling by LP, particularly the importance of the marginal values, are discussed in the following chapter.
CHAPTER 2

THE CONCEPT OF LINEAR PROGRAMMING

Linear Programming (LP) is not a new modelling technique: it has been used routinely for over forty years to describe different productive and economic systems, and also problems in scheduling and distribution. The mathematics of linear programming are well established and presented in number of books (e.g. Dantzig, 1963; Goddard, 1963; Hadley, 1962; Hillier and Lieberman, 1967; Kim, 1971; Wagner, 1969), while computer packages for solving large LP models are well developed and widely available (e.g. Dash Assoc., 1993). Therefore, the purpose of this chapter is not to explain the theory and solution procedures in LP; instead, it reviews the notation and formulation of LP problems. More attention is given to interpreting the solution of LP models, illustrated by a simple example, with emphasis on marginal or dual values because of their importance in the context of LCA. An introduction to sensitivity analysis, as part of interpreting the LP solution, is also outlined. The final part of this chapter presents the concept of multiobjective linear programming.

2.1 Introduction

The methodology of obtaining an optimal solution of a system has been named the system approach to problem solving. Mathematical modelling is just one phase of the system approach to problem solving. In addition to predicting the response of the system, a mathematical model can be used to evaluate the relative importance of subsystems and external factors that influence the output. In summary, mathematical models provide a means of improving decision-making capabilities and hence the basis for improved system performance.

Linear programming is one specialised mathematical decision-making aid. It can be applied to many problems in the real world, not because the world is linear but because it is a powerful problem-solving technique. Like other mathematical methods, care should be taken in interpreting the results of an LP model; they can only help us explain data and examine theories about the way things work or should work. If the data are incomplete or inappropriate, results of linear programming are likely to confuse rather than clarify a decision.
The term linear programming was first used by George Dantzig in 1947 to refer to specific problems of optimisation which assume that both constraints and objective function are linear (Dantzig, 1963). As with other branches of Operations Research, the first applications of LP are found in military planning activities. Soon after that, LP came into wide use in industry, with the most fruitful utilisation in the petroleum, petrochemical and food industries (extensive reference lists can be found in Dantzig, 1963 and Williams, 1967). Although it is somewhat more complex than the other linear techniques, such as Regression and Input-Output analysis, it was accepted readily by industry for several reasons:

- it provided a novel view of operations;
- it was able to account for the internal structure of the system; and
- it helped to improve the efficiency of industrial operations rather than merely describing their performance.

Since then, LP has been applied in a number of different areas, including transport, energy, finance, agriculture, health etc. However, more recently, with the development of a general methodology for LCA, a possibility for a new application of LP has been recognised (Azapagic and Clift, 1994; Azapagic and Clift, 1995a-c) and accepted by SETAC (Huppes, 1994). In addition to being able to model the internal structure of the system, it is able to solve some of the unresolved problems in LCA, such as allocation of environmental burdens. Also, it can be used as a tool in the Improvement Assessment stage for identifying and assessing options for improving a system's performance. In the light of potential application in LCA, the relevant characteristics of LP, following a definition of basic terms, are discussed below.

### 2.2 Linear Programming Defined

A linear programming model in general has the form:

Maximise (or minimise)  \[ F = \sum_{i=1}^{I} c_i x_i \]  \hspace{1cm} (2.1)

subject to  \[ x_i \geq 0 \hspace{1cm} i = 1,2,...,I \]  \hspace{1cm} (2.2)

\[ \sum_{i=1}^{I} a_{ij} x_i = e_j \hspace{1cm} j = 1,2,...,K \]  \hspace{1cm} (2.3)
Chapter 2

and

\[ \sum_{i=1}^{I} a_{j,i} x_i \leq e_j \quad j = K+1, \ldots, J \]  

(2.4)

where (2.1) represents an objective function, eqns. (2.2) and (2.4) inequality constraints, and eqn. (2.3) equality constraints of the model described by activities or variables, \( x_i \). The operational level of the activities is defined by the input or output coefficients, \( a_{j,i} \), and limited by the right-hand side coefficients or parameters, \( e_j \). The objective function is usually a measure of economic performance such as profit or cost, utility, turnover, or return on investment, and is maximised or minimised accordingly, subject to certain constraints. Some of the most common types of constraints are defined by material balance relationships, productive capacity, raw material availabilities, quality requirements, market demand and so on.

There are three types of LP models. Infeasible models have no solution and indicate that there are self-contradictory or overrestrictive constraints in the model. Unbounded models can be optimised without limit, i.e. the objective function can be improved without limits. Constraints in these models are not restrictive and, therefore, non-binding. The third type of models are called solvable. A solution to these models which satisfies all constraints simultaneously is called a feasible solution. The feasible solution which gives the best value (extremum) of the objective function is the optimal solution. The optimum always lies on an extreme point of the feasible region, i.e. on some constraint or at the intersection of several constraints and not in the inner part of the constrained region. However, in a given problem it is possible to have more than one optimal solution; they are called alternative optimal solutions. Since LP models belong to a convex type of programming problem, where the objective function and the linear constraints form a convex set (Edgar and Himmelblau, 1988), a local optimum is also a global optimum. This is an important property of LP models, because it allows for a very efficient solution technique: there is a relatively small number of extreme points in comparison to the infinite range of feasible solutions. The simplex method, one of the most practical and efficient mathematical techniques of solving linear programming models, exploits this characteristic of LP problems by searching only the extreme points. This is best illustrated by an example.

2.3 Linear Programming Model and Solution - An Example

A hypothetical example, a solution of which can be represented graphically, will be used for illustration of LP and for interpreting the solution. Suppose that a system produces
Product 1 and Product 2 from two raw materials both of which can be used as feeds. The outputs of the products are related to the two input activities, $x_1$ and $x_2$, describing inputs of the raw materials by the mass balance relationships:

Product 1:  \[ x_1 + 4x_2 < 70 \]  \hspace{1cm} (2.5)

Product 2:  \[ x_1 + 0.16x_2 = 10 \]  \hspace{1cm} (2.6)

where  \[ x_1 \geq 0, \ x_2 \geq 0 \]

The right-hand sides of the constraints represent production limitations for the products. Suppose that production of Product 1 is constrained to a maximum of 70 (unspecified) units, which is in LP terms defined by the inequality constraint (2.5). The production of Product 2 is specified exactly and cannot exceed 10 units; this is represented by the equality constraint (2.6).

In addition, suppose that the plant is subject to a capacity constraint of 100 units. This is represented by the following inequality constraint:

Capacity:  \[ 6x_1 + 2x_2 \leq 100 \]  \hspace{1cm} (2.7)

To provide the energy requirements for the process, a maximum of 40 units of heat can be supplied, so that there is a heat supply constraint of the form:

Heat:  \[ 2x_1 + 1.6x_2 < 40 \]  \hspace{1cm} (2.8)

Suppose that objective function in this case is to maximise the profit and is defined by the equation:

Maximise:  \[ F = 0.5x_1 + x_2 \]  \hspace{1cm} (2.9)

The types of questions that LP helps answer in this and similar problems is related to finding the optimum operating point in the system that maximises the profit and uses the optimum amount of resources, subject to the certain constraints. It also helps identify, for instance, which capacity is in most need of being increased, how much spare (or slack) capacity there is, which resources should be used, and so on.
A graphical illustration of the model is shown in Figure 2.1, where the constraints and objective function are depicted in terms of the two input activities. The space defined by the activities or decision variables is referred to as a decision space. The feasible region is convex and is in the shaded area delineated by points OABC, which means that the optimal solution could lie either at 0, A, B or C, as defined by the objective function.

The optimal solution for the objective function defined in the above example is found at the intersection of constraints (2.5), (2.6) and (2.8), as represented by point B. The solution of the model is given in Table 2.1. The profit amounts to 19.38 units and is determined by the Product 1 and Product 2 constraints, which are said to be active or binding, while the Capacity constraint does not influence the solution, i.e. it is non-binding or non-active. The Heat constraint is also non-binding, but, because of the way it is defined, it happens to pass through the optimum point. At the optimum, all constraints are converted into equalities by introducing slack variables, which can be defined as the surplus or the amount by which the left hand side of the inequality differs from the constraint value on the right hand side. For instance, there is a surplus of 23.75 units of capacity, or in other words, capacity limit could be reduced by 23.75 units without any effect on the profit. When a constraint is binding it is always satisfied as an equality at the optimal solution and its slack value is equal to zero: e.g. Product 1 output. However, if a constraint is non-binding, then it may or may not be satisfied as an equality at the optimum. If it is not satisfied as an equality, then its corresponding slack variable is always non-zero. This is the case with the constraint on capacity: it is non-binding, and its slack value is 23.75.

Fig. 2.1 Graphical presentation of solution for the LP example
Table 2.1 Optimal solution for the LP example
(F= 19.38; x₁= 7.50; x₂= 15.63)

<table>
<thead>
<tr>
<th>Constraints</th>
<th>Value at optimum</th>
<th>Slack value</th>
<th>Dual value</th>
<th>Right-hand side coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product 1</td>
<td>70.00</td>
<td>0.00</td>
<td>0.24</td>
<td>70.0</td>
</tr>
<tr>
<td>Product 2</td>
<td>10.00</td>
<td>0.00</td>
<td>0.26</td>
<td>10.0</td>
</tr>
<tr>
<td>Capacity</td>
<td>76.25</td>
<td>23.75</td>
<td>0.00</td>
<td>100.0</td>
</tr>
<tr>
<td>Heat</td>
<td>40.00</td>
<td>0.00</td>
<td>0.00</td>
<td>40.0</td>
</tr>
</tbody>
</table>

Furthermore, if the non-binding constraint is satisfied as an equality at the optimal solution, as is the case with the Heat constraint, then its slack value, similar to the binding constraints, is always equal to zero. Such constraints are also said to be redundant and the system overspecified. The binding and non-binding constraints can be distinguished at the optimal solution by their dual values: the dual values of the former are always non-zero (e.g. Product 1 and Product 2 constraints), while the non-binding constraints have zero dual values (e.g. Capacity and Heat). Dual values are discussed in the next section. Their mathematical formulation, together with the Kuhn-Tucker conditions, is given in Appendix 2.

2.4 Dual values

As already mentioned in the previous section, each active or binding constraint of an LP model at optimum solution has a non-zero dual or marginal value or shadow price. The interpretation of a dual or marginal value is the effect of an incremental or marginal change in the right-hand side of the constraint on the optimal value of the objective function. The value of the objective function at the optimum is, therefore, a function of the right-hand side coefficients:

\[ F = f(e_1, e_2, ..., e_j) \quad (2.10) \]

In the case of marginal changes in these coefficients, the corresponding marginal change in the objective function is equal to:

\[ \Delta F = \sum \Delta e_i \frac{\partial f}{\partial e_i} \]

\[ \Delta F = \sum \Delta e_i \cdot \text{dual value of } e_i \]

In nonlinear programming, these values are equivalent to the Lagrange multipliers.
Chapter 2

\[ dF = \sum_{j=1}^{J} \left( \frac{\partial F}{\partial e_j} \right)_{e_1, e_2, \ldots, e_j, e_{j+1}, \ldots, e_J} \, de_j \]  \hspace{1cm} (2.11)

where the partial derivative:

\[ \lambda_j = \left( \frac{\partial F}{\partial e_j} \right)_{e_1, e_2, \ldots, e_j, e_{j+1}, \ldots, e_J} \]  \hspace{1cm} (2.11a)

represents dual or marginal value and is interpreted as a change in the optimum value of the objective function, \( F \), with an incremental or marginal change in the right-hand side of one constraint, \( e_j \), while the values of the right-hand sides of other constraints are held constant. This implies that coefficients or parameters \( e_j \) are independent, i.e. that they can in principle be subject to independent incremental changes. In the example used above, the constraint on Product 1 output has a dual value of 0.24, which means that the profit would increase by this value if the output of Product 1 increased by a marginal value, say 1 unit, while the values of other constraints are kept unchanged. According to the Complementary Slackness Theorem (see Appendix 2), a non-binding constraint, such as Capacity in this example, will have a dual value of zero associated with it, which indicates that a small change in a right-hand side coefficient will have no effect on the objective function. Therefore, there is no point in increasing the capacity, since the value which is already available is not fully used.

The most important characteristic of the dual values is that they are valid only for the optimal solution and for differential or marginal changes to that solution. The reason for this is that the dual values depend on which constraints are binding. Therefore, for a marginal change in the right-hand side coefficients, the marginal values will remain constant, so that eqn. (2.11) can be integrated to give:

\[ F = \sum_{j=1}^{J} \left( \frac{\partial F}{\partial e_j} \right)_{e_1, e_2, \ldots, e_j, e_{j+1}, \ldots, e_J} \, e_j \]  \hspace{1cm} (2.12)

or, substituting eqn. (2.11a) into eqn. (2.12):

\[ F = \sum_{j=1}^{J} \lambda_j e_j \]  \hspace{1cm} (2.12a)

which represents the objective function of the dual LP model corresponding to the primal defined by eqns. (2.1)-(2.4) (for a definition of the dual LP model, see Appendix 2).
By moving away from the optimal solution too far, a new set of constraints can become binding and hence change the dual values. Therefore, it is only valid to interpret the dual values as referring to the effect of small changes in one right-hand side coefficient while all the others are kept constant. This means that we could not make small changes in two right-hand side coefficients simultaneously and conclude that the effect on the objective function will necessarily be the sum of the dual values.

The dual values depend on the state of the system as defined by which constraints are active. If the state of the system changes, so do the dual values. Suppose that requirements on the heat change so that instead of maximum value of 40 units an exact amount of 40 units has to be supplied. In LP terms this means that, instead of an inequality, the constraint on heat supply can be expressed by an equality:

$$2x_1 + 1.6x_2 = 40$$ \hspace{1cm} (2.13)

while all other constraints are unchanged. The optimal solution remains the same (point B in Fig. 2.1), but the dual values change: Product 2 is not a binding constraint anymore and its dual value is zero; the constraint on heat is now active and has a dual value of 0.16. In addition to this, the dual value of the Product 1 constraint has decreased from 0.24 to 0.19, although the constraint itself remained unchanged. The results of these variations in the LP example are given in Table 2.2. If all constraints except Capacity are equalities, the dual values default to the values in the optimal solution shown in Table 2.1.

<table>
<thead>
<tr>
<th>Constraints</th>
<th>Value at optimum</th>
<th>Slack value</th>
<th>Dual value</th>
<th>Right-hand side coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product 1</td>
<td>70.00</td>
<td>0.00</td>
<td>0.19</td>
<td>70.0</td>
</tr>
<tr>
<td>Product 2</td>
<td>10.00</td>
<td>0.00</td>
<td>0.00</td>
<td>10.0</td>
</tr>
<tr>
<td>Capacity</td>
<td>76.25</td>
<td>23.75</td>
<td>0.00</td>
<td>100.0</td>
</tr>
<tr>
<td>Heat</td>
<td>40.00</td>
<td>0.00</td>
<td>0.16</td>
<td>40.0</td>
</tr>
</tbody>
</table>

Table 2.2 Dual values as a result of change of the state of the system

\((F_1 = 19.38; x_1=7.50; x_2=15.63)\)
Thus, this simple example illustrates how the dual values and, accordingly, the effect of the constraints on the objective function, can change depending on the state of the system. As already mentioned, the dual values are valid only for the optimal solution and for the function the system has been optimised on. In the example presented here, that was the profit. However, in the context of LCA, objective functions are defined as the environmental burdens or impacts. Depending on the objective of the study, these objective functions can either be used for calculating and allocating environmental burdens and impacts in the Inventory Analysis or Impact Assessment stage. In addition, the system can be optimised on the environmental objectives to identify possibilities for better performance in the Improvement stage. These applications of LP in LCA are described in more detail in Chapters 3 and 4.

2.5 Sensitivity Analysis

The above interpretation of the dual values is valid only within a certain range of the right-hand side coefficients, which is usually calculated for the optimal solution. The lower and upper ranges are the levels of the activity below or above which the dual value changes. The lower and upper ranges for Product 2 constraint are, for instance, 3 and 10 respectively, which means that the right-hand side of this constraint can take any value in the interval (3,10) and the dual value will remain the same. The changes of the right-hand side coefficients of a binding constraint change the values of the activities in the optimal solution, as well as the value of the objective function. However, if the coefficients of the objective function are changed within the permitted range, in an analogous manner to change of the right-hand side coefficients, then the optimal values of the activities will not change, although the optimal value of the objective function may change.

In this way it is possible to find the ranges over which the right hand sides can vary without the optimal basis changing, the ranges over which the dual values hold, and the activities which limit these changes. The application of sensitivity analysis is, however, somewhat limited because it is valid only within the permitted range and only if one change at a time in coefficients is made. Furthermore, sensitivity analysis does not provide an answer to what the new optimal solution would be if these parameters changed. One of the obvious ways to find new optimal solutions would be to change the parameters of interest and solve the linear programming model again. However, in most cases it is possible to use the basic solution and specify the range over which the parameters should be changed. The simplex iterations are continued until a new optimal solution is found. This is called parametric programming and it represents an efficient
means of examining the effects of more than one change of the parameters of interest outside the permitted ranges. Parametric programming can be particularly useful for generating solutions to multiobjective linear programming, which is introduced in the next section.

2.6 Multiobjective Linear Programming

Multiobjective linear programming (MOLP) deals with optimisation problems with two or more objective functions. It differs from the classical, single-objective linear programming only in the number and the way of expressing the objective functions. The multiobjective linear programming problem can in general be expressed by:

Maximise (or minimise) \( F(x) = [F_1(x), F_2(x), \ldots, F_Q(x)] \) (2.14)

subject to constraints given by eqns. (2.2)-(2.4) and where \( F_1, F_2, \ldots, F_Q \) are individual objective functions defined by eqn. (2.1) and \( x \) is the I-dimensional vector of decision variables. The system is simultaneously optimised on all objective functions to find a range of optimal solutions. This is a very important distinction from single-objective LP, where the optimisation yields only one optimum solution. In multiobjective LP the notion of optimality is different - the solution that optimises one of the objective functions will not, in general, optimise any other objective. A concept of noninferiority is introduced instead. It is also known as "nondominance" by mathematical programmers, "efficiency" by statisticians and economists, and "Pareto optimality" by welfare economists. A feasible solution to a multiobjective programming problem is noninferior if there exists no other feasible solution that will yield an improvement in one objective without causing a degradation in at least one other objective.

This concept is illustrated for two objective functions in Fig. 2.2. The area inside of the shape, called the objective space, and its boundaries represent feasible solutions. The noninferior solutions will always lie on the boundary of the objective space, since a feasible solution which leads to an improvement in both objectives simultaneously can be found by moving from the interior solution to the one on the border. For instance, consider an inferior solution at point A in Fig. 2.2: both objective functions, \( F_1 \) and \( F_2 \), which are being maximised, can be improved by moving from A to B or C. Solution B gives more \( F_1 \) than the solution C without worsening the value of \( F_2 \) and similarly, solution C increases the value of \( F_2 \) without decreasing the value of \( F_1 \).
Multiobjective LP is further illustrated by the example given earlier in this chapter. In addition to maximising the profit, the problem is now to minimise some environmental burden. The additional objective function is assumed to be given by:

\[ F_2 = x_1 + 2x_2 \]  

The noninferior solutions lie on the boundary of the feasible region \( F_0 \) in objective space. In this particular example, as shown in Fig. 2.3, the feasible objective space is represented by the \( OABC \) curve. The points A, B and C are images of corresponding points in decision space. Thus, point A in Fig. 2.1 corresponds to point A in Fig. 2.3: the difference is that the former is defined through the values of the decision variables and the latter through the values of the objective functions corresponding to the same decision variables. Indeed, there is a close relationship between decision and objective space: \( F_0 \) is a transformation of \( F_D \), and its shape depends on the objective functions. For a mathematical representation of noninferiority, see Appendix 2.

It has already been pointed out that the noninferior solutions lie on the boundary of the objective space and not in the inner part of it. However, not all points on the boundary of the objective space will be noninferior. For two-objective problems, it is quite easy to identify the inferior solutions; for three and more objectives various methods, one of which is described in the following section, can be used. All noninferior solutions are optimal and, without further information, none can be said to be better or worse than any other. For instance, moving from A to B increases profit by 1.88 units, or by 10%, but it also increases emissions by 11.25 units, or by 24.3%. Moving from A to C decreases emissions by 71%, but at the same time, the profit is diminished by 71%. It is obvious
that at each noninferior solution one objective must be sacrificed in order to gain an improvement in the other. The question is then: which solution is the best and how do we choose it? This dilemma, often present in multiobjective decision-making situations, is discussed in Chapter 4, where it is also illustrated how a compromise between conflicting objectives can be found by using multiobjective linear programming.

2.6.1 Methods in Multiobjective Linear Programming

Multiobjective programming is not specific to linear programming - rather, LP is used as one of the techniques to generate solutions for the multiobjective problem. Since this work is concerned with application of LP to LCA, the discussion that follows will be focused on the methods in multiobjective linear programming only. For an overview of mathematical programming with multiple objectives see, for instance, Hwang et al. (1980) and Haimes and Li (1988).

There are several approaches to solving multiobjective linear programming models. For the reasons explained in Chapter 4, the "generating" techniques have been used in this thesis. Generating techniques include several methods, e.g. the weighting method, the
constraint method, the noninferior set estimation method, multiobjective simplex algorithm etc. (Cohon, 1978). Each of the methods has its advantages and disadvantages, a brief discussion of which is given in Appendix 4. Due to the software used in this work, it was possible to use only two of these methods, i.e. either the weighting or the constraint method. Since the constraint method, also known as the $\varepsilon$-constrained method, provides complete control of the spacing and coverage of the noninferior set it has been chosen for generation of noninferior solutions and will be briefly outlined here.

As given in the preceding section, a multiobjective problem with $Q$ objectives can be expressed as:

Maximise $F(x) = [F_1(x), F_2(x), ..., F_Q(x)]$

subject to $f_j(x) \leq e_j \quad j = 1,2,\ldots,J$

and $x \in X$

where $f_j(x)$ includes the constraints (2.2)-(2.4), $x$ is the $I$-dimensional vector of decision variables and $X$ is the feasible decision region. In the constraint method, the problem is transformed into:

Maximise $F_h(x)$

subject to $f_j(x) \leq e_j \quad j = 1,2,\ldots,J$

and $F_q(x) \geq \varepsilon_q \quad q=1,2,\ldots,h-1,h+1,\ldots,Q$

where the $h$th objective is arbitrarily chosen for maximisation, and all other objective functions of the problem are converted into constraints (see Appendix 2). In other words, multiobjective linear programming problem is transformed into a single objective problem, which is then solved in the familiar manner, i.e. by the simplex method. By changing the constrained levels, $\varepsilon_q$, from $\varepsilon_{q_{\text{min}}}$ to $\varepsilon_{q_{\text{max}}}$, the solution of the problem (2.17) will generate noninferior points. However, care has to be taken to choose $\varepsilon_q$ so that the feasible solutions to the resulting single objective problem still exist. The upper bound on $\varepsilon_q$, $\varepsilon_{q_{\text{max}}}$, is identified by maximising a problem on $F_q$ with other objective functions ignored; the lower bound, $\varepsilon_{q_{\text{min}}}$, is obtained when the system is optimised on the other objective function and $F_q$ is ignored. For more than two objectives, the procedure is
repeated for each of them in turn. The algorithm for optimisation by the constraint method is given in Appendix 2.

The dual values (see Appendix 2) associated with the constrained objectives of the problem (2.17) represent the trade-offs between the objectives - they indicate how much of one objective function has to be given up in order to improve the value of the other. For the active constrained objectives the corresponding multipliers are non-negative, whereas for the non-binding constraints, i.e. objective functions, they are equal to zero. For the active constrained objective function $F_q(x) \geq e_q$, the dual value is equal to:

$$w_{hq} = -\frac{\partial F_h}{\partial F_q}$$  \hspace{1cm} (2.18)

It shows that, if the objective $F_q$ is active and therefore $w_{hq} > 0$, an improvement in the $F_h$ objective can only happen at the expense of the $F_q$ objective. This means that, for the objectives satisfied as inequalities, the solutions with strictly positive dual values correspond to the noninferior set. However, if the constrained objectives are expressed as equalities, then the dual values which correspond to the noninferior set can be either positive or negative. The Kuhn-Tucker conditions for the constraint method for MOLP are given in Appendix 2.

2.7 Concluding Remarks

In this chapter, the concept of linear programming (LP) has been introduced. The intention was not to discuss the theory of LP but to review the notation and formulation of LP problems. In the view of possible applications of LP to Life Cycle Assessment, a particular emphasis has been given to interpreting the solutions of LP models. This has included a discussion of the marginal values, which can be used to allocate the burdens in multiple-function systems. This application of LP in LCA is presented in the next chapter.

Another use of LP in LCA, proposed in this work, includes multiobjective optimisation of a system in the LCA context. To introduce it, a general concept of multiobjective LP has also been discussed in this chapter. Its application to LCA is the subject of Chapters 4 and 7.
CHAPTER 3

ALLOCATION OF ENVIRONMENTAL BURDENS
IN MULTIPLE-FUNCTION SYSTEMS

Allocation of environmental burdens is one of the methodological problems in Life Cycle Assessment. It refers to the problem of associating environmental burdens, such as resource depletion, emissions to air and water and solid waste to each functional input or output of a multiple-function system. It is recognised as one of the issues in LCA that remain to be resolved (Consoli, 1993). Many different methods are being used at present, a review of which is given in the introductory part of this chapter; however none of them solves the problem of allocation in a satisfactory way. Therefore a new approach for solving the allocation problem in multiple-function systems, based on whole system modelling by linear programming is proposed in this work. This approach is presented and discussed in the remaining part of this chapter.

3.1 Introduction

There are three types of multiple-function systems, as shown schematically in Fig. 3.1, where allocation of environmental burdens can be relevant:

a) Multiple-input systems (waste treatment processes),
b) Multiple-output systems (co-production), and
c) Multiple-use or "cascaded use" systems ("open-loop recycling").

In multiple-input systems, such as combined waste treatment processes, a number of different materials are treated in the same system. These input materials have different composition and therefore properties which determine the total environmental burdens from the system. The allocation problem in these systems is, therefore, related to allocating the burdens between different inputs into the system. For example, if waste plastic is incinerated, chlorine enters the process as PVC but the emissions of chlorinated organic compounds (including dioxins) are not necessarily to be attributed only to the input of PVC, but to other parameters as well, such as for instance the calorific value of the waste. Similar problems occur in multiple-output or co-product systems, which produce more than one functional output. An example of a co-product system is a naphtha cracker producing ethylene, propylene, butenes and pyrolysis gasoline. The problem of allocation is then to find a procedure which would assign to each of the
a) Multiple-input system: combined waste treatment

b) Multiple-output system: co-production

c) Multiple-use or "cascaded use" systems: "open-loop recycling"
products only those environmental burdens for which it is responsible. The situation is even more complicated in multiple-use or "cascaded use" systems, where products can be reprocessed and reused in another systems. This is in the LCA context termed "open-loop recycling". For instance, broken PET bottles can be melted and reused as a feed for a manufacture of another plastic container (e.g. crate) which is, at the end of its life, reprocessed and used as a raw material for carpet fibres. Here, the problem is to allocate the environmental burdens among the PET bottle, the crate and the carpet systems so as to reflect both use and production of recycled materials.

In general, there are two ways to deal with the allocation problem: it can either be avoided by expanding system boundaries or disaggregating the system or solved by one of the many methods proposed by previous authors. Both ways are reviewed and discussed in the following section; however, first some definitions and distinctions are introduced.

### 3.1.1 Foreground and Background Systems

There is a distinction between foreground and background systems in LCA, which is particularly relevant in solving the problem of allocation. It is also crucial in setting the system boundaries. The foreground system is often defined as the system of primary concern in the study, delivering a functional unit specified in the Goal Definition and Scoping stage of the study. The background system is a system which supplies energy and materials to the foreground system, via a homogeneous market so that individual plants and operations cannot be identified. A schematic representation of background and foreground system is shown in Fig. 3.2.

Differentiation between foreground and background systems is also important for deciding on what kind of data to use in an LCA - marginal or average. Marginal data correspond to a specific process or technology while average data are related to a mix or a set of mixes of different technologies or processes. One of the main aims of LCA is comparison of some changes around an existing situation, be it a small change in a product composition or technology or a complete change to a different product or technology. Hence, changes in a system can either be marginal, average or discrete. In order to decide what kind of allocation is appropriate for a given situation it is important to decide what kind of changes will be considered in a system. The choice between marginal or average changes to a system, depending on the goal of the study, can be summarised as follows (SETAC, 1996):
1. Marginal changes to a specific technology are relevant when the performance of the specific process is analysed and when changes around the system of interest are incremental. This would be the case when analysing a waste incinerator operation in relation to marginal changes in chlorine content or calorific value, while other variables are held constant. It is also relevant in the co-product and recycling systems, where outputs can be changed independently of each other and the system is analysed on marginal changes in these outputs. This approach has been proposed by Azapagic and Clift (1994) and Clift and Azapagic (1996) and will be explained further below.

2. Average changes to a specific technology are applicable to comparing a new product to the existing one that would lead to a change of existing specific technology to a different one. In this case, the aim of the study is to compare the average behaviour of a new and existing technology, rather than marginal changes in the behaviour of the existing technology. In the case of a waste incinerator, an example would be assessing...
the average changes in the system that would occur due to the addition of the air emission control equipment to the incinerator.

3. Average changes to an average technology mix are relevant if different processes or products with similar function are compared. In this case, average changes due to a shift to a different technology mix are considered; e.g. comparing different waste management options, such as incineration and recycling.

4. Discrete changes in technology mix are applicable when fundamental changes in society are considered that would influence a large number of technologies. One such discrete change would be, for instance, a shift to a chlorine-free economy which would mean phasing out all products that contain chorine and introducing a completely new mix of technologies for producing alternative products.

The four types of changes that can be considered in LCA and their relation to background and foreground systems are shown in Table 3.1 (SETAC, 1996). For foreground systems, the preferred choice is between "marginal changes to a specific technology" or "average changes of a specific technology", or "discrete changes in a technology mix", depending on the goal and scope of the study. In the absence of better data, foreground systems may be analysed on "average change of an average technology mix". Background systems are always to be treated as "average changes of the average technology mix", except in cases where better data are not available, so that "average changes of a specific technology" is the only other option left.

**3.2 Procedures for Allocation in Multiple-function Systems**

As already mentioned, there are two ways to treat the problem of allocation. The allocation can be either:

1. avoided by expanding the system boundaries or disaggregating the given process into different subprocesses, or
2. solved by disaggregating and allocating by one of the suitable methods or by allocating directly without disaggregating.

The current draft of the relevant International Standard (ISO 14041) recognises these approaches, and recommends that the former should be used in preference to the latter wherever possible.
Chapter 3

Table 3.1 Possible approaches to representing technologies in foreground and background systems

<table>
<thead>
<tr>
<th>Change</th>
<th>Foreground systems</th>
<th>Background systems</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Marginal change to a specific technology</td>
<td>Goal and scope dependent</td>
<td>Never</td>
</tr>
<tr>
<td>2. Average change of a specific technology</td>
<td>Goal and scope dependent</td>
<td>Only in the absence of better data</td>
</tr>
<tr>
<td>3. Average change of an average technology mix</td>
<td>Only in the absence of better data</td>
<td>Always</td>
</tr>
<tr>
<td>4. Discrete change of a technology mix</td>
<td>Goal and scope dependent</td>
<td>Never</td>
</tr>
</tbody>
</table>

3.2.1 Avoiding Allocation in Multiple-function Systems

One of the procedures for dealing with the problem of allocation is to avoid it by broadening the system boundaries and introducing several functional units (Heintz and Baisnee, 1992; Ekvall, 1994; Heijungs, 1994). For instance, if System I produces products A and B and System II produces only product C and A is to be compared with C (Fig. 3.3a), then allocation can be avoided in two ways. The system can be broadened so that an alternative way of producing B is added to System II. The comparison is now between System I with A+B and System II and III with C+B (see Fig. 3.3b).

Another way to broaden the system boundaries is to subtract burdens arising from the alternative way of producing B from System I, so that only A is now compared to C, as illustrated in Fig. 3.3c. The latter approach is also known as the "avoided burdens" or "avoided impacts" method, and has mostly been used for systems where a co-product can replace one or more other products, e.g. heat from co-generation to substitute heat from oil. In this case it is argued that production of these other products will no longer be needed, and hence the environmental burdens that would otherwise come from this production will be avoided (Pedersen and Christiansen, 1993; Clift and Doig, 1995). The environmental burdens allocated to the main product in the system are then calculated as the total environmental burdens in the system minus the avoided environmental burdens of the co-products. In some cases the resulting burdens can be negative. For instance, Lindfors et al. (1995) illustrate this approach for the example of a refrigerator.
Chapter 3

a) Systems for comparison

b) Expanding system boundaries

c) Avoided burdens approach

Fig. 3.3 Avoiding allocation by system enlargement
which produces heat during its life time and so reduces the demand for heat produced from other sources. The emissions and resource demand avoided through substitution of fuel for refrigerator heat are included in the system as a credit for the refrigerator. The analysis shows that the net emissions of CO$_2$, SO$_2$, NOx, CO, HC and particulates are negative, i.e. heat from the refrigerator is more beneficial than that from, for example, oil. The same authors illustrate the avoided burdens approach for the open-loop recycling systems. If some parts of the refrigerator, e.g. steel and aluminium, are recycled and used in other products, the system boundaries can be expanded to include the life cycle of the products containing recycled metals from the refrigerator. A similar approach to allocation in open-loop recycling is also proposed by Fava et al. (1991), Vigon et al. (1993), Fleischer (1994) etc.

The avoided burdens method has also been applied to waste incineration (Tillman et al., 1991; Vigon et al., 1993). Doig and Clift (1995) have applied this approach to waste-to-energy systems, where avoided burdens are associated with the background activities. For instance, supplying electricity by coal-fired plants is no longer required because of the recovery of materials and energy from waste in the foreground system.

Avoiding allocation is an appealing way to deal with this problem; however, there are some difficulties in applying it. Although broadening system boundaries will imply a more complete and accurate model of a system, its main drawback is that, by including other functional units, the system becomes more complicated. The avoided allocation approach has the same problem, with an additional constraint: this method is suitable only if the co-product (or waste) can replace another product. Another way of avoiding allocation would be to disaggregate a given process into a number of subprocesses and relate the environmental burdens from each sub-process only to a product which they produce. However, this approach cannot avoid allocation completely because there will always be processes which are common for several products and some kind of allocation will still be necessary.

### 3.2.2 Solving Allocation in Multiple-function Systems

If allocation cannot be avoided, then an appropriate method has to be chosen to allocate the burdens in a multiple-function system. A number of different approaches have been proposed for solving the allocation problem so far and most of them are based on some physical quantity or economic value. Physical quantities used include mass, energy or exergy content, volume and molecular mass. Mass has probably most often been used as
a basis for allocation (e.g. Boustead, 1989, 1992, 1994; Consoli, 1993; Vigon et al., 1993). For instance, if a system produces 1000 kg of one product and 500 kg of another, then two thirds of the releases are attributed to the first and the rest to the second product. Allocation by volume, molecular mass and energy content is done in a similar manner (Boustead, 1994; Eggels, 1994). Methods based on economic value usually include market value (gross sales value) of products, or expected economic gain (Huppes, 1994). So, for example, if the market value of one product is £100 and the other £50, the environmental burdens are split between them in the ratio 2/1. Similar to allocation on the basis of gross sales value is allocation based on expected economic gain. However, expected economic gain is equal to gross sales value minus total production and distribution costs, so that the two methods are closely related. In some cases allocation is also done on a purely arbitrary basis so that, for instance, 100% of the burdens are allocated to one part of the system and 0% to the others or it is done on an equal basis between different parts of the system, i.e. 50%:50%. The latter method is in particular used to allocate the burdens in open-loop recycling systems (e.g. Fava et al., 1991; Vigon et al., 1993).

However, none of these methods offers a general solution to the allocation problem. Allocation on a mass basis, although the easiest to apply, can sometimes be impractical and illogical. This is particularly the case when one product outweighs the other: the product most responsible for the environmental burdens will be the one with the highest mass, although its production in the system may be less "burden intensive". Similar criticisms apply to allocation based on volume, molecular mass or energy content. Economic value of the product is not the most appropriate basis for allocation either: although in most cases it reflects the use of energy and therefore the associated burdens, allocation on this basis covers only one aspect of the system. Another reason for not using the market value for allocation is its variability with time (sometimes up to 50% in a short time period) and difficulty in determining a real value of a product. Allocation on an equal basis to all parts of the system or 100% to one and 0% to others cannot be seriously considered for allocation because it is highly arbitrary but so are the other above mentioned methods: they are all chosen arbitrarily without considering any causalities in the system.

The importance of causality in LCA is quite obvious: one of the main aims of LCA is comparison of some changes around an existing situation and causality is always related to the question of what would happen if some conditions in the system were changed. More recently, it has been agreed that allocation in multiple-function systems must be based on natural causal relationships, i.e. physical, chemical, biological or technical,
between functional units and the environmental burdens, provided that this type of causality exists (Clift, 1994; Huppes and Frischknecht, 1995; ISO 14041, 1996). This means that if there are natural causal relationships between the parameters related to the functional units and the environmental burdens, then a change in one of the system parameters will cause a certain change in the burdens, with other parameters held constant. An example where natural causation applies is a naphtha cracker in which it is possible to change the ratio of the functional outputs, i.e. one output can increase while others remain unchanged. The type of the changes considered (marginal or average) will, in turn, as discussed in Section 3.1.1, depend on the goal and scope of the study and the questions to be answered by LCA.

It is not always obvious what kind of natural causality exists in the system. In some cases physical causation, based on mass or energy content, is more relevant than chemical or technical or vice versa. This means that in some cases the burdens will be allocated on the basis of a simple physical quantity, e.g. mass; however, the point here is that the choice of allocation parameter is based on the natural causation involved as distinct from arbitrarily choosing one of these quantities regardless of whether a natural causation exists and what kind of causality that is. In order to identify the type of causality to be used as the basis for allocation in a particular situation, the system operation must be well understood and detailed data on the subprocesses in the system must be available.

However, in some systems, the ratio between two or more functional units and their parameters in the system may be fixed; examples of this arise in the chemical industry, where the ratio of sodium hydroxide (NaOH) and chlorine (Cl₂) produced by electrolysing brine is constant, and in agricultural production, where ratios are defined by the physical and chemical structure of a plant crop (e.g. rapeseed oil and residue) or an animal (e.g. beef and leather). In these cases there is no possibility of varying one functional output while keeping the other constant. Consequently, allocation cannot be done on the natural causation principle and in that case socio-economic relationships, as the only other relevant choice, should be used instead. These relationships can be translated into some economic measure, such as gross sales value or expected economic gain.

The most appropriate approach to allocation in any specific case depends on the system being examined and on the goal of the study. Before the allocation procedure is selected, the system and its behaviour should be well studied and understood. One of the ways to do this is to disaggregate the multiple-function system to subprocesses and then allocate the burdens on the natural causation principle or on a socio-economic basis, as
appropriate. The level of disaggregation will depend on the detail of the data available; if there are insufficient data then allocation has to be done without system disaggregation. SETAC recommends the following four-step procedure as a guide to performing the allocation (Huppes and Frischknecht, 1995):

1. disaggregate the system and split-off subprocesses of a system that operate for one of its functions only; then
2. split-off subprocesses of a system that operate for two or more of its functions but where the proportion that belongs to each of the functions is quantifiable; then
3. establish natural causation principles for the rest of the system, and allocate the burdens, and finally
4. apply socio-economic relationships for the parts of the system where natural principles do not apply.

International Standard ISO 14041 adopts a similar procedure for allocation, except that there steps 1 and 2 are combined in one stage.

Clearly, there are many ways to deal with the problem of allocation in LCA; however, whichever the method used, it has to reflect the real behaviour of the system under study. Realistic representation of the system behaviour relies on a realistic description of the causal relationships, i.e. changes in the system due to a change in the system parameters. This means that changes in the system must be represented by a realistic system model. Thus far, system analysis in the context of LCA has been based on representing the system as linear and unconstrained\(^1\), i.e. it is assumed that changes in the burdens are directly proportional to the changes in the system parameters. However, this is necessarily an approximation because real systems are usually nonlinear. Nevertheless, the linear approach to system analysis is appropriate if the goal of the LCA study is to consider marginal changes around an existing state, as outlined in Section 3.1.1, so that the system can be linearised around this state. However, it cannot be used in general to describe average or macroscopic changes in the system, because they may be nonlinear.

This work is concerned with system analysis where incremental changes to a specific system are the goal of the study, so that the linear representation of a system model is appropriate. A general method for allocation based on natural causality for this type of system model has been developed in this work and this is presented next.

\(^1\)In this context, "unconstrained" means that the system operation is not subject to constraints on for instance market demand, material availability or productive capacity.
3.3 Marginal Allocation in Multiple-function Systems

Total environmental burdens from a multiple-function system depend, in general, on the properties of materials and processes in a system, i.e. on the state of the productive system. This means that, in a system which can be described on the basis of natural causality, a change in either material properties or process operation will cause a change in the environmental burdens. If a change in the state of the system, related to a change in a material property, causes a change in the total burden, then the burden is said to be material-related or product-related, depending on whether it refers to multiple-input or multiple-output and multiple-use systems, respectively. An example of a material-related burden is the total emission of dioxins from a waste incinerator which can increase by increasing the total chlorine content in the waste material being processed. However, if the environmental burdens change as a result of a change of the property of the process (e.g. temperature, pressure, capacity etc.), the burdens are said to be process-related. For a waste incinerator, for example, a change in incineration temperature can cause a change in the burdens. Thus the total burdens are, in general, related to the material (or product) and process properties by:

\[ B_j = f[u_1, u_2, \ldots, u_N, v_1, v_2, \ldots, v_M] \]  

(3.1)

where \( B_j \) is environmental burden \( j \) and \( u_1, u_2, \ldots, u_N \) and \( v_1, v_2, \ldots, v_M \) are the material (or product) and process properties, respectively. If incremental or marginal changes in a system are considered, then the corresponding changes in the environmental burdens are given by:

\[ dB_j = \left( \frac{\partial B_j}{\partial u_1} \right)_{u_2, \ldots, u_N, v_1, \ldots, v_M} du_1 + \left( \frac{\partial B_j}{\partial u_2} \right)_{u_1, u_3, \ldots, u_N, v_1, \ldots, v_M} du_2 + \ldots + \left( \frac{\partial B_j}{\partial u_N} \right)_{u_1, u_2, \ldots, u_{N-1}, v_1, \ldots, v_M} du_N \]

\[ \quad + \left( \frac{\partial B_j}{\partial v_1} \right)_{u_1, u_2, \ldots, u_N, v_2, \ldots, v_M} dv_1 + \left( \frac{\partial B_j}{\partial v_2} \right)_{u_1, u_2, \ldots, u_N, v_3, \ldots, v_M} dv_2 + \ldots + \left( \frac{\partial B_j}{\partial v_M} \right)_{u_1, u_2, \ldots, u_N, v_1, \ldots, v_{M-1}} dv_M \]  

(3.2)

The partial derivatives:

\[ U_n = \left( \frac{\partial B_j}{\partial u_n} \right)_{u_1, u_2, \ldots, u_{n-1}, u_{n+1}, \ldots, u_N, v_1, \ldots, v_M} \]

(3.3)

\[ V_m = \left( \frac{\partial B_j}{\partial v_m} \right)_{u_1, u_2, \ldots, u_N, v_1, \ldots, v_{m-1}, v_{m+1}, \ldots, v_M} \]  

(3.4)

\( B_j \) may be an intensive variable, i.e. burden per quantity of waste treated, and in this case the \( u \) and \( v \) must be intensive variables, such as composition or calorific value. If \( B_j \) is an extensive variable, e.g. total quantity of some emission, then the \( u \) and \( v \) must also be extensive variables, such as total mass, total calorific value or total chlorine content of the waste processed.
are defined in the usual way: they represent an incremental or marginal change in burden \( B_j \) with a change in one of the material or process properties, while all other properties are held constant. For instance, if \( B_j \) is the total dioxin emission and \( u_n \) is the chlorine content in the waste material being incinerated, then derivative (3.3) represents the change in dioxin emissions resulting from an incremental change in the total chlorine content in the waste, without changing any other properties of the material or the process. Similarly, for the process-related burdens, if derivative (3.4) is related to the temperature \( (v_m) \) in the waste incinerator, then it represents a change of total dioxin emissions with the change in temperature only, with all other parameters kept constant. For a marginal change, the derivatives (3.3) and (3.4) will remain constant, i.e. the properties of the system will not change. Thus, eqn. (3.2) is after integration equal to:

\[
B_j = \left( \frac{\partial B_j}{\partial u_1} \right) u_1 \ldots u_N v_1 \ldots v_m u_1 + \ldots + \left( \frac{\partial B_j}{\partial u_N} \right) u_1 \ldots u_N v_1 \ldots v_m + \ldots + \left( \frac{\partial B_j}{\partial v_1} \right) u_1 \ldots u_N v_1 \ldots v_m v_1 + \ldots + \left( \frac{\partial B_j}{\partial v_M} \right) u_1 \ldots u_N v_1 \ldots v_M v_m
\]

(3.5)

This is consistent with use of Taylor’s theorem to linearise the function. The constant of integration is neglected here because it can be eliminated by appropriate scaling. This is possible because the function \( B_j \) is linear and homogenous to degree one.

In a simplified notation, equation (3.5) can be written as:

\[
B_j = \sum_{n=1}^{N} u_n + \sum_{m=1}^{M} V_m v_m
\]

(3.6)

Equation (3.6) relates total burdens in the system to the material and process properties through the marginal allocation coefficients, \( U_n \) and \( V_m \). If the system is modelled by LP with \( B_j \) defined as the objective function, then these coefficients are equal to the marginal or dual values at the solution of the LP model, as shown below.

As given in Chapter 2, an LP model of a system has the general form:

Maximise  
\[
F = \sum_{i=1}^{I} c_i x_i
\]

subject to  
\[
x_i \geq 0 \quad i = 1,2,\ldots,I
\]

\[
\sum_{i=1}^{I} a_{ji} x_i = e_j \quad j = 1,2,\ldots,K
\]
where eqn. (2.1) represents an objective function, usually a measure of economic performance, and eqns. (2.2)-(2.4) are the constraints in the system. In the context of LCA, the LP model will, in general, have the same form, with the constraints (2.3) and (2.4) now encompassing all activities from extraction of the primary materials from the earth to final disposal. However, the objective functions in this context are the environmental burdens, rather than an economic objective, as represented by:

Minimise

\[ B_j = \sum_{i=1}^{1} b_{c_{j,i}} x_i \]  \hspace{2cm} (3.7)

where \( b_{c_{j,i}} \) is burden \( j \) from process or activity \( x_i \). The objective functions can also be the environmental impacts, as defined by:

Minimise

\[ E_k = \sum_{j=1}^{J} e_{c_{k,j}} B_j \]  \hspace{2cm} (3.8)

where \( e_{c_{k,j}} \) represents the relative contribution of burden \( B_j \) to impact \( E_k \), as defined by the "problem-oriented" approach (Heijungs et al., 1992). To simplify the explanations, the following discussion will deal with allocation of the burdens only, although exactly the same kind of analysis can be applied to allocation of environmental impacts.

From the analysis of the marginal values in Chapter 2, it follows that at the optimum solution of the LP model, the total burden \( B_j \) is related to the marginal values by:

\[ B_j = \sum_{j=1}^{J} \lambda_j e_j \]  \hspace{2cm} (3.9)

where \( \lambda_j \) is a marginal or dual value of the \( j \)th constraint, i.e. it represents the change in the total burden with the change in coefficient \( e_j \). If this coefficient is related to the material or process property or both:

\[ e_j = u_n \quad \text{or} \quad e_j = v_m \]  \hspace{2cm} (3.10)

then \( \lambda_j \) is equivalent to derivative \( U_n \) or \( V_m \).
This is, indeed, the most important link between marginal allocation and LP - dual values evaluated at the solution of the LP model represent the marginal allocation coefficients, which relate changes in the burden to the incremental changes in one of the material or process properties while all other properties of a multiple-function system are held constant. Therefore, in a system in which natural causal relationships exist, marginal values represent a realistic description of these relationships and thus closely reflect changes in the behaviour of the system with changes in the system parameters. In this way, whole system modelling by LP serves as a tool for establishing natural causation principles in multiple-function systems. This kind of allocation is, therefore, consistent with step 3 of the four-step procedure outlined in the preceding section.

Equation (3.5) implies that all material and process properties are independent parameters, i.e. they can in principle be subject to independent incremental changes. This means that marginal allocation is appropriate only if a natural causation principle can be used to describe the system. However, as noted above, in some systems it is not possible to change one property of the system independently of other parameters. The marginal values in these systems cannot be interpreted in a meaningful way and the allocation has to be done on the socio-economic basis, as recommended in step 4 of the four-step procedure.

The above analysis implies that the burdens can be both material- (or product-) and process-related. This kind of analysis is applicable where the response of the system with respect to incremental changes in both material and process properties is the goal of the LCA study. However, in some cases, the analysis will be limited to the incremental changes in material or product properties only with the process ones kept constant, or vice versa, so that eqn. (3.6) reduces to:

\[
B_j = \sum_{n=1}^{N} U_n u_n = \sum_{j=1}^{J} \lambda_j e_j \quad (3.13)
\]

or

\[
B_j = \sum_{m=1}^{M} V_m v_m = \sum_{j=1}^{J} \lambda_j e_j \quad (3.14)
\]
for the material- (or product-) and process-related burdens, respectively. As already pointed out, the same kind of analysis applies to allocation of environmental impacts, so that in general, eqn. (3.9) can be written as:

\[ E_k = \sum_{j=1}^{J} \lambda_j e_j \]  

(3.15)

with

\[ \lambda_j = \left( \frac{\partial E_k}{\partial u_n} \right)_{u_1, \ldots, u_{n-1}, u_{n+1}, \ldots, u_N, v_1, \ldots, v_{m-1}} \]  

(3.16)

or

\[ \lambda_j = \left( \frac{\partial E_k}{\partial v_m} \right)_{u_1, \ldots, u_N, v_1, \ldots, v_{m-1}, v_{m+1}, \ldots, v_{m+l}} \]  

(3.17)

i.e. marginal value \( \lambda_j \) is now related to a change in environmental impact \( E_k \) with a change in one of the properties of the system.

Marginal allocation of the environmental burdens based on the causality principle will now be illustrated using examples of different multiple-function systems.

3.3.1 Example 1 - Allocation in Multiple-input Systems

Multiple-input processes, typically found in waste treatment systems, represent a case where allocation of environmental burdens can become a particular problem, because the burdens have to be allocated between different inputs and their parameters. Therefore, this section is an attempt to illustrate how allocation can be solved by analysis of marginal changes in the multiple-input system parameters, using waste incineration as an example (Clift and Azapagić, 1995). As already mentioned, this kind of allocation is appropriate in systems where independent marginal changes to a specific technology are the goal of the study.

The independent parameters used to describe a waste incineration process are here taken to be: the total mass of waste processed (M), total chlorine content in the mass M of waste (H), lower calorific value of the mass M of waste (L) and the combustion temperature (T). The examples developed here concentrate on the case where one environmental burden - emission of dioxin (B) - is critical. However, in general, many burdens can be considered, including both emissions and resource usages. As a limitation to the analysis, a system in which all the independent parameters can in principle be subject to independent incremental changes is considered here. This would exclude, for
example, analysis of an incinerator which is already working at maximum throughput (i.e. $M$ cannot be increased) but where the interest is in the effects of changing the characteristics of the waste passing through it.

The functional unit in this example is one tonne of waste processed. The values of the independent parameters are, therefore, expressed per tonne of waste processed:

$$b = B/M; \quad h = H/M; \quad l = L/M$$  \hspace{1cm} (3.18)

The total emission is related to the total waste processed by:

$$B = f[M, H, L, T]$$  \hspace{1cm} (3.19)

Consider now incremental changes in the system and the corresponding changes in dioxin emission, which can be described by the total differential:

$$dB = \left(\frac{\partial B}{\partial M}\right)_{H, L, T} dM + \left(\frac{\partial B}{\partial H}\right)_{L, T, M} dH + \left(\frac{\partial B}{\partial L}\right)_{T, M, H} dL + \left(\frac{\partial B}{\partial T}\right)_{M, H, L} dT$$  \hspace{1cm} (3.20)

The partial derivatives in eqn. (3.20) are defined in the way described in Section 3.3. Thus $(\partial B/\partial M)_{H, L, T}$ represents the change in emission resulting from an incremental change in mass of waste processed, without changing the total chlorine in the waste ($H$) or its total calorific value ($L$) or the processing conditions ($T$). It corresponds, for example, to the effect of adding a small quantity of inert chlorine-free non-combustible solid - for example a glass container - to the waste processed. Similarly, $(\partial B/\partial H)_{L, T, M}$ describes the effect on emission of changing the chlorine content without changing the mass or calorific value or the operating conditions: e.g. substituting a piece of PVC for an equal mass of a chlorine-free waste with equal calorific value. $(\partial B/\partial L)_{T, M, H}$ describes the effect of changing the calorific value without changing mass or total chlorine - replacing a fragment of inert glass by an equal mass of chlorine-free combustible, for example. Finally, $(\partial B/\partial T)_{M, H, L}$ represents the effect of changing the processing operating conditions but still treating exactly the same waste. To simplify the notation, the following symbols will be used:

$$b_M = \left(\frac{\partial B}{\partial M}\right)_{H, L, T}; \quad b_H = \left(\frac{\partial B}{\partial H}\right)_{L, T, M}; \quad b_L = \left(\frac{\partial B}{\partial L}\right)_{T, M, H}; \quad b_T = \left(\frac{\partial B}{\partial T}\right)_{M, H, L}$$  \hspace{1cm} (3.21)

By substituting (3.21) into eqn. (3.20):
\[ dB = b_M dM + b_H dH + b_L dL + b_T dT \]  \hspace{1cm} (3.22)

Thus the parameters \( b_M, b_H, b_L \) and \( b_T \) are the marginal allocation coefficients, corresponding to the dual values in an LP model, which relate changes in the dioxin emission to incremental changes in the waste stream and the process conditions. For a marginal change in the system, the marginal allocation coefficients will remain constant, so that, by analogy with eqn. (3.2), eqn. (3.22) after integration becomes:

\[ B = b_M M + b_H H + b_L L + b_T T \]  \hspace{1cm} (3.23)

The total dioxin emission is therefore allocated to both material and process properties, i.e. the burden is in general both material- and process-related.

Consider now a case in which the waste processing technology and its operating conditions are kept unchanged, i.e. \( dT = 0 \). Equation (3.23) then becomes:

\[ dB = b_M dM + b_H dH + b_L dL \]  \hspace{1cm} (3.24)

or after integration:

\[ B = b_M M + b_H H + b_L L \]  \hspace{1cm} (3.25)

If the dioxin emission is expressed per tonne of functional unit, i.e. waste processed, then by substituting the term (3.18) into eqn. (3.25):

\[ b = b_M + b_H h + b_L l \]  \hspace{1cm} (3.26)

Equations (3.25) and (3.26) show that the total dioxin emission is allocated to the properties of the waste stream, i.e. the burden is material-related.

To take this analysis further, let us assume that for constant treatment conditions (T) and specific calorific value (l), the dioxin emission per tonne of waste processed (b) varies with the chlorine fraction in the waste (h) as shown schematically in Figure 3.4 (Eggels and van der Ven, 1994). When the chlorine content is large, so that chlorine is present in excess and does not limit dioxin emissions, \( b \) approaches an asymptotic value \( b_{\text{max}} \) which depends on the process used, i.e. on the type of combustion plant, combustion temperature, conditions in the gas cleaning system, etc. Usually \( b_{\text{max}} \) is set in practice by regulations on the permissible emissions from the plant.
From equation (3.26), the dioxin to be allocated to chlorine content is given by the gradient of the curve in Figure 3.4. Under conditions at point 1 (which corresponds to the current composition of municipal solid waste throughout Europe), the chlorine content is sufficiently high that incinerators are effectively operating at the asymptote. Thus changes in chlorine content have virtually no effect on dioxin emissions: the gradient is very small and $b_H \to 0$. Equations (3.25) and (3.26) then simplify to:

$$B \approx b_M M + b_L L \quad (3.27)$$

and

$$b \approx b_M + b_L l \quad (3.28)$$

Eggels and van der Van (1994) have also argued that dioxin emissions depend on the lower calorific value of the waste rather than its mass; i.e. that $b_M$ is also very small. Given the definition of $b_M$ - see the first term in (3.21) - this conclusion is perhaps not surprising. It implies that adding inert non-combustible material to the waste has no effect on dioxin levels. The system model then reduces to:

$$B \approx b_L L \quad (3.29)$$

and

$$b \approx b_L l = b_{\text{max}} \quad (3.30)$$
From eqn. (3.30):

\[ b_L = \frac{b_{max}}{l} \]  

(3.31)

so that after substituting (3.31) into (3.29):

\[ B \approx b_{max} M \]  

(3.32)

Equations (3.30) and (3.32) indicate that the dioxin emission is now a process-related burden because it depends primarily on \( b_{max} \) which in turn reflects the process technology and its operating conditions.

The situation is, however, quite different for the conditions in the region of point 2 in Fig. 3.4. The gradient of the curve in Figure 3.4 is now significant, i.e. \( b_H \) is no longer vanishingly small. Figure 3.5 shows the variation of the total dioxin emission, \( B \), with, for instance, the total lower heating value of the waste incinerated, \( L \). When equation (3.32) applies, \( B \) simply varies linearly with \( L \) for all chlorine content, \( h \), which corresponds to the conditions in the region of point 1 in Fig. 3.4. In the region of point 2, however, the total dioxin emission, \( B \), now depends both on the total calorific value and on the total chlorine content of the waste processed (or the average concentration in the waste) so that:

\[ B = b_H H + b_L L \]  

(3.33)

and

\[ b = b_H h + b_L l \]  

(3.34)
and the burden is again material-related.

By definition, allocation on a marginal basis is only appropriate if the system parameters can be changed independently. If this is not the case, for example, if the total mass of waste which can be processed in an interval of time is limited by the maximum possible plant throughput then M, H and L (or uN and vM in general) cannot be varied independently. If the system conditions are described by an LP model, then its (optimum) operation always lies at the intersection of active constraints (see Chapter 2). System conditions can then only be changed by changing the values of constraints, for instance by modifying the plant to increase throughput, or by shifting to the intersection of a different set of constraints. In that case, the burdens are allocated to the active constraints at the operating point of interest, as will now be demonstrated in the example of multiple-output systems.

3.3.2 Example 2 - Allocation in Multiple-output Systems

Multiple-output or co-product systems represent another case where the problem of allocation is encountered: the burdens have to be allocated between different functional outputs produced in the same system. In this section, an illustration of how the marginal approach to solving the problem of allocation can be applied to these systems is presented. Again, the emphasis is on systems where the goal of the study is to consider marginal changes to a specific technology and where the functional outputs of the system can be changed independently.

In Chapter 2, a hypothetical example of a system producing two products, Product 1 and Product 2, was considered to introduce the basic concepts of LP. Here, the same example is used in the context of LCA, to illustrate the marginal allocation through LP in multi-output systems. The system is described by the following constraints:

Product 1: \[ x_1 + 4x_2 < 70 \] \hspace{1cm} (2.5)

Product 2: \[ x_1 + 0.16x_2 = 10 \] \hspace{1cm} (2.6)

Capacity: \[ 6x_1 + 2x_2 < 100 \] \hspace{1cm} (2.7)

Heat: \[ 2x_1 + 1.6x_2 < 40 \] \hspace{1cm} (2.8)
For a more detailed description of the system see Chapter 2. Since this example is considered here in the context of LCA, the system boundary is drawn to include all activities from extraction of primary resources through refining and transport to the production of two products (Fig. 3.6). The use and the disposal phases of the life cycle are not considered here, i.e. the concern is with "cradle-to-gate" analysis, rather than a true "cradle-to-grave" approach. The functional units of the system are quantities of Product 1 and Product 2. As already mentioned, it is assumed that outputs of Product 1 and Product 2 can be changed independently of each other so that output of Product 2 is being changed and that of Product 1 is kept constant. The emphasis is on the two activities representing inputs to the production stage, respectively Raw material 1, represented by activity $x_1$, and Raw material 2, represented by activity $x_2$. It is assumed that both may be used as alternative feedstock for producing these products. Note that activities $x_1$ and $x_2$ also include the activities associated with the extraction and transport of primary resources.

In the example used in Chapter 2, the objective function was defined as profit (eqn. (2.9)). In the context of LCA, the objectives are defined as environmental burdens or impacts. In this hypothetical example, only two burdens are considered: one is associated with resource extraction or depletion:

$$B_1 = x_1 + 2x_2$$

(3.35)

while the other represents atmospheric emissions such as carbon dioxide (CO$_2$):

$$B_2 = x_1 + 15x_2$$

(3.36)
Figure 3.7 Allocation by the marginal approach for the co-product example

Figure 3.7 shows the constraints in terms of two input activities. In order to illustrate the effect of marginal values on the value of the burdens graphically, sets of contours corresponding to constant values of the environmental burdens are also shown in the figure. Point B represents the optimum solution of the system identified in Chapter 2. At the solution, only two constraints in the system are active: Product 1 and Product 2, which means that only these constraints will have non-zero marginal values (see Table 3.2). This means, in turn, that the environmental burdens are allocated between Products 1 and 2 only; i.e. the burdens are product-related in this case. Process-related burdens are zero because the constraints that describe the process, i.e. the capacity and heat, are non-active, so that they do not constrain the system operation. The total burdens are thus equal to:

\[ B_1 = b_{1,1}P_1 + b_{1,2}P_2 \]  
\[ B_2 = b_{2,1}P_1 + b_{2,2}P_2 \]

Therefore, for a system with \( L \) different outputs or functional units \( P_i \) (\( i=1,...,L \)), each generating \( b_{j,i} \) units of environmental burden \( j \), the total generation of product-related burden \( B_j \) (\( j=1,...,J \)) is given by, in general:
Chapter 3

with the process-related burdens equal to zero. The marginal change in burden $B_j$ allocated to output $P_1$ is then defined as:

$$B_j = \sum_{i=1}^{l} b_{j,i} P_i$$ (3.39)

with the output of other products kept constant. It is obvious that eqns. (3.37) and (3.40) are equivalent to eqns. (3.13) and (3.3), respectively.

Consider now the effect of increasing output of Product 2 by one unit while the output of Product 1 is kept constant. This corresponds to changing the right-hand side coefficient of the constraint (2.6), i.e. shifting the line representing this constraint in Figure 3.7. For a marginal change, the same constraints remain active, so that the solution of the system moves from point B to point B'. The total environmental burdens also change. In this case, $B_1$ increases from 38.75 units to 39.27, because the marginal value of the burden, $b_1$, allocated to output of Product 2 is positive and equal to 0.52. However, the same change causes $B_2$ to decrease from 241.88 to 239.01 units, because the marginal value, $b_2$, allocated to output of Product 1 is negative and equal to -2.87. This is possible because most of the burden arises from activity $x_2$ which is reduced by the increase in Product 2 output. Similarly, if the output of Product 2 is decreased by the same marginal value, the environmental burden $B_1$ decreases while $B_2$ increases. This is represented by point B'' in Fig. 3.7.

The above analysis shows that allocated environmental burdens, as determined by marginal values, can be either positive or negative. Clearly, in this example Product 2 contributes more to resource depletion than Product 1. The situation is quite opposite for the emission of CO$_2$: not only is the contribution of Product 2 less than that of Product 1, its marginal value is also negative. This means that an increase of its production would lead to a decrease in total CO$_2$ emissions. Thus, in addition to solving the problem of allocation, marginal analysis can also be useful in environmental management of a productive system because it indicates possible places for system improvement. Environmental management of a system is the subject of the following chapter.

Continuing with the analysis of marginal allocation in the co-product system, it is now interesting to see what happens to the marginal values if the state of the system changes, i.e. if the system is operated in a different way. Suppose that the heat requirement on the
Table 3.2 Marginal allocation in the example of the co-product system 
\( (x_1=7.50; x_2=15.63) \)

<table>
<thead>
<tr>
<th>Constraints</th>
<th>Value at optimum</th>
<th>Slack value</th>
<th>( b_1 ) (( B_1=38.75 ))</th>
<th>( b_2 ) (( B_2=241.88 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product 1</td>
<td>70.00</td>
<td>0.00</td>
<td>0.48</td>
<td>3.86</td>
</tr>
<tr>
<td>Product 2</td>
<td>10.00</td>
<td>0.00</td>
<td>0.52</td>
<td>-2.87</td>
</tr>
<tr>
<td>Capacity</td>
<td>76.25</td>
<td>23.75</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Heat</td>
<td>40.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

process changes, so that instead of maximum value of 40 units, an exact amount of 40 units of heat has to be supplied, i.e. eqn. (2.8) becomes:

Heat:

\[
2x_1 + 1.6x_2 = 40
\]  

(2.13)

With all other requirements on the system unchanged, the marginal values and, therefore, the allocated burdens for the new system conditions are quite different from those in Table 3.2. Although the values of the constraints at the optimum are the same as in the above case, the system operation is now determined by a different set of active constraints, i.e. Product 1 and Heat, instead of Product 1 and Product 2. Therefore, the marginal values of the constraints are now different and they are shown in Table 3.3. Since Product 2 and Capacity are non-active constraints their marginal values are equal to zero so that they do not contribute to the total burdens from the system. Thus, the burdens are allocated to the production of Product 1 and to the heat requirements in the process, which means that burdens are both product- and process-related. They are equal to:

\[
B_1 = b_{1,1}P_1 + b_{1,2}H
\]  

(3.41)

\[
B_2 = b_{2,1}P_1 + b_{2,2}H
\]  

(3.42)

or, in general:

\[
B_j = \sum_{i=1}^{L} b_{j,i}P_i + \sum_{p=1}^{P} b_{j,p}R_p
\]  

(3.43)

where \( R_p \) represents values of the process-related parameters. The process-related marginal burden, \( b_{j,p} \), is equal to:

\[
b_{j,p} = \frac{\partial B_j}{\partial R_p}
\]  

(3.44)
Chapter 3

Table 3.3 Change of marginal values with change in the state of the system

\( (x_1=7.50; x_2=15.63) \)

<table>
<thead>
<tr>
<th>Constraints</th>
<th>Value at optimum</th>
<th>Slack value</th>
<th>( b_1 ) (( B_1=38.75 ))</th>
<th>( b_2 ) (( B_2=241.88 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product 1</td>
<td>70.00</td>
<td>0.00</td>
<td>0.38</td>
<td>4.44</td>
</tr>
<tr>
<td>Product 2</td>
<td>10.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Capacity</td>
<td>76.25</td>
<td>23.75</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Heat</td>
<td>40.00</td>
<td>0.00</td>
<td>0.31</td>
<td>-1.72</td>
</tr>
</tbody>
</table>

Equations (3.43) and (3.44) are equivalent to eqns. (3.13) and (3.4), respectively. Thus, this simple example illustrates the general point that the allocated environmental burdens depend on the state of the system, as defined by the way in which the system is operated. This approach to allocation offers more accurate description of a product system because it reflects behaviour of the system through the changes that occur in its operation. The marginal allocation approach is illustrated in Chapter 6 by applying it to a real co-product system using data obtained in a field study.

3.3.3 Example 3 - Allocation in Multiple-use Systems

At the end of their useful life, some products can be reprocessed and reused to fulfil the same function as before, or alternatively they can be reused in another productive system with a different function. In the former, closed-loop recycling systems, the problem of allocation does not occur because both recycled and virgin materials are used in the same system. However, in the latter, open-loop recycling systems, products (i.e. materials) are passed from one system to another, taking part of the burdens from the upstream to the downstream system in the cascade. Therefore, the burdens have to be allocated among these systems. The main problem here is to allocate the burdens so as to reflect the behaviour of the system in the most realistic way. Similar to other multiple-function systems, it is argued here that, for marginal changes in the system, the behaviour of the multiple-use systems can also be modelled by linear programming and the marginal values of the model can be used for allocation of the burdens in these systems. By describing all considered cascades of uses in LP terms, the burdens are allocated among the cascades taking into account both the use and the production of recycled materials. In this way, different cascades are "credited" or "penalised" for recycling, depending on the burdens associated with the reprocessing of recycled materials. This is the aspect that
some allocation methods fail to take into account: recycling does not "come for free" and is normally associated with additional burdens. Therefore, the argument that systems which use recycled materials should always be credited is not universally valid. Furthermore, allocation in open-loop recycling based on whole system modelling avoids double accounting of burdens, which occurs when both the product and the subsequent recycled material are "charged" for producing the burdens.

To illustrate the approach, consider a simplified open-loop recycling system with three cascaded uses, as shown schematically in Fig. 3.8. Product 1 ($x_1$) in the first system is produced from virgin materials ($x_4$) only, and at the end of its useful life 50% of it is collected and reprocessed to be reused in the second system for Product 2 ($x_2$). The rest of Product $x_1$ is landfilled as waste ($x_7$). Product $x_2$ is, therefore, made from 50% virgin material ($x_5$) and 50% material recycled from the first system ($x_{10}$). At the end of its useful life, 50% is recycled and used in System III while the rest is landfilled ($x_8$). Product 3 ($x_3$) is, therefore, also made of 50% recycled product $x_2$ ($x_{11}$) and 50% virgin material ($x_6$) and after use is discarded as waste ($x_9$). If total production of each product is 100 units, then the LP model describing this system is defined by the following constraints:

\[
x_1 = 100
\]
\[
x_2 = 100
\]
\[
x_3 = 100
\]
\[
x_1 - x_4 = 0
\]
\[
x_1 - x_7 - x_{10} = 0
\]
\[
x_{10} - x_5 = 0
\]
\[
x_2 - x_5 - x_{11} = 0
\]
\[
x_2 - x_8 - x_{11} = 0
\]
\[
x_{11} - x_6 = 0
\]
\[
x_3 - x_6 - x_{11} = 0
\]
\[
x_3 - x_9 = 0
\]
For simplicity, consider one burden only, e.g. CO₂, which is taken to be product-related; to keep the argument clear, process-related burdens are not considered. Suppose that each activity associated with the virgin materials generates the same amount of CO₂, i.e. 0.05 units per unit of virgin material. In addition, activities associated with the reprocessing of the recycled materials each produce 0.02 units CO₂/unit, so that the environmental objective function of the system is defined by:

\[ B = 0.05 \cdot x_4 + 0.05 \cdot x_5 + 0.05 \cdot x_6 + 0.02 \cdot x_{10} + 0.02 \cdot x_{11} \]  

(3.56)

At the solution of the LP model, given in Table 3.4, the marginal values of the active constraints, i.e. eqns. (3.45)-(3.47), represent the CO₂ emissions allocated between three systems, i.e. the products. The marginal allocated burdens are, therefore, equal to 0.050, 0.035 and 0.035 for products x₁, x₂ and x₃, respectively. This means that the first use in the cascade (System I) is allocated the CO₂ emissions that are equal to CO₂ generated by the virgin material used for the production of x₁. The first system, therefore, gets no credit in CO₂ emission for producing the recyclable material; however, its total waste is reduced by the amount of material being recycled. The other two uses in the cascade are both credited for using the recycled material: because they displace the production of virgin materials, the recycled materials are taken to be "burden-free", i.e. their burdens are equal to zero. However, since recycling itself is associated with additional burdens,
Table 3.4 Marginal allocation in the open-loop recycling example

<table>
<thead>
<tr>
<th>Constraints</th>
<th>Value at optimum</th>
<th>1. $b_{CO_2}$ †</th>
<th>2. $b_{CO_2}$ †</th>
<th>3. $b_{CO_2}$ †</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$x_{10} = 50%$; $x_{11} = 50%$</td>
<td>$x_{10} = 50%$; $R_2 = 90%$</td>
<td>$R_1 = 50%$; $R_2 = 90%$</td>
</tr>
<tr>
<td>Product 1</td>
<td>100</td>
<td>0.050</td>
<td>0.050</td>
<td>0.050</td>
</tr>
<tr>
<td>Product 2</td>
<td>100</td>
<td>0.035</td>
<td>0.035</td>
<td>0.038</td>
</tr>
<tr>
<td>Product 3</td>
<td>100</td>
<td>0.035</td>
<td>0.023</td>
<td>0.023</td>
</tr>
</tbody>
</table>

† eqn. (3.56) † eqn. (3.57)

the system has also to be penalised for it, i.e. these burdens are added to those generated by the virgin materials. Therefore, the burden allocated to the second and the third use in the cascade is equal to 0.035. If the reprocessing of the recycled materials was not accounted for, the burdens allocated to these subsystems would be equal to 0.025, thus reflecting only the burdens associated with the life cycle of the virgin materials and their percentage (50%) in products $x_1$ and $x_2$, respectively. Similar to the allocation in the first use, there is no credit to the second use for producing the recyclable materials, so that double accounting of the burdens is avoided.

It is now interesting to see how the marginal burdens change with a change in the way the system is operated, e.g. with changing recycling ratios. For instance, if the percentage of the material recycled into the third cascade is increased from 50% to 90%, while all other parameters are kept constant, the marginal burdens allocated to this subsystem decrease from 0.035 to 0.023 (Table 3.4, Case 2), while the burden allocated to the other parts of the system remains the same. At the same time, the total emissions of $CO_2$ decrease from 12 units (Case 1) to 10.8. In this particular example, increasing the rate of recycling decreases the total burdens so that it is desirable to increase the total recycling rate as much as possible.

However, in some systems that may not be the case because the recycling process may generate more burdens than the production of virgin materials. Suppose, for example, that in this hypothetical example, the virgin material $x_3$ can be replaced by an alternative virgin material $x_5$ with unit emission of $CO_2$ equal to 0.035. However, this plant is situated in a remote area, so that the burden associated with transport of the recycled material $x_{10}$ to the manufacturing site is increased. Suppose that the total emission of $CO_2$ from recycling $x_{10}$ is now equal to 0.04 so that eqn. (3.56) becomes:
If the recycling ratios are kept the same as in Case 2 (Table 3.4), then the burdens allocated to the first and the third subsystems remain the same, while the burden in the second increases from 0.035 to 0.038. Since the burden associated with $x_5$ is equal to 0.035, this in fact means that the emissions of CO$_2$ are higher in the system with recycling than without it. It is, therefore, better in this case not to use the recycled material.

Thus this simple example illustrates once again that marginal allocated burdens depend, in general, on the way the system is operated and not just on the structure of the system. For simplicity, the product-related burdens were considered only; in a real case study the process-related burdens would be also included in the system model, so that the marginal allocated burdens would reflect the changes in the behaviour of the system as closely as possible.

### 3.4 Concluding Remarks

Allocation of environmental burdens in multiple-function systems is one of the unresolved problems in LCA. More recently, after much debate, the LCA experts agreed that allocation can be dealt with in two ways: it can either be avoided by expanding system boundaries or solved by one of the suitable methods for allocation. Although avoiding allocation is an appealing way to deal with this problem, there are many cases where this approach cannot be used and some kind of allocation is still required.

Many different methods for allocation have been proposed so far, however, none of them offers a general solution to the problem. In most cases these methods are chosen arbitrarily without considering any causality in the system. The importance of causality in LCA is obvious: one of the main aims of LCA is comparison of some changes around an existing situation and causality is always related to the question of what would happen if some conditions in the system changed. The type of the changes considered, as discussed in this chapter, can be marginal, average or discrete. They will, in turn, depend on the goal and scope of the study and questions to be answered by LCA, a fact that most of the previously proposed methods fail to take into account. Therefore, a new approach to solving the allocation problem is needed. One such approach is proposed in this work and that has been the subject of this chapter.
A novel marginal approach to allocation developed here is based on whole system modelling. It applies to system analysis where marginal changes to a specific system are the goal of the study. The allocation coefficients are evaluated by representing the whole system as a Linear Programming (LP) model. They are equivalent to the marginal values calculated at the solution of the LP model. It is argued that marginal values represent a realistic description of the causal relationships and thus closely reflect changes in the behaviour of the system with changes in the system parameters.

This approach has been illustrated on three simple examples of multiple-function systems: combined waste treatment, co-product and recycling. A further illustration of the marginal approach is given in Chapter 6 by applying it to a real co-product system, using data obtained in a field study. However, prior to presenting LCA case study in Chapter 5 and the results of marginal allocation in Chapter 6, the following chapter introduces multiobjective optimisation as a tool for environmental management of a product system in the context of LCA.
CHAPTER 4

MULTIOBJECTIVE DECISION-MAKING
AND LIFE CYCLE ASSESSMENT

Life Cycle Assessment belongs to a class of multiobjective decision problems: it quantifies and identifies a number of environmental burdens and potential impacts of processes and products and so provides a basis for comparison among different alternatives. The main questions asked in multiobjective-decision problems, and therefore in LCA, is: how do we choose between different alternatives and how do we identify the best one for a given situation? Unfortunately, there are no easy answers to these questions and often we have to use different modelling techniques and tools in order to provide the answers. A number of these techniques and tools have been developed to help in multiobjective decision-making processes and some of them are discussed below. In this work, Multiobjective Linear Programming (MOLP) has been chosen as a specific tool. This chapter, therefore, after introducing multiobjective problems and some of the methods for solving them, focuses on possible application of MOLP in the decision-making process, with particular emphasis on its use in the LCA context. It is argued that this method provides a more effective approach to environmental management of product system by offering a range of alternative optimal solutions and enabling decision-makers to choose the best practicable environmental option not entailing excessive cost.

4.1 Classification of Multiobjective Programming Methods

The methods available for multiobjective decision-making can be classified in two general groups: single decision-maker and multiple decision-maker problems. The first group relates to those situations in which there is a single decision-maker or a group of decision-makers that share the same interests and preferences about the conflicting objectives of a multiobjective problem. The second group involves the situations in which there are many decision-makers and interest groups or "stakeholders", each of which has different or conflicting preferences and objectives. There are many different ways to approach these two groups of situations and their relationship is illustrated in Fig. 4.1.

Methods used in single decision-maker problems are classified into techniques with and without articulating preferences. The latter are also known as generating techniques, in which a decision-maker chooses the best compromise solution from a range of generated noninferior solutions. Although these techniques generate noninferior solutions without
prior articulation of preferences by decision-makers, in order to choose the best compromise solution, some articulation of preferences is still necessary. However, it is deferred until all noninferior solutions have been identified and analysed. The noninferior solutions are found in objective and decision spaces by optimising on a number of objective functions, subject to certain constraints. If both the constraints and the objectives are linear, then Multiobjective Linear Programming (MOLP) can be used to generate the noninferior solutions. The methods for generation of the noninferior set of MOLP models include the constraint method, the weighting method, the noninferior set estimation method, and the multiobjective simplex method (Cohon, 1978; Hwang et al., 1980). In this work the constraint method has been used, the basics of which are given in Chapter 2 and Appendix 2. A brief review of other methods for generating the noninferior solutions can be found in Appendix 4a.

The techniques that incorporate preferences (preference-oriented methods) require that decision-makers articulate their preferences in advance or during the analysis. In iterative preference-oriented methods, the preference information articulated during the analysis is used by the analyst to find a better set of noninferior solutions, for which a decision-maker identifies new preferences. This process is repeated until the decision-maker is satisfied. These methods include the step method, and local approximation of the multiattribute utility function (see Appendix 4a). In noniterative preference-oriented methods, the decision-maker is required to articulate preferences in advance of the
analysis and the best compromise solution is defined without generating the noninferior set of solutions. Among noniterative preference methods, the multiattribute utility function is the most commonly used; others include various geometrical definitions of "best", including goal-programming, etc.

The techniques for the multiple decision-makers category of multiobjective problems comprise aggregation of individual preferences, counselling techniques and predicting outcomes from multiple-interest decision problems. The aggregation techniques are based on the assumption that preferences or interests of a group can be expressed as a combination of each individual's preference or interest. This approach is mostly favoured in welfare economics (e.g. Pareto analysis and subconsequently cost-benefit analysis), as discussed in the following section. Distinct from the aggregationists' view of group interests are techniques based on counselling, by which the interests of a group are defined and articulated by a decision-maker who represents that group. The theories of some modern welfare economists and multiobjective generating techniques are more or less consistent with this view of multiple decision-making processes.

The third group of methods used in multiple decision-making processes are those that predict outcomes from these processes. They differ from the aggregation and counselling methods in the goal they are trying to achieve: those two approaches are prescriptive in nature, i.e. they attempt to identify what should be done, while the prediction of an outcome is an analysis with prescriptive consequences, i.e. what will be done. In predicting the outcomes, what really seems to matter are powerful stakeholders that are assumed to control decision-making process, so that the centre of the analysis are the participants with the power of effective action. Methods used for predicting the outcomes of decision-making processes are, for instance, game theory and various models of voting procedures. A brief overview and definitions of the above mentioned methods are given in Appendix 4a; a more detailed account of the methods used in multiobjective decision-making processes can be found in Cohon (1978) and Hwang et al. (1980).

Clearly, there is a range of different methods for solving multiobjective problems. Although a distinction has been made between single and multiple decision-maker problems, the methods used overlap between them. The selection of a particular method will depend on the problem and the decision-making context. In this work, a generating method has been chosen for the following reasons. Firstly, generating methods do not require a priori articulation of preferences, so that the whole noninferior set of solutions
can be explored. The emphasis is then on the range of choices from the set of noninferior solutions, rather than explicit definition of preferences before analysing all the trade-offs among objectives. Although the choice of the best compromise solution will still imply certain preferences and value judgements, at least the choice will be made from all possible noninferior solutions, unlike for instance, in the multiattribute utility function method where, due to the way the utility function is assessed, the bulk of noninferior solutions can be ignored.

Secondly, generating methods can be applied in a wide range of decision-making contexts. In the case of single decision-makers, the generating methods provide information on the trade-offs between different objectives, to show explicitly what can be gained and what lost by choosing each alternative. Where there are multiple decision-makers with conflicting interests, this technique can still help to resolve disputes by generating different alternative solutions. Decision makers who understand the trade-offs and the alternatives are more likely to understand the interests of other parties and, therefore, to compromise. A further reason for choosing this approach is that objectives do not have to be aggregated into a single objective, as is the case with methods which aggregate individual preferences. This is particularly relevant in the LCA context, because it avoids the controversial and debatable concept of aggregation of environmental impacts into a single environmental impact function in the Valuation stage. Furthermore, by being able to trade-off incommensurable objectives, e.g. environmental impacts and economic requirements, this approach avoids the well known problems encountered, for instance, in cost-benefit analysis, i.e. reducing individual preferences to a market value or trying to express quality of the environment in financial terms.

As already pointed out, methods for solving multiobjective problems provide a decision-maker with a set of noninferior solutions. The noninferior state is achieved if no objective can be improved without worsening the value of any other objective (for a mathematical formulation see Chapter 2). If analysed more closely, it is obvious that this definition is identical to the Pareto optimality concept. This concept marked the beginning of a new school of thought in economics - new welfare economics - and has been influencing decision-making process ever since. In order to fully understand the contribution welfare economists have made to the analysis of multiobjective problems and decision-making processes, the history of welfare economies, with emphasis on the Pareto analysis, is briefly introduced in the following section.
4.2 Pareto Analysis - A History of Welfare Economics

Welfare economics, although historically divided into several periods, focuses on the general problem: how should resources be allocated for the production and consumption of goods so as to maximise social welfare? The question asked remained the same until today; what changed over time, however, was the definition of "social welfare" and the approaches to solving this problem.

The era of welfare economics started in the 18th century with the original welfare economists, who adopted the ideas of Bentham\(^1\) and other utilitarians. For them, social welfare was a summation of the "utility" of each individual. Utilitarianism defined utility as the ultimate goal of all economic activity relating it to the pleasure or satisfaction derived by an individual from being in a particular situation or consuming goods or services. Bentham described it as that which appears "to augment or diminish the happiness of the party whose interest is in question". Utilitarians assumed that utility was measurable and that interpersonal comparisons of utilities were possible. They also maintained that utilities of some individuals should be sacrificed if that meant greater overall utility, i.e. better social welfare.

New welfare economics is based on the work of Pareto (1971\(^2\)) in which utility was replaced by the indifference curve, on which different combinations of social states yield the same level of utility, and Pareto's "optimality" condition was formulated. There are still various interpretations of Pareto's thought, but there is consensus as to what constitutes a Pareto optimum: a social state is Pareto optimal if no individual can be made better off without making at least one other individual worse off. In other words, if such a state is reached it is not possible to increase the utility of some individuals or groups without diminishing that of others. Pareto also argued that interpersonal comparisons of individual utilities could not be made and that maximum utility of a community was not the simple summing of the single individuals' utilities, as the original welfare economists believed.

A Pareto optimum curve, represented by the social-welfare function (see Appendix 4a) and related to the utilities of a two-individual society, is shown in Fig. 4.2. All points on the curve are Pareto optimal since more of individual 2's utility \(U_2\) can be gained only by sacrificing some of the utility \(U_1\), e.g. by moving from B to C in the figure. Point A,

---

\(^1\)Jeremy Bentham (1748-1832), English philosopher, founder of utilitarianism

\(^2\)Originally published in 1909
below the curve, is not a Pareto optimal social state since both individuals can be made better off by moving to state B. In general, there is a continuum of Pareto-optimal social states, and no individual state can be considered better than any other in the absence of further value judgements. Pareto recognised here that when the optimum is reached, movements along the Pareto curve involve resorting to considerations foreign to economics, in order to "decide on grounds of ethics, social utility, or something else, which individuals it is advisable to benefit, which to sacrifice" (Tarascio, 1966).

The practical use for the concept of Pareto optimality was in evaluation of a movement from a present inferior social state to a new Pareto optimal one. However, some cases, such as movement from A, which is not Pareto optimal, to C, which is, cannot be evaluated in a strict sense since the utilities of the two individuals are not both increased.

![Fig. 4.2 A welfare frontier for a two-individual society](Cohon, 1978)

Economists found a solution to this problem, or thought they did, in the work of Kaldor and Hicks, who proposed "compensation tests" to allow evaluation of movement from an existing to a new social state, such as from A to C in Fig. 4.2. The general idea was that those who benefited by such a movement would have gained enough to compensate the losers and would still have positive gains left over (Cohon, 1978; Rees, 1990). For example, compare individual 2's gain, FG, due to a shift from A to C, to individual 1's loss, DE. The utilities of both individuals would improve after the redistribution since
person 2 can compensate person 1 for the loss DE. However, in order to do this, it is necessary to compare the interpersonal utilities, i.e. to decide on the importance of each persons' utility. The "compensation tests" laid the foundation for Cost-Benefit Analysis (CBA), probably the tool most exploited by neoclassical economists in the decision-making process, particularly in the area of public investments. CBA is based on the idea of maximum net gain: it reduces aggregate social welfare to the monetary unit of net economic benefit. So for example, given several alternatives, the CBA approach would favour the one in which the difference between benefits and costs is the greatest. More recently, CBA has been applied in environmental decision-making. The most widely applied, and even more criticised, technique is "contingent valuation". In it, participants are asked to say how much they would be prepared to pay to protect an environmental asset ("willingness to pay") or how much they would be willing to accept for loss of that asset ("willingness to accept") (see e.g. Pearce, 1983; Turner et al., 1994). The argument that followers of this neoclassical approach to environmental economics use is that "preserving and improving the environment is never a free option; it costs money and uses up real resources" (Pearce et al., 1989).

Limitations and difficulties of this approach have been recognised both by its proponents and the critics. So Pearce (1992), one of the distinguished advocates of CBA, writes about the credibility problem of CBA admitting that there is "the fairly widespread belief that valuation techniques do not give rise to 'real' values". This has lead, on the one hand, to lack of belief in the accuracy of valuation techniques used in CBA and, on the other hand, to objections with regard to the neoclassical paradigm on which valuation rests.

Critics (e.g. Jacobs, 1991; Adams, 1993; Clift, 1994a) have pointed out that CBA has serious difficulties in dealing with problems of intergenerational equity and sustainability and in valuing the natural environment. Critics have also shown that contingent valuation is based on individual preferences which may not provide firm foundations for environmental decision-making. Furthermore, the results of the analysis largely depend on the way the questions are asked, and whether the participants are familiar with the asset in question. It is more likely that people who know nothing about the asset will place a nil value on it, although the life of others may depend on it. Also, the values that

3Neoclassical economics is a loose amalgam of subschools of thought that emerged toward the end of the nineteenth century and was revolving around Alfred Marshall in England, Leon Walras in France (later on followed by Pareto) and Carl Menger in Austria. The common feature of neoclassical economics is that it reduces many broad categories of market phenomena to considerations of individual choice, subject to the constraints of technical knowledge, social practice, and scarcity of resources.
people place on things strongly depend on self-interest, which does not help resolving conflict between opposing parties.

To summarise, the above mentioned economic approaches to decision-making, be it "original welfare", "new welfare" or "neoclassical" ones, face at least three problems: the measurement of individual preferences, the interpersonal comparison of those preferences, and their aggregation into a social preference function. All these operations imply ethical value judgements, with probably the least acceptable one being the expression of individual preferences and values in monetary terms. Indeed, the controversial techniques of pricing nonmonetary objectives, such as environmental quality, and aggregating noncommensurables into a single utility function provide a strong motivation for using multiobjective analysis in environmental decision-making.

Furthermore, these approaches cannot provide information for decision-making on a local level: for example, they cannot advise engineers on how to modify a process in order to improve its environmental performance. Multiobjective LP, on the other hand, does exactly this: it can optimise the operation of a system with environmental, technical, economic and other aspects taken into account. If applied in the LCA context, it can optimise the whole life cycle of a process or product and so provide a more efficient approach to environmental management of a system. Application of MOLP to LCA is the subject of the next section.

4.3 Multiobjective Linear Programming and LCA

As an environmental management tool in decision-making, LCA has two main objectives. The first is to quantify and evaluate the environmental performance of a product or a process from "cradle to grave" and so help decision-makers to choose between alternative products and processes. Another objective of LCA is to provide a basis for assessing potential improvements in the environmental performance of a product system. The importance of the latter objective can be twofold, depending on the objective of the LCA study. If LCA is performed in order to compare supply and demand patterns or alternative processes in a system, it can help identify the best possible choices in this respect. However, if LCA is performed for a specific process or a product, then this objective can be of particular importance to engineers, because it can advise them on how to modify a system to decrease its overall environmental impacts. In order to achieve environmental improvements in the system in the optimum way, a suitable optimisation technique must be used. The optimisation problem in the LCA
context will inevitably be a multiobjective one and, immediately, one can think of at least a dozen different programming methods that could be used here (see section 4.1). As already explained, in this work multiobjective linear programming has been chosen. Its potential in environmental optimisation of a product system in the context of LCA is explained below.

4.3.1 Environmental Optimisation of Product System

The analysis in Chapter 3 concentrated on solving the problem of allocation in the Inventory Analysis and Impact Assessment stages of LCA, by using LP. System analysis in that context is limited to analysing processes or products on the basis of marginal changes around their current performance, i.e. optimisation of the system is not a goal of the analysis. In the Improvement Assessment stage, however, the objective of system analysis is to identify possibilities for improving the environmental performance. Therefore, this is the stage where the optimisation of a system is necessary and where multiobjective LP can be used as an optimisation tool.

Application of multiobjective linear programming in LCA is a novel approach to analysing and managing the environmental performance of a product system (Azapagić et al., 1996a, 1996b; Azapagić and Clift, 1995c, 1996c). In general, a MOLP model of a system formulated using LCA can be defined by $n$ mass balance equations:

$$\sum_{i=1}^{n} a_{ji} x_i = 0$$  \hspace{1cm} (4.1)

with constraints on demand $D_i$ of products $P_i$, supply $S_m$ of primary and raw materials $R_m$ and capacity $C_i$ of process $i$:

$$P_i \leq D_i, \; R_m \leq S_m, \; x_i \leq C_i$$  \hspace{1cm} (4.2)

The objective functions of the system are represented by a number of environmental burdens, $B_j$,

Minimise

$$B_j = \sum_{i=1}^{n} b_{ji} x_i$$  \hspace{1cm} (4.3)

or impacts, $E_k$: 

69
Minimise

\[ E_k = \sum_{j=1}^{J} e_{k,j} B_j \]  

where \( b_{k,j} \) represents emission coefficient from process \( x_j \), and \( e_{k,j} \) represents the relative contribution of burden \( B_j \) to impact \( E_k \). As already mentioned, there are a number of methods for generating noninferior solutions of a MOLP model (see Appendix 4a); for the reasons explained in Chapter 2, the constraint method has been used in this work. The general procedure for generating noninferior solutions of a MOLP model, using the constraint method, can be summarised in four steps:

1. Formulate a system in LP terms and define environmental burdens and impacts, as determined by LCA, as objective functions;
2. Perform minimisation on each objective function to create a pay-off table with lower and upper bounds on the feasible region;
3. Convert all objectives but one into constraints and repeat minimisation with the right-hand side coefficients of the objectives-constraints ranging from lower to upper bound for each constraint;
4. Create a table with noninferior Pareto solutions or, if possible, illustrate the noninferior curve graphically.

In the context of LCA, the objective functions represent environmental burdens or environmental impacts. The system is optimised on all of these functions simultaneously in order to find a range of environmental optima of the system. As noted above, this approach avoids having to articulate preferences at outset, aggregating objective functions, or applying Valuation to different environmental impacts.

The environmental optima are found on the multidimensional noninferior or Pareto surface. Hence, local and global system improvements are found by first moving the system to conditions on the Pareto surface, and then moving along it. By definition, as shown in sections 4.2 and 2.6, none of the objective functions at the Pareto optimum can be improved without worsening the value of any other objective function. Therefore, some trade-offs between objective functions are necessary in order to reach the preferred optimum solution in a given situation. For example, if \( \text{CO}_2 \) and \( \text{SO}_2 \) emissions are optimised simultaneously, the resulting Pareto optimum does not necessarily mean that these functions are at their minima achieved when the system is optimised on each of them separately. The Pareto optimum, however, does mean that the set of best possible options has been identified for a system in which both emissions should be reduced.
One of the possible ways to choose the "best" solution is to consider a graphical representation of the noninferior set and then choose the best compromise solution on the basis of the trade-offs. However, this approach is limited to two or three objective functions at most, because graphical representation becomes less than helpful with more than three objectives. Another way to look at it is to express the values of objectives at noninferior solutions in terms of the percentage that they are away from their individual optima. If all objectives are considered to be of the same importance, than the best compromise solution could be that which equalises the percentage by which all objectives differ from their optimum values. However, should any of the objectives be considered more important than the others, then other techniques that allow ordering of preferences (see Appendix 4a) could be used to identify the best compromise solution. Although this implies expressing preferences for the objectives, these preferences are at least articulated in the post-optimal analysis of all noninferior solutions and their trade-offs, as distinct from expressing preferences and aggregating the objectives prior to identifying all noninferior solutions. These considerations are now illustrated by a hypothetical example of a system to produce polymer materials used for packaging.

4.3.1.1 An example of Environmental Optimisation of Product System

The system under consideration is based around the commodities polypropylene (PP), expanded polystyrene (EPS), high density polyethylene (HDPE), low density polyethylene (LDPE) and high intensity polystyrene (HIPS). The flow diagram of the system is shown in Fig. 4.3. All operations and activities from the extraction of raw materials from the earth up to production of polymer products are included in the model. However, the use and disposal stages in the life cycle are not considered here ("cradle-to-gate" approach). The goal of the LCA study in this example is to identify possibilities for improvements in the environmental performance of the system by identifying the optimum production of the products, subject to certain constraints. The LP model of the polymers system is defined by the mass balance equations:

\[ \sum_{i=1}^{40} a_{ji} x_i = 0, \quad j=1,\ldots,69 \]  

where the coefficients \( a_{ji} \) are given in Appendix 4b (Table A4.2). Other constraints include market demand for the five thermoplastic products, \( P_i \), and supply of oil, \( S_m \), expressed in tonnes/year:
The objective functions of the system are the environmental burdens and the impacts, as defined by eqns. (4.3) and (4.4). Depending on whether the objective of the study is to analyse the performance of the system at the Inventory or the Impact Assessment level, multiobjective optimisation can be performed on either environmental burdens or environmental impacts. Suppose that at this stage of the analysis, the objective of this study is to quantify the burdens only, so that the system can be optimised on a number of
objective functions as defined by the total burdens. However, to illustrate the approach graphically, the system is optimised on two objectives only, for example on CO$_2$ and NO$_x$, as given by:

$$B_j = \sum_{i=1}^{35} b_{c_i} x_i \quad j=1,2$$ (4.12)

The $b_{c_i}$ coefficients are listed in Table A4.2 in Appendix 4b. If the four-step procedure for multiobjective optimisation given above is followed, the system is first optimised on each objective function to create a pay-off table (Table 4.1). The optimum value of CO$_2$ is 1801266 t/yr and the corresponding calculated emissions of NO$_x$ are 17880 t/yr. The optimisation on NO$_x$ gives an optimum value of 14282 t/yr, with the corresponding calculated emissions of CO$_2$ equal to 2138495 t/yr. The system is then optimised on the NO$_x$ function and the CO$_2$ objective is converted to a constraint, with the values of right hand sides, $\varepsilon_q$, ranging from minimum of 1801266 t/yr to its maximum value of 2138495 t/yr (see the pay-off table). As already pointed out in Chapter 2, in the two-objective problems, $\varepsilon_q$ can be chosen arbitrarily; in problems with three and more objective functions, however, $\varepsilon_q$ has to be chosen in a more systematic way (e.g. by using eqn. (A2.19)), because higher-dimensional problems will usually lead to some infeasible constrained problems.

<table>
<thead>
<tr>
<th></th>
<th>$F_{CO_2}$ (t/yr)</th>
<th>$F_{NO_x}$ (t/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^1$</td>
<td>1801266</td>
<td>17880</td>
</tr>
<tr>
<td>$x^2$</td>
<td>2138495</td>
<td>14282</td>
</tr>
</tbody>
</table>

A number of successive minimisations have been performed and the resulting noninferior or Pareto curve$^4$ with a selection of noninferior solutions is drawn in Fig. 4.4. Points A and G in the figure correspond to the single optimisations, while all other points shown on the curve are obtained in the multiobjective minimisations. The production rates of the five products obtained in the optimisations are given in Fig. 4.5. It is obvious from Fig. 4.4 that the solution that minimises CO$_2$ maximises NO$_x$ and vice versa. The best compromise solution can then be selected by examining the trade-offs between the two objectives and understanding how much of one objective has to be given up in order to

$^4$Actually, the "curve" is piecewise-linear.
gain in the other. For instance, if the emissions of NOx are to be decreased from its maximum of 17880 t/yr at point A to 15011 t/yr at point C, the emissions of CO₂ will increase from the minimum value of 1801266 to 1840000 t/yr. In other words, for a decrease in NOx emissions of 16%, the CO₂ emissions increase by 2%. This state of the system is achieved through a reduction in the PP production by 47% and an increase in the production of LDPE of 39% (Fig. 4.5). The production of EPS is also reduced by 53%, while other productions remain unchanged. By moving along the noninferior curve towards point G, the emissions of CO₂ are further increased and those of NOx are decreased. However, it can be noticed that, due to a different set of active constraints at each solution, the slope of the curve changes significantly at points C, D and E. For the noninferior solutions found on the curve EG, the slope is almost equal to zero which means that a very small reduction in the NOx emissions, causes a much larger increase in the CO₂. Thus, for example, to reduce the emissions of NOx at point G by 0.4% relative to point E, the corresponding emissions of CO₂ have to increase by 11%. Again, these changes are a result of a change in the production rate of the polymers, as shown in Fig. 4.5.

By trading-off solutions on the noninferior curve, a decision-maker can choose the one that is the most suitable for a given situation. If both objectives are considered to be of the same importance, then it is better to look for the system improvements on the curve segment delineated by points AE in Fig. 4.4, because higher reduction in NOx can be achieved at relatively low increase in CO₂. However, if this is not the case, then any solution on the noninferior curve could be chosen as the most appropriate for a particular decision-making situation.

![Graph showing noninferior curve for environmental optimisation of the polymers system](image)

**Fig. 4.4 Noninferior curve for environmental optimisation of the polymers system**
4.3.2 Economic and Environmental Optimisation of Product System

Multiobjective LP optimisation on environmental objective functions generates environmental optimum solutions and so identifies places in the life cycle of a system where improvements can be made. Hence, MOLP serves as a tool for managing and improving the environmental performance of productive systems. However, system improvements are usually not carried out on the basis of environmental LCA only; other factors - technical, financial and social - have to be considered as well. This can be of particular relevance, for instance, to the chemical process industries, which face problems of having to keep total costs down while at the same time complying with environmental legislation and other socio-economic requirements. So, in addition to optimisation on the environmental objective functions, the system has to be optimised on socio-economic objectives. In this way, MOLP can help identify acceptable solutions which represent the compromise between these conflicting objectives and so lead to an improved performance of a system throughout its life cycle.

To illustrate this, consider the polymers system again. In Section 4.3.1 objective functions were defined as environmental burdens and impacts. Here, in addition to these, an economic objective function, represented by profit is also defined. Following the four-step procedure given in Section 4.3.1, the system can then be optimised on all objective functions simultaneously to obtain the Pareto or noninferior surface. Again, in order to illustrate the procedure and results graphically, only two objective functions are
considered here: global warming potential (GWP), and profit (PR), defined by the following equations:

Minimise \[ GWP = \sum_{j=1}^{2} ec_{GWP,j} B_j \] (4.13)

Maximise \[ PR = \sum_{l=1}^{5} pr_l P_l \] (4.14)

where \( ec_{GWP,j} \) is a GWP coefficient of burden \( B_j \) and \( pr_l \) is a hypothetical profit coefficient for each commodity \( P_l \). The burdens contributing to the GWP considered here are \( \text{CO}_2 \) and \( \text{VOCs} \), with the corresponding \( ec_{GWP} \) values equal to 1 and 11, respectively (Table A5.15). The hypothetical profit factors \( pr \) are taken to be 550, 780, 500, 500, and 865 £/t for PP, EPS, HIPS, HDPE, and LDPE, respectively.

![Fig. 4.6 Noninferior curve for economic and environmental optimisation of the polymers system](image)

The results of multiobjective optimisation are shown in Figs 4.6 and 4.7. Points A and E in Fig. 4.6 represent the minimum value of GWP and maximum value of PR respectively, with the system optimised on each objective function individually. All other points on the curve are obtained in the multiobjective optimisation by converting the profit objective function into a constraint and minimising GWP. Although all solutions obtained in the
optimisations are noninferior, not all of them will represent appropriate operation for a given situation. So, for example, the value of GWP at point A is minimum, but so is the profit and the production of EPS and HIPS is zero (Figs 4.6 and 4.7). Solution B increases GWP from the minimum by 4.2% and decreases profit from its maximum by 4.0% and EPS is not produced. At solution D all commodities are produced but GWP exceeds its minimum by 17.3%. Similar to the curve in Fig. 4.5, the slope of the curve in Fig. 4.6 changes due to a different set of active constraints at each solution, as defined by the output of the polymer products.

Which of these solutions will be chosen as the best compromise depends on the circumstances and the objectives of the study. As already mentioned, if all objectives are considered to be of equal importance to decision-makers one of the possible ways to choose the best compromise solution is to identify the one that equalises the percentage by which both objectives differ from their optimum values. If the "equal percentage-approach" is chosen, then the best compromise solution in this example is found at point B, where both GWP and PR differ from their optimum values by approximately 4% (Fig. 4.8). Should some objectives be considered more important than the others, the decision-makers can identify the most preferred alternative by using one of the methods with articulating preferences (see Appendix 4a).

This example illustrates the value of MOLP optimisation in not being prescriptive; it offers a set of alternative options for system improvements, rather than a single optimum
solution. Single objective models, such as CBA for example, dictate the use of a single measure of efficiency and provide only one solution for decision makers. Decision-makers like to decide and multiobjective optimisation allows them to do so. Another attractive feature of this approach to system analysis is the ability to identify the environmental optima of the system and trade it off with the economic and technical constraints. On the practical level, this enables a decision-maker to identify and choose the Best Practicable Environmental Option (BPEO) not entailing excessive cost.

4.4 Concluding Remarks

As a management tool in environmental decision-making, LCA has two main objectives. The first is to quantify and evaluate the environmental performance of a product or a process from "cradle to grave" and so help decision-makers to choose between alternative products and processes. Another objective of LCA is to provide a basis for assessing potential improvements in the environmental performance of a product system. Two main problems are associated with these objectives of LCA. First, in many cases there will be a number of options and possibilities for improvements and it may not always be obvious which of them represents the optimum solution. Therefore, some kind of system optimisation will be necessary. Secondly, there may exist more than one optimum solution for improving the system's performance, in which case the issue becomes that of choosing the best compromise option from a number of optimum
solutions. This is a typical problem encountered in multiobjective decision-making and is not specific to LCA. It is related to expressing individual preferences and prioritising the options in order to identify the most preferred solution for a particular situation.

The optimisation problem in the LCA context is inevitably a multiobjective one, and that is one of the reasons that multiobjective linear programming has been chosen for this work as a specific multiobjective optimisation method. Although MOLP is not a new technique, it has not been applied to LCA before. Its main advantage over other methods which have been used in LCA is that generating optimum solutions does not require a priori articulation of preferences, so that the whole noninferior set of solutions can be explored. The emphasis is then on the range of choices from the set of noninferior solutions, rather than explicit definition of preferences before analysing all the trade-offs among objectives. Although the choice of the best compromise solution will still imply certain preferences and value judgements, at least the choice will be made from all possible noninferior solutions, unlike other methods where the bulk of noninferior solutions may be ignored.

This is particularly relevant in the LCA context, because it enables avoiding the controversial and debatable concept of aggregation of environmental impacts into a single environmental impact function in the Valuation stage. Furthermore, by being able to trade-off incommensurable objectives, e.g. environmental impacts and economic requirements, this approach avoids the well known problems encountered, for instance, in cost-benefit analysis, i.e. reducing individual preferences to a market value or trying to express quality of the environment in financial terms.

The simplified example presented in this chapter is chosen to illustrate the potential of the MOLP approach in LCA. In Chapter 7, this approach is applied to a real case study of boron products, where it is shown that the system can be optimised simultaneously on a number of environmental and economic objective functions to give a range of optimum solutions for better performance of the whole system. Prior to discussing these results, the LCA case study of the boron products system is presented in the following chapter.
CHAPTER 5

LCA OF A MULTI-OUTPUT PRODUCT SYSTEM
-A CASE STUDY-

The discussion in the preceding two chapters focused on the theory of whole system modelling by LP in the context of LCA. This and the two following chapters concentrate on the application of this theoretical framework to a real case study. As an introduction, the LCA case study of five boron products is presented in this chapter. It is followed in Chapter 6 by discussion of the results of applying the "marginal allocation" approach to the system. The environmental and economic optimisation of the whole system by multiobjective LP, as part of the Improvement Assessment stage, is the subject of Chapter 7.

This chapter is intended to be a self-contained LCA study, and is, therefore, written following the recommendations laid out by SETAC (Consoli, 1993). It includes the results of the Inventory and Impact Assessment stages only; the results of the Improvement Assessment stage are part of Chapter 7. The environmental burdens and impacts presented here are calculated by formulating the whole system as an LP model. Since the application of LP is more relevant for the allocation of environmental burdens and for the improvements in the system, the mathematical formulation of the model and related discussion are presented in Chapters 6 and 7.

5.1. Goal Definition and Scoping

5.1.1 Purpose

There are several objectives to be achieved by undertaking this case study and they include:

a) evaluation of the environmental performance of the boron system producing the following five products:
   1. disodium tetraborate decahydrate or 1OMol borate (Na₂B₄O₇·10H₂O), hereafter called 10Mol,
   2. disodium tetraborate pentahydrate or 5Mol borate (Na₂B₄O₇·5H₂O), hereafter called 5Mol,
   3. orthoboric acid (H₃BO₃), hereafter called BA,
   4. disodium tetraborate or anhydrous borax (Na₂B₄O₇), hereafter called AB, and
Chapter 5

5. boric oxide or anhydrous boric acid (B₂O₃), hereafter called ABA;
b) solving the problem of allocation of environmental burdens in the boron system;
c) identification of possibilities for environmental improvement of the system by optimising the system on environmental performance; and
d) identification of possibilities for improving both environmental and economic performance in order to provide decision-makers with the necessary information for effecting these improvements.

This study is carried out with the intention that the results will be used internally by the company, in order to improve the overall performance of the system. Potentially, the results could be used externally; in that case a further refinement of the data used for the background system, in particular for the raw materials, would be necessary.

5.1.2 Scope

The system under consideration in this study, defined as that which produces five boron products, is divided into foreground and background subsystems, shown schematically in Fig. 5.1. The foreground system, defined as the system of primary concern in the study, delivering a functional unit specified in the goal of the study is represented by the central process, i.e. production of the five boron products. This system, which is located in California, USA, includes all activities from extraction of boron from the earth up to the packing of the boron products. On-site cogeneration of energy as well as the transport in the mine are also within this subsystem. Data for the foreground system are based on the actual operation of this system during one year. The data are therefore considered to be marginal because they correspond to a specific process and technology. The background subsystem, defined as a system which supplies energy and materials via a homogeneous market to the foreground system, represents all other activities associated with the system under study, including extraction of primary materials from the earth, to producing and transporting fuels and raw materials to be used in the foreground subsystem. The data for the background system are taken from the available databases, i.e. PEMS (1994) and SimaPro2 (1995) and adapted to the US conditions (average data). Greater detail on the data and the subsystems are given in the Inventory stage.

Excluded from the system boundaries are:

- use and disposal stages in the boron products life cycle ("cradle to gate" approach);
- manufacture of ancillary materials, such as chemicals for water treatment (NaCl, NaOH and HCl), sodium dithionite (Na₂S₂O₄), hydrogen peroxide (H₂O₂), and flocculants;
- manufacture and maintenance of capital equipment (except for the maintenance of the trucks in the mine);
- heating and lighting and their maintenance;
- on-site transport (with the exception of transport in the mine).

5.1.3 Functional unit

Three types of functional units have been identified, depending on the intended use of the study (Table 5.1). Firstly, the functional unit has been chosen under the assumption that the study will be used internally only, in which case the engineers and operators are concerned with assessing and improving the system performance on the basis of its actual operation. Therefore, the interest here lies in analysing the effect of marginal changes around an existing state of the system. Hence, the functional unit has been defined as "operation of the system for one year", represented by the production of the five boron products. The product outputs used in this study are shown in Table 5.1. However, they are not based on actual production or sales figures but represent a possible scenario for this system and therefore create a valid basis for discussion in this and subsequent chapters.

Secondly, it has been assumed that the results of the study will eventually be used externally. In this case, two types of functional units have been identified: 1000 kg of each product or 1000 kg of B₂O₃ equivalent in each product. The former type of functional unit is applicable to a comparison of the same products delivering the same function but produced in different product systems. The functional unit based on the amount of B₂O₃ equivalent can be used when comparing two different products with the same function, produced either in the same or different systems.

5.1.4 Data-quality Assessment

5.1.4.1 Marginal Data

The marginal data relate to the foreground processes and are obtained directly from the company. The data used are representative of the operation which has not changed...
significantly in the previous years. Production data, including data on mass and energy, are a result of the process measurements, while the environmental data represent the best estimates obtained using the methodology recommended by the US Environmental Protection Agency (1995).

5.1.4.2 Average data

Average data refer to the background system and are taken from the available databases (PEMS and SimaPro2). They include life cycles of the fuels, i.e. natural gas and diesel fuel, and the materials, i.e. sulphuric acid, soda ash or trona, and explosives, used in the foreground system. Data for the packaging for the boron products are also taken from these databases. These data represent the average mix of technologies. Although they correspond to the US conditions, they are not necessarily representative technologies for producing these reactants. Particular care has to be taken with the data for sulphuric acid because of its significant consumption in the process. If the study is going to be used externally it is recommended to obtain these data directly from the supplier, in effect bringing sulphuric acid production into the foreground system.

![Fig. 5.1 Schematic representation of the background and foreground subsystems](image)
Table 5.1 Functional units in relation to the intended use of the study

<table>
<thead>
<tr>
<th>Intended use of the study</th>
<th>Internal</th>
<th>External</th>
<th>External</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Functional unit</strong> =&gt; <strong>Product</strong></td>
<td>&quot;Operation of the system for one year&quot; (ton*/yr)</td>
<td>&quot;1000 kg of each product&quot; (ton*)</td>
<td>&quot;1000 kg of B₂O₃ equivalent&quot; (ton*)</td>
</tr>
<tr>
<td>10Mol</td>
<td>81000</td>
<td>1000</td>
<td>3030</td>
</tr>
<tr>
<td>5Mol</td>
<td>810000</td>
<td>1000</td>
<td>2305</td>
</tr>
<tr>
<td>BA</td>
<td>150000</td>
<td>1000</td>
<td>1958</td>
</tr>
<tr>
<td>AB</td>
<td>16000</td>
<td>1000</td>
<td>1593</td>
</tr>
<tr>
<td>ABA</td>
<td>5000</td>
<td>1000</td>
<td>1000</td>
</tr>
</tbody>
</table>

*Note: "ton" means "short ton"; 1 ton = 2000 lb = 9.071874·10² kg = 0.9071874 tonnes

5.2 Inventory Analysis

5.2.1 Defining the System and the Boundaries

The whole product system considered in this study is divided into foreground and background subsystems, as explained in Section 5.1.2 and shown in Fig. 5.1. The foreground system relates to the central process under the study, i.e. to the production of the five boron products. This system is situated in the north-western Mojave Desert in California, about 90 miles north-east of Los Angeles and 3 miles north of the town Boron. A view of the foreground system from the air is shown in Fig. 5.2.

The foreground system consists of two main parts located at the same site: the mine and the plant. Boron minerals, borax (Na₂B₄O₇·10H₂O) and kernite (Na₂B₄O₇·4H₂O), are extracted in the mine, crushed and transported to the adjacent plant. 5Mol and 10Mol borates are produced by dissolving borax and kernite in water. Na-borates are then separated from insolubles, crystallised and dried to produce powder products. Boric acid (BA) is produced in a separate plant, by reacting kernite ore with sulphuric acid. The rest of the process is similar to the 5Mol and 10Mol production. Anhydrous borax (AB) and anhydrous boric acid (ABA) are made in high-temperature furnaces from 5Mol borate and BA, respectively. All products are then either packed or shipped in bulk. Electric energy and the steam for the system are provided by the on-site natural gas cogeneration facility, which meets around 98-99% of the electricity and most of the steam demand. The additional steam is provided by the steam plant which is also fired by natural gas. The overburden from the mine and the gangue from the process are stocked in piles
Fig. 5.2 Air view of the foreground system

Scale: 1:20000
north and south of the mine. The waste water from the refinery is discharged into self-contained ponds which are either covered or are being reclaimed. All activities, from extraction of raw materials to the production of the boron products and materials used, are included in the system. However, the use and disposal phases of the products are not considered in this study ("cradle to gate" approach).

Therefore, the foreground system is broken down into the four main subsystems: Mining, Production, Packing & Shipping, and Cogeneration. Each of them are further split into smaller subsystems so that the overall flow diagram of the foreground system looks as shown in Fig. 5.3. The broken lines around the subsystems represent the system boundaries. The Production subsystem is further broken down into the Secondary crusher, Primary process, Boric acid, Anhydrous boric acid and Anhydrous borax subsystems. The Cogeneration is split into two parts: Cogeneration plant and Steam plant. The background system is also divided into the subsystems which include extraction of primary resources, production, and transport of Sulphuric acid, Soda ash, Natural gas, Diesel fuel and Packaging. Detailed description of all these subsystems, their flow diagrams and inventory data are given in Appendix 5.

5.2.4 Data collection

5.2.4.1 Foreground system

All data for the foreground system were collected from US Borax Inc., the producer of the boron products. Most of the process data, including energy data, are taken from the Monthly Operations Report and the "login" process sheets, supplied by the company. Some of the data are obtained from the process engineers and the plant manager.

The data on air emissions are obtained from the database compiled by US Borax over five years. The database contains data on emissions of total suspended particles (Dust or TSP), particulate matter \( \leq 10\mu \) (PM10), carbon monoxide (CO), sulphur dioxide (SO\(_2\)), nitrogen oxides (as NO\(_2\)) and hydrocarbons (as CH\(_4\)). Data on emissions, calculated using US EPA methodology (1985), are estimated for each part of the plant and mobile equipment and reported to the EPA quarterly. These data serve as a basis on which the "Permits to operate" the plant are issued. In addition, emissions of CO\(_2\) from the foreground system are calculated based on the stoichiometry of combustion.
Water emissions and solid waste data are taken from the reports prepared for the Lahontan Regional Water Quality Control Board. The water emission data include effluent discharges to the ponds and concentrations of arsenic, boron, and antimony in the effluent. Data on solid waste comprise mass and volume of the overburden and the process gangue.

Operating data for the cogeneration plant are obtained from the project documentation and the Monthly Operations Report. Air emissions data are a result of a continuous flue gas monitoring programme.
The data on transport activities in the mine are taken from the Monthly Operations Report. The air emissions related to transport are taken from the Borax database, except for the emissions of CO₂ which are calculated using the SimaPro2 database.

Data on the materials from the background system, used in the foreground system, are obtained from the Monthly Operations Report and the Purchasing Department.

5.2.4.2 Background system

Most data for the background system are obtained from the PEMS database (1994). Additional data for some subsystems, such as explosives, ammonia, and nitric acid production, are taken from SimaPro2 (1994). Since these databases are compiled using mostly European data, they had to be updated to reflect US conditions. This has been possible for usage of energy and related fuels for which data are also available in the database. However, data on different processes for materials used in the foreground system are more difficult to update to US conditions because they represent the average technology mixes, so that this has not been done.

Data on transport of materials to the foreground system, i.e. distances and types of transport, were obtained from the Borax Purchasing Department and the suppliers. Average transport distances within different subsystems of the background system are assumed to be 300 km. Environmental burdens from transport are calculated using the PEMS database with the transport and road types adjusted to the US conditions. A more detailed account of the background system is given in Appendix 5.

5.2.5 Allocation - Materials

5.2.5.1 Foreground system

This case study is concerned with analysis of marginal changes to a specific technology. As a result, a general "marginal allocation" approach for multiple-function systems, based on whole system modelling by linear programming (see Chapters 2 and 3), is applied to allocate the environmental burdens in the foreground system. The results of applying this approach to the case study and the comparison with other allocation methods are presented in Chapter 6 and are, therefore, not discussed here.
5.2.5.2 Background system

It has been argued in Chapter 3 that the background systems can be analysed on "average changes of the average technology mix" so that average data can be used. Therefore, the data on the background system in this study are taken from the available databases where processes represent an average mix of different technologies. Since the allocation of environmental burdens in these databases is done on the mass basis, this approach has been adopted for the background system.

5.2.6 Allocation - Energy

5.2.6.1 Foreground system

The on-site cogeneration facility generates both electricity and steam. About a quarter of the electricity is used by US Borax, while the rest is exported to other consumers. All steam produced by the plant is consumed on site. For the electricity used by Borax and the steam produced together with that amount of electricity, no allocation is needed: they are both part of the foreground system and the environmental burdens are calculated accordingly. However, the problem is to allocate the burdens between the electricity exported from the system and the rest of the steam used by Borax. In this study the "avoided burdens" allocation approach has been used, the results and discussion of which are presented in Chapter 6.

5.2.6.2 Background system

The burdens associated with the background energy systems in this study are, as for material supplies, adopted from the databases used in this study, with all fuel and energy types corresponding to US conditions, assessed as "average changes of the average technology mix".

5.2.7 Results of the Inventory Analysis

As already stated, this study has been carried out under the assumption that its results will be used internally. The results presented here, therefore, correspond to the functional unit defined as "operation of the system for one year", with outputs equal to total production (see Table 5.1). The environmental burdens are calculated for each
subsystem and the results are presented both in tables and graphs, showing the contribution of each subsystem to the total burdens. Results shown in the graphs are split into two categories: resource requirements and emissions to air, water and land. All units are in short tons/yr (materials and burdens) and MJ/yr (energy). Prior to discussing these results, the main assumptions, already mentioned elsewhere, are summarised as follows:

- electricity used in the foreground system is generated by the Cogeneration plant and no other source of electricity is used;
- trona and Soda ash are assumed to be sodium carbonate (Na$_2$CO$_3$) produced by the Solvay process;
- explosive is assumed to be ammonium nitrate (NH$_4$NO$_3$);
- transport: -foreground system - waste from the tyres used for the mine trucks is included in the system boundary as mass and volume of the disposed tyres;
  -background system - average distances in the background system are taken to be 300 km;
- packaging: disposal at the end of life is not included in the system boundary.

5.2.7.1 Mining

A flow diagram of the Mining subsystem is shown in Fig. A5.1. The environmental burdens arising from the mine are a result of electricity consumption, fuel usage and the ore handling. A small proportion of the burdens is related to the use of natural gas for the maintenance of mobile equipment (e.g. for welding). Other burdens in the mine are associated with the production of fuel, natural gas and soda ash.

5.2.7.1.1 Blasting and Transport

The detailed data on environmental burdens arising from the Blasting and transport subsystem are shown in Appendix 5, Table A5.1. In Figs. 5.4 - 5.5 the Inventory results are aggregated to show resource requirements and emissions from the five main activities identified in this subsystem: Explosive manufacture and transport to Borax, Drilling, Blasting, Stripping, and Mine transport. Analysis of the figures shows that the main contributors to resource depletion and emissions are the Explosive and Mine transport subsystems, mainly associated with production of the explosive and the fuel. Drilling and Stripping activities are responsible for a smaller proportion of the environmental burdens, generated mostly by the production of electricity used in these processes.
5.2.7.1.2 Primary Crusher

The environmental burdens from the Primary crusher area come from the Primary crusher, Kernite hydration and Soda ash production and transport subsystems. Inspection of the results in Table A5.2 and Figs. 5.6-5.7 shows, however, that the burdens are mostly associated with the Soda ash production and transport to Borax. Some environmental burdens arise during the production of electricity used in the Primary crusher, while a considerably smaller proportion is a result of ore handling and crushing in the primary crusher and during kernite hydration.

5.2.7.1.3 Total Mining

Total environmental burdens from the Mine are shown in Table A5.3 and Figs. 5.8-5.9. In order to preserve as much transparency as possible, the results are shown for the four main subsystems: Blasting, Mine transport, Primary crusher and Other activities. The materials and activities included in these subsystems are listed below.

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Includes the environmental burdens of:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2 Blasting:</td>
<td>Explosive manufacture and transport to Borax; Drilling; Electricity production for Drilling; Blasting; Stripping; Electricity production for Stripping.</td>
</tr>
<tr>
<td>1.4 Mine transport:</td>
<td>Diesel fuel production, transport to Borax and fuel use.</td>
</tr>
<tr>
<td>1.5 Primary crusher:</td>
<td>Primary crusher; Electricity production for Primary crusher; Kernite hydration; Blending (ore).</td>
</tr>
<tr>
<td>Other activities:</td>
<td>Water for dust suppression; Natural gas for the truck shop; Natural gas production; Other fuel (petrol) production and use.</td>
</tr>
</tbody>
</table>

On average, the Primary crusher subsystem shows the highest contribution to resource consumption, in particular coal, other non-renewables, renewable resources, and ancillaries. Most of these are attributed to soda ash production, with the exception of other non-renewables which mainly represent depletion of borax and kernite ore in the mining process (here assigned to the Primary crusher subsystem). Blasting is the second largest contributor to primary resource usage, mainly because of the explosive and electricity production. Mine transport shows a significant depletion of oil reserves (70%) in comparison to other subsystems, with minor contributions to coal, renewable resources, water and air consumption. It is less obvious which process is the most responsible for the emissions, although it appears that the Primary crusher is the largest source of the emissions. The contribution of Mine transport is also more significant than in the case of resource requirements, in particular with respect to emissions of CO, NOx, SO2, TDS and Oil and greases.
Fig. 5.4 Blasting and transport: Comparison of resource requirements

Fig. 5.5 Blasting and transport: Comparison of emissions to air, water and land
Fig. 5.6 Primary crusher:
Comparison of resource requirements

Fig. 5.7 Primary crusher:
Comparison of emissions to air, water and land
Fig. 5.8 Total mining:
Comparison of resource requirements

Fig. 5.9 Total mining:
Comparison of emissions to air, water and land
5.2.7.2 Secondary Crusher

There are three main activities within this subsystem, associated with Ore stockpile, Secondary crusher, and Ore bins (Fig. A5.4). Their corresponding environmental burdens are shown in Table A5.4 and Figs. 5.10-5.11. The environmental burdens associated with the Secondary crusher area arise mostly from electricity production. A small amount of dust emissions is a result of ore handling and crushing in this area.

---

Fig. 5.10 Secondary crusher:  
Comparison of resource requirements

---

Fig. 5.11 Secondary crusher:  
Comparison of emissions to air, water and land
5.2.7.3 Primary Process

The Primary process (Fig. A5.5) is the main user of energy in the foreground system; electricity, natural gas and the steam are all responsible for the environmental burdens in this subsystem. Some air emissions are also created by product handling in the Dryers area.

The Primary process is divided into six subsystems: Dissolvers, Thickeners, 5Mol, 10Mol and Cold 10Mol Crystallisers, and Product dryers areas (Figs. A5.6-11). The Product dryers area is further broken down into 5Mol - Rotary dryer, 5Mol (Penta) - Fluid Bed (FB) dryer and 10Mol - Wysmont (tray) dryer. The burdens for all these subsystems are listed in Table A5.5. For the graphical presentation, however, the results are aggregated into a smaller number of subsystems, as given in Figs. 5.12-5.13. Almost all the data were available in a disaggregated form, with the exception of the electricity consumption in the Crystallisers and Dryers area. These were reported together as "Granulation" and are listed here under the Dryers area. The environmental burdens shown in the figures include the subsystems and activities listed below.

The greatest consumption of primary resources in the Primary process is associated with the Rotary dryer area, which contributes approximately 50% to this category. The main reason for this is high electricity and natural gas consumption and the burdens related to their productions. The Thickeners, Fluid bed and Wysmont dryer areas consume the other 50% of the resources, the exception being water usage where the Dissolvers play a more significant role (35%). The situation is similar with the emissions; however, the Other activities account for all emissions of metals to air and water, waste water, TDS and TSS.

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Include environmental burdens of:</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2.1 Dissolvers</td>
<td>Dissolvers area; Electricity production.</td>
</tr>
<tr>
<td>2.2.2 Thickeners</td>
<td>Thickeners area; Electricity production.</td>
</tr>
<tr>
<td>2.2.6.1 5Mol- Rotary Dryer</td>
<td>Rotary dryer area; Crystallisers area, Electricity and Natural Gas production; Gas use.</td>
</tr>
<tr>
<td>2.2.6.2 5Mol- Fluid Bed Dryer (FB)</td>
<td>FB dryer area; Electricity and Natural Gas production; Gas use.</td>
</tr>
<tr>
<td>2.2.6.3 10Mol- Wysmont Dryer</td>
<td>Wysmont dryer area; Electricity and Natural Gas production, Gas use.</td>
</tr>
<tr>
<td>Other</td>
<td>Water and Electricity that could not be assigned to any of the subsystems; Cooling towers; Discharges to ponds.</td>
</tr>
</tbody>
</table>
Chapter 5

2.2.1. Dissolvers
2.2.2. Thickeners
2.2.6.1. 5Mol - Rotary Dryer
2.2.6.2. 5Mol - FB Dryer
2.2.6.3. 10Mol - FB Dryer
Other

Fig. 5.12 Primary process:
Comparison of resource requirements

Fig. 5.13 Primary process:
Comparison of emissions to air, water and land
5.2.7.4 Boric Acid Plant

Flow diagrams for the Boric acid (BA) plant are shown in Figs. A5.12-A5.19. Similar to the Primary process, environmental burdens from the Boric acid plant stem from the use of electricity, gas and steam. Additional burdens are generated by the ore and product handling in the Reactor and Dryers area. Production of sulphuric acid, as well as fuel and gas, add to the total emissions associated with this part of the foreground system.

The environmental data for BA were available for the Reactor and Dryers area only; all other data are reported as aggregated and listed under Other activities (Table A5.6), which include electricity and water consumption for the whole plant and various emissions to water and air. As shown in Figs. 5.14-5.15, sulphuric acid and electricity production are the main sources of resource depletion and emissions to air. Emissions to water, however, are generated in almost equal proportions by these two and the Other activities.

5.2.7.5 Anhydrous Boric Acid Plant

A flow diagram of the ABA plant is shown in Fig. A5.20. The Inventory results are listed in Table A5.7 and shown graphically in Figs. 5.16-5.17. The only burden associated with the Feed bin and Hammer mills and Separating bins subsystems is dust; Total furnace includes emissions from natural gas combustion and its production. The burdens that were reported in the aggregated form for the whole plant are listed under Other activities. Since the ABA production is an energy intensive process, it is not surprising that the main sources of environmental burdens in this subsystem are the gas and electricity consumption. A proportion of the dust is also generated from the material handling in the feed bin and hammer mills.

5.2.7.6 Anhydrous Borax Plant

The main sources of environmental burdens associated with the Anhydrous borax (AB) plant are also electricity and gas consumption and their production. In fact, since the AB and ABA processes are almost identical (Fig. A5.21), the contributions of electricity and natural gas usage to the total burdens from the AB are very similar to those from the ABA plant (see Table A5.8 and Figs. 5.18-5.19). The only difference between AB and ABA is that the main source of the dust from the AB plant is the Hammer mills subsystem.
Chapter 5

Fig. 5.14 Boric acid:
Comparison of resource requirements

Fig. 5.15 Boric acid:
Comparison of emissions to air, water and land
Fig. 5.16 Anhydrous boric acid: Comparison of resource requirements

Fig. 5.17 Anhydrous boric acid: Comparison of emissions to air, water and land
Chapter 5

5.1 Furnace feed  5.2 Total Furnace  5.5 Hammer Mills

bin

Elec for AB  Other

Fig. 5.18 Anhydrous borax:
Comparison of resource requirements

Fig. 5.19 Anhydrous borax:
Comparison of emissions to air, water and land

101
5.2.7.7 Packing and Shipping

The only energy used in the Packing and shipping process is electricity, so that related burdens derive from its production. In addition, some dust is generated during packing of the products. However, most environmental burdens in this subsystem are related to the production of packaging. The environmental burdens of different packaging used in the system are compared in Figs. A5.33-A5.34. The basis for comparison is the amount of packaging needed for 1000 kg of a product. It is apparent from the figures that the highest burdens are associated with the paper bags for ABA, with 25kg and 50lb bags for ABA following closely. The environmentally most favourable packaging in the system are the PP bags.

5.2.7.7.1 Refinery Packing and Shipping

This subsystem comprises packing and shipping of 5Mol, 10Mol, AB and ABA products (Fig. A5.22). Most environmental burdens stem from production of the packaging, and some are attributed to the on-site electricity consumption and material handling. The environmental burdens for all these subsystems are listed in Table A5.9; Figs. 5.20-5.21 illustrate their contributions to the total resource depletion and emissions. 10Mol and 5Mol packing and shipping are responsible for 40% and 50% of the burdens respectively, the exception being Electricity production, which contributes the most to the gas reserves depletion and emissions of NOx, VOC, and metals to water.

5.2.7.7.2 Boric Acid Packing and Shipping

The data on environmental burdens from the packing and shipping of BA, except for the dust generation during the material handling, were not available in a disaggregated form; they are accounted for in the BA Plant. Therefore, all the burdens shown in Table A5.9 and Figs. 5.22-5.23 are a result of the packaging production. Only about 20% of the dust is emitted from the Day bins and the Silos.

5.2.7.8 Steam Plant

This subsystem is related to the environmental burdens associated with the steam production in the Steam plant (Fig. A5.24). The analysis of the results in Table A5.10 and Figs. 5.24-5.25 shows that most environmental burdens in this system result from the production of Natural gas, with some contribution from Electricity production. Emissions of CO2 and dust are mainly a result of the use of natural gas for steam generation.
Fig. 5.20 Refinery packing and shipping: Comparison of resource requirements

Fig. 5.21 Refinery packing and shipping: Comparison of emissions to air, water and land
Fig. 5.22 Boric acid packing and shipping: Comparison of resource requirements

Fig. 5.23 Boric acid packing and shipping: Comparison of emissions to air, water and land
Chapter 5

Fig. 5.24 Steam plant:
Comparison of resource requirements

Fig. 5.25 Steam plant:
Comparison of emissions to air, water and land

[Graphs showing resource and emission comparisons]
5.2.7.9 Cogeneration Plant

The Cogeneration plant is divided into two subsystems: Cogeneration of electricity with steam, and Steam cogeneration (Fig. A5.25). The former is related to the electricity used by Borax and the steam cogenerated with it; the latter corresponds to the steam that is cogenerated with the electricity exported from the system. Since the electricity exported to other users is not part of this system and all cogenerated steam is used by Borax, the burdens have to be allocated between these two co-products. In this case, the "avoided burdens" allocation approach has been used (see Chapter 6). The comparison of the contributions of these two subsystems, as well as the Gas production, to the total environmental burdens is illustrated in Figs. 5.26-5.27 and Table A5.11. The Natural gas production is responsible for the primary resource consumption, except for the water and air, used for the cogeneration of electricity and steam. Emissions of CO$_2$, Dust and Waste water are generated by the same source; all other emissions result mainly from the Natural gas production. The contribution of the Cogenerated steam to the total emissions is negligible.

5.2.7.10 Total Environmental Burdens

The total environmental burdens of the whole product system are listed in Table A5.12. Comparison of the contribution of different subsystems to the total burdens is illustrated in Figs. 5.28-5.29. All subsystems considered are the same as discussed above; the only difference is the Total steam production in the system, which in addition to steam produced by the Steam plant, includes the Steam cogeneration, as defined in the preceding section. The reason for including the Steam cogeneration in the Total steam production is that the electricity use and production have already been accounted for in all subsystems, so that Cogeneration plant is not shown as a separate system in this study.

Primary process and Steam production are, in total, responsible for 80% of nuclear and hydro-electricity and gas and water consumption. The BA plant consumes around 60% of the coal reserves in the system, almost all of which is used in the life cycle of sulphuric acid. Packaging and shipping are the main users of renewable resources (paper bags), while oil reserves and other non-renewables (i.e. borax and kernite ore) are used in the Mining operations. All other subsystems have considerably smaller shares in the total resource consumption.
Chapter 5

Fig. 5.26 Cogeneration:
Comparison of resource requirements

Fig. 5.27 Cogeneration:
Comparison of emissions to air, water and land
Chapter 5

1. Mining
2.1 Secondary Crushed
2.2 Primary Process
3. Boric Acid
4. Anhydrous Boric Acid
5. Anhydrous Borax
6. Packaging & Shipping
7. Total Steam Production

Fig. 5.28 Total environmental burdens:
Comparison of resource requirements

Fig. 5.29 Total environmental burdens:
Comparison of emissions to air, water and land
Chapter 5

The Mining activities are the main source of emissions of metals and dust to air, Landfill volume and weight. The SO₂ emissions and other emissions to air, TSS and TDS are a result of the BA production. All these burdens are generated in the foreground system, with the exception of SO₂ which is a part of the sulphuric acid life cycle. Most of the remaining air emissions, i.e. CO, CO₂, NOₓ and VOC, come from the Total steam production, while the Primary process is the main source of the halides and emissions of waste water and metals to water. The COD and BOD are attributed to the Packaging production, because of the high water consumption in this process.

5.3. Impact Assessment

Impact Assessment undertaken in this study includes the results of Classification, Characterisation and Normalisation steps only. Valuation has not been carried out in this study because it is argued that this stage should be avoided if possible. Instead, it is proposed to use the Pareto approach, discussed in Chapter 4 and illustrated for the boron case study in Chapter 7.

5.3.1 Classification and Characterisation

The environmental burdens identified and quantified in the Inventory stage are aggregated into impact categories following the Problem oriented approach; the Medium oriented approach (Critical volumes) is not considered in this study because of the lack of classification factors for some impact categories. The categories of the Problem oriented approach considered in this study are Abiotic reserve depletion, Global warming potential (direct and indirect), Ozone depletion, Acidification, Nutrification, Photochemical oxidants creation potential (or Photochemical smog), Human toxicity, Aquatic ecotoxicity and Landfill volume. The impacts are calculated following the most widely adopted methodology, proposed by Heijungs et al. (1992). The Classification factors are listed in Table A5.15 in Appendix 5.

The total environmental impacts of the borate products system, calculated from the Inventory Analysis, are listed in Table A5.13 and shown graphically in Fig. 5.30. The Abiotic reserve depletion, mainly related to the depletion of Na₂B₄O₇ reserves, is a result of the Mining operations. Global warming potential (direct and indirect), as well as Ozone depletion and Nutrification are contributed to by the Primary process and Steam production in almost the same proportions (30% each), with BA share of 20% and AB,
Chapter 5

ABA, and Packaging and shipping amounting to the remaining 20%. Furthermore, the responsibility of the BA for Acidification is higher (35%) than that of the Primary process and Steam production (25% each). The Photochemical smog (mostly VOCs) is generated mainly by the Steam production (70%); a small proportion is related to the Primary process and the BA. Human toxicity arises from the Mine, Primary process, BA and Steam production.

Aquatic toxicity is, on the other hand, almost entirely the responsibility of the Primary process, with a minute contribution from BA production. However, it should be noted that Aquatic toxicity is not considered to be an impact here, because 99.9% of it represents arsenic in the waste water, discharged into the self-contained ponds. The ponds are either lined and covered or are scheduled for reclamion. In either case, the waste water does not leave the system and the risk of leaching into the ground water is minimum. To ensure that this is the case, the quality of the ground water around the ponds is being continuously monitored and supervised by the EPA and so far no accidents were reported.

The Landfill volume is mostly generated in the Mine, i.e. 90% of that is overburden. The remaining 9% is gangue from the Primary process and the BA, while approximately 1% comes from the other life cycles in the system. Although stripping of over 10 million tons of the overburden a year represents a significant disturbance that will continue in the future, it is worth mentioning that the California Surface Mine and Reclamation Act (SMARA) of 1976 requires surface mines to reclaim all post-act disturbance to an appearance similar to the nearby natural setting (Dames and Moore, 1994). In accordance with this requirement, the reclamation plan for the overburden area has already been prepared (Dames and Moore, 1994) and the activities required for revegetation have already started. It is estimated that it will take several years before the conditions are restored to the original ones. This temporary loss of aesthetic landscape has not been included as an impact. However, because it is important to show all material flows in the system, it has been included in the Inventory Analysis as discussed in Section 5.2.7.10.

5.3.2 Normalisation

Since LCA is global in nature, it is sometimes argued that the total environmental impacts should be normalised to the total world impacts during a certain period. These results, which represent impacts from the whole system normalised to the annual world
impacts, are shown in Table A5.14 and Fig. 5.31. Landfill volume has not been included as an impact in the Normalisation stage, because of the lack of data on this impact world-wide. Contribution to the annual creation of the Photochemical smog is 0.003% and the increase of the annual fossil fuel depletion and acidification caused by this system represents 0.0017% and 0.0015% respectively. Other impacts contribute, on average, \(8\times10^{-4}\) percent to the total world impacts in one year. It should be pointed out, however, that these results should be interpreted with caution because of the lack of reliable data on total world annual environmental impacts. The Normalisation effect scores are just an indication of what a potential global impact of the system might be, and no final conclusions should be based on it.

![Fig. 5.30 Total environmental impacts of the boron products system](image_url)

![Fig. 5.31 Environmental impacts normalised to the world annual impacts](image_url)
5.4 Concluding Remarks

The LCA study of the boron product system has been carried out and its results are presented in this chapter. The considerations here included identifying and quantifying the environmental burdens and impacts as part of the Inventory and Impact Assessment stages.

In the Inventory stage, it has been found that Primary process and Steam production are, in total, responsible for 80% of nuclear and hydro-electricity and gas and water consumption. The BA Plant accounts for around 60% of the coal usage associated with the system, almost all of which is used in the life cycle of sulphuric acid. Packaging and shipping are the main users of renewable resources (paper bags), while oil reserves and other non-renewables (i.e. borax and kernite ore) are used in the Mining operations. All other subsystems have considerably smaller shares in the total resource consumption.

The Mining activities are the main source of emissions of metals and dust to air, Landfill volume and weight. The $\text{SO}_2$ emissions and other emissions to air, TSS and TDS are a result of the BA production. Most of other air emissions, i.e. $\text{CO}$, $\text{CO}_2$, $\text{NO}_x$ and $\text{VOC}$, come from the Steam production, while the Primary process is the main source of the halides and emissions of waste water and metals to water. The COD and BOD are attributed to the Packaging production, because of the high water consumption in this process.

The results of the Impact Assessment stage reveal that Abiotic reserve depletion, mainly related to depletion of $\text{Na}_2\text{B}_4\text{O}_7$ reserves, is a result of the Mining operations. Global warming potential, as well as Ozone depletion and Nutrification arise from the Primary process and Steam production in almost the same proportions (30% each), with BA contributing 20% and AB, ABA, and Packaging and shipping accounting to the remaining 20%. Furthermore, the responsibility of the BA for Acidification is higher (35%) than that of the Primary process and Steam production (25% each). Photochemical smog (mostly VOCs) is generated mainly by the Steam production (70%); a small proportion is related to the Primary process and the BA. Human toxicity arises from the Mine, Primary process, BA and Steam production.

The analysis and discussion in this chapter has concentrated on the comparison of environmental burdens and impacts of the processes in the boron products system. At this stage there is no indication as to which products are the most responsible for the resource depletion and the emissions. This will we will find out in the following chapter.
CHAPTER 6

ALLOCATION OF ENVIRONMENTAL BURDENS
IN THE BORON PRODUCTS SYSTEM

In the preceding chapter, the results of LCA of the boron products system were presented to show the contribution of different subsystems to the total environmental burdens and impacts of the whole system. This chapter illustrates how these burdens and impacts can be allocated to the co-products in the boron system by using Linear Programming (LP) and the marginal allocation approach. Furthermore, it illustrates the point made in Chapter 3 that the values of allocated burdens and impacts are not fixed—they change depending on the way the system is operated. To illustrate this, an example of how the allocated burdens change from product-related to process-related burdens is presented. It is argued that allocation through whole system modelling provides a more realistic approach to system analysis by being able to account for changes in the operating modes of systems and the corresponding changes in the environmental burdens. This is generally not possible with the other allocation methods, also examined in this chapter. In addition, allocation in the cogeneration of steam and electricity, as a special case of a multiple-function system within the boron products system, is also discussed.

6.1 Allocation in the Boron Products System

Because the method used for allocation of the burdens will, in general, depend on the goal of the study, different approaches to allocation in a co-product system are possible. In general, the goal of the study could be a comparison of a product with the same or an alternative product produced in a different system. A goal of the study could also be the evaluation of environmental performance of the co-products in the system either in order to compare their performance and identify possibilities for improvements, or to provide LCA data for the co-products which are subsequently used in different productive systems. As discussed in Chapter 3, depending on the goal, allocation can then either be avoided by system enlargement or disaggregation or it can be solved by applying appropriate natural causation principles or economic relationships. These different allocation methods are discussed here in the context of the boron products system and their relevance with respect to the possible goals of the study is examined accordingly. Given a real goal of the study in this work, particular attention is directed to allocation
on the basis of natural causality, established by linear programming and marginal analysis. To illustrate different approaches to allocation and their implications, the discussion is focused on allocation in the foreground system only while allocation in the background system is not considered. For further explanation and definition of the foreground and background systems, see Chapter 5 and Appendix 5.

6.1.1 Avoiding Allocation by Expanding System Boundaries

If the goal of the study is comparison of one of the co-products from the boron system (System I in Fig. 6.1), for example 5Mol borate, with 5Mol (or an alternative product) produced in another, single-output system (System II), then allocation can be avoided in two ways. System II can be enlarged so that an alternative way of producing the rest of the products from System I, i.e. 10Mol borate, Boric Acid (BA), Anhydrous Borax (AB) and Anhydrous Boric Acid (ABA), is added to System II. The comparison is now between System I, here represented by the boron system, and System II + III.

While this approach to allocation, or rather avoiding allocation, is perhaps feasible for smaller systems producing at most two co-products, it becomes extremely cumbersome and impractical in the case of more complicated systems, such as the one considered in this example. In many cases this approach will mean gathering a significant amount of additional data; in some cases, data on alternative production of the co-products will not be available, or there will be few or none alternative processes. Therefore, in cases similar to the one presented here this approach to allocation is not useful.
Furthermore, this method does not provide data on the contribution of individual co-products to the total environmental burdens and impacts; it only enables comparison of one system with other productive systems. Therefore, if the goal of the study is to compare performance of the co-products in the system or to provide LCA data for systems in which the co-products are used as raw materials, avoiding allocation by expanding system boundaries is not relevant.

Another way to enlarge the system boundaries is to subtract burdens arising from the alternative way of producing the co-products, i.e. 10Mol, BA, AB, and ABA in System III from those in System I. In that case only 5Mol is compared to 5Mol in System II, as illustrated in Fig. 6.2. The latter approach is known as the "avoided burdens" or "avoided impacts" approach (see Chapter 3). This method is in principle similar to the one discussed above, so that for the boron system it offers no particular advantage for the same reason: it is difficult and impractical to collect the data for additional products produced in an alternative way. However, this approach can be useful in cases where the production of co-products in one system displaces their production elsewhere, so that the environmental burdens that would otherwise arise from this production are avoided. This is a common case in systems for co-generation of electricity and heat, where co-production of heat displaces the need for burning fuels to produce that amount of heat in another system. The "avoided burdens" approach is illustrated later in this chapter on the example of the Cogeneration plant.

Fig. 6.2 Avoiding allocation by the "avoided burdens" approach
6.1.2 Avoiding Allocation by System Disaggregation

One of the possible ways to avoid allocation, as outlined in the four-step procedure in Chapter 3, is to disaggregate a multiple-function system into a number of subprocesses and split-off subprocesses of the system that operate for one of its functions only. For instance, the boron system can be disaggregated into subprocesses that are dedicated for production of BA, AB and ABA only. However, the production of 5Mol and 10Mol is more difficult to disaggregate because their production lines are coupled from early in the process. Therefore, disaggregation cannot avoid allocation completely because there will always be processes which are common for all functions and some kind of allocation will still be necessary.

It is important to emphasise that avoiding allocation by system disaggregation is only possible in a system for which detailed process data are available. If that is the case, then this approach should be used in order to minimise the allocation problem, regardless of the goal of the study. The importance of system disaggregation for allocation is demonstrated in Section 6.4.

6.1.3 Allocation on the Basis of Natural Causality

If it is not possible to avoid allocation, then allocation must be based on the natural causality principle, if such a principle can be applied to the system. This means that if there are natural causal relationships between the parameters related to the functional units and the environmental burdens, then it must be possible to change the system parameters independently, i.e. one parameter can be changed while other parameters are kept constant. Depending on which parameter change causes a change in the burdens or impacts in a multiple-function system, the burdens are said to be either product- (or material-) or process-related. Furthermore, the changes considered in the system, which can either be marginal, average or discrete, depend in general on the goal of the LCA study and questions to be answered by LCA (see Chapter 3). Marginal changes are relevant when the performance of a specific process in the foreground system is analysed and when changes around the system of interest are incremental. However, if the goal of the study is to consider average or discrete changes of a specific or average technology, then marginal analysis is not relevant and the changes should be analysed by appropriate system modelling, which may not necessarily be linear. This study is concerned with the marginal changes around an existing state of a specific system in which natural causality exists. Therefore, the necessary conditions for using the marginal analysis, which is an
integral part of LP, for allocating the burdens in the multiple-function system are satisfied. Prior to presenting and discussing the marginal allocation results, the LP model of the boron system is defined and the main principles of marginal analysis are revisited.

6.2 Linear Programming Model of the Boron Products System

The LP model of the boron products system is composed of six submodels, which correspond to the subsystems defined in Chapter 5: Mine and Secondary crusher, Primary process, Boric acid, Anhydrous boric acid, Anhydrous borax, and Packing and shipping. The unit operations of the Cogeneration and Steam plant subsystems are not modelled; however their environmental burdens are included in the model. The constraints of the model are defined by the material balances, market demand, primary and raw materials availability, productive capacities and heat requirements in the system:

Mass balance constraints: \[ \sum_{j=1}^{J} a_{j,i} x_{i} = 0, \quad j=1,2,...,J \] (6.1)

Market demand constraints: \[ P_{l} \leq D_{l}, \quad l=1,2,...,L \] (6.2)

Primary material availability: \[ R_{m} \leq S_{m}, \quad m=1,2,...,M \] (6.3)

Productive capacity constraints: \[ x_{i} \leq C_{i}, \quad i=1,2,...,I \] (6.4)

Heat requirements: \[ H_{z} \leq Q_{z}, \quad z=1,2,...,Z \] (6.5)

where production \( P_{l} \) is limited by the product demand \( D_{l} \), primary and raw materials consumption \( R_{m} \) is determined by the supply \( S_{m} \), activities or processes \( x_{i} \) are subject to the capacity limit \( C_{i} \) and the heat requirement \( H_{z} \) is constrained by the heat availability \( Q_{z} \). Since the discussion in this chapter is related to the functional unit defined as the operation of the system for one year, the product demand \( D_{l} \) is taken to be equal to the total output of each product for one year (see Chapter 5). The objective functions of the system are defined as environmental burdens and impacts, respectively:

Minimise \[ B_{j} = \sum_{i=1}^{I} b_{c,j,i} x_{i}, \quad j=1,2,...,J \] (6.6)

Minimise \[ E_{k} = \sum_{j=1}^{J} e_{c,k,j} B_{j}, \quad k=1,2,...,K \] (6.7)
where coefficients $b_{cj,i}$ represent the resource depletion and emissions from activity $x_i$ and $e_{cij}$ is a coefficient that relates burdens to the impacts, as defined by the "problem oriented" approach (Heijungs et al., 1992). A more detailed description and the printout of the LP model are given in Appendix 6. A large scale LP software, XPRESSMIP (Dash Associates, 1993), has been used to solve the system model and to calculate marginal values at the model solution.

6.3 LP and the Marginal Allocation Approach Revisited

In allocation on the marginal basis, the burdens and impacts are allocated among different system parameters through the marginal allocation coefficients. It has been explained in Chapter 3 that these coefficients are equal to the marginal or dual values obtained when the objective functions of an LP model are defined as the environmental burdens and impacts and the model is solved for each of these objectives. It is important to note here that the model is not optimised at this stage; it is only solved for each environmental objective in order to calculate the total burdens and impacts and their corresponding marginal values. At the LP solution, the burdens and impacts are related to the marginal values by:

$$B_j = \sum_{j=1}^{J} \lambda_j e_j$$  (3.9)

or

$$E_k = \sum_{j=1}^{J} \lambda_j e_j$$  (3.15)

where $\lambda_j$ is a marginal value that relates change in the environmental objective function, $B_j$ or $E_k$, due to a change in one system parameter $e_j$, while all other parameters are held constant:

$$\lambda_j = \left( \frac{\partial B_j}{\partial e_j} \right)_{e_1,\ldots,e_{j-1},e_{j+1},\ldots,e_J}$$  (6.8)

or

$$\lambda_j = \left( \frac{\partial E_j}{\partial e_j} \right)_{e_1,\ldots,e_{j-1},e_{j+1},\ldots,e_J}$$  (6.9)
Therefore, the marginal values obtained by whole system modelling by LP embody the natural causal relationships.

In a co-product system the parameter $e_j$, and therefore the allocated burdens and impacts $\lambda_j$, can either be product- or process-related, depending on which parameter change causes a change in the burdens or impacts. Since changes considered in the system depend in general on the goal of the study, and the allocated burdens are dependent on the type of changes analysed, it follows that the allocated burdens will also depend on the goal of the study. If the goal is to analyse changes in the burdens and impacts due to a change in a product output, then the burdens are allocated among the products and are considered to be product-related. For the boron products system described by eqns. (6.1)-(6.7), this in LP terms means that eqns. (6.2) are active constraints. From the discussion in Chapter 2, it is clear that their marginal values and, therefore, the product-related allocated burdens and impacts are non-zero. In that case the operation of the system is constrained solely by the product outputs; the availability of raw and primary materials is unlimited and the capacities of the operating units do not constrain the production. Therefore, constraints related to the process, i.e. eqns. (6.3)-(6.5), are non-active and the process-related burdens and impacts are zero.

For the product-related burdens and impacts, each system parameter $e_j$ is defined by the output of product $D_i$ (eqn. (6.2)), so that the allocated burdens and impacts are equal to:

$$\lambda_j = b_{j,1} = \left(\frac{\partial B_j}{\partial D_1}\right)_{D_{i,1}=D_{i,2}=D_{i,3}=\ldots=D_{i,L}}$$  \hspace{1cm} (6.10)

or

$$\lambda_j = e_{i,k,1} = \left(\frac{\partial E_k}{\partial D_1}\right)_{D_{i,1}=D_{i,2}=D_{i,3}=\ldots=D_{i,L}}$$  \hspace{1cm} (6.11)

Total burdens and impacts are then given by:

$$B_j = \sum_{i=1}^{L} b_{j,i} P_i$$  \hspace{1cm} (6.12)

and

$$E_k = \sum_{i=1}^{L} e_{i,k,1} P_i$$  \hspace{1cm} (6.13)

Thus, the burdens and impacts related to the activities $x_i$, as defined by eqns. (6.6) and (6.7), have been translated into the burdens related to the products $P_i$. It is then possible
to find out how much each product in this multi-output system contributes to each 
environmental burden and impact. These results are presented in Section 6.4.1.

However, if the goal of the study is to consider the effects of changes in the process-
related parameters, then the burdens and impacts are allocated among these parameters 
and treated as process-related. In that case, some or all of the process constraints (6.3)-
(6.5) become active, so that the process-related parameter $e_j$ is defined by the material 
availability $S_m$, capacity limit $C_i$ or heat availability $Q_z$. The allocated burdens are then 
equal to:

$$\lambda_j = b_{j,m} = \left( \frac{\partial B_j}{\partial S_m} \right)_{S_1,\ldots,S_m-1,S_{m+1},\ldots,S_M}$$  \hspace{1cm} (6.14)$$
or

$$\lambda_j = b_{j,i} = \left( \frac{\partial B_j}{\partial C_i} \right)_{C_1,\ldots,C_{i-1},C_{i+1},\ldots,C_i}$$  \hspace{1cm} (6.15)$$
or

$$\lambda_j = b_{j,z} = \left( \frac{\partial B_j}{\partial Q_z} \right)_{Q_1,\ldots,Q_{z-1},Q_{z+1},\ldots,Q_z}$$  \hspace{1cm} (6.16)$$

where some or all marginal burdens can be non-zero, depending on which constraints are 
active. The total burdens are then equal to:

$$B_j = \sum_{m=1}^{M} b_{j,m} S_m + \sum_{i=1}^{I} b_{j,i} C_i + \sum_{z=1}^{Z} b_{j,z} Q_z$$  \hspace{1cm} (6.17)$$

Similarly, the allocated impacts are equal to:

$$\lambda_j = e_{j,m} = \left( \frac{\partial E_k}{\partial S_m} \right)_{S_1,\ldots,S_m-1,S_{m+1},\ldots,S_M}$$  \hspace{1cm} (6.18)$$
or

$$\lambda_j = e_{j,i} = \left( \frac{\partial E_k}{\partial C_i} \right)_{C_1,\ldots,C_{i-1},C_{i+1},\ldots,C_i}$$  \hspace{1cm} (6.19)$$
or

$$\lambda_j = e_{j,z} = \left( \frac{\partial E_k}{\partial Q_z} \right)_{Q_1,\ldots,Q_{z-1},Q_{z+1},\ldots,Q_z}$$  \hspace{1cm} (6.20)$$

and the total impacts are defined by:
Finally, in a study where changes in both parameters are of interest, the allocated burdens are considered to be both product- and process-related, and the total burdens and impacts are given by:

\[ B_j = \sum_{l=1}^{L} b_{j,l} P_l + \sum_{m=1}^{M} b_{j,m} S_m + \sum_{i=1}^{I} b_{j,i} C_i + \sum_{z=1}^{Z} b_{j,z} Q_z \quad (6.22) \]

and

\[ E_k = \sum_{l=1}^{L} e_{k,l} P_l + \sum_{m=1}^{M} e_{k,m} S_m + \sum_{i=1}^{I} e_{k,i} C_i + \sum_{z=1}^{Z} e_{k,z} Q_z \quad (6.23) \]

These different cases of marginal allocation in the boron products system are presented and discussed in the following sections.

### 6.4 The Results of Marginal Allocation in the Boron Products System

The results of marginal allocation of the burdens and impacts in the boron products system are summarised in Figs. 6.3 and 6.4, respectively. They correspond to the functional unit defined as "operation of the system for one year". However, it is interesting to note that the allocated marginal burdens remain the same if the functional unit is defined as "1000 kg of each product" or "1000 kg of B₂O₃ equivalent" (for definitions of the functional units, see Chapter 5). The only difference is obviously related to the total burdens and impact from the system, due to a different output from the system as defined by the functional unit.

For further discussion in this chapter, only several representative examples of the burdens and impacts are considered. The results of the marginal approach are also compared with allocation on an arbitrary basis, such as mass or financial value. The remaining results of marginal allocation of environmental burdens and impacts can be found in Appendix 6 (Figs. A6.1-A6.28).

In order to show the environmental burdens and impacts associated with the packaging, boron products, i.e. 10Mol, 5Mol, BA, AB, ABA, have been further classified into nineteen different product categories depending on the type of packaging and they are listed in Table 6.1.
Chapter 6

Fig. 6.3 Environmental burdens in the boron products system allocated by the marginal approach

Fig. 6.4 Environmental impacts of the boron products system allocated by the marginal approach
# Table 6.1 Product categories with respect to a different type of packaging

<table>
<thead>
<tr>
<th>Product</th>
<th>Production* (ton/yr)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>10mol-bulk</td>
<td>24000</td>
<td>10Mol, Wysmont dryer, bulk shipping</td>
</tr>
<tr>
<td>10mol-25kg&amp;50lb bags</td>
<td>26000</td>
<td>10Mol, Wysmont dryer, in 25kg and 50lb paper bags</td>
</tr>
<tr>
<td>10mol-50kg&amp;100lb bags</td>
<td>17000</td>
<td>10Mol, Wysmont dryer, in 50kg and 100lb paper bags</td>
</tr>
<tr>
<td>10mol-PP bags</td>
<td>14000</td>
<td>10Mol, Wysmont dryer, in PP bags</td>
</tr>
<tr>
<td><strong>TOTAL 10 Mol</strong></td>
<td><strong>81000</strong></td>
<td><strong>Total 10 Mol shipped</strong></td>
</tr>
<tr>
<td>Pentabulk</td>
<td>91000</td>
<td>5Mol Fluid Bed dryer, bulk shipping</td>
</tr>
<tr>
<td>5mol-bulk</td>
<td>650000</td>
<td>5Mol, Rotary dryer, bulk shipping</td>
</tr>
<tr>
<td>5mol-25kg&amp;50lb bags</td>
<td>27000</td>
<td>5Mol, Rotary dryer, packed in 25kg&amp;50lb paper bags</td>
</tr>
<tr>
<td>5mol-50kg&amp;100lb bags</td>
<td>32000</td>
<td>5Mol, Rotary dryer, packed in 50kg&amp;100lb paper bags</td>
</tr>
<tr>
<td>5mol-PP bags</td>
<td>10000</td>
<td>5Mol, Rotary dryer, packed in PP bags</td>
</tr>
<tr>
<td><strong>TOTAL 5 Mol</strong></td>
<td><strong>810000</strong></td>
<td><strong>Total 5Mol shipped</strong></td>
</tr>
<tr>
<td>BA-bulk</td>
<td>72000</td>
<td>BA shipped in bulk</td>
</tr>
<tr>
<td>BA-25kg&amp;50lb bags</td>
<td>26000</td>
<td>BA packed in 25kg and 50lb bags</td>
</tr>
<tr>
<td>BA-50kg&amp;100lb bags</td>
<td>12000</td>
<td>BA packed in 50kg and 100lb bags</td>
</tr>
<tr>
<td>BA-PP bags</td>
<td>40000</td>
<td>BA packed in PP bag</td>
</tr>
<tr>
<td><strong>TOTAL BA</strong></td>
<td><strong>150000</strong></td>
<td><strong>Total BA shipped</strong></td>
</tr>
<tr>
<td>AB-25kg&amp;50lb bags</td>
<td>2000</td>
<td>AB packed in 25kg and 50lb bags</td>
</tr>
<tr>
<td>AB-50kg&amp;100lb bags</td>
<td>9000</td>
<td>AB packed in 50kg and 100lb bags</td>
</tr>
<tr>
<td>AB-PP bags</td>
<td>5000</td>
<td>AB packed in PP bags</td>
</tr>
<tr>
<td><strong>TOTAL AB</strong></td>
<td><strong>16000</strong></td>
<td><strong>Total AB shipped</strong></td>
</tr>
<tr>
<td>ABA-25kg&amp;50lb bags</td>
<td>2000</td>
<td>ABA packed in 25kg and 50lb bags</td>
</tr>
<tr>
<td>ABA-50kg&amp;100lb bags</td>
<td>2000</td>
<td>ABA packed in 50kg and 100lb bags</td>
</tr>
<tr>
<td>ABA-PP bags</td>
<td>1000</td>
<td>ABA packed in PP bags</td>
</tr>
<tr>
<td><strong>TOTAL ABA</strong></td>
<td><strong>5000</strong></td>
<td><strong>Total ABA shipped</strong></td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>1062000</strong></td>
<td><strong>Total products shipped</strong></td>
</tr>
</tbody>
</table>

*Production figures shown in this table are not based on actual production or sales figures in any particular year, but represent a possible production scenario.

## 6.4.1 Product-related Burdens and Impacts

The results and discussion presented in this section are relevant if the goal of the study is to analyse changes in the burdens and impacts due to a change in the product-related parameters, here defined by the product outputs.

### 6.4.1.1 Carbon Dioxide

The results of marginal allocation of the CO₂ emissions among different products are shown in Fig. 6.5. As explained in Section 6.3, the marginal allocated burdens relate a marginal change of the output of one of these products to a change in the emission of CO₂, with other outputs held constant. For instance, an average marginal value of 0.176 t/t for 10Mol means that one ton increase in the production of 10Mol will cause a total increase of CO₂ emissions by 0.176 tons.
Figure 6.5 shows that ABA and AB account for most of the total CO₂ emissions of 252918 t/yr, with the average marginal values for the packed and bulk products equal to 1.942 and 0.980 t/t, respectively. The high emissions allocated to these two products result from the fact that they are produced in an energy intensive process (see Chapter 5). In addition, their environmental burdens include burdens associated with the production of BA and 5Mol used as respective feed in these processes. The CO₂ emissions allocated to other products are considerably smaller, and range from 0.414 t/t for BA to 0.206 t/t and 0.176 t/t for 5Mol and 10Mol, respectively. Among 5Mol products, Penta, produced in the Fluid bed dryer, has the highest allocated CO₂ emissions because of the higher energy consumption in this dryer in comparison to the Rotary dryer. If compared with regard to the packaging, products packed in polypropylene (PP) bags have the lowest allocated emissions of CO₂, and those packed in 25 kg and 50 lb paper bags, the highest.

Let us now compare the results of the marginal allocation approach with the arbitrary allocation methods most commonly used in co-product systems: mass and market value bases (Fig. 6.6). To illustrate the importance of system disaggregation, allocation on the mass basis for both aggregated and disaggregated systems is considered. If allocation on the mass basis is done without system disaggregation, the total burdens are allocated among different products in proportion to the mass of their outputs. In that case all products in the boron system would have the allocation factor of 0.238 t/t (total CO₂ emissions of 252918 t/yr divided by the total production of 1062000 t/yr). This
represents a total average difference of 49% from the emissions allocated by the marginal values, with the highest average difference of 85% for ABA. The reason is that, in allocation on the mass basis without system disaggregation, it is assumed that all processes for production of co-products are the same, which they obviously are not. These results illustrate the point that allocation done in this way does not reflect the natural causality but is carried out arbitrarily and, therefore, cannot allocate the burdens realistically in complex industrial multiple-function systems. A similar conclusion emerges if allocation is based on the $\text{B}_2\text{O}_3$ content in different products. Although it seems that the $\text{B}_2\text{O}_3$ allocation coefficients increase as the degree of processing of different products increases to give a higher boron content (Fig. 6.6), this increase is actually linearly related to the content of $\text{B}_2\text{O}_3$ in the products and not to the burdens associated with their processing.

However, the situation is quite different if allocation on the mass basis is done after the system has been disaggregated to take into account differences in the processes for producing different products. In that case allocation on marginal and mass bases gives the same results (Fig. 6.6). This means that in case of the CO$_2$ emissions, physical causation determines the behaviour of the system and allocation based on a physical quantity, i.e. mass, is relevant. Therefore, in some cases it is correct to allocate the burdens on the basis of a physical quantity; however, the point here is that the choice of allocation parameter is based on natural causation involved rather than on arbitrary choice of one physical quantity for allocation regardless of whether and what kind of causality exists in the system. A correct type of causality can only be identified if the system operation is well understood and detailed data on subprocesses in the system...
available. In whole system modelling by LP, the correct type of causality is identified by the model itself, so that the possibility of arbitrary approach to allocation is eliminated.

Allocation on the basis of market value (here taken to be gross selling price of the products, respectively: 10 Mol: $239.5; 5 Mol: $255.4; BA: $527.4; AB: $612.5; ABA: $1535.2) does not give satisfactory results in this case. Although the burdens allocated on the marginal and market value bases are quite similar for 10 Mol, 5 Mol and BA, the difference is much larger for AB and ABA (51% and 38%, respectively). This implies that the "external" costs of the environmental burdens are not proportional to the current economic values of these products. Therefore, allocation by financial value can give misleading results and should not be used in systems where natural causality exists.

6.4.1.2 Sulphur Dioxide

The emissions of SO₂ allocated among the products through marginal values are shown in Fig. 6.7. It is interesting to note that the relative contributions of different products to the total SO₂ emissions are quite different from those for CO₂. The highest emissions of 0.013 t/t are still allocated to ABA; however, the reason is now the emissions of SO₂ from the life cycle of BA rather than high energy consumption in the ABA process, as was the case for the CO₂ emissions. The main source of the SO₂ emissions of 6.03E⁻³ t/t allocated to BA is the life cycle of sulphuric acid used in the BA plant. 5 Mol and 10 Mol contribute to the total emissions by on average 3.56E⁻⁴ t/t and 3.08E⁻⁴ t/t, respectively, while emissions of SO₂ allocated to AB amount to 1.51E⁻³ t/t. About 50% of the SO₂ emissions allocated to the latter three products stem from the activities related to transport in the mine.

![Fig. 6.7 SO₂ emissions allocated by the marginal approach](image-url)
The comparison of SO\(_2\) emissions allocated by the marginal approach with the mass and market value approaches is shown in Fig. 6.8. If the burden is allocated arbitrarily on the mass basis without system disaggregation, the results differ from those obtained by the marginal analysis of natural causality by, on average, 70%. Similar results are obtained in allocation on the basis of B\(_2\)O\(_3\) content. However, if allocation on the mass basis is done by disaggregating the system, the allocated burdens are equal to the marginal values calculated by whole system modelling. Similar to the emissions of CO\(_2\), the type of causality involved is physical and mass emerges from that causality as a relevant allocation parameter.

If the burdens are allocated on the basis of market value, the average difference from the results obtained by the marginal approach is 55% (Fig. 6.8). We recall that in the case of CO\(_2\) emissions, the source of which was mainly the energy consumption, the results of allocation by financial value for 5Mol and 10Mol were not significantly different from those obtained by the marginal allocation method (3%). However, for the SO\(_2\) emissions, which do not result from the energy use, the difference between two approaches is on average 65%. This is another illustration of the point, that, although allocation based on the market value will sometimes reflect direct energy use and the related burdens, it cannot take into account the burdens arising elsewhere in the system.

![Graph showing comparison of different allocation methods for SO\(_2\) emissions]

**Fig. 6.8 Comparison of different allocation methods for SO\(_2\) emissions**
6.4.1.3 Total Solid Waste

This environmental burden is mostly related to the overburden from the mine. The burdens allocated by the marginal values are graphically presented in Fig. 6.9. The most responsible for the total solid waste are ABA and AB, contributing on average 18.71 and 13.75 t/t, respectively. The marginal allocated burdens for other products range from 9.55 t/t for 10Mol to 10.04 t/t for BA.

It is now interesting to compare the marginal allocation approach for this burden with the mass and market value approaches. Allocation of solid waste on the mass basis without disaggregation for 5Mol, 10Mol and BA gives similar results to those obtained by marginal allocation (Fig. 6.10). This can only be explained by a coincidence, because the difference in the results between these two approaches for AB and ABA is respectively 30% and 50%. The reason for this is that allocation on the marginal basis for AB and ABA also includes solid waste arising from life cycles of their respective feeds, i.e. 5Mol and BA. In allocation on the mass basis without system disaggregation it is not possible to account for this effect. Therefore, this is another illustration of how this approach breaks down in complex industrial systems where some of the products are used for production of other co-products in the system.

![Solid waste allocated by the marginal approach](image-url)
If the burden is allocated on the basis of $B_2O_3$ content, however, the results seem to be more closely related to those of the marginal approach (Fig. 6.10). Although the $B_2O_3$ allocation coefficients are not exactly equal to the marginal values, they at least follow the same trend. This is to be expected because the amount of the overburden is dependent on the total $B_2O_3$ required in the process, which should be reflected in the allocation coefficients. However, the main reason for the difference in the results, particularly for 10 Mol, is related to additional solid waste arising in the individual processes. Similar to the allocation on the mass basis, this allocation approach cannot take this effect into account.

However, if the burden is allocated on the mass basis with disaggregated data, the results are the same as in allocation on the marginal basis. This illustrates yet again the importance of system disaggregation and identification of correct causality in the system.

Allocation based on the market value, as shown in Fig. 6.10, does not give satisfactory results. The difference between this approach and the marginal value approach is on average 35%, up to 65% for ABA. Similar to the $SO_2$ emissions, this example also demonstrates that allocation by financial value cannot account correctly for the burdens that are not energy-related but arise from other activities in the system.
6.4.1.4 Abiotic Resource Depletion

The marginal allocation results, shown in Fig. 6.11, include depletion of fossil fuels (i.e. gas, oil and coal) and of boron minerals (i.e. borax and kernite). However, fossil fuels contribute 0.2% to the total resource depletion so that this impact is mostly related to the depletion of the borax and kernite reserves. The allocated abiotic reserve depletion ranges from $8.43 \times 10^{-9}$ for the ABA to $3.01 \times 10^{-9}$ 1/t-product for 10Mol. Similar to the solid waste, the resource depletion allocated to ABA and AB stems from the impact allocated to BA and 5Mol; the higher impact for the former two products is a result of the stoichiometry of the process.

![Fig. 6.11 Abiotic reserve depletion allocated by the marginal approach](image)

The marginal allocation results for depletion of resources are compared with the mass and market value approaches in Fig. 6.12. In allocation on the mass basis without system disaggregation, the difference between this and marginal allocation approach ranges from 5% for 5Mol to 54% for ABA. The reason why this allocation method is unrealistic in the case of resource depletion is that abiotic resource depletion, here mostly related to depletion of borax and kernite ores, depends on the quality of ore, i.e. the $B_2O_3$ content. Since different products contain different amount of this mineral, they need different amount of borax or kernite ore. Clearly, mass allocation without system disaggregation is not able to account for this effect. However, the situation is quite different if the impact is allocated on the basis of $B_2O_3$ content. In that case, the marginal values and the coefficients based on $B_2O_3$ content are approximately equal. The
negligible difference is related to a relatively small proportion of fossil fuel depletion. This means that physical causality exists for this impact as well, and is represented by the B$_2$O$_3$ content. Similarly, if allocation is done on the mass basis with system disaggregation, the results are equal to those obtained by whole system modelling using marginal analysis. The reason for this is that mass allocation in a disaggregated system is able to take into account different B$_2$O$_3$ content in different minerals. However, if allocation is done on the basis of market value it is again clear that this method has little relevance when energy consumption is not the main source of the impacts.

As already mentioned, the burdens and impacts considered here are chosen as representative cases of allocation in the boron system. As illustrated by these examples, the causality that determines system behaviour is physical and allocation on the mass basis with disaggregated data is a realistic approach. Further analysis of the system shows that the same causality exists for the other burdens and impacts, so that they can be allocated in the same way.

These findings now beg a question: why use whole system modelling and LP if allocation on the mass basis in a disaggregated system gives the same results? The answer to this is simple: allocation on the mass basis may hold for the state of the system where the burdens are product-related; however, if the operating conditions of the system change and the burdens also become process-related, the type of causality may
change and the mass basis may not be relevant anymore. Since marginal allocation is based on a system model, it represents a description of the causal relationships and so helps identify the true causality in the system. Therefore, allocation by whole system modelling provides a more realistic approach to system analysis by being able to account for changes in the operating conditions of a system and the corresponding changes in the environmental burdens. This is illustrated in the following section.

6.4.2 Allocation as a Function of the Operating State of the System

In order to illustrate changes in the causality and therefore in the allocated burdens with the operating conditions of the system, two distinct allocation cases are considered. The first case examines changes in the state of the system with the ratio of kernite to borax ore. Since the goal of the study is still related to the analysis of marginal changes in the burdens and impacts with the product outputs for different ore ratios, the burdens are considered to be product-related. In the second example, changes in the burdens are caused by the process-related parameters, represented for example by the capacity of the dryer for 5Mol and the heat requirement in the system. In both cases, the marginal allocation approach is compared with allocation on the mass and market value bases.

6.4.2.1 Changes in the State of the System - Product-Related Burdens

In this example, the ratio of kernite to borax ore (K/B) related to the B$_2$O$_3$ content is varied from 0.15 to 0.45. It is compared with the average K/B ratio of 0.205 used in the current operations. So, what changes in the state of the system will the change of the ore mix induce? Firstly, with an increased kernite to borax ratio, the B$_2$O$_3$ content increases. This means that less ore will be required in the production process which will, therefore, reduce the mining activities and the related environmental burdens. Furthermore, the ratio of the borates to insolubles in the dissolvers and thickeners will change, causing changes in their operating conditions. These will, in turn, change the environmental burdens from the system, i.e. gangue, energy requirements, etc. However, these changes will affect directly only those products for which the ore mix is a variable of the process, i.e. 5Mol and 10Mol. It is also to be expected that, through 5Mol, the burdens allocated to AB will change. The marginal burdens of BA and ABA should remain the same for all K/B ratios.

As an illustration, the allocated emissions of CO$_2$ for different K/B ratios are considered. As expected, only the marginal values of 5Mol and 10Mol change with the K/B ratio.
(Fig. 6.13), while those of ABA and BA remain unchanged. In addition, the burdens allocated to AB also change in proportion to those of 5Mol. As shown in Figs. 6.13, with an increase in the kernite to borax ratio the burdens allocated to 5Mol and 10Mol decrease, thus causing a decrease in the total emissions of CO₂ from the system.

A similar kind of analysis applies to the other mining-related activities, for instance, the consumption of diesel fuel. By increasing K/B from 0.15 to 0.45, the fuel consumption allocated to the 5Mol and 10Mol decreases by, on average, 5% (Fig. 6.14), which causes the total consumption of fuel in the mine to decrease by 4%. Reduction in oil consumption means reduced environmental burdens; however, it also means reduced production costs. Thus, whole system modelling by LP, in addition to solving the problem of allocation of environmental burdens, has other practical implications, such as identifying places for improvement of environmental and, equally important, economic performance of the system. The potential of applying LP to the Improvement stage of LCA is the subject of the next chapter.

Although discussion in this section considers changes in the state of the system with the operating conditions, the changes in the burdens remain product-related. Therefore, the same type of causality remains valid so that comparison of marginal allocation results with the mass and market value approaches shows the same trends as in the preceding section and is not repeated here. However, the situation is quite different when the burdens change to become process-related, as demonstrated in the following section.

![Fig. 6.13 Changes in the allocated CO₂ emissions with change of the state of the system](image)
6.4.2.2 Changes in the State of the System - Process-related burdens

In the examples presented so far, the goal of the study was related to the analysis of changes in the burdens and impacts with the product-related parameters such as, for instance, total product output. This section examines how product-related burdens change to become both product- and process-related in a study where changes in both parameters are of interest. In addition, a case in which the burdens are only process-related is also discussed. As an illustration, the process-related parameters are represented by the capacity of the rotary dryer for 5Mol and the heat requirement in the system. Although the allocation results are shown for the emissions of CO₂ only, similar observations apply to the other burdens and impacts in the system.

Suppose that the goal of the study is to analyse changes in the burdens with changes in the product outputs and the available process capacity of the rotary dryer in a system where these parameters can be changed independently. This would exclude, for example, analysis of the system in which the rotary dryer is already working at maximum throughput (i.e. production of 5Mol cannot be increased). This goal of the study is relevant in situations where the operation of the system is not only determined by the product demand but also by the productive capacities. In LP terms this means that the active constraints of the model are related to the output of the products and the capacity of the dryer, described by eqns. (6.2) and (6.4). This means that marginal values of these
constraints, and therefore the allocated burdens, will be non-zero and that the burdens will be both process- and product-related, as defined by eqn. (6.22).

The results of marginal allocation for these conditions are shown in Table 6.2 and Fig. 6.15 (Case 2). When compared with the product-related burden (Case 1), it is obvious that only emissions allocated to 5Mol from the rotary dryer change, decreasing from an average 0.196 t/t to 0.136 t/t, while other allocated emissions remain the same. The difference of 0.060 t/t of CO₂ is allocated to the capacity of the dryer. The fact that the burden previously allocated to 5Mol is now shared between 5Mol and the capacity of the dryer is not surprising because the capacity is a process parameter related to the production of 5Mol. However, without whole system modelling it is not possible to determine the proportion in which the allocation should be made. In allocation on the mass basis without system disaggregation, the emissions would be partitioned in equal proportions among all product- and process-related parameters to give an allocation factor of 0.095 t/t (total CO₂ emissions of 252918 t/yr divided by the sum of the total product output of 1062000 t/yr and the capacity of the dryer of 1584000 t/yr). This represents a total average difference of 85% from the emissions allocated by the marginal values, with the highest average difference of 95% for ABA. If on the other hand, the burden is allocated on the mass basis in a disaggregated system, the results correspond to the marginal allocated burdens obtained by whole system modelling. This is therefore an indication that the causality in the system has not changed and the mass basis (with disaggregation) is still a relevant allocation parameter.

Allocation on the market value basis is not possible in this case because the market that relates process-related burdens, here represented by the capacity, to a financial value does not exist. This is yet another reason why allocation on the basis of market value should not be used in systems where natural causality exists.

Consider now a case where the system operation is, in addition to the product demand and capacity of the rotary dryer, determined by the heat requirement in the system, so that the active constraints in the system are eqns. (6.2), (6.4), and (6.5). The marginal allocated burdens are, as shown in Table 6.2 and Fig. 6.15 (Case 3), quite different from those obtained in the previous two cases: all product-related emissions decrease by an average 56%, and the heat-related burden is now equal to 0.0007 t/TJ. This is perhaps to be expected: the heat constraint is related to all products so that by allocating some of the burden to it, the contribution of the products to the total emissions should change. However, what is unexpected at first is the character of the change: the relative contributions of the products are no longer the same and 5Mol now contributes to the
Table 6.2 Changes in the allocated CO₂ emissions with changes in the state of the system

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10Mol-bulk</td>
<td>0.156</td>
<td>0.156</td>
<td>0.033</td>
<td>0.033</td>
<td>0.033</td>
<td>0.000</td>
</tr>
<tr>
<td>10Mol-25kg&amp;50lb</td>
<td>0.192</td>
<td>0.192</td>
<td>0.069</td>
<td>0.069</td>
<td>0.069</td>
<td>0.000</td>
</tr>
<tr>
<td>10Mol-50kg&amp;100lb</td>
<td>0.189</td>
<td>0.189</td>
<td>0.066</td>
<td>0.066</td>
<td>0.066</td>
<td>0.000</td>
</tr>
<tr>
<td>10Mol-PP bags</td>
<td>0.165</td>
<td>0.165</td>
<td>0.042</td>
<td>0.042</td>
<td>0.042</td>
<td>0.000</td>
</tr>
<tr>
<td>10Mol average</td>
<td>0.176</td>
<td>0.176</td>
<td>0.052</td>
<td>0.052</td>
<td>0.052</td>
<td>0.000</td>
</tr>
<tr>
<td>Penta-bulk</td>
<td>0.247</td>
<td>0.247</td>
<td>0.032</td>
<td>0.032</td>
<td>0.032</td>
<td>0.000</td>
</tr>
<tr>
<td>5Mol-bulk</td>
<td>0.177</td>
<td>0.117</td>
<td>0.044</td>
<td>0.000</td>
<td>0.042</td>
<td>0.000</td>
</tr>
<tr>
<td>5Mol-25kg&amp;50lb</td>
<td>0.212</td>
<td>0.153</td>
<td>0.080</td>
<td>0.000</td>
<td>0.077</td>
<td>0.000</td>
</tr>
<tr>
<td>5Mol-50kg&amp;100lb</td>
<td>0.209</td>
<td>0.149</td>
<td>0.076</td>
<td>0.000</td>
<td>0.073</td>
<td>0.000</td>
</tr>
<tr>
<td>5Mol-PP bags</td>
<td>0.184</td>
<td>0.124</td>
<td>0.051</td>
<td>0.000</td>
<td>0.049</td>
<td>0.000</td>
</tr>
<tr>
<td>5Mol avg.(excl. Penta)</td>
<td>0.196</td>
<td>0.136</td>
<td>0.063</td>
<td>0.000</td>
<td>0.055</td>
<td>0.000</td>
</tr>
<tr>
<td>BA-bulk</td>
<td>0.395</td>
<td>0.395</td>
<td>0.165</td>
<td>0.165</td>
<td>0.165</td>
<td>0.000</td>
</tr>
<tr>
<td>BA-25kg&amp;50lb</td>
<td>0.431</td>
<td>0.431</td>
<td>0.201</td>
<td>0.201</td>
<td>0.201</td>
<td>0.000</td>
</tr>
<tr>
<td>BA-50kg&amp;100lb</td>
<td>0.427</td>
<td>0.427</td>
<td>0.198</td>
<td>0.198</td>
<td>0.198</td>
<td>0.000</td>
</tr>
<tr>
<td>BA-PP bags</td>
<td>0.404</td>
<td>0.404</td>
<td>0.174</td>
<td>0.174</td>
<td>0.174</td>
<td>0.000</td>
</tr>
<tr>
<td>BA average</td>
<td>0.414</td>
<td>0.414</td>
<td>0.185</td>
<td>0.185</td>
<td>0.185</td>
<td>0.000</td>
</tr>
<tr>
<td>AB-25kg&amp;50lb</td>
<td>0.991</td>
<td>0.906</td>
<td>0.025</td>
<td>-0.048</td>
<td>0.021</td>
<td>0.000</td>
</tr>
<tr>
<td>AB-50kg&amp;100lb</td>
<td>0.987</td>
<td>0.902</td>
<td>0.021</td>
<td>-0.052</td>
<td>0.017</td>
<td>0.000</td>
</tr>
<tr>
<td>AB-PP bags</td>
<td>0.962</td>
<td>0.877</td>
<td>-0.005</td>
<td>-0.077</td>
<td>-0.008</td>
<td>0.000</td>
</tr>
<tr>
<td>AB average</td>
<td>0.980</td>
<td>0.900</td>
<td>0.010</td>
<td>-0.060</td>
<td>0.010</td>
<td>0.000</td>
</tr>
<tr>
<td>ABA-25kg&amp;50lb</td>
<td>1.955</td>
<td>1.955</td>
<td>0.227</td>
<td>0.227</td>
<td>0.227</td>
<td>0.000</td>
</tr>
<tr>
<td>ABA-50kg&amp;100lb</td>
<td>1.952</td>
<td>1.952</td>
<td>0.224</td>
<td>0.224</td>
<td>0.224</td>
<td>0.000</td>
</tr>
<tr>
<td>ABA-PP bags</td>
<td>1.918</td>
<td>1.918</td>
<td>0.190</td>
<td>0.190</td>
<td>0.190</td>
<td>0.000</td>
</tr>
<tr>
<td>ABA average</td>
<td>1.940</td>
<td>1.940</td>
<td>0.210</td>
<td>0.210</td>
<td>0.210</td>
<td>0.000</td>
</tr>
<tr>
<td>Capacity</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.024</td>
<td>-0.046</td>
<td>-0.008</td>
<td>0.000</td>
</tr>
<tr>
<td>Heat</td>
<td>0.000</td>
<td>0.000</td>
<td>0.0007</td>
<td>0.0007</td>
<td>0.0007</td>
<td>0.0007</td>
</tr>
</tbody>
</table>

total CO₂ emissions more than AB. Furthermore, 5Mol produced in the rotary dryer has higher allocated burden than 5Mol (Penta) produced in the fluid bed dryer. Finally, the burdens allocated to AB-PP bags and the capacity are now negative, which means that increasing either output of AB or capacity would in effect decrease the total emissions of CO₂.

Similar results are obtained if operation of the system is again determined by the same constraints, but where production of 5Mol from the rotary dryer is not limited by the market demand and can vary according to the process requirements. This means that at the LP solution, the equations that describe the output of 5Mol products are non-active,
so that their marginal values, and therefore the allocated burdens, are zero (Case 4 in Table 6.2). The interpretation of these results is that production of 5Mol can increase by a marginal amount with no effect on the total CO₂ emissions. In addition, since the burdens allocated to all AB products are negative, a unit increase in either of the AB products will cause an average of 0.06 tons decrease in CO₂. Thus, for this state of the system, it would be better from the environmental point of view to increase the production of AB or perhaps 5Mol, while keeping other outputs constant. It can also be noticed that the burden allocated to the capacity has changed from a negative to a positive value of 0.046 t/t.

Furthermore, if the state of the system changes to become constrained by the products output and heat constraints only, the allocated burdens change again. As shown in Table 6.2 (Case 5), average contributions of both 5Mol and AB to the total CO₂ increase to 0.055 t/t and 0.010 t/t, respectively. The burden allocated to the capacity constraint is zero, while all other burdens are unchanged.

And finally, in cases where the interest lies in the effect on the burdens of changes in the capacity and heat constraints, the burdens are allocated to these two process parameters and the products are no longer responsible for the emissions (Table 6.2, Case 6). This means that the burdens have been shifted from the products to the processes and are now considered to be process-related only. This is possible because the state of the system is now determined by a different set of active constraints, which define the allocated
burdens in the system. Therefore, the burden allocated to the capacity of the rotary dryer changes to become 0.016 t/t while the heat burden remains unchanged at 0.0007 t/TJ. However, in addition to these, constraints related to the kernite availability (eqn. (6.3)) and capacities of the AB and ABA plants (eqns. (6.4)) have also become active. Their burdens have changed from zero to 0.163 t/t, -0.054 t/t, and -0.060 t/t, respectively.

The results shown for Cases 3-6 illustrate how the environmental burdens change with the state of the system, as defined by which constraints are active. They prove the point made earlier on in this Chapter that the allocated burdens are not fixed but change to reflect changes in the system parameters that determine its operation. Since the effect on the burdens of changes in the parameters is a direct consequence of the causality in the system, the question is: do changes in the state of the system cause a change in the causality (or vice versa), and if they do, how do we know? To answer it, let us compare the marginal allocation results for these cases with allocation on the mass basis with disaggregation. It has been demonstrated in the previous examples that the burdens allocated on the mass basis in a disaggregated system correspond to those obtained by the marginal allocation. The reason for that is that the causality in the system was physical and mass was a relevant allocation parameter. However, examination of the above examples demonstrates it is no longer possible to apply the same causality principle. Since the state of the system is now defined by a number of process- and product-related parameters, the causal relationships in the system have become too complex to be determined by a simple allocation on the mass basis. This therefore indicates that the causality in the system has changed and allocation on the mass basis is not relevant anymore.

However, the question is now: what kind of causality is present in the system then and which parameters are relevant for allocation in these cases? Because of the complex interactions between different parts of the industrial systems, this question cannot be answered without whole system modelling. The allocated burdens depend on the state of the system which, on the other hand, depends on which constraints are active. As shown in these examples, it is not possible to know in advance which constraints will be active and therefore determine the state of the system. This demonstrates the value of whole system modelling: by being able to account for the complex relationships between different parts of the system, it can determine the type of the causality in the system and allocate the burdens accordingly. In addition, whole system modelling can indicate places in the system where improvements can be made and thus aid the environmental system management. Whole system modelling and the improvements of the systems performance are the subject of the following chapter.
6.5 Allocation in the Cogeneration Plant

Although the Cogeneration plant constitutes a part of the foreground boron products system, because of the lack of detailed data, it has not been possible to allocate the burdens in this subsystem in the same way as in the rest of the foreground system, i.e. by the marginal approach. Therefore, allocation in the Cogeneration plant is considered here as a separate example of a co-product system.

As given in Chapter 5, the Cogeneration plant comprises two subsystems: Cogeneration of electricity with steam, and Steam cogeneration. The former is related to the electricity used in the foreground system and the steam cogenerated with it; the latter corresponds to the steam cogenerated with the electricity exported from the system. In Cogeneration of electricity with steam, no allocation is needed because both steam and electricity are part of the boron system. However, the allocation problem arises in the Steam cogeneration subsystem, because some of the electricity leaves the boundaries of the boron system. Therefore, it is necessary to allocate the burdens between electricity exported from the system (443480 MW) and steam cogenerated with this electricity and used within the system (952127 t). The rest of the discussion in this Chapter is related to allocation of the burdens in the Steam cogeneration subsystem.

As discussed in this and Chapter 3, allocation in multiple-function systems can either be avoided by system enlargement or disaggregation or it can be solved by applying an appropriate natural causation principle or economic relationships. Since no detailed data on the cogeneration process are available, system disaggregation is not possible in this case. The same is true for allocation by natural causation: without a detailed knowledge of the process it is not possible to build the model of the system and therefore the natural causation principles cannot be determined. However, it is known that natural causation in the system exists, because it is possible to change the outputs of the products independently. Hence, following the recommendations of the four step procedure introduced in Chapter 3, allocation on the basis of economic relationships should not be used in this case. This leaves only one method to consider, i.e. avoiding allocation by expanding system boundaries. The results of allocation by this approach are presented in the rest of this chapter. In addition, this method is also compared with allocation by heat content, one of the arbitrary methods most commonly used in cogeneration systems.
6.5.1 Avoiding Allocation by Expanding System Boundaries

As discussed in Section 6.1.1, allocation in co-product systems can be avoided in two ways, depending on the goal of the study. If the goal is a comparison of steam production from Steam cogeneration (System I in Fig. 6.16) with steam production in another single-output system (System II), then System II can be enlarged so that an alternative way of producing electricity in System I, is added to System II. The comparison is now between System I, here represented by the Steam cogeneration, and Systems II + III.

However, the goal of the study in this work is to provide data on the burdens and impacts allocated to the steam used in the boron foreground system. Since allocation by system enlargement enables only comparison between different systems and does not provide data on the contribution of individual co-products to the total environmental burdens and impacts from Steam cogeneration, this approach is therefore not relevant here.

The second way of avoiding allocation by expanding system boundaries is to use the "avoided burdens" approach (see Chapter 3). In that case, the burdens arising from the alternative way of producing electricity in System III are subtracted from those produced in Steam cogeneration (System I) so that only steam production is now compared to the steam in System II, as illustrated in Fig. 6.17a. However, because of the goal of the study in this case, comparison with System II is not relevant and therefore only Systems I - III need to be considered (Fig. 6.17b).

Fig. 6.16 Avoiding allocation in the cogeneration system by system enlargement
The above analysis shows that the avoided burdens approach is the only allocation method applicable for the Steam cogeneration subsystem. However, before this method can be applied, it is necessary to define System III, i.e. an alternative source of electricity. If the electricity were not cogenerated, it would be produced in the on-site power plant and natural gas would be used for its generation. Steam would in that case be produced in the Steam Plant. Thus, a power plant fired by natural gas is defined as System III.

Another possibility to allocate the burdens between steam and electricity is to consider the burdens that are avoided by producing steam in the Steam cogeneration subsystem, rather than by the Steam plant where the steam would be generated otherwise. However, prior to applying this approach, some kind of allocation between electricity and steam produced in Steam cogeneration is necessary. In this example, the burdens in the Steam cogeneration subsystem are allocated on the arbitrary basis, here taken to be heat content. This is one of the methods most commonly used for allocation in the cogeneration systems, whereby the burdens are allocated on the basis of the heat content.
of steam and electricity, with the efficiency of conversion of the thermal energy supplied by fuel taken into account (Boustead, 1992). The heat delivered by 952127 t of steam (p=1.31·10^6 Pa and T=463.7 K) and 443480 MW of electricity are respectively equal to 2.41·10^9 MJ and 1.60·10^9 MJ and their respective efficiencies of thermal energy conversion are 49.5% and 31.3%. Since energy conversion in the production of electricity is only 63% as efficient as in steam generation ((31.3/49.5)*100), 63% of the environmental burdens are allocated to the electricity and the rest to the steam. As illustrated in Fig. 6.18, the avoided burdens are calculated by subtracting the burdens arising from the Steam Plant (System II) from the burdens allocated to the cogenerated steam on the heat content basis (System I). In both systems natural gas is used as a fuel.

These two options for allocation by the avoided burdens approach are summarised in Table 6.3 (Methods 1 and 2), along with allocation by heat content (Method 3). Comparison of their results is shown in Figs. 6.19a-6.19b. As the figures indicate, Method 1 gives on average the least environmental burdens because the system is credited for cogenerating heat with electricity. If the Cogeneration plant did not exist, the alternative to cogeneration would be the electricity produced by natural gas in a power plant and steam produced in the Steam plant. By cogenerating steam with the electricity, these activities are displaced and the burdens associated with these processes are avoided. Therefore, it is correct to credit the system for reducing the burdens that would otherwise be generated.

Method 2 also credits the system for avoiding the burdens; however, it involves allocation on the arbitrary basis, i.e. heat content, and is hence not recommended. This also applies to allocation on the heat content basis only, which in this case gives the highest allocated burdens.
Chapter 6

a) Resource requirements

b) Emissions to air, water and land

Fig. 6.19 Comparison of different allocation methods in the steam cogeneration system
Table 6.3 Allocation methods considered for the steam cogeneration

<table>
<thead>
<tr>
<th>Allocation method</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method 1: Avoided burdens approach</td>
<td>Burdens of (443480MW electricity + 952127 t steam) by cogeneration</td>
</tr>
<tr>
<td>Method 2: Avoided burdens approach &amp; allocation on the heat content basis</td>
<td>Burdens of 443480MW electricity by natural gas - Burdens of 952127 t steam by cogeneration</td>
</tr>
<tr>
<td>Method 3: Allocation on the heat content basis</td>
<td>Burdens of 952127 t steam by cogeneration</td>
</tr>
</tbody>
</table>

Since the contribution of Steam cogeneration to the total emissions from the boron system is considerable for some categories (see Chapter 5), it is now interesting to see how different allocation methods in this subsystem influence the total environmental burdens. As shown in Figs. 6.20a-6.20b, the results of the study are quite sensitive to the allocation method used for the cogeneration. The highest difference in the results occurs for the burdens to which Steam cogeneration contributes the most. In the resource consumption category, allocation by Methods 2 and 3 increases the use of nuclear and hydro-electricity by 60% relative to the avoided burdens approach; gas reserves allocated by these two methods increase by 30% and 50%, respectively. For the emissions to air, in comparison to allocation by Method 1, the other two methods increase the total burdens by on average 15% and 35%, respectively. The differences for the emissions to water and land are considerably less, the reason being that the burdens from the Steam cogeneration do not contribute much to these categories.

6.6 Concluding Remarks

The analysis in this chapter demonstrates the importance of allocation in multiple-function systems for the outcome of an LCA study. In many cases different allocation methods will give different results, and this may influence the final conclusions of the study. It is thus important that, before an allocation method is chosen, different possibilities for allocation are examined and choice of the most appropriate method made so as to reflect the real behaviour of the system under study as closely as possible. Wherever feasible, the behaviour of the system should be described by a whole system model. As shown in this chapter, the advantage of whole system modelling is that it relates changes in the state of the system to the system parameters and so allocates the burdens realistically among different products or processes. In addition, once the
a) Resource depletion

b) Emissions to air, water and land

Fig. 6.20 Sensitivity analysis:
Change of the total burdens compared to the "avoided allocation" approach
burdens have been allocated to indicate which parts of the system are the most responsible for the total burdens, system modelling can help identify the possibilities for improving the performance of the system. These considerations are the subject of the following chapter.
CHAPTER 7

MULTIOBJECTIVE OPTIMISATION
OF THE BORON PRODUCTS SYSTEM

The LCA results presented in Chapters 5 and 6 provide valuable information on the contribution of individual processes and products to the total environmental impacts of the boron products system. On the basis of these results decision-makers can then identify possibilities for improving the environmental performance of the system. This chapter illustrates how improvements can be achieved in the optimum way by using multiobjective linear programming. Since system improvements cannot be carried out on the basis of environmental LCA only, the boron products case study is used to show how both environmental and economic performance can be optimised to find the best compromise solution. The approach used here is based on the methodology developed and discussed in Chapter 4.

7.1 Optimisation on Environmental Performance

7.1.1 Identification of the Improvement Options

As stated in Chapter 5, one of the aims of carrying out the LCA of the boron products system is to identify and evaluate possibilities for improving its environmental performance. Hence, the first step is to identify the subsystems that contribute the most to the total burdens and impacts from the system. The efforts to improve its performance are then aimed at these subsystems to achieve the maximum decrease in the total impacts on the environment. The analysis of the improvement possibilities will usually identify several different alternatives and it may not always be obvious which alternative or their combination is the best for a particular operating state of the system. Therefore, some kind of system optimisation will usually be necessary. As discussed in Chapter 4, the optimisation problem in the LCA context is inevitably multiobjective and therefore a multiobjective optimisation technique must be used. In this work, Multiobjective Linear Programming (MOLP) has been chosen as a specific technique and its application to environmental optimisation of the boron product system within the Improvement Assessment stage of LCA is presented and discussed in this section.
The results of the Inventory and Impact Assessment stages, presented in Chapter 5 (see Figs. 5.28-5.30), indicate that several subsystems contribute to most of the burdens and impacts. They are Mining, Primary process, Steam production, Boric acid plant, and Packaging and shipping. Therefore, these subsystems are the first to be considered for targeted system improvements. Other subsystems, such as Anhydrous borax plant and Anhydrous boric acid plant, could also be included in the improvement analysis; however, improving their performance at this stage would not reduce the impacts from the system significantly, so that for the further analysis presented here these subsystems are not considered.

There are a number of possibilities for improvements to the above mentioned subsystems; however, lack of data at present restricts the analysis here to the most immediate options. To illustrate the potential of multiobjective LP as a tool in the Improvement Assessment stage, several alternatives for improving the environmental performance of the system are considered, as explained below.

In the Mining subsystem, a significant part of the burdens and impacts is attributed to transport within the mine. Therefore, one of the possibilities to reduce the burdens from this subsystem is to consider conveyors as an alternative means for transport of the ore. In theory, introducing the conveyors would not necessarily mean phasing out the truck transport, but keeping the trucks in a stand-by mode so that they could be used in cases of, for instance, conveyor break-down. In reality, this would probably be more difficult to achieve, because of the problems of logistics (e.g. fuel supply) and labour availability (e.g. truck drivers), but for the analysis in this work it is assumed that both alternatives could be "activated" at any time. For an illustration, it is supposed that belt conveyors would be used and the electricity to drive the motors would be provided from the Cogeneration plant. An estimated amount of 11200 MW/yr of electrical energy (Sharpe, 1995) has been used to calculate the environmental burdens related to the use of conveyors (see Table A6.1). As mentioned in Chapter 6, another possibility considered for reducing the burdens from the Mine and therefore from the boron system, is to identify the optimum kernite to borax ratio, subject to the process constraints.

The burdens from the Primary process are mainly energy related, and a significant proportion is attributed to the Dryers area. There are a number of possibilities to reduce the burdens from this area; however, in this work only two of them are considered. Since 5Mol can be produced in both Rotary and Fluid bed dryers, the most immediate option is to optimise their use so that only dryers with the least environmental impacts in the system are in operation. This option is also easy to implement because it does not require
any major changes in the process. The second option for improvements in the Primary process concerns plans, already considered by the Company, to install low-NOx burners in the dryers.

Furthermore, Steam production, which includes the Steam cogeneration and Steam plant subsystems, has been identified as one of the significant contributors to the burdens from the boron system. As explained in Chapters 5 and 6, the Steam cogeneration subsystem corresponds to the steam that is cogenerated with the electricity exported from the system. Since the steam can be produced in both Cogeneration and Steam plants, one of the possibilities to reduce the burdens is to identify the best option for generating the steam.

The final improvement option taken into consideration here is related to Packaging and shipping. Since most of the burdens from this subsystem arise from life cycle of the packaging, the system is optimised to identify the type of packaging that causes the least environmental burdens.

These alternatives for the environmental improvements are then built into the LP model (see Appendix 6) and a number of optimisations are performed. The system is first optimised on each environmental objective function, defined as the burdens and impacts, to identify the best practicable environmental option (BPEO) in the system and these results are compared with the existing operations. The system is then simultaneously optimised on a number of environmental objective functions to identify the optimum solutions within the improved operations of the system. These results are presented next.

7.1.2 Environmental Improvements in the System

As already mentioned, the system defined in Chapter 6 is first optimised on each environmental objective function, defined as environmental burdens or impacts and given by:

Minimise

\[ B_j = \sum_{i=1}^{1} b_{c_{j,i}} x_i, \quad j=1,2,...,J \]  \hspace{1cm} (6.6)

Minimise

\[ E_k = \sum_{j=1}^{J} e_{c_{k,j}} B_j, \quad k=1,2,...,K \]  \hspace{1cm} (6.7)

subject to the constraints:
Mass balance constraints: \[ \sum_{i=1}^{J} a_{ij} x_i = 0, \quad j=1,2,...,J \] (6.1)

Market demand constraints: \[ p_l \leq d_l, \quad l=1,2,...,L \] (6.2)

Primary material availability: \[ r_m \leq s_m, \quad m=1,2,...,M \] (6.3)

Productive capacity constraints: \[ x_i \leq c_i, \quad i=1,2,...,I \] (6.4)

Heat requirements: \[ h_z \leq q_z, \quad z=1,2,...,Z \] (6.5)

The model is therefore similar to that used for allocation of environmental burdens, except that the market constraints are now different: instead of being defined by the current operations, they are determined by the market demand projected on the basis of the trends in the last few years (see Appendix 6, eqns. (A6.9)). Other changes to the model presented in Chapter 6, are related to the alternatives for improving the systems performance, as discussed in the previous section. A more detailed account and the printout of the LP model are given in Appendix 6.

The results of the single-objective optimisations on the environmental burdens and impacts are compared with the existing operations in Figs. 7.1 and 7.2. Fig. 7.1 indicates that environmental optimisation of the boron products system offers a potential for an average reduction of the burdens of 11.5%, with the highest reduction of 43.5% for Oil reserves. The environmental impacts follow similar trends: the average reduction in the optimised system is 20% while Photochemical oxidants creation potential is decreased by 62%. At the same time, the total production is reduced by only 0.5% in comparison to the current operations. On closer inspection of the optimisation results presented in Tables A7.1 and A7.2, the reason for these significant improvements becomes apparent. Firstly, the ratio of kernite to borax ore has increased from the current value of 0.205 to the optimum value of 0.4. Since increasing the kernite to borax ratio increases \( B_2O_3 \) content, the total amount of ore required in the production process is reduced. As discussed in Chapter 6, this has a direct effect on reducing the extent of the mining operations and the related environmental burdens from the mine. Moreover, the increased kernite to borax ratio also causes a decrease of the insolubles to borates ratio in the dissolvers and thickeners. This, in turn, results in reduced gangue, energy requirements, and other related environmental burdens from the Primary process.
Fig. 7.1 Improvements in the boron system: environmental burdens

Fig. 7.2 Improvements in the boron system: environmental impacts
Reductions of the environmental burdens from the Primary process are also achieved by producing Penta (i.e. 5Mol) in the Rotary dryer, instead of the Fluid bed dryer as in the current operations. The main reason for choosing the Rotary dryer as a better environmental option for the production of 5Mol lies in lower energy requirements (see Table A6.1). A reduction in the burdens and impacts from the dryers due to this change amounts to 60% per unit of 5Mol. A further reduction of up to 85% per unit of product in the NOx emissions (and the corresponding impacts) from the dryers, is also achieved by installing low NOx burners in the dryers.

A further decrease in the burdens and impacts in the optimised operations is achieved through different transportation means in the mine. However, unlike the other improvement options discussed so far, it is more difficult to decide which type of transport is a better choice. In minimising gas consumption, for example, transport by trucks is a more environmentally acceptable solution (see Table A7.1), because the electricity used to drive the conveyors is generated by the gas. Optimisation on fuel consumption, on the other hand, favours the use of conveyors because of the reduced need for diesel fuel.

Therefore, to choose the best practicable environmental option in the system with conflicting objectives, some kind of trade-offs among them is necessary. To ensure that the whole noninferior set of solutions is explored before a choice of the "best" solution is made, the system has to be optimised on all environmental objectives simultaneously. As discussed in Chapter 4, the emphasis is then on the range of choices from the set of noninferior or Pareto solutions, rather than on a priori choice of the best solution before analysing all the trade-offs among objectives. This is illustrated in the next section.

7.1.3 Multiobjective Optimisation on the Environmental Performance

To identify the feasible region for multiobjective optimisation, the system has first to be optimised on each environmental objective function and the values of all other functions calculated in turn (see Appendix 2). Depending on whether the objective of the study is to analyse the performance of the system at the Inventory or the Impact Assessment level, multiobjective optimisation can be performed on either environmental burden or impact objective functions. In this study, the interest is in identifying possibilities for reducing both the burdens and the impacts. Therefore, the first part of the discussion in this section is related to optimisation on the burdens and in the second part the possibilities for reducing the impacts are explored. The results of the single-objective
optimisations, performed to define the feasible region, are shown in the pay-off Tables A7.3 and A7.4.

Analysis of the pay-off Table A7.3 shows that values of some burdens, such as Nonrenewable resources, TDS and TSS do not change in the optimisations, while the values of some, such as Metals (Air), TSP, Waste water and Landfill weight, change by less than 3%. Since their optimisation would not bring significant improvements in the environmental performance of the system, they are not considered further in the multiobjective optimisation. Further analysis of Table A7.3 also shows that optimisation on the Gas objective minimises the Nuclear and Hydro-electricity, VOC, Halide and Metals (water) objectives and vice versa, while optimisation on, for instance, NOx gives the approximate minima of all other functions. Therefore, to reduce the computational burden, which in some cases may be prohibitive due to the computer time required, the system can be optimised on two objectives only, for example Gas and NOx, and other functions will be optimised accordingly. The constraint method, in which one of the functions is arbitrarily chosen for the optimisation and all other objectives are converted to constraints (see Appendix 2), has been used for generating the optimum solutions.

The noninferior curve, showing trade-offs between these two functions, is shown in Fig. 7.3. The values of the objective functions have been normalised by dividing them by their respective optimum values, Gas* and NOx*, obtained in the single-objective optimisations. It is apparent from the results shown in Fig. 7.3 and Table A7.5a that a decrease in the NOx emissions is associated with an increase in gas consumption and vice versa.

![Fig. 7.3 Noninferior curve for multiobjective optimisation on Gas and NOx objective functions](image_url)
Chapter 7

Fig. 7.4 Selected noninferior solutions for multiobjective optimisation on Gas and NOx objective functions

Thus, at point A, Gas is at the optimum and so are the Nuclear and Hydro-electricity, VOC, Halide and Metals (water) objectives; however, the NOx emissions are nearly 6% above the optimum (Fig. 7.4). Other burdens are also at the maximum at this point. For instance, Oil is 41% above the minimum; the CO emissions increase by 9.5% and SO_{2} and Renewables are approximately 5% away from their optimum values. These changes in the optimum solutions are mainly related to a change in the transportation means in the mine, while other improvement options remain the same in all optimisations. So, at solution A, transport in the mine by trucks is a preferred environmental option. On the other hand, solution at point F gives minimum values for the NOx, CO, CO_{2}, SO_{2} emissions and consumption of oil. However, the gas consumption increases by 5% from its minimum and renewable resources are 3% above the optimum. Similar increases are observed for VOC and Halides, while Nuclear and Hydro-electricity are around 30% above the optimum. At this solution, the conveyors are chosen as a better environmental option (see Table A7.5a). Obviously, all other solutions shown in Figs. 7.3 and 7.4 are also optimal, and the decision-maker can identify the most preferred alternative by using any of the methods for articulating preferences discussed in Chapter 4 and Appendix 4.

It is now interesting to see what happens if the system is simultaneously optimised on the environmental impact functions. The pay-off Table A7.4 shows that the value of the Resource depletion objective function does not change in the optimisations, and it is not
considered further here. The table also shows that optimisation on Global warming potential - indirect (GWPI) minimises Global warming potential - direct (GWP), Acidification, Nutrification and Human toxicity, while optimisation on the Photochemical oxidants creation potential (POCP) objective gives the optimum value of Ozone depletion (OD). Therefore, to identify and explore all noninferior solutions, it suffices to optimise the system on two objectives only, for instance GWPI and POCP, and the other objectives will be optimised accordingly.

The resulting noninferior curve of the optimisation on these two objectives is shown in Fig. 7.5. Similar to the optimisation on the burdens, the values of the impact objective functions have been normalised by dividing them by their respective optimum values, GWPI* and POCP*, obtained in the single-objective optimisations. At point A', GWPI and other related functions are at the minimum; however, POCP is 65% above the optimum (Fig. 7.6). These changes in the optimum solutions are mainly related to a change in the transportation means and to the source of steam. So for instance, at solution A', transport by the conveyors and steam produced in the cogeneration plant are chosen as better environmental options. On the contrary, the best environmental options at point E' are transport by the trucks and steam produced in the steam plant. This solution yields the minimum values of POCP and OD; however, other functions reach their maximum values. As in the optimisation on the burdens, the best compromise solution can be chosen by using any of the decision-making techniques discussed in Chapter 4.

![Fig. 7.5 Noninferior curve for multiobjective optimisation on GWPI and POCP objective functions](image-url)
The considerations so far concentrated on the environmental optimisation of the system and possible improvements in its performance. However, although some importance has been attached to the production requirements, the system has not been optimised on it, nor has any other indication of the economic performance of the system been given. In practice, environmental improvements cannot be carried out on the basis of the environmental LCA only; other factors, such as the economics of the system, have to be taken into account as well. Therefore, it is demonstrated in the rest of this chapter that both environmental and economic performance can be optimised to help identify the best compromise solution for the improvements in the system.

7.2 Optimisation on Economic and Environmental Performance

As outlined in Chapter 5, one of the objectives of this work is optimisation of the system on environmental and economic performance to identify a range of possibilities for minimising total environmental burdens and impacts from the system, while maximising production subject to total product demand and keeping the production costs at
minimum. The information obtained would then serve as a basis for a decision on effecting improvements in the system.

Therefore, the objective functions of the LP model, in addition to environmental burdens and impacts, include total production and the production costs, defined by:

Maximise

\[ P = \sum_{i=1}^{L} P_i \]  

(7.1)

subject to the product demand, \( D_t \), as defined by eqn. (6.2). The cost objective function is given by:

Minimise

\[ C = \sum_{i=1}^{I} c_i x_i \]  

(7.2)

Because the foreground system considered in this work is over 30 years old and is fully depreciated, the cost objective function is defined by the direct production costs only and the capital costs are not considered. The break-down of the costs in the system is given in Table A6.1, Appendix 6.

As already discussed, optimisation on environmental performance can be carried out either at the Inventory or Impact Assessment level. For a further illustration and discussion of the approach, optimisation at the Impact Assessment level has been chosen, because it gives a more general description of the overall environmental impact of the system. As demonstrated in the previous section, optimisation on GWP, for instance, optimises GWPI, Acidification, Nutrification and Human toxicity objectives; similarly optimisation on OD minimises POCP. Hence, as in the preceding section, it suffices to optimise the system on two environmental objectives only, for instance GWP and OD, to ensure optimum values of the other impact functions. These functions are then optimised simultaneously with the total production and costs function. The obtained results are discussed below.

In order to explain the approach on a simpler example, the system is first optimised on three objectives only (i.e. GWP, P and C) and other functions are ignored. In the final part of this chapter, in addition to these three objectives, multiobjective optimisation is also performed on OD, to give a range of noninferior solutions which define a multidimensional Pareto surface.
7.2.1 The Three-objective Optimisation

The three-dimensional objective space ABCD representing the noninferior surface obtained in optimisations on the GWP, P and C objectives is shown in Fig. 7.7. To illustrate the noninferior set more clearly, the objective space has been projected to show the noninferior subsets for the two objectives with constant values of the third function. Fig. 7.8 shows the projection on the Production-Cost plane with the GWP objective varying parametrically. Similarly, Fig. 7.9 is the projection on the GWP-Costs plane with production as a parameter.

Point A in Figs. 7.7 - 7.9 represents the minimum costs; however the production is at the minimum and GWP is 31% above its optimum. The Kuhn-Tucker multipliers, \( \lambda \), equal to 140 and -51 for GWP and Production, respectively (Table A7.8), indicate that at this solution, the effect of GWP on costs is much more pronounced than that of the production. This also means that a ton decrease in the GWP objective, is associated with a cost increase of $140; similarly, if the production were to increase by one ton, the resulting increase in the costs would be equal to $51\(^1\). The environmentally preferred options at point A are transport in the mine by the trucks and steam produced in the Steam plant.

---

\(^1\)Note that because the system is being minimised, the effect of the Kuhn-Tucker multipliers is opposite to their sign.
Fig. 7.8 Noninferior subsets for optimisation on GWP, P and C, for constant GWP

Fig. 7.9 Noninferior subsets for optimisation on GWP, P and C, for constant P
By moving from point A along the noninferior curve for constant GWP, both costs and production increase, to reach their maximum feasible values at point B. Here, the Costs function is 4% above its optimum value. The effect of the GWP and Production objectives on the costs reverses here, so that P influences changes in the costs much more than GWP. If Production is increased by one ton, $446 of the Costs objective have to be given up. Similarly, one ton change in the GWP is associated with a cost change of $170. At this solution, steam is generated by both Steam and Cogeneration plants, however, the contribution of the later to the total steam production is only 6%. As opposed to the solution at point A, the preferred transportation means in the mine are the conveyors. The reason for this is a change in the state of the system associated with the relaxation of the production constraint.

Furthermore, if for instance the system were to be operated at point C, GWP would be 3.3% above its optimum value obtained in the single-objective optimisation. The production would be at the minimum, and the costs would increase by 14%. The effect of GWP and Production on Costs is similar to that found at point A, except that an improvement in GWP of one ton would worsen the values of the Costs objective by $54, while a ton increase in P would result in $170 increase in the costs. These changes in the system are due to the different environmental options chosen at this solution. Here, 93% of the steam is generated by the Cogeneration plant and the rest is produced in the Steam plant. The conveyors still remain the best transport option in the mine.

However, if for example, point D were to be chosen as the best compromise solution, then for the same value of GWP as at point C, the production would reach the maximum; however, costs would have to increase by 17%. It can be noticed here that both GWP and Production exhibit similar effect on the Costs: a decrease in GWP by one ton increases Costs by $5241. If the production is increased by one ton, the costs increase by $5388 (see Table A7.8). At this solution, the best environmental option is defined by truck transport in the mine and steam production in the Cogeneration plant.

It is now interesting to find out what improvement options exist if the system is optimised on OD, GWP, P and C objective functions.

7.2.2 The Four-objective Optimisation

As discussed previously, optimisation on OD minimises POCP and optimisation on GWP gives optimum solutions with respect to the other objective functions. Therefore, the system can be optimised on these two environmental objectives, to give optimum values
of the other impact functions. These functions are then optimised simultaneously with the total production and costs functions to create multidimensional noninferior surface on which each solution is optimal so that the surface represents the possibilities for improving both economic and environmental performance of the system.

As an illustration, the three-dimensional surfaces projected in the OD-GWP-C space for different constant values of the P objective, are shown in Figs.7.10a-7.10d. The surface delineated by points A, B, C, D in Fig.7.10a represents the noninferior solutions for a constant production of 1.8% below the optimum. At solution B, for instance, the costs are at the minimum; however, GWP and OD are 31% and 27% above their optimum values. This solution corresponds to point A in Fig. 7.7, obtained in the three-objective optimisation. There, GWP and C are respectively 0.7% and 0.3% lower than at point B; however, OD is 3.4% higher. As at point A, transport by the trucks and steam production in the Steam plant are also identified as the best environmental options at solution B (see Table A7.10d). Furthermore, it is noticeable that the effect of OD on the costs is much higher than the effect of the other two objectives: one kg decrease in OD causes costs to increase by $974. For the same change in GWP and P, the costs increase by respectively $0.134 and $0.054.

If the operating state of the system moves, for example, from point B to A, it is possible to reduce the value of GWP by 1%. However, this improvement is carried out at the expense of OD and C, which increase by 3% and 0.8%, respectively. Trucks and Steam plant are still the best environmental options in the system, except that about 2% of the steam is now also produced in the Cogeneration plant (Table A7.10d).

A more extreme change occurs if the system is operated around solution C in Fig. 7.10a. There, the costs are 14.5% above the minimum and OD and GWP are respectively 6.9% and 1.3% higher than their optimum values. The ore is transported by the conveyors and 97% of the steam is generated in the Cogeneration plant. Furthermore, for the same GWP, a 0.1% increase in the costs brings the value of OD down to the minimum at point D. The steam is again produced in the Cogeneration plant and trucks are the best environmental option for transport in the mine. If compared to the three-objective optimisation, operating state at point C in Fig. 7.7 falls in between points C and D in Fig. 7.10a.

Because of a relatively low value of the OD objective, it would be wrong to talk of marginal changes in this objective in terms of tons. Therefore, the changes are expressed in kg.
Fig. 7.10a Noninferior surface for optimisation on OD, GWP, P and C for $P/P^*=0.982$

Fig. 7.10b Noninferior surface for optimisation on OD, GWP, P and C for $P/P^*=0.987$
Fig. 7.10c Noninferior surface for optimisation on OD, GWP, P and C for $P/P^*=0.992$

Fig. 7.10d Noninferior surface for optimisation on OD, GWP, P and C for $P/P^*=1.00$
Similar trade-offs among C, OD and GWP are noticed for $P/P^*=0.987$, i.e. for production 1.3% below the optimum (Fig. 7.10b). At points $A_2$, $B_2$ and $C_2$, OD and GWP remain almost the same as at solutions $A_1$, $B_1$ and $C_1$ in Fig. 10a, while costs increase by on average 0.5%. However, at point $D_2$, OD is no longer at the minimum (1.9% above) and Costs increase by 1% in relation to the values obtained for solution $D_1$. At the same time, GWP is 1.2% higher than the optimum. These changes are a result of the combined transport of ore by conveyors and trucks, as opposed to transport by trucks only which was the best environmental option at point $D_1$. Moreover, the effect of OD on the cost objective function at point $D_2$ reaches its maximum of $23241.2/\text{kg}$ while the effect of the same change in GWP or $P$ is only $4.379$ and $0.745$, respectively.

As production increases to reach the maximum and the requirements on the other objectives become stricter, the noninferior space becomes progressively more narrow and therefore offers a more limited choice of the noninferior solutions (Figs. 7.10c and 7.10d). This is due to the stricter constraints imposed on operations with respect to each of the objectives. For instance, if the system is operated anywhere on the boundary between points $C_1$ and $D_1$ (Fig. 7.10a), the noninferior solutions with respect to OD range from 0-6.9% above the minimum. Compared to this, the choice of the noninferior solutions between points $C_4$ and $D_4$ (Fig. 7.10d) is significantly more limited and ranges from 0-1.2% above the optimum. Solutions outside the noninferior surfaces are either infeasible or inferior.

Clearly, all points on the noninferior surfaces shown in Figs. 7.10a-7.10d are optimal. They have been generated in the multiobjective optimisation without prior articulation of preferences. However, in order to choose the best compromise solution, some articulation of preferences is now necessary. If, for instance, all objectives are considered to be of equal importance then, as discussed previously, one of the possible ways to choose the best compromise solution is to identify the operation at which all objectives differ from their optima by approximately the same percentage. However, should there be stronger preferences for some objectives then the others, any other noninferior solution that satisfies the criteria set by the decision-makers could be chosen as the best compromise option for improving the performance of the system.

7.3. Concluding Remarks

It has been demonstrated in this chapter that multiobjective LP can successfully be combined with LCA to assist in the decision-making process for improving both
environmental and economic performance of product systems. The value of multiobjective optimisation in the context of LCA lies in offering a range of alternatives for environmental improvements of the system rather than a single optimum solution, which may be optimum but not appropriate for a particular situation. This enables decision-makers to identify their preferences after analysing all the trade-offs among the objectives. Therefore, multiobjective LP provides a more effective approach to environmental management of product systems by offering a range of alternative optimal solutions and enabling decision-makers to choose the Best Practicable Environmental Option (BPEO) not entailing excessive cost.

Furthermore, generating methods such as MOLP can be applied in a wider range of decision-making contexts. In the case of single decision-makers, the generating methods provide information on the trade-offs between different objectives, to show explicitly what can be gained and what lost by choosing each alternative. Where there are multiple decision-makers with conflicting interests, this technique can help to resolve disputes by generating different alternative solutions. Decision makers who understand the trade-offs and the alternatives are more likely to understand the interests of other parties and, therefore, to compromise.

The discussion in this and Chapter 4 also indicates that it is not possible to avoid subjective value judgement in the problems with the conflicting objectives: if the best compromise solution is to be identified and agreed upon by all interested parties, some kind of subjective valuation has to be carried out. However, the point here is that the valuation is deferred until all noninferior solutions have been identified and analysed. The choice is then made from the noninferior solutions which have been generated without aggregating the objectives. This is particularly relevant in the LCA context, because it enables avoiding the aggregation of environmental impacts into a single environmental impact function in the Valuation stage.
CHAPTER 8

CONCLUSIONS

This research has addressed some of the methodological problems in Life Cycle Assessment. In particular, it has focused on solving the problem of allocation of environmental burdens and impacts in multiple-function systems. For this, whole system modelling by Linear Programming (LP) has been proposed. Furthermore, this work has concentrated on the possibility of applying multiobjective LP in LCA as a tool for identifying the best practicable environmental options for improving the performance of a system. Finally, it has been applied to a specific case study to demonstrate the potential of LCA as a tool in environmental decision-making.

The objectives of this work have been met in that:

1. a novel, marginal approach, has been developed to solve the allocation problem in multiple-function systems on the general level, based on whole system modelling by linear programming (Chapter 3);

2. a general theoretical framework developed in this work has been applied to a case study of the boron products system (Chapter 5) to demonstrate that the marginal approach can successfully solve the allocation problem in multiple-function systems (Chapter 6);

3. the potential for the use of multiobjective LP in LCA as a tool for identifying the optimum options for improving the performance of a system over its whole life cycle has been demonstrated (Chapter 4);

4. it has been shown on a case study of the boron products that LCA combined with MOLP represents a powerful tool for environmental system analysis and management (Chapter 7);

5. these general considerations represent a contribution to the development of the LCA methodology.
8.1 General Conclusions

A number of general conclusions can be made as a result of this work:

1. In order to help resolve some of the methodological problems, such as allocation of environmental burdens in multiple-function systems, a whole system modelling approach in LCA is necessary.

2. Given that LCA is based on linear models of human economic activities and the environment, LP is an appropriate tool for whole system modelling in LCA.

3. Allocation in LCA may be encountered wherever there is a system or process delivering more than one function. Combined waste treatment, co-product, and recycling systems are all examples of these systems or processes.

4. Allocation on an arbitrary basis, such as mass flow, has to be avoided.

5. Depending on the goal of the study, the allocation procedure should follow the recommendations of SETAC and ISO. It should either be:
   i) avoided by expanding system boundaries or disaggregating the process into different sub-processes; or
   ii) solved by disaggregating and allocating by a suitable allocation method.

6. Although avoiding allocation by system extension is an appealing way to deal with the allocation problem, its main drawbacks are that it is not always possible to apply it and the system becomes more complicated because of the need for additional data on other subsystems to be included in the system boundary.

7. Avoiding allocation by system expansion is applicable only when specific processes are analysed and data on their performance are available. Rearranging the expanded system according to the “avoided burdens” approach has advantages, particularly in waste management, in enabling comparison between alternatives.

8. Avoiding allocation by system disaggregation can be helpful where describing the system in greater detail shows that the processes for different functional units can be separated. For other, simple systems there is a risk that subdivision can lead to allocation on arbitrary bases.
9. System disaggregation cannot be used to avoid allocation when the system includes processes which are necessarily common to different functional units. System expansion is not practical where alternative ways to provide the functional outputs are unavailable or not practised. System expansion is not likely to be helpful in studies whose goal is to provide LCA data for systems which use the co-products as inputs. In these cases, it is necessary to allocate the burdens by a suitable allocation method (see 5.ii)).

10. Where there is a natural causality between functional units and environmental burdens, allocation should always be based on these relationships. This means that it must be possible to change the value or delivery of any functional unit while keeping the delivery of other functions unchanged.

11. The type of changes considered in the system can be marginal, average or discrete and they in general depend on the goal of the study and questions to be answered by LCA.

12. It is not always obvious what kind of causality is present in the system. In order to establish it, the system behaviour must be well understood and detailed data on the subprocesses in the system must be available. This approach to allocation usually requires the process or system to be described by a realistic system model.

13. In some cases, allocation by causality using a system model may lead to a simple basis for allocation, such as mass flow. However, the basis must emerge from the analysis, rather than being an arbitrary a priori assumption.

14. System behaviour can be described by whole system modelling using linear programming. In a system where natural causal relationships exist, and where marginal changes to a specific system are the goal of the study, the marginal values calculated at the solution of the LP model are equivalent to the allocation coefficients. Since the marginal values are a result of system modelling, they represent a realistic description of causal relationships and thus closely reflect changes in behaviour of the system. Therefore, whole system modelling by LP serves as a tool for establishing natural causation principles in multiple-function systems.

15. The marginal allocation approach applies only to a system analysis where marginal changes to a specific system are of interest. Since marginal values are valid only for small changes around an existing state, they cannot be used to describe average or
discrete changes in the system, because they may be nonlinear. In that case, the system model has to be solved again to identify a new state of the system. Marginal values evaluated for that new state can again be used to allocate the burdens at that particular state of the system.

16. Where the functions of a multiple-function system cannot be varied independently, allocation by causality cannot be implemented. It is then necessary to allocate the burdens on the basis of socio-economic relationships, such as financial value of the functional units.

17. LCA provides a basis for assessing and identifying the options for potential improvements in the environmental performance of a product system. Since in many cases there will be more than one option, the optimum improvement possibilities must be identified by system optimisation.

18. The optimisation problem in the LCA context is inevitably multiobjective, so that multiobjective LP can be used as a specific optimisation method. Its main advantages over other multiobjective optimisation techniques are as follows:

- it does not require \textit{a priori} articulation of preferences, so that the whole noninferior set of solutions can be explored;
- it generates a range of alternative optimum solutions and so enables decision-makers to choose the best compromise solution;
- it provides the information on trade-offs between different objectives to show explicitly what can be gained and what lost by choosing different alternatives;
- the objectives do not have to be aggregated into a single measure of performance, which enables the Valuation stage to be avoided.

19. Multiobjective LP can be used in a wide range of decision-making contexts. In addition to environmental objectives, it can incorporate economic and social criteria in the environmental system analysis, thus enabling choice of the Best Practicable Options (BPEO) not entailing excessive costs.

\textbf{8.2 Specific Conclusions}

The specific conclusions of this research are related to the boron products case study:
1. Mining operations consume most of the oil reserves and other nonrenewables (i.e. borax and kernite ore). They are also the main source of emissions of metals and dust to air, and solid waste.

2. The Primary process is responsible for 40% of nuclear and hydro-electricity and gas and water consumption. It is also the main source of the halides and emissions of waste water and metals to water.

3. The Boric acid plant consumes around 60% of the coal reserves in the system, almost all of which is used in the life cycle of sulphuric acid. The SO$_2$ emissions and other emissions to air, TSS and TDS are also a result of the boric acid production.

4. Packaging and shipping are the main users of renewable resources (paper bags). The COD and BOD are attributed to the Packaging production, because of the high water consumption in this process.

5. Steam production uses 40% of total nuclear and hydro-electricity, and gas and water reserves. The CO, CO$_2$, NO$_x$ and VOC emissions also come from the Steam production.

6. Mining operations are mostly responsible for the abiotic reserve depletion.

7. Primary process and Steam production contribute to the global warming potential, ozone depletion and nutrification in almost the same proportions (30% each), with the Boric acid plant share of 20% and anhydrous borax, anhydrous boric acid, and Packaging and shipping amounting to the remaining 20%.

8. The contribution of the boric acid for acidification is higher (35%) than that of the Primary process and Steam production (25% each).

9. Steam production contributes to the most of the photochemical oxidants creation potential (70%);

10. Mine, Primary process, Boric acid plant and Steam production are accountable for the human toxicity.
11. Single-objective environmental optimisation of the boron products system offers a potential for reducing most of the burdens by 11.5%. The average possible reduction in the impacts is 20%, with Photochemical smog reduction of 62%.

12. The improvements are achieved through several options considered in this work:
   - increasing borax to kernite ratio from 0.205 to 0.400;
   - producing all 5Mol, including Penta, in the Rotary dryer instead of the Fluid bed dryer;
   - production of all steam in the Cogeneration plant;
   - changing the transportation means in the mine from the trucks to the conveyors.

13. Multiobjective optimisation on economic and environmental performance offers a range of optimum solutions for improvements in the system. Each solution has a different potential for improving both environmental and economic performance.

### 8.3 Recommendations for Future Work

With regard to the LCA methodology, it is recommended that the following be done:

1. Future LCA studies should always observe the allocation procedure recommended by SETAC and ISO. Allocation on arbitrary bases must not be used.

2. LCA data-bases must be re-evaluated to ensure that they are based on the consistent approach to allocation. This should be carried out as part of a systematic harmonisation and quality-control of LCA data. It is recognised that this represents a major task. However, it is essential if the results of LCA studies are to be made fully representative, reliable and independent of the individual practitioner carrying out any study.

3. Further research should be carried out to determine how allocation by economic value can be implemented without introducing short-term variations in allocation in response to price volatility.

4. A linear approach in LCA has to be re-examined. The relationship between the burdens and functional units is not always linear and nonlinearities have to be introduced. Therefore, future research should concentrate on these aspects of system analysis in the LCA context.
5. Data bases must become more widely available and they must be coupled with the user-friendly, PC-based system modelling and optimisation software in order to make LCA less time consuming procedure requiring less resources.

8.4 Concluding Remarks

The methodology of Life Cycle Assessment is still developing. Amongst the issues which need to be resolved before it can become a widely accepted environmental management tool are:
- consistent approaches to allocation of burdens and impacts in multiple-function systems, and to modelling complex systems with constraints;
- system optimisation and systematic selection of the Best Practicable Environmental Option (BPEO).

It is hoped that the work in this dissertation has contributed to solving these problems, by demonstrating the practical value of the whole system modelling approach by multiobjective linear programming.
REFERENCES


References


References


References


References


References


APPENDIX 1

A1. Impact Assessment

A1.1 Problem-oriented Approach

This section gives an overview of the calculation procedure to estimate the contributions of burdens identified in the Inventory stage to the different impact categories (Heijungs et al., 1992). The numerical values of the classification factors used for calculating the impacts are given in Appendix 5.

A1.1.1 Abiotic Resource Depletion

Abiotic resource depletion (ARD) includes depletion of non-renewable resources, i.e. fossil fuels, metals and minerals. The effect score is calculated by:

\[ E_1 = \sum_{j=1}^{J} \frac{B_j}{e_{c_{1,j}}} \]  

(A1.1)

where \( B_j \) is the quantity of a resource used and \( e_{c_{1,j}} \) represents total estimated reserves of that resource.

A1.1.2 Global Warming Potential

Global warming potential (GWP) represents total emissions of the greenhouse gases, \( B_j \), (i.e. \( \text{CO}_2, \text{N}_2\text{O}, \text{CH}_4 \) and other VOCs) multiplied by their respective GWP factors, \( e_{c_{2,j}} \):

\[ E_2 = \sum_{j=1}^{J} e_{c_{2,j}} B_j \text{ (t)} \]  

(A1.2)

where \( B_j \) represents emission of greenhouse gas \( j \). GWP factors, \( e_{c_{2,j}} \), for different greenhouse gases are expressed relative to the global warming potential of \( \text{CO}_2 \), which is therefore defined to be unity. The values of GWP depend on the time horizon over which the global warming effect is assessed. GWP factors for shorter times (20 and 50 years) provide an indication of the short-term effects of greenhouse gases on the climate, while GWP for longer periods (100 and 500 years) are used to predict the cumulative effects of these gases on the global climate. GWP is often expressed as direct and indirect; the
former is related to the direct contribution of the greenhouse gases to global warming, while the latter includes GWP of CO, NOx and hydrocarbons (HCs). These gases cause tropospheric ozone formation, which also acts as a greenhouse gas.

A1.1.3 Ozone Depletion Potential

The ozone depletion potential (ODP) category indicates the potential of emissions of chlorofluorohydrocarbons (CFCs) and chlorinated HCs for depleting the ozone layer and is expressed by:

$$E_3 = \sum_{j=1}^{J} e_{3j} B_j \quad (t) \quad (A1.3)$$

where $B_j$ is the emission of ozone depleting gas $j$. The ODP factors $e_{3j}$ represent depletion potential of the emissions relative to the ozone depletion potential of CFC-11.

A1.1.4 Acidification Potential

Acidification potential (AP) is based on the contributions of $SO_2$, NOx, HCl, NH$_3$, and HF to the potential acid deposition, i.e. on their potential to form H$^+$ ions. AP is calculated according to the formula:

$$E_4 = \sum_{j=1}^{J} e_{4j} B_j \quad (t) \quad (A1.4)$$

where $e_{4j}$ represents the acidification potential of gas $j$ expressed relative to the AP of $SO_2$, and $B_j$ is its emission per functional unit.

A1.1.5 Nutrification Potential

Nutrification or Eutrophication potential (NP) is defined as the potential to cause over-fertilisation of water and soil, which can result in increased growth of biomass. It is calculated as:

$$E_5 = \sum_{j=1}^{J} e_{5j} B_j \quad (t) \quad (A1.5)$$

where $B_j$ are the emissions of species such as NOx, NH$_4^+$, N, PO$_4^{3-}$, P, and COD and $e_{5j}$ are their respective nutrification potentials. NP is expressed relative to PO$_4^{3-}$. 

A1-2
A1.1.6 Photochemical Oxidants Creation Potential

Photochemical oxidants creation potential (POCP), or Photochemical smog, is expressed relative to the POCP of ethylene and is calculated by:

\[ E_6 = \sum_{j=1}^{J} e_{6,j} B_j \quad (t) \]  
(A1.6)

Where \( B_j \) are the emissions of different contributory species, primarily VOCs, classified into the following categories: alkanes, halogenated HCs, alcohols, ketones, esters, ethers, olefins, acetylenes, aromatics and aldehydes; \( e_{6,j} \) are their respective classification factors for photochemical oxidation formation.

A1.1.7 Human Toxicity Potential

Human toxicity potential (HTP) is calculated by adding human toxic releases to three different media, i.e. air, water and soil:

\[ E_7 = \sum_{j=1}^{J} e_{7,jA} B_{jA} + \sum_{j=1}^{J} e_{7,jW} B_{jW} + \sum_{j=1}^{J} e_{7,jS} B_{jS} \quad (t) \]  
(A1.7)

Where \( e_{7,jA} \), \( e_{7,jW} \), and \( e_{7,jS} \) are human toxicological classification factors for the effects of the toxic emission to air, water and soil, respectively. \( B_{jA} \), \( B_{jW} \) and \( B_{jS} \) represent the respective emissions of different toxic substances into the three media. The toxicological factors are calculated using the acceptable daily intake or the tolerable daily intake of the toxic substances. The human toxicological factors are still at an early stage of development so that HTP can only be taken as an indication and not as an absolute measure of the toxicity potential.

A1.1.8 Ecotoxicity Potential

Ecotoxicity potential (EP) is divided into aquatic and terrestrial ecotoxicity, which are calculated as:

\[ E_{8A} = \sum_{j=1}^{J} e_{8,jA} B_{jA} \quad (m^3) \]  
(A1.8)
where $e_{c,j,t}$ and $e_{c,j,t}$ represent the ecotoxicity classification factors of different toxic substances and $B_{j,A}$ and $B_{j,T}$ are their respective emissions to the aquatic and terrestrial ecosystems. EP is based on the maximum tolerable concentrations of different toxic substances in water and soil. Similar to the HTP, classification factors for EP are still developing, so that EP can only be used as an indication of potential ecotoxicity.

A1.2 Medium-oriented Approach (Critical Volume)

The critical volume approach is based on the regulatory standards for the emissions of different substances to air and water and is calculated according to the formulae:

$$CVA = \sum_{j=1}^{J} \frac{B_{j,A}}{MC_A} \quad (m^3) \quad (A1.10)$$

$$CVW = \sum_{j=1}^{J} \frac{B_{j,w}}{MC_w} \quad (m^3) \quad (A1.11)$$

where $CVA$ and $CVW$ represent the volumes of air and water that would be necessary to dilute the emissions to such an extent that the maximum allowable concentration is not exceeded. $B_{j,A}$ and $B_{j,w}$ are the emissions to air and water and $MC_A$ and $MC_w$ are maximum allowable concentrations in these media.
A2.1 Duality in Linear Programming

Associated with every linear programming problem, called the "primal", is another linear programming problem, called the "dual" problem. It is possible to use the dual LP problem to obtain a solution to the primal one. If a primal problem is defined as:

Max \( F = c_1x_1 + c_2x_2 + \ldots + c_1x_1 \)
subject to
\[ a_{11}x_1 + a_{12}x_2 + \ldots + a_{11}x_1 \leq e_1 \]
\[ a_{21}x_1 + a_{22}x_2 + \ldots + a_{21}x_1 \leq e_2 \]
\[ \vdots \]
\[ a_{j1}x_1 + a_{j2}x_2 + \ldots + a_{j1}x_1 \leq e_j \] (A2.1)

then its corresponding dual problem is created as follows:

Min \( Z = e_1\lambda_1 + e_2\lambda_2 + \ldots + e_j\lambda_j \)
subject to
\[ a_{11}\lambda_1 + a_{21}\lambda_2 + \ldots + a_{j1}\lambda_j \geq c_1 \]
\[ a_{12}\lambda_1 + a_{22}\lambda_2 + \ldots + a_{j2}\lambda_j \geq c_2 \]
\[ \vdots \]
\[ a_{1j}\lambda_1 + a_{2j}\lambda_2 + \ldots + a_{jj}\lambda_j \geq c_i \] (A2.2)

The objective function is now being minimised instead of maximised and its coefficients are the right-hand sides of the primal problem. The constraints of the dual are formed by transposing coefficients in the constraints of the primal model and changing the direction of inequalities. The important theorem associated with the primal and dual models in LP is the Duality Theorem which states that:

**Duality Theorem:** If feasible solutions to the primal and dual system exist, there exists an optimum solution for both systems and \( \text{Min } Z = \text{Max } F \).

Another related theorem states that:

**Complementary Slackness Theorem:** If \( (x_1^*, \ldots, x_j^*, F^*) \) is a feasible solution to the primal and \( (\lambda_1^*, \ldots, \lambda_j^*, Z^*) \) is a feasible solution to the dual and the following is true for the dual:
Appendix 2

\[ c_i^* = c_i - \sum_{j=1}^{J} \lambda_j^* a_{ji} \geq 0 \text{ and } \sum_{j=1}^{J} \lambda_j^* e_j = Z^* \] (A2.3)

\( \forall i = 1, 2, \ldots, I, \) then a necessary and sufficient condition for optimality of both solutions is:

\[ c_i^* = 0 \text{ for } x_i^* > 0 \] (A2.4)

This theorem is important for the complementary slackness in the primal and dual systems. It can be restated in the form: every slack variable, \( x_{n+i} \), of the primal and its corresponding dual variable must satisfy the following relationship:

\[ \lambda_j x_{n+i} = 0 \] (A2.5)

which requires that \( \lambda_j = 0 \) if \( x_{n+i} > 0 \) and \( x_{n+i} = 0 \) if \( \lambda_j = 0 \). The same is true for the slack of the dual and its corresponding variable in the primal.

For the proof of the Duality and Complementary Slackness Theorems see Dantzig (1963).

A2.2 The Kuhn-Tucker Conditions

The Kuhn-Tucker conditions (Kuhn and Tucker, 1951) are important for finding noninferior solutions in Multiobjective LP (MOLP). In this section the Kuhn-Tucker conditions for single objective LP models will be presented. Section A2.3 then introduces these conditions for MOLP. For a more detailed account, see Cohon (1978).

The LP model, defined by:

Maximise

\[ F = \sum_{i=1}^{I} c_i x_i \]

subject to

\[ \sum_{i=1}^{I} a_{ji} x_i \leq e_j \quad j = 1, 2, \ldots, J \] (A2.6)

and

\[ x_i \geq 0 \]
can, for simplicity, be expressed as:

Maximise $F(x)$

subject to $f_j(x) \leq e_j \quad j = 1, 2, ..., J$ (A2.7)

and $x \in X$

where $f_j(x)$ includes the non-negativity restriction, $x$ is the $I$-dimensional vector of decision variables and $X$ is the feasible decision region. The Kuhn-Tucker conditions state that if $x^*$ is an optimal solution to (A2.7), then there exist multipliers $\lambda_j \geq 0$, $\forall j = 1, 2, ..., J$ and:

$$x^* \in X \quad (A2.8)$$

$$\lambda_j f_j(x^*) = 0 \quad j = 1, 2, ..., J \quad (A2.9)$$

$$\nabla F(x^*) - \sum_{j=1}^{J} \lambda_j \nabla f_j(x^*) = 0 \quad (A2.10)$$

Equation (A2.8) requires $x^*$ to be feasible, and (A2.9) is a complementary slackness statement. Equation (A2.10) correlates the gradient of the objective functions at $x^*$ with the negative of the gradients of the binding constraints at $x^*$, where:

$$\nabla = \left[ \frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, ..., \frac{\partial}{\partial x_I} \right] \quad (A2.11)$$

The condition (A2.10) implies that movement from $x^*$ along any direction that increases the value of the objective function must be infeasible, so that the direction of improvement must be opposite from the direction of feasibility. Thus, $-\nabla f_j(x)$ points toward feasibility (the minus sign is because of "\leq" constraints in (A2.7)). For an optimum solution of a linear programming problem conditions (A2.8)-(A2.10) are necessary and sufficient.

**A2.3 Kuhn-Tucker Conditions for Noninferior Solutions in MOLP**

The multiobjective linear programming problem can be written as:
Maximise

\[
F(x) = [F_1(x), F_2(x), ..., F_Q(x)]
\]

subject to

\[
f_j(x) \leq \epsilon_j \quad j = 1, 2, ..., J
\]

and

\[
x \in X
\]

where the objective function is now a Q-dimensional vector. The Kuhn-Tucker conditions for noninferiority in MOLP are then:

\[
x^* \in X
\]

\[
\lambda_j f_j(x^*) = 0 \quad j = 1, 2, ..., J
\]

\[
\sum_{q=1}^{Q} w_q \nabla F_q(x^*) - \sum_{j=1}^{J} \lambda_j \nabla f_j(x^*) = 0
\]

where \( w_q \) and \( \lambda_j \) are the multipliers such that \( w_q \geq 0 \) and \( \lambda_j \geq 0 \). These conditions differ from the conditions for the scalar or single-objective optimisation only in the last condition. The first term in (A2.10) has been replaced by a non-negative linear combination of the gradients of the Q objective functions. The conditions (A2.13)-(A2.15) are necessary for noninferiority. They are also sufficient if the \( F_q(x) \) are concave for \( q = 1, 2, ..., Q \); \( X \) is convex and \( w_q > 0 \), \( \forall q \). For a fuller explanation see Cohon (1978).

### A2.4 The Kuhn-Tucker Conditions and the Constraint Method in MOLP

The third Kuhn-Tucker condition in (A2.15) can be rewritten as:

\[
w_h F_h(x^*) + \sum_{q=1, q \neq h}^{Q} w_q \nabla F_q(x^*) - \sum_{j=1}^{J} \lambda_j \nabla f_j(x^*) = 0
\]

where \( x^* \) is the noninferior solution, \( w_q \geq 0 \), \( q = 1, 2, ..., Q \) and \( \lambda_j \geq 0 \), \( \forall j = 1, 2, ..., J \). If \( w_h > 0 \), then (A2.16) can be interpreted as a third condition for optimality of the single-objective problem:
Appendix 2

Maximise \( w_h F_h(x) \)

subject to
\[
f_j(x) \leq e_j \quad j = 1, 2, \ldots, J
\]

and
\[
F_q(x) \geq \varepsilon_q \quad q = 1, 2, \ldots, h-1, h+1, \ldots, Q
\]

Since the solution of the LP problem does not change if the objective function is divided by a positive number (and the assumption is that \( w_h > 0 \)), \( w_h F_h(x) \) can be divided by \( w_h \) so that the problem in (A2.17) reduces to:

Maximise \( F_h(x) \)

subject to
\[
f_j(x) \leq e_j \quad j = 1, 2, \ldots, J
\]

and
\[
F_q(x) \geq \varepsilon_q \quad q = 1, 2, \ldots, h-1, h+1, \ldots, Q
\]

The set of equations (A2.18) provides a theoretical basis for the constraint method.

A2.5 Algorithm for the Constraint Method in MOLP (Cohon, 1978)

Step 1: Pay-off table
1. Solve \( Q \) single-objective optimisation problems to find the optimal solution for each of the \( Q \) objectives. Optimal solution for the \( q \)th objective is denoted as \( x^q = (x_1^q, x_2^q, \ldots, x_i^q) \).
2. Compute the value of each objective at each of the \( Q \) optimal solution: \( F_1(x^q), F_2(x^q), \ldots, F_Q(x^q), q = 1, 2, \ldots, Q \). This gives \( Q \) values for each of the \( Q \) objectives.
3. Construct a payoff table with rows corresponding to \( x^1, x^2, \ldots, x^Q \) and the columns equal to the number of objectives (Table A2.1).
4. Identify the largest and the smallest numbers in the \( q \)th column and denote them by \( M_q \) and \( n_q \) respectively. Repeat for \( q = 1, 2, \ldots, Q \).

\[\begin{array}{|c|c|c|c|c|}
\hline
& F_1(x^1) & F_2(x^1) & \cdots & F_Q(x^1) \\
\hline
x^1 & F_1(x^1) & F_2(x^1) & \cdots & F_Q(x^1) \\
\hline
x^2 & F_1(x^1) & F_2(x^2) & \cdots & F_Q(x^2) \\
\hline
\cdots & \cdots & \cdots & \cdots & \cdots \\
\hline
x^Q & F_1(x^Q) & F_2(x^Q) & \cdots & F_Q(x^Q) \\
\hline
\end{array}\]

Table A2.1 Pay-off table for a multiobjective problem
Appendix 2

Step 2: Constraints
Convert a MOLP problem, such as (A2.12), to its corresponding constrained problem (A2.18).

Step 3: Right-hand side coefficients
The $\varepsilon_{\text{min}}$ and $\varepsilon_{\text{max}}$ represent the upper and lower bounds for the $q$th objective: $\varepsilon_{\text{min}} \leq \varepsilon_q \leq \varepsilon_{\text{max}}$. Choose the number of different values of $\varepsilon_q$ and denote them by $r$.

Step 4: Optimisation
Solve the constrained problem in Step 2 for every combination of values for the $\varepsilon_q$, $q=1,2,\ldots,h-1,h+1,\ldots,Q$, where:

$$\varepsilon_q = n_q + \left[\frac{t}{(r-1)}\right](M_q - n_q), \quad t = 0,1,2,\ldots,(r-1) \quad (A2.19)$$
APPENDIX 4a

A4.1 Single Decision-Maker Problems

A4.1.1 Techniques Without Preferences - Generating Techniques

Techniques without preferences do not require preferences to be articulated by decision-makers prior to identification of the noninferior set of solutions; the articulation of preferences is deferred until the range of choice is identified. Different techniques are used to generate noninferior solutions from a previously formulated multiobjective problem. This appendix concentrates on the methods used in multiobjective programming and some of these are briefly reviewed below.

A4.1.1.1 The Constraint Method

In the constraint method, all objectives but one are converted into the constraints and the problem is then optimised on one objective function only in order to generate the noninferior solutions. The values of the constrained objectives are varied systematically until all noninferior solutions are generated. In general, a multiobjective problem with Q objectives can be expressed as:

Maximise \[ F(x) = [F_1(x), F_2(x), \ldots, F_Q(x)] \] (A4.1)

subject to \[ f_j(x) \leq e_j \quad j=1,2,\ldots,J \] (A4.1a)

and \[ x \in X \] (A4.1b)

where \( f_j(x) \) includes the nonnegativity restriction, \( x \) is the \( I \)-dimensional vector of decision variables and \( X \) is the feasible decision region. In the constraint method, the problem is transformed into:

Maximise \[ F_h(x) \] (A4.2)

subject to \[ f_j(x) \leq e_j \quad j=1,2,\ldots,J \] (A4.2a)

and \[ F_q(x) \geq \varepsilon_q \quad q=1,2,\ldots,h-1,h+1,\ldots,Q \] (A4.2b)
where the \( h \)th objective is arbitrarily chosen for maximisation, and all other objective functions of the problem are converted into constraints. In other words, the multiobjective linear programming problem is transformed into a single objective problem, which can be solved by using, for instance, the simplex method for linear problems. For more details on this method see Chapter 2 and Appendix 2.

**A4.1.1.2 The Weighting Method**

Weighting the objectives to obtain noninferior solutions is the oldest multiobjective solution technique. It is based on attaching weights to the objective functions, which is equivalent to the identification of a desirable trade-off between the objectives. In general, for a multiobjective problem all objectives can be weighted by attaching different weights, \( w_j \), to them, so that the problem (A4.1) becomes:

\[
\text{Maximise} \quad F(x, w) = w_1 F_1(x) + w_2 F_2(x) + \cdots + w_Q F_Q(x) \quad (A4.3)
\]

subject to (A4.1a)-(A4.1b).

The multiobjective problem is, therefore, converted into a single objective problem and the noninferior solutions can then be generated by using well known techniques for LP, such as the simplex method. The set of noninferior solutions is generated by arbitrarily changing the weights attached to the objectives. To ensure that the optimal solution of the weighted problem (A4.4) is a noninferior solution of the multiobjective problem (A4.1), the weights have to be nonnegative; if they are negative then this is equivalent to transforming the original maximisation problem to a minimisation one. One of the possible problems with this method, however, is that some of the noninferior solutions can be missed if the incremental changes of the weights are too high: the resulting solutions may still be feasible, but they may not necessarily be noninferior. The weighting and the constraint methods are related in such a way that the marginal values of the objectives converted to the constraints in the constraint method can be used as the weights to obtain the same noninferior solution by the weighting method.

**A4.1.1.3 The Noninferior Set Estimation (NISE) Method**

The NISE method (Cohon, 1978) is based on finding a number of noninferior extreme points and evaluating the properties of the line segments between them: if the line segment is noninferior, then moving in a direction out from the segment is infeasible; if
the line segment is inferior, then there are noninferior points in the outward direction. Noninferior points in this method are found by using the weighting method, with the values chosen so that the next noninferior point is the feasible solution farthest out in a direction normal to the line segment connecting two adjacent noninferior solutions.

A4.1.4 Multiobjective Simplex Method

The multiobjective simplex method generates the exact representation of the noninferior set by using the simplex method. In this method all objectives are optimised simultaneously, without converting the multiobjective problem into a single objective one. This is done by moving mathematically from one extreme noninferior point to adjacent noninferior extreme points until all noninferior solutions have been found. The multiobjective simplex method is a complex mathematical problem that is not yet entirely solved.

A4.1.2 Techniques with preferences

In the techniques with preferences, the best compromise solution is identified by articulation of decision-makers' preferences prior or during the analysis. In general, these techniques can be noniterative or iterative, and some of them are presented in the following sections.

A4.1.2.1 Non-iterative Methods

A4.1.2.1.1 Multiattribute Utility Function

A utility function is a mathematical function that associates a single number, called utility, with each alternative so that all alternatives may be ordered. For instance, if there are two alternatives, A and B, the utility function enables expressing preference of A over B, B over A or indifference between A and B. An ordinal utility function enables alternatives to be ranked in order of preference, but does not indicate the degree to which one alternative is preferred to another. A cardinal utility function, in addition to the order of preference, also indicates a level of preference, e.g. A is preferred to B and is 5 units of utility more desirable than B. In the rest of this section, only cardinal utility functions will be dealt with so that they will be referred to simply as utility functions.

Utility functions are used to express the preferences of a decision-maker for various objectives or attributes, as they are often called in the utility theory literature. Where there are many attributes or objectives of an alternative, the utility function has multiple
arguments and is referred to as a multiattribute utility function. Every alternative implies a value for each objective or attribute, and a multiattribute utility function associates a single number, a utility, with the combination of values for the objectives. The general multiobjective problem given by (A4.1)-(A4.1b) is converted into:

Maximise \[ U(F(x)) = U[F_1(x), F_2(x), \ldots, F_Q(x)] \] (A4.4)

subject to (A4.1a) - (A4.1b). Equation (A4.4) represents a general form of a multiattribute utility function where the objectives \( F \), which are the arguments of the utility function, are themselves functions of the decision variables \( x \). The utility function translates the values of \( Q \) objectives into a single number that represents the utility or degree of preference which that combination of objectives yields. The utility function \( U \) can be of many forms. The common form assumes additivity and linearity in the utility function. Additivity means that the utility function is additively separable with respect to the objectives, so that it has the form:

\[ U = \sum_{q=1}^{Q} U_q F_q \] (A4.5)

Additivity of the utility function implies that the rate of trade-off between two attributes may depend upon the values of those two attributes, but will not depend upon the values of other attributes. Linearity is a special case of additivity. It implies that each attribute can be quantified in terms of a common scale of measurement, say money, and that a rate of trade-off between two attributes is constant.

Determination of the utility function in a multiobjective problem is not an easy task. A number of methods for assessing the utility function have been developed over the past years and some of them include simple additive weighting, weighted product, median ranking method, the Analytic Hierarchy Process (AHP), Multiattribute Utility Theory (MAUT) etc. (Yoon and Hwang, 1995). The choice of the method will depend on a given multiattribute situation and sophistication of decision-makers. Some comparative studies have shown that results obtained by using different methods are quite comparable, demonstrating that the choice of the method for assessing the multiattribute
utility function is not crucial for the outcome of the process; what matters, however, is the generation of appropriate attributes.

A4.1.2.1.2 Methods Based on Geometrical Definition of "Best" - Goal Programming

These methods are based on identifying an ideal solution, in a real situation almost always unattainable, and then defining a maximum acceptable distance from that solution. Different mathematical methods can be applied to find the feasible solution that is closest to the ideal solution, one of which is Goal programming.

Goal programming is probably the best known multiobjective method, and is often confused with multiobjective linear programming, which it clearly is not. The method requires decision-makers to set goals for each objective that they want to attain. A preferred solution is then defined as the one which minimises the deviations from the set goals. If the goal for the qth objective is \( G_q \), the goal programming problem is to minimise the distance, \( d \), from the goals:

\[
\text{Minimise } \quad d = \sum_{q=1}^{Q} |G_q - F_q(x)| \quad (A4.6)
\]

subject to (A4.1a) - (A4.1b). Formulation (A4.6) is equivalent to:

\[
\text{Minimise } \quad \sum_{q=1}^{Q} (d_q^+ + d_q^-) \quad (A4.7)
\]

where

\[
G_q - F_q(x) = d_q^- - d_q^+ \quad (A4.7a)
\]

and

\[
d_q^-, d_q^+ \geq 0 \quad (A4.7b)
\]

subject to (A4.1a) - (A4.1b). Since both positive and negative deviations, \( d_q^+ \) and \( d_q^- \), are being minimised, they can never both be nonzero, i.e. one of them will always be zero.

There is a number of modifications of the goal programming method defined by (A4.6). One of these attaches weights to the objectives to indicate their relative importance, which is equivalent to attaching weights to the positive and negative deviations related to these functions, i.e.:
Minimise \[ \sum_{q=1}^{Q} (w_q^+d_q^+ + w_q^-d_q^-) \] \hspace{1cm} (A4.8)

with the constraints (A4.1a) - (A4.1b) and (A4.7a)-(A4.7b) unchanged. Whichever the method, however, the sensitivity analysis should be conducted on the goals and the weights to examine the change in the solution as the decision parameters change.

A4.1.2.2 Iterative methods

A4.1.2.2.1 Step method

The step method is based on the geometric notion of "best", i.e. the minimum distance from an ideal solution, with modification of this criterion derived from a decision maker's reactions to a generated solution. The method begins with the construction of a pay-off table, similar to Table A2.1 for multiobjective LP, given in Appendix 2. The table is constructed by optimising (taken to be maximising here) each objective individually, where the maximum of objective \( q \) is \( M_q \). The basic problem in the step method is then to minimise the distance, \( d \), of a solution from the ideal solution:

Minimise \[ d \] \hspace{1cm} (A4.9)

subject to \[ \pi_q [M_q - F_q] - d \leq 0, \quad d \geq 0 \] \hspace{1cm} (A4.10)

and (A4.1a) - (A4.1b). The pay-off table is used to develop weights \( \pi \) on \( d \) (see Cohon, 1978 and Hwang, 1980). After the weights have been found, the original problem given by (A4.9) and (A4.10) is solved and the solution is shown to the decision-makers. If they are satisfied with the solution, then the process is terminated; if not new goals are set and the whole procedure is repeated until the "best" solution is found.

A4.1.2.2.2 Local Approximation of Multiattribute Utility Function

This iterative technique assumes an underlying utility function that is approximated locally as the algorithm proceeds. The optimum solutions are found by moving from an initial feasible solution towards the optimum solution by following directions of steepest ascent, i.e. directions that provide the maximum rate of increase in the objective function. The algorithm is divided into two parts: determination of the best direction and the step size along that direction. Both parts of the problem require involvement of the decision maker, and the procedure is repeated iteratively until a satisfactory solution has been found.
Appendix 4

A4.2 Multiple Decision-Maker Problems

A.4.2.1 Aggregating Techniques

The aggregating techniques for multiple decision-making problems are attributed to modern welfare economists and development of a so-called "social-welfare function". The social-welfare function represents an explicit expression of the optimal social state, obtained by aggregating individual utilities. For instance, if there are \( p \) individuals and \( n \) commodities, then the social-welfare function can be expressed as:

\[
W[U_1(q_1), U_2(q_2), \ldots, U_p(q_p)]
\]

where \( q_p = (q_{p1}, q_{p2}, \ldots, q_{pn}) \) is an \( n \)-dimensional commodity consumption vector of an individual \( p \). The basic element of this function is the utility or satisfaction \( U_p(q_p) \) of each member of society realised as a result of consumption of the commodities. Mathematically, the social-welfare function is similar to multiattribute utility function, and can be approximated by the weighted sum of utilities, i.e.:

\[
\sum_{p=1}^{p} \alpha_p U_p(q_p)
\]

in which relative weights \( \alpha_p \) can be different for different individuals. This approach is very similar to that of the old welfare economists, except that they assumed that the weights \( \alpha \) were the same for all individuals. Thus, the aggregating techniques assume that measurability and comparability of the utilities are possible and that individual preferences can be added to arrive at a social-welfare function.

A.4.2.2 Counselling Techniques

The counselling approach to multiple decision-making problems is based on the view that society's preferences cannot be derived from an aggregation of individual preferences.
Instead, the identification of best compromise solutions is based on preferences that public decision-makers articulate, as advised by the interest groups they represent. Depending on the context of decision-making process, some methods used in single decision-making problems, i.e. preference based methods or generating techniques, could be applied to identify the best compromise solution in these situations.

A.4.2.3 Predicting Outcomes

This decision-making context is relevant in situations where each interest group wants to gain as much of their own interests as possible. The goal of these methods is a prediction of the decision which will be chosen, given the power and bargaining abilities of the participants. There are many methods that can be used for predicting the outcome of a decision-making process; one of the most widely used is game theory, which is briefly explained below.

A4.2.3.1 Game Theory

The basis of game theory is the interaction between the participants in the decision-making situation: the utility gained by a decision-maker depends on the nature of these interactions. The easiest way to explain this concept is to consider a so-called two-person zero-sum game, i.e. a game in which a gain to one player means a loss to another. In this case, there are only two players and each of them has a set of strategies to play a game. However, neither of them knows in advance which strategy the other will choose. For instance, consider the game of "odds and evens" in which each player can put out one or two fingers. Each player, X and Y, has two options, as follows:

- X1: Player X puts out one finger
- X2: Player X puts out two fingers
- Y1: Player Y puts out one finger
- Y2: Player Y puts out two fingers

Each pair of strategies results in a gain of £1 to one player and loss of £1 to another. If X wins on odds and Y on evens, the possible combinations are shown in the pay-off table A4.1. If player X selects option X1 and Y selects option Y1, then player X looses £1 and player Y gains £1 because the total sum is even.
Table A4.1 Pay-off table for the two-person zero-sum game

<table>
<thead>
<tr>
<th>Player Y</th>
<th>Y₁</th>
<th>Y₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>X</td>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

The concept of game theory can also be applied in situations where there are more than two players and more than two options. In these situations, as opposed to zero-sum games, the gains and losses of the players are not necessarily the same. The strategy for choosing the best option in these situations is that of maximin: player X should choose a strategy that will be best in the worst situation, i.e. the strategy that will maximise the minimum pay-off to X.
APPENDIX 4b

A4.3 Printout of the LP program for the example of the polymers system

LET R=70 ! No of rows
LET C=35 ! No of columns
LET N=5 ! No of cost coefficients

VARIABLES
x(C)

TABLES
COE(R, C)
COST(N)

DATA
COST (1) = 550,780,500,500,865

CONNECT excel
DISKDATA -c
COE=c:\polmatr4.xls (B5: AH75) ! Reads data from Table A4.2; B5:B75=x1 to AF5:AF75=x33

DISCONNECT

CONSTRAINTS

pp: \[
\text{SUM}(n=1; m=1:C) \ COE(1,m)\times(m) < 416000
\]
propylene: \[
\text{SUM}(n=2; m=1:C) \ COE(2,m)\times(m) = 0
\]
steam: \[
\text{SUM}(n=3; m=1:C) \ COE(3,m)\times(m) = 0
\]
naphtha: \[
\text{SUM}(n=4; m=1:C) \ COE(n,m)\times(m) = 0
\]
gas: \[
\text{SUM}(n=5; m=1:C) \ COE(n,m)\times(m) = 0
\]
 crude oil: \[
\text{SUM}(n=6; m=1:C) \ COE(n,m)\times(m) = 0
\]
nahoh: \[
\text{SUM}(n=7; m=1:C) \ COE(n,m)\times(m) = 0
\]
eps: \[
\text{SUM}(n=8; m=1:C) \ COE(n,m)\times(m) < 33000
\]
ps: \[
\text{SUM}(n=9; m=1:C) \ COE(n,m)\times(m) = 0
\]
styrene: \[
\text{SUM}(n=10; m=1:C) \ COE(n,m)\times(m) = 0
\]
aromatics: \[
\text{SUM}(n=11; m=1:C) \ COE(n,m)\times(m) = 0
\]
ethylen: \[
\text{SUM}(n=12; m=1:C) \ COE(n,m)\times(m) = 0
\]
na2co3: \[
\text{SUM}(n=13; m=1:C) \ COE(n,m)\times(m) = 0
\]
naci: \[
\text{SUM}(n=14; m=1:C) \ COE(n,m)\times(m) = 0
\]
caco3: \[
\text{SUM}(n=15; m=1:C) \ COE(n,m)\times(m) = 0
\]
coa: \[
\text{SUM}(n=16; m=1:C) \ COE(n,m)\times(m) = 0
\]
hips: \[
\text{SUM}(n=17; m=1:C) \ COE(n,m)\times(m) < 85000
\]
polybuta: \[
\text{SUM}(n=18; m=1:C) \ COE(n,m)\times(m) = 0
\]
hdpe: \[
\text{SUM}(n=19; m=1:C) \ COE(n,m)\times(m) < 32000
\]
lde: \[
\text{SUM}(n=20; m=1:C) \ COE(n,m)\times(m) < 550000
\]
toluene: \[
\text{SUM}(n=21; m=1:C) \ COE(n,m)\times(m) = 0
\]
electricity: \[
\text{SUM}(n=22; m=1:C) \ COE(n,m)\times(m) = 0
\]
heat: \[
\text{SUM}(n=23; m=1:C) \ COE(n,m)\times(m) = 0
\]
extrac oil: \[
\text{SUM}(n=24; m=1:C) \ COE(n,m)\times(m) = 0
\]
fuel oil: \[
\text{SUM}(n=25; m=1:C) \ COE(n,m)\times(m) = 0
\]
othercrack: \[
\text{SUM}(n=26; m=1:C) \ COE(n,m)\times(m) = 0
\]
gas extrac: \[
\text{SUM}(n=27; m=1:C) \ COE(n,m)\times(m) = 0
\]
coal extrac: \[
\text{SUM}(n=28; m=1:C) \ COE(n,m)\times(m) = 0
\]
Appendix 4

*************OBJECTIVE FUNCTIONS******************************
PR: COST(1)*x(1) + COST(2)*x(9) + COST(3)*x(28) + &
COST(4)*x(29) + COST(5)*x(27) $
GWP: x(34) + 11*x(35) $

*************x(1)=PP; x(9)=EPS; x(28)=HDPE; x(29)=LDPE; x(27)=HIPS*****

*************x(34)=CO2; x(35)=VOC
ne: SUM(n=29:29, m=1:C) COE(n, m)*x(m) $ 
he: SUM(n=30:30, m=1:C) COE(n, m)*x(m) $ 
coalres: SUM(n=31:31, m=1:C) COE(n, m)*x(m) $ 
oilres: SUM(n=32:32, m=1:C) COE(n, m)*x(m) $ 
gasres: SUM(n=33:33, m=1:C) COE(n, m)*x(m) $ 
othnonren: SUM(n=34:34, m=1:C) COE(n, m)*x(m) $ 
renewres: SUM(n=35:35, m=1:C) COE(n, m)*x(m) $ 
ancilaries: SUM(n=36:36, m=1:C) COE(n, m)*x(m) $ 
water: SUM(n=37:37, m=1:C) COE(n, m)*x(m) $ 
air: SUM(n=38:38, m=1:C) COE(n, m)*x(m) $ 
other: SUM(n=39:39, m=1:C) COE(n, m)*x(m) $ 
 wastewater: SUM(n=40:40, m=1:C) COE(n, m)*x(m) $ 
co: SUM(n=41:41, m=1:C) COE(n, m)*x(m) $ 
co2: SUM(n=42:42, m=1:C) COE(n, m)*x(m) $ 
co2renew: SUM(n=43:43, m=1:C) COE(n, m)*x(m) $ 
nox: SUM(n=44:44, m=1:C) COE(n, m)*x(m) $ 
s02: SUM(n=45:45, m=1:C) COE(n, m)*x(m) $ 
voc: SUM(n=46:46, m=1:C) COE(n, m)*x(m) $ 
dust: SUM(n=47:47, m=1:C) COE(n, m)*x(m) $ 
halide: SUM(n=48:48, m=1:C) COE(n, m)*x(m) $ 
otherair: SUM(n=49:49, m=1:C) COE(n, m)*x(m) $ 
oils: SUM(n=50:50, m=1:C) COE(n, m)*x(m) $ 
heavymet: SUM(n=51:51, m=1:C) COE(n, m)*x(m) $ 
landwght: SUM(n=52:52, m=1:C) COE(n, m)*x(m) $ 
othopnlo: SUM(n=53:53, m=1:C) COE(n, m)*x(m) $ 
otherox: SUM(n=54:54, m=1:C) COE(n, m)*x(m) $ 
othwater: SUM(n=55:55, m=1:C) COE(n, m)*x(m) $ 
tds: SUM(n=56:56, m=1:C) COE(n, m)*x(m) $ 
tss: SUM(n=57:57, m=1:C) COE(n, m)*x(m) $ 
cod: SUM(n=58:58, m=1:C) COE(n, m)*x(m) $ 
bod: SUM(n=59:59, m=1:C) COE(n, m)*x(m) $ 
landvlum: SUM(n=60:60, m=1:C) COE(n, m)*x(m) $ 
specwast: SUM(n=61:61, m=1:C) COE(n, m)*x(m) $ 
solidind: SUM(n=62:62, m=1:C) COE(n, m)*x(m) $ 
steamcra: SUM(n=63:63, m=1:C) COE(n, m)*x(m) = 0 
oilref: SUM(n=64:64, m=1:C) COE(n, m)*x(m) = 0 
oilaval: SUM(n=65:65, m=1:C) COE(n, m)*x(m) = 1000000

!Burdens used to calculate GWP
co2: SUM(n=42:42, m=1:C) COE(n, m)*x(m) - x(34) = 0
voc: SUM(n=46:46, m=1:C) COE(n, m)*x(m) - x(35) = 0

GENERATE
### Table A4.2 LP matrix for the example of the polymers system

<table>
<thead>
<tr>
<th>m⇒ n</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>x₁</strong></td>
</tr>
<tr>
<td>1 PP</td>
</tr>
<tr>
<td>2 Propylene</td>
</tr>
<tr>
<td>3 Steam</td>
</tr>
<tr>
<td>4 Naphtha</td>
</tr>
<tr>
<td>5 Gas</td>
</tr>
<tr>
<td>6 Crude oil</td>
</tr>
<tr>
<td>7 NaOH</td>
</tr>
<tr>
<td>8 EPS</td>
</tr>
<tr>
<td>9 PS</td>
</tr>
<tr>
<td>10 Styrene</td>
</tr>
<tr>
<td>11 Aromatics</td>
</tr>
<tr>
<td>12 Ethylene</td>
</tr>
<tr>
<td>13 Na2CO3</td>
</tr>
<tr>
<td>14 NaCl</td>
</tr>
<tr>
<td>15 CaCO3</td>
</tr>
<tr>
<td>16 Coal</td>
</tr>
<tr>
<td>17 HIPS</td>
</tr>
<tr>
<td>18 Polybutad.</td>
</tr>
<tr>
<td>19 HDPE</td>
</tr>
<tr>
<td>20 LDPE</td>
</tr>
<tr>
<td>21 Toluene</td>
</tr>
<tr>
<td>22 Electricity</td>
</tr>
<tr>
<td>23 Heat</td>
</tr>
<tr>
<td>24 Extrac oil</td>
</tr>
</tbody>
</table>

**Notes:**
- The matrix represents the LP model for the polymers system.
- Each row corresponds to a variable, and each column corresponds to an input.
- The entries represent the coefficients of the variables in the objective function or constraints.

*Appendix 4*
Table A4.2 Continued

<table>
<thead>
<tr>
<th>m =&gt;</th>
<th>x₁</th>
<th>x₂</th>
<th>x₃</th>
<th>x₄</th>
<th>x₅</th>
<th>x₆</th>
<th>x₇</th>
<th>x₈</th>
<th>x₉</th>
<th>x₁₀</th>
<th>x₁₁</th>
<th>x₁₂</th>
<th>x₁₃</th>
<th>x₁₄</th>
<th>x₁₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>Other crack.</td>
<td>0.386</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.00178</td>
<td></td>
<td>-0.169</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>Gas Extrac.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>Coal Extrac.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>Nucl. Elec.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>Hydro Elec.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>Coal reserv.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>Oil reserv.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-1.034</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>Gas reserv.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>Oth. Nonren.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-1.2</td>
</tr>
<tr>
<td>35</td>
<td>Renew. Res.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>Ancilaries</td>
<td>-0.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>Water</td>
<td>-100.4</td>
<td>-69.45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-4.1996</td>
<td>-5.3998</td>
<td></td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>Air</td>
<td>-0.087</td>
<td>-0.863</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.13494</td>
<td>-4.93E-3</td>
<td>-0.6114</td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>Other</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>Waste Water</td>
<td>100.74</td>
<td>69.45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.1979</td>
<td>5.4121</td>
<td>66</td>
<td>9.538</td>
</tr>
<tr>
<td>41</td>
<td>CO</td>
<td>3.60E-05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.10E-06</td>
<td>1.47E-05</td>
<td>7.30E-05</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>CO₂</td>
<td>1.186</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.20031</td>
<td>8.05E-03</td>
<td>0.0735</td>
<td>0.20</td>
</tr>
<tr>
<td>43</td>
<td>CO₂ Renew.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>NOₓ</td>
<td>8.5E-5</td>
<td>7.0E-03</td>
<td>4.71E-04</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.48E-04</td>
<td>2.40E-05</td>
<td>2.02E-04</td>
<td>6.0E-5</td>
</tr>
<tr>
<td>45</td>
<td>SO₂</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.50E-03</td>
<td></td>
<td>1.42E-03</td>
<td></td>
</tr>
<tr>
<td>46</td>
<td>VOC</td>
<td>8.0E-4</td>
<td>9.70E-05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8.53E-04</td>
<td>7.74E-07</td>
<td>2.01E-04</td>
<td>0.0412</td>
</tr>
<tr>
<td>47</td>
<td>Dust</td>
<td>1.0E-5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.025</td>
</tr>
</tbody>
</table>

Appendix 4
### Table A4.2 Continued

<table>
<thead>
<tr>
<th>m⇒</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( x_4 )</th>
<th>( x_5 )</th>
<th>( x_6 )</th>
<th>( x_7 )</th>
<th>( x_8 )</th>
<th>( x_9 )</th>
<th>( x_{10} )</th>
<th>( x_{11} )</th>
<th>( x_{12} )</th>
<th>( x_{13} )</th>
<th>( x_{14} )</th>
<th>( x_{15} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n \downarrow )</td>
<td>PP</td>
<td>Propyl.</td>
<td>Stm. Crac.</td>
<td>Naphtha</td>
<td>Fuel oil</td>
<td>Oil Ref.</td>
<td>Oil distr.</td>
<td>Oil well</td>
<td>EPS</td>
<td>PS</td>
<td>Styrene</td>
<td>Aromatics</td>
<td>Ethylene</td>
<td>NaOH</td>
<td>Na2C</td>
</tr>
<tr>
<td>48</td>
<td>Halide</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>49</td>
<td>Other Air</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>Oils</td>
<td>3.60E-05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>Heavy Metal</td>
<td>9.94E-06</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>52</td>
<td>Land Wght.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>53</td>
<td>Oth.Open L.</td>
<td>0.107</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>54</td>
<td>Other Out.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>Other Water</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>56</td>
<td>TDS</td>
<td>4.40E-04</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>57</td>
<td>TSS</td>
<td>1.00E-05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>58</td>
<td>COD</td>
<td>5.90E-04</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>59</td>
<td>BOD</td>
<td>1.90E-04</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>Land Volum.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>61</td>
<td>Spec. Waste</td>
<td>0.0001</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>62</td>
<td>Solid W/Ind.</td>
<td>0.0038</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>63</td>
<td>Steam Crac.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>64</td>
<td>Oil refinery</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>65</td>
<td>Oil Avalab.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Values are in arbitrary units.

A4-14
<table>
<thead>
<tr>
<th>$m$</th>
<th>$x_{16}$</th>
<th>$x_{17}$</th>
<th>$x_{18}$</th>
<th>$x_{19}$</th>
<th>$x_{20}$</th>
<th>$x_{21}$</th>
<th>$x_{22}$</th>
<th>$x_{23}$</th>
<th>$x_{24}$</th>
<th>$x_{25}$</th>
<th>$x_{26}$</th>
<th>$x_{27}$</th>
<th>$x_{28}$</th>
<th>$x_{29}$</th>
<th>$x_{30}$</th>
<th>$x_{31}$</th>
<th>$x_{32}$</th>
<th>$x_{33}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>PP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Propylene</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Steam</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Naphtha</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Gas</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.04</td>
</tr>
<tr>
<td>6</td>
<td>Cru oil</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>NaOH</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>EPS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>PS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Styrene</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.97</td>
</tr>
<tr>
<td>11</td>
<td>Aromatics</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Ethylene</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Na2CO3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>NaCl</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>CaCO3</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Coal</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>HIPS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>Polybutad</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>HDPE</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>LDPE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>Toluene</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>Electric</td>
<td>-0.338</td>
<td>-0.003</td>
<td>-0.096</td>
<td>0.205</td>
<td>0.309</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.679</td>
</tr>
<tr>
<td>23</td>
<td>Heat</td>
<td>-0.236</td>
<td>-0.114</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>Extrac. oil</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>m =&gt; n</td>
<td>$x_{16}$</td>
<td>$x_{17}$</td>
<td>$x_{18}$</td>
<td>$x_{19}$</td>
<td>$x_{20}$</td>
<td>$x_{21}$</td>
<td>$x_{22}$</td>
<td>$x_{23}$</td>
<td>$x_{24}$</td>
<td>$x_{25}$</td>
<td>$x_{26}$</td>
<td>$x_{27}$</td>
<td>$x_{28}$</td>
<td>$x_{29}$</td>
<td>$x_{30}$</td>
<td>$x_{31}$</td>
<td>$x_{32}$</td>
<td>$x_4$</td>
</tr>
<tr>
<td>-------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>49 Other Air</td>
<td>0.11</td>
<td>4.1E-05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50 Oils</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>51 Heavy Metal</td>
<td>2.0E-05</td>
<td>1.6E-04</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>52 Land weight</td>
<td>3.70E-05</td>
<td>2.61E-03</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>53 Oth. Open. L.</td>
<td>0.31</td>
<td>0.011</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>54 Other</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>55 Other water</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>56 TDS</td>
<td>8.0E-05</td>
<td>0.001</td>
<td>0.023</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>57 TSS</td>
<td>2.0E-05</td>
<td>6.97E-4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>58 COD</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>59 BOD</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60 Land. volum.</td>
<td>4.80E-05</td>
<td>8.66E-03</td>
<td>0.00294</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>61 Spec W.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>62 Solid W/Ind</td>
<td>0.131</td>
<td>0.426</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>63 Steam crac.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>64 Oil refin.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>65 Oil Aval.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix 5 is given in Volume II
A6.1 LP Model of the Boron Products System

The LP model of the boron system described here is related to the functional unit defined as "operation of the system for one year" (see Chapter 5). For a functional unit defined as "1000 kg of each product" or "1000 kg of B₂O₃ equivalent", the model remains the same except that the right hand sides of the constraints related to process capacities, inputs of primary and raw materials and outputs of products change accordingly. The environmental burdens and cost coefficients used in the LP model are given in table A6.1.

A6.1.1 Mass Balance Constraints

A6.1.1.1 Mine and Secondary Crusher

The activities in the mine and the secondary crusher are described by the mass balance equations:

\[
\sum_{i=1}^{32} a_{ji}(M)x_j(M) = 0, \quad j=1,\ldots,23 \quad (A6.1)
\]

related to the activities which include blasting, stripping, transport, primary and secondary crushing, kernite hydration, blending, and stock-piling. There are 23 such equations and 32 variables.

A6.1.1.2 Primary Process

There are 531 mass balance equations and 1592 variables in the Primary process (PP) subsystem. The mass balances include three components of the 324 streams in the PP: anhydrous borax (S), insolubles (I) and water content (W). The constraints are described by the following equations:

\[
\sum_{i=1}^{1592} a_{ji}(P)x_j(P) = 0, \quad j=1,\ldots,531 \quad (A6.2)
\]

where \(P_i\) is related to the flow of S, I, W or to the total flow of stream \(i\). The system is divided into six areas (as described in Chapter 5) and each operation unit is described by appropriate mass balance constraints.
A6.1.1.3 Boric Acid Process

The operations in the Boric acid (BA) plant are split into eight areas (see Chapter 5) with total of 129 streams. Six components are included in calculating composition of the streams: liquid and solid boric acid (BA and BAS), liquid and solid sodium sulphate (SSL and SSS), water (WF), and insolubles (IF). The BA system is described by 352 material balance constraints and 933 variables:

\[
\sum_{i=1}^{933} a_{ij}(BAP) x_i(BAP) = 0, \quad j=1,\ldots,352
\]  

(A6.3)

where BAP is related to the flow of BA, BAS, SSL, SSS, WF, IF or to the total flow of stream \(i\).

A6.1.1.4 Anhydrous Borax and Anhydrous Boric Acid

These two processes are essentially the same and each is described by 10 mass balance equations, as defined by:

\[
\sum_{i=1}^{9} a_{ij}(AB) x_i(AB) = 0, \quad j=1,\ldots,10
\]  

(A6.4)

\[
\sum_{i=1}^{31} a_{ij}(ABA) x_i(ABA) = 0, \quad j=1,\ldots,10
\]  

(A6.5)

where AB and ABA represent material flows in the AB and ABA processes, respectively.

A6.1.1.5 Packing and Shipping

This subsystem includes activities related to packing of the boron products in five different packagings and bulk shipping of some of them. It is described by 30 mass balance equations and 51 variables:

\[
\sum_{i=1}^{51} a_{ij}(PS) x_i(PS) = 0, \quad j=1,\ldots,30
\]  

(A6.6)
A6.1.2 Primary and Raw Material Availability Constraints

Supply of primary resources and raw materials is defined by:

\[
\begin{align*}
R_{\text{Kern}} & \leq 700000 \\
R_{\text{Tinc}} & \leq 1900000 \\
R_{\text{Expl}} & \leq 2000 \\
R_{\text{Trona}} & \leq 20000 \\
R_{\text{SA}} & \leq 130000 \\
R_{\text{NG}} & \leq 60000 \\
R_{\text{Diesel}} & \leq 7000
\end{align*}
\]  

(A6.7)

where R represents inputs in the foreground system of kernite and tincal ores, explosives, soda ash (trona), sulphuric acid, natural gas and diesel fuel (t/yr), respectively.

A6.1.3 Market Demand Constraints

To allocate the burdens among different products in the system, the market demand constraints are defined by the functional unit of the system, which is related to the total output of the products in one year (t/yr):

\[
\begin{align*}
P_{5\text{mol}} & \leq 800000 \\
P_{10\text{mol}} & \leq 81000 \\
P_{\text{BA}} & \leq 150000 \\
P_{\text{AB}} & \leq 16000 \\
P_{\text{ABA}} & \leq 5000
\end{align*}
\]  

(A6.8)

In order to show the environmental burdens associated with packaging, the output of each product is further specified according to the type of packaging used to contain it (see Chapter 6, Table 6.1).

In the optimisation part of the work, constraints (A6.8) are replaced by the projected market demand of the products, based on the trends in the last few years:

\[
\begin{align*}
P_{5\text{mol}} & \leq 826000 \land P_{5\text{mol}} > 818000 \\
P_{10\text{mol}} & \leq 83000 \land P_{10\text{mol}} > 71000
\end{align*}
\]
Appendix 6

P_{BA} \leq 145000 \land P_{BA} > 134000 \quad (A6.9)
P_{AB} \leq 17500 \land P_{AB} \leq 14500
P_{ABA} = 6700

A6.1.4 Productive Capacity Constraints

These constraints are defined by the available processing capacity (t/yr) of the dryers in the primary and the boric acid processes and furnaces in the AB and ABA plants:

Fluid bed dryer: \quad x_{169}(P) \leq C_{FB}
Rotary dryer: \quad x_{179}(P) \leq C_{RD}
Wysmont (tray) dryer: \quad x_{239}(P) \leq C_{WD} \quad (A6.10)
Boric acid dryer: \quad x_{56}(BAP) \leq C_{BAD}
AB Plant: \quad x_{3}(AB) \leq C_{AB}
ABA Plant: \quad x_{3}(ABA) \leq C_{ABA}

A6.1.5 Heat Constraints

This constraint is related to the total heat requirement in the system, as defined by:

H \leq 2.55 \cdot 10^9 \quad (A6.11)

The heat requirement H is determined by the lower heating value and the consumption of natural gas necessary for generation of this amount of heat (TJ/yr).

A6.1.6 Objective Functions

A6.1.6.1 Environmental Objective Functions

There are ninety one burdens and ten impacts defined as objective functions:

Minimise \quad B_j = \sum_{i=1}^{2628} b_{i,j} \cdot x_i, \quad j=1,\ldots,91 \quad (A6.12)

Minimise \quad E_k = \sum_{j=1}^{91} e_{k,j} \cdot B_j, \quad k=1,\ldots,10 \quad (A6.13)

A6-4
where some of the coefficients $b_{c_{i,j}}$ and $e_{c_{k,j}}$ are zero. Coefficient $b_{c_{j,i}}$ are given in Table A6.1; coefficients $e_{c_{k,j}}$ can be found in Appendix 5 (Table A5.13).

**A6.1.6.2 Economic Objective Functions**

The economic objective functions are used in the multiobjective optimisation and are defined as the production costs $C$ and total production $P$ of the products, respectively:

Minimise  
$$C = \sum_{i=1}^{2628} c_{i} x_{i}$$  \hspace{1cm} (A6.14)

Maximise  
$$P = \sum_{i=1}^{5} P_{i}$$  \hspace{1cm} (A6.15)

where $P_{1}$ is the output of product $l$. The cost coefficients $c_{i}$ are listed in table A6.1.
A6.2 LP model of the boron product system

LET C=324 ! TOTAL NO OF STREAMS IN PRIMARY PROCESS
LET C1=200 ! MIXING STREAMS IN THE PRIMARY PROCESS
LET CBAP=257 ! TOTAL NUMBER OF STREAMS IN BORIC ACID PLANT
LET C1BAP=4 ! MIXING STREAMS IN THE BORIC ACID PLANT

VARIABLES

EXPL ! TOTAL EXPLOSIVE
ANOB ! AMMONIUM NITRATE FOR OVERBURDEN
EMOB ! EMULSION FOR OVERBURDEN
ANTI ! AMMONIUM NITRATE FOR TINCAL
EMTI ! EMULSION FOR TINCAL
ANKE ! AMMONIUM NITRATE FOR KERNITE
EMKE ! EMULSION FOR KERNITE
BLAST ! TOTAL BLASTED
OBBL ! OVERBURDEN BLASTED
TIBL ! TINCAL BLASTED
KEBL ! KERNITE BLASTED
OBMI ! OVERBURDEN MINED
OBSTR ! OVERBURDEN STRIPPED
STRIP ! TOTAL STRIPPED
TIMI ! TINCAL MINED
TISP ! TINCAL - STOCK PILE
KEMI ! KERNITE MINED
KESP ! KERNITE - STOCK PILE
TRANS ! TOTAL TRANSPORTED
TRUCK ! TRANSPORT BY TRUCKS
CONVEY ! TRANSPORT BY CONVEYORS
PRCRIIN ! PRIMARY CRUSHER INPUTS
PRCROUT ! PRIMARY CRUSHER OUTPUTS
KEPP ! KERNITE FOR PRIMARY PROCESS
KEBA ! KERNITE FOR BORIC ACID
KEHD ! KERNITE HYDRATED
KWAT ! WATER FOR KERNITE HYDRATION
TIKE ! TINCAL AND KERNITE FOR PRIMARY PROCESS
TRON ! TRONA
ORSP ! TOTAL ORE FOR PRIMARY PROCESS - STOCK PILE
TIKECR ! TINCAL AND KERNITE TO THE SECONDARY CRUSHER
ORESP ! UNUSED ORE IN STOCK PILE AFTER THE SECONDARY CRUSHER
TIMI ! TINCAL TO PRIMARY PROCESS

W(C) ! WATER COMPONENT OF A STREAM
S(C) ! BORON COMPONENT OF A STREAM
I(C) ! INSOLUBLE COMPONENT OF A STREAM
WM(C1) ! WATER COMPONENT OF A STREAM FORMED BY MIXING WITH OTHER STREAMS
SM(C1) ! BORON COMPONENT OF A STREAM FORMED BY MIXING WITH OTHER STREAMS
IM(C1) ! INSOLUBLE COMPONENT OF A STREAM FORMED BY MIXING WITH OTHER STREAMS
PP(C) ! TOTAL FLOW OF A PP STREAM
WYSM ! WYSMONT DRYERS
FBDR ! FBDRYER
ROTARD ! ROTARY DRYER
LNBWYS ! WYSMONT DRYERS WITH LOW-NOx BURNERS
Appendix 6

LNBROT ! ROTARY DRYER WITH LOW-NOx BURNERS
LNBFBD ! FLUID BED DRYER WITH LOW-NOx BURNERS

**********BAP VARIABLES**********

BA(CBAP) ! LIQUID BORIC ACID COMPONENT OF A STREAM
BAS(CBAP) ! SOLID BORIC ACID COMPONENT OF A STREAM
WF(CBAP) ! WATER COMPONENT OF A STREAM
IF(CBAP) ! INSOLUBLE COMPONENT OF A STREAM
SSL(CBAP) ! SOLUBLE SODIUM SULPHATE (LIQUID) COMPONENT OF A STREAM
SSS(CBAP) ! SOLUBLE SODIUM SULPHATE (SOLID) COMPONENT OF A STREAM
SA ! SULPHURIC ACID
AB ! ANHYDROUS BORON
SS ! TOTAL SOLID AND LIQUID FRACTION OF SOLUBLE SODIUM SULPHATE
WR ! WATER CONTENT IN ORE
BASC56 ! TOTAL BORIC ACID IN STREAM #56
BAM(C1BAP) ! BORIC ACID (LIQUID) COMPONENT OF A STREAM FORMED BY MIXING WITH OTHER STREAMS
BASM(C1BAP) ! BORIC ACID (SOLID) COMPONENT OF A STREAM FORMED BY MIXING WITH OTHER STREAMS
WFM(C1BAP) ! WATER FRACTION OF A STREAM FORMED BY MIXING WITH OTHER STREAMS
IFM(C1BAP) ! INSOLUBLE FRACTION OF A STREAM FORMED BY MIXING WITH OTHER STREAMS
SSLM(C1BAP) ! SODIUM SULPHATE (LIQUID) COMPONENT OF A STREAM FORMED BY MIXING WITH OTHER STREAMS
SSSM(C1BAP) ! SODIUM SULPHATE (SOLID) COMPONENT OF A STREAM FORMED BY MIXING WITH OTHER STREAMS
BAP(CBAP) ! TOTAL FLOW OF A BAP STREAM

***********AB VARIABLES**********

FM ! FIVE MOL FOR AB
FMFB ! FIVE MOL TO FEED BINS
ABCR ! AB FROM CHILL ROLLS
ABHM ! AB TO HAMMER BINS
ABHM ! AB FROM HAMMER BINS
ABREJ ! AB REJECT
ABPR ! AB PRODUCT
ABFM ! AB CONTENT IN FIVE MOL
WFMOL ! WATER FRACTION IN FIVE MOL

**********ABA VARIABLES**********

BAAB ! BORIC ACID FOR ABA
ABABA ! ABA CONTENT IN BORIC ACID
BAFB ! BORIC ACID TO FEED BINS
BAFFB ! BORIC ACID FROM FEED BINS
ABAF ! ABA FROM FURNACE
ABACR ! ABA FROM CHILL ROLLS
ABAHM ! ABA IN HAMMER MILLS
ABAPR ! ABA IN TEST BINS
ABALOS ! ABA LOSS
ABAPROD ! ABA PRODUCT
WABA ! WATER FRACTION IN BORIC ACID

**********SHIPPING VARIABLES**********

FMSHIP ! FIVE MOL SHIPPED
FMAB ! FIVE MOL FOR AB
FMSC ! FIVE MOL SCREENED
FMREJ ! FIVE MOL REJECT
FMS ! FIVE MOL BULK
FMP ! FIVE MOL PACKED
TMP ! TEN MOL PACKED
TMREJ ! TEN MOL REJECT
Appendix 6

TMST ! TEN MOL STORED
TMBULK ! TEN MOL BULK
FMFULK ! FIVE MOL BULK
PENTS ! PENTA BULK
PENTREJ ! PENTA REJECT
PPP ! TOTAL PRIMARY PRODUCTS PACKED
ABP ! AB PACKED
ABS ! AB STORED
ABBSH ! AB SHIPPED
ABBULK ! AB BULK
ABAP ! ABA PACKED
ABAS ! ABA STORED
ABABSH ! ABA SHIPPED
BASH ! BA SHIPPED
BAREJ ! BA REJECT
BADYB ! BA DAY BINS
BAABA ! BA FOR ABA
BASIL ! BA IN SILOS
BABS ! BA BULK
BABP ! BA PACKED
BASTOR ! BA STORED
BABSH ! BA SHIPPED
FM2550 ! FIVE MOL IN 25KG & 50LB BAGS
FM50100 ! FIVE MOL IN 50KG & 100LB BAGS
FMPPBAG ! FIVE MOL IN PP BAGS
TM2550 ! TEN MOL IN 25KG & 50LB BAGS
TM50100 ! TEN MOL IN 50KG & 100LB BAGS
TMPPBAG ! TEN MOL IN PP BAGS
AB2550 ! AB IN 25KG & 50LB BAGS
AB50100 ! AB IN 50KG & 100LB BAGS
ABPPBAG ! AB IN PP BAGS
ABA2550 ! ABA IN 25KG & 50LB BAGS
ABA50100 ! ABA IN 50KG & 100LB BAGS
ABAPPBAG ! ABA IN PP BAGS
BA2550 ! BA IN 25KG & 50LB BAGS
BA50100 ! BA IN 50KG & 100LB BAGS
BAPPBAG ! BA IN PP BAGS
FMSHIP1 ! TOTAL FIVE MOL SHIPPED (INCL. PENTA)
TMSHIP ! TOTAL TEN MOL SHIPPED
ABSHIP ! TOTAL AB SHIPPED
ABASHIP ! TOTAL ABA SHIPPED
BASHIP ! TOTAL BA SHIPPED
TOTSH ! TOTAL PRODUCTS SHIPPED

*****************************************************************************
ENVIRONMENTAL BURDENS*****************************************************************************

LET D=52 ! NOS OF COLUMNS FOR ENVIRONMENTAL BURDENS (EXCEL MATRIX)
LET E=96 ! NO ENVIRONMENTAL BURDENS (EXCEL MATRIX)
X(D)
ENVB(E) ! VARIABLES RELATED TO THE ENVIRONMENTAL BURDENS
COGEN ! VARIABLE RELATED TO THE COGENERATION PLANT
STMLT ! VARIABLE RELATED TO THE STEAM PLANT
COST ! COST VARIABLE
TABLES
COE(E,D) ! COEFFICIENTS RELATED TO THE ENV. BURDENS
COE1(E,D) ! COEFFICIENTS RELATED TO THE COSTS
CONNECT excel
DISKDATA -c ! THIS PART OF THE MODEL FOR ALLOCATION
Appendix 6

COE = C:\BORAX\ENVCOMB7.XLS (B5:BA100)  !READ ENV. BURDEN COEFFICIENTS
COEI = C:\BORAX\ENVCOMB7.XLS (B101:AY101) !READ COST COEFFICIENTS
DISCONNECT

!THIS PART OF THE MODEL FOR THE OPTIMISATION
COE = C:\BORAX\ENVCOMB9.XLS (B5:BF100)  !READ ENV. BURDEN COEFFICIENTS
COEI = C:\BORAX\ENVCOMB9.XLS (B101:BF101) !READ COST COEFFICIENTS
DISCONNECT
DISCONNECT

CONSTRAINTS

!**********************************************************************ECONOMIC OBJECTIVE FUNCTIONS**********************************************************************
PRODUCTION:  TOTSH $
COSTS:  COST $

!**********************************************************************ENVIRONMENTAL OBJECTIVE FUNCTIONS**********************************************************************
!**********************************************************************INVENTORY STAGE - THE BURDENS**********************************************************************
STEAMBOR:  STMLT+COGEN $
ELECBO:  ENVB(2) $
GASBO:  ENVB(3) $
FUELBOR:  ENVB(4) $
TROCE:  ENVB(5) $
TEXTREN:  ENVB(6) $
TNUCL:  ENVB(7) $
TOTHY:  ENVB(8) $
TCOAL:  ENVB(9) $
TOIL:  ENVB(10) $
TGAS:  ENVB(11) $
TBORKER:  ENVB(12) $
TIRON:  ENVB(13) $
TBAUXITE:  ENVB(14) $
TNACL:  ENVB(15) $
TLimest:  ENVB(16) $
TOTHNR:  ENVB(17) $
TNONRE:  ENVB(18) $
TRENEW:  ENVB(19) $
TANCIL:  ENVB(20) $
TPROCW:  ENVB(21) $
TSTMOT:  ENVB(22) $
TWATER:  ENVB(23) $
TAIR:  ENVB(24) $
TOTHER:  ENVB(25) $
TCO:  ENVB(26) $
TCO2NO:  ENVB(27) $
TCO2RE:  ENVB(28) $
TCO2:  ENVB(29) $
TNOX:  ENVB(30) $
TSO2:  ENVB(31) $
THC:  ENVB(32) $
TCH4:  ENVB(33) $
TALDEH:  ENVB(34) $
TCHLORH:  ENVB(35) $
TCLF:  ENVB(36) $
TOTVOC:  ENVB(37) $
TVOC:  ENVB(38) $
TASAIR:  ENVB(39) $
THGAI:  ENVB(40) $

A6-9
TOTHME: ENVB(41) $  
TMETAIR: ENVB(42) $  
TTSP: ENVB(43) $  
TPM10: ENVB(44) $  
TCL2: ENVB(45) $  
TF2: ENVB(46) $  
THCI: ENVB(47) $  
THF: ENVB(48) $  
THALIDE: ENVB(49) $  
TMERCAP: ENVB(50) $  
THERBIC: ENVB(51) $  
TINSECT: ENVB(52) $  
TNH3: ENVB(53) $  
TOTHERA: ENVB(54) $  
TOTHERA: ENVB(55) $  
TPROCWA: ENVB(56) $  
TSTMWA: ENVB(57) $  
TOTHW: ENVB(58) $  
TWSTWA: ENVB(59) $  
TASWAT: ENVB(60) $  
TSBWAT: ENVB(61) $  
TCRWAT: ENVB(62) $  
TCUWAT: ENVB(63) $  
TFEWAT: ENVB(64) $  
THGWAT: ENVB(65) $  
TNIWAT: ENVB(66) $  
TPBWAT: ENVB(67) $  
TZNWAT: ENVB(68) $  
TOTHME: ENVB(69) $  
TMETWA: ENVB(70) $  
TCHLORID: ENVB(71) $  
TFLUORID: ENVB(72) $  
TNITRAT: ENVB(73) $  
TPHOSP: ENVB(74) $  
TSULPH: ENVB(75) $  
TOTHERT: ENVB(76) $  
TTDS: ENVB(77) $  
TTSS: ENVB(78) $  
TOILSGRE: ENVB(79) $  
TAMMO: ENVB(80) $  
TCYLORS: ENVB(81) $  
TCYANID: ENVB(82) $  
TBWATER: ENVB(83) $  
TPESTIC: ENVB(84) $  
TPHENOL: ENVB(85) $  
TMISCEL: ENVB(86) $  
TACID: ENVB(87) $  
TALKALI: ENVB(88) $  
TMISC: ENVB(89) $  
TCOD: ENVB(90) $  
TBOD: ENVB(91) $  
TLANDFIL: ENVB(92) $  
TOPENL: ENVB(93) $  
TOTHSS: ENVB(94) $  
TLDNFV: ENVB(95) $  
TLDFM3: ENVB(96) $
APPENDIX 6

/*------------------------------IMPACT ASSESSMENT STAGE - THE IMPACTS------------------------------*/

RESDEPL:
8.72*ENVB(9) + 6.62*ENVB(10) + &
1.19*ENVB(11) + 4.4707*ENVB(12) +
400*ENVB(35) + 5000*ENVB(36) + &
11*ENVB(37) +

GWPDIR:
ENVB(29) + 11*ENVB(33) + &

GWPIND:
3*ENVB(26) + 40*ENVB(30) + &
11*ENVB(32) +

OZONE:
0.5*ENVB(35) + 0.4*ENVB(36) + &
0.005*ENVB(37) +

ACIDIF:
0.7*ENVB(30) + ENVB(31) + &
0.88*ENVB(47) + 1.6*ENVB(48) + &
1.88*ENVB(53) +

NUTRIF:
0.13*ENVB(30) + 0.42*ENVB(73) + &
ENVB(74) + 0.33*ENVB(80) + &
0.022*ENVB(90) +

POCP:
0.416*ENVB(32) + 0.007*ENVB(33) + &
0.443*ENVB(34) + 0.007*ENVB(37) +

HUMTOX:
0.012*ENVB(26) + 0.78*ENVB(30) + &
1.2*ENVB(31) + 1.7*ENVB(32) + &
0.98*ENVB(35) + 0.022*ENVB(36) + &
4700*ENVB(39) + 120*ENVB(40) + &
0.48*ENVB(46) + 0.48*ENVB(48) + &
0.78*ENVB(50) + 1.7*ENVB(52) + &
0.02*ENVB(53) + 1.4*ENVB(60) + &
0.57*ENVB(62) + 0.02*ENVB(63) + &
0.0036*ENVB(64) + 4.7*ENVB(65) + &
0.057*ENVB(66) + 0.79*ENVB(67) + &
0.0029*ENVB(68) + 0.041*ENVB(72) + &
0.00078*ENVB(73) + 0.000041*ENVB(74) + &
0.0017*ENVB(80) + 0.29*ENVB(81) + &
0.057*ENVB(82) + 0.14*ENVB(84) + &
0.048*ENVB(85) +

AQTOX:
2E08*ENVB(60) + 10E08*ENVB(62) + &
2E09*ENVB(63) + 5E11*ENVB(65) + &
3.3E08*ENVB(66) + 2E09*ENVB(67) + &
3.8E08*ENVB(68) + 5E07*ENVB(79) + &
6E07*ENVB(81) + 1.3E09*ENVB(84) + &
5.9E09*ENVB(85) +

/*------------------------------COSTS------------------------------*/

DIRCOST(n=1:1): SUM(m=1:D) COE(n,m) * X(m) - COST = 0

/*------------------------------CALCULATION OF ENVIRONMENTAL BURDENS------------------------------*/

ENVBS1(n=1:1): SUM(m=1:42) COE(n,m) * X(m) - ENVB(n) = 0

ENVBS(n=2:E): SUM(m=1:D) COE(n,m) * X(m) - ENVB(n) = 0

/*------------------------------ASSIGN VARIABLES FOR READING FROM THE EXCEL FILE------------------------------*/

X1: X(1) - EXPL = 0
X2: X(2) - BLAST = 0
X3: X(3) - STRIP = 0
X4: X(4) - TRANS = 0
X5: X(5) - PRCROUT = 0
X6: X(6) - KEHD = 0
X7:  X(7) - TIKE = 0
X8:  X(8) - TIMI-KEPP-KEBA = 0
X9:  X(9) - TIKECR = 0
X10: X(10) - PP(151) = 0
X11: X(11) - PP(155)-PP(30)-PP(324)-PP(28) = 0

***********************************************************************
X12: X(12) - PP(49) = 0
X13: X(13) - PP(94) = 0
X14: X(14) - PP(77) = 0

***********************************************************************
X12: X(12) - FBDR = 0
X13: X(13) - ROTARD = 0
X14: X(14) - WYSM = 0

***********************************************************************
X15: X(15) - PP(49)-PP(94)-PP(77) = 0
X16: X(16) - BAP(106) = 0
X17: X(17) - BAP(2) = 0
X18: X(18) - BAP(34) = 0
X19: X(19) - FMFB = 0
X20: X(20) - ABCR = 0
X21: X(21) - ABHM = 0
X22: X(22) - ABPR = 0
X23: X(23) - BAFB = 0
X24: X(24) - ABACR = 0
X25: X(25) - ABAHM = 0
X26: X(26) - ABAPROD = 0
X27: X(27) - FMSHIP1-TMSHIP-ABSHIP-ABASHIP = 0
X28: X(28) - FM2550 = 0
X29: X(29) - FM50100 = 0
X30: X(30) - FMPPBAG = 0
X31: X(31) - TM2550 = 0
X32: X(32) - TM50100 = 0
X33: X(33) - TMPPBAG = 0
X34: X(34) - AB2550 = 0
X35: X(35) - AB50100 = 0
X36: X(36) - ABPPBAG = 0
X37: X(37) - ABA2550 = 0
X38: X(38) - ABA50100 = 0
X39: X(39) - ABAPBAG = 0
X40: X(40) - BA2550 = 0
X41: X(41) - BA50100 = 0
X42: X(42) - BAPPPBAG = 0
X43: X(43) - STMPLT = 0
X44: X(44) - S(151) = 0
X45: X(45) - S(61) = 0
X46: X(46) - S(73) = 0
X47: X(47) - S(29) = 0
X48: X(48) - KEPP = 0
X49: X(49) - BADYB = 0
X50: X(50) - TRON = 0
X51: X(51) - PP(1) - PP(6) = 0
X52: X(52) - S(1) - S(6) - AB = 0
X53: X(53) - COGEN = 0
X54: X(54) - CONVEY = 0
X55: X(55) - LNBWYS = 0
X56: X(56) - LNBFBF = 0
X57: X(57) - LNBROT = 0
Appendix 6

WYSMO: \[ \text{LNBWYS} - PP(77) = 0 \]
FBD: \[ \text{LNFBBD} - PP(49) = 0 \]
ROTR: \[ \text{LNBRROT} - PP(94) = 0 \]

TRANSP: \[ X(54) + X(4) - \text{TRANS} = 0 \]
STPLT: \[ \text{STMLT} + \text{COGEN} - 1.03309*\text{ENVB}(1) = 0 \]

|****************************INPUTS INTO PP- FOR ALLOCATION***********************|
|**BX1:** \( \text{S}(1) < 641987 \)
|**BX6:** \( \text{S}(6) < 0.205*\text{S}(1) = 0 \)
|**WAT 172:** \( \text{KWAT} - 0.417071*\text{KEPP} = 0 \)

|****************************INPUTS INTO PP- FOR OPTIMISATION*********************|
|**BX1:** \( \text{S}(1) > 530000 \)
|**BX6:** \( \text{S}(6) > 210000 \)

|****************************INPUTS INTO BAP- FOR ALLOCATION************************|
|**KERN:** \( \text{AB} < 168136 \)

|****************************INPUTS INTO BAP- FOR OPTIMISATION************************|
|**KERN:** \( \text{AB} > 155000 \)

|****************************OUTPUTS FROM SHIPPING- FOR ALLOCATION********************|
|**SHIP1:** \( \text{BASTOR} = 0 \)
|**SHIP2:** \( \text{BABS} = 72000 \)
|**SHIP3:** \( \text{PENTS} = 91000 \)
|**SHIP4:** \( \text{FMS} = 650000 \)
|**SHIP5:** \( \text{TM} = 24000 \)
|**SHIP6:** \( \text{FM2550} = 27000 \)
|**SHIP7:** \( \text{FM50100} = 32000 \)
|**SHIP8:** \( \text{FMPPBAG} = 10000 \)
|**SHIP9:** \( \text{TM2550} = 26000 \)
|**SHIP10:** \( \text{TM50100} = 17000 \)
|**SHIP11:** \( \text{TMPPBAG} = 14000 \)
|**SHIP12:** \( \text{AB2550} = 2000 \)
|**SHIP13:** \( \text{AB50100} = 9000 \)
|**SHIP14:** \( \text{ABPPBAG} = 5000 \)
|**SHIP15:** \( \text{ABA2550} = 2000 \)
|**SHIP16:** \( \text{ABA50100} = 2000 \)
|**SHIP17:** \( \text{ABAPPBAG} = 1000 \)
|**SHIP18:** \( \text{BA2550} = 26000 \)
|**SHIP19:** \( \text{BA50100} = 12000 \)
|**SHIP20:** \( \text{BAPPBAG} = 40000 \)

|****************************OUTPUTS FROM SHIPPING- FOR OPTIMISATION********************|
|**bastor:** \( \text{BASTOR} = 0 \)
|**babulk:** \( \text{BABS} < 67000 \)
|**babul:** \( \text{BABS} > 63000 \)
|**fmpent:** \( \text{PENTS} + \text{FMS} < 755000 \)
|**frpen:** \( \text{PENTS} + \text{FMS} > 755000 \)
|**tmbulk:** \( \text{TM} < 25000 \)
|**tmbul:** \( \text{TM} > 20000 \)
|**fm2550:** \( \text{FM2550} < 28000 \)
|**fm255:** \( \text{FM2550} > 25000 \)
|**fm50100:** \( \text{FM50100} < 33000 \)
|**fmpent:** \( \text{PENTS} + \text{FMS} < 30000 \)
|**fmpent:** \( \text{PENTS} + \text{FMS} > 30000 \)
|**tmppbag:** \( \text{FMPPBAG} < 10000 \)
|**fmpbba:** \( \text{FMPPBAG} > 8000 \)
|**tm2550:** \( \text{TM} < 27000 \)
|**tm255:** \( \text{TM} > 25000 \)
|**tm50100:** \( \text{TM} < 17000 \)
|**tm5010:** \( \text{TM} > 15000 \)
|**tmppbag:** \( \text{TMPPBAG} < 14000 \)
Appendix 6

tmppba: TMPPBAG > 11000
ab2550: AB2550 < 2500
ab255: AB2550 > 2000
ab50100: AB50100 < 10000
ab5010: AB50100 > 8000
abppbag: ABPPBAG < 5000
abppba: ABPPBAG > 4500
aba2550: ABA2550 = 2200
aba50100: ABA50100 = 2000
abappbag: ABAPPBAG = 2500
abappba: ABAPPBAG = 2000


!*********************************************************************
!*********************************************************************
EXP: EXPL - ANOB - EMOB - ANTI - EMTI - ANKE - EMKE = 0
EXP1: ANOB - 0.520395*EXPL = 0
EXP2: EMOB - 0.213920*EXPL = 0
EXP3: ANTI - 0.144773*EXPL = 0
EXP4: EMTI - 0.059154*EXPL = 0
EXP5: ANKE - 0.048820*EXPL = 0
BLA1: BLAST - OBBL - TIBL - KEBL = 0
BLA2: EXPL - 0.000127551*BLAST = 0
BLA3: OBBL - 3.555444*TIBL - 3.555444*KEBL = 0


!*********************************************************************
!*********************************************************************
STR: OBMI - OBBL - OBSTR = 0
STR2: OBSTR - 0.104946*OBMI = 0
STR3: STRIP - OBBL - OBSTR - TIBL - KEBL = 0

!*********************************************************************
!*********************************************************************
TR: TIBL - TIMI - TISP = 0
TR2: KEBL - KEMI - KESP = 0
TR3: TRANS - OBMI - TIMI - KEMI = 0


!*********************************************************************
!*********************************************************************
PRCR1: PRCRIN - TIMI - KEMI = 0
PRCR2: PRCROUT - TIMI - KEPP - KEBA = 0


!*********************************************************************
!*********************************************************************
KERH1: KEHD - KEPP - KWA = 0
KERH2: KEMI - KEPP - KEB = 0


!*********************************************************************
!*********************************************************************
BL1: TIKE - TIMI - KEHD - TRON = 0
BL2: TRON - 0.006222*TIKE = 0


!*********************************************************************
!*********************************************************************
ORS: ORSP - TIKE - KEB = 0


A6-14
Appendix 6

SC:  
| TIKECR - TIKE = 0 |

ORBIN:  
| TIKECR - PP(100) - ORESP - TRON = 0 |

ORE COMPOSITION - LINKING WITH THE PP AND BAP

ORC1:  
| TIMI - S(1) - W(1) - I(1) = 0 |

ORC2:  
| KEPP - S(6) - W(6) - I(6) = 0 |

ORC3:  
| KEBA - AB - WF(1) - IF(1) = 0 |

WAT:  
| W(172) - KWAT = 0 |

PRIMARY PROCESS

ORE BLEND INTO DISSOLVER

MASS BALANCES

WORE100:  
| W(100) - W(1) - W(6) - W(172) = 0 |

SORE100:  
| S(100) - S(1) - S(6) = 0 |

IORE100:  
| I(100) - I(1) - I(6) = 0 |

COMPOSITION-TINCAL

STINCl:  
| 1.084640*W(1) - S(1) = 0 |

WTINCl:  
| 0.968208*S(1) - I(1) = 0 |

COMPOSITION-KERNITE

SKERN6:  
| 1.780000*W(6) - S(6) = 0 |

WKERN6:  
| 0.685393*S(6) - I(6) = 0 |

DISSOLVER

MASS BALANCES

DISEFL:  
| WM(151) - W(129) - W(162) - W(100) = 0 |

DISS151:  
| WM(151) - W(163) - W(151) = 0 |

S151:  
| S(151) - S(129) - S(100) = 0 |

I151:  
| I(151) - I(100) = 0 |

COMPOSITION

WATDiss:  
| W(162) - 0.005635*W(151) - 0.005635*S(151) - 0.005635*I(151) = 0 |

VAPDiss:  
| W(163) - 0.001794*W(151) = 0 |

TYROCK SCREENS

MASS BALANCES-MIXING

WTYROUT:  
| WM(152) - W(151) - W(65) - W(4) = 0 |

STYROUT:  
| SM(152) - S(151) - S(4) = 0 |

ITYROUT:  
| IM(152) - I(151) = 0 |

MASS BALANCES-SEPARATION

WTYR152:  
| WM(152) - W(164) - W(2) - W(152) = 0 |

STYR152:  
| SM(152) - S(2) - S(152) = 0 |

ITYR152:  
| IM(152) - I(2) - I(152) = 0 |

FIX WATER INPUT TO THE TOTAL FLOW IN TYROCK SCREEN EFFLUENT

W65:  
| W(65) - 0.011138*W(152) - 0.011138*S(152) - 0.011138*I(152) = 0 |

W4:  
| W(4) - 0.028559*W(152) - 0.028559*S(152) - 0.028559*I(152) = 0 |

VAPOUR FLOW

VAPTYR:  
| W(164) - 0.007234*WM(152) = 0 |

COMPOSITION OF TYROCK EFFLUENT

INERTYR152:  
| I(152) - 0.056396*W(152) = 0 |

TOTAL FLOW AND WATER AND SODIUM BORATE IN GANGUE

WGANG2:  
| W(2) - 0.306122*I(2) = 0 |

SGANG2:  
| S(2) - 0.054422*I(2) = 0 |

TOT2:  
| PP(2) - 0.098672*PP(152) = 0 |

DISSOLVER LOSS

MASS BALANCES

WTYRSPL:  
| W(152) - W(12) - W(154) = 0 |

STYRSPL:  
| S(152) - S(12) - S(154) = 0 |

ITYRSPL:  
| I(152) - I(12) - I(154) = 0 |

FIXED COMPOSITION

WLOSI2:  
| W(12) - 0.007191*W(152) = 0 |

SLOSI2:  
| S(12) - 0.007191*S(152) = 0 |
ILS12: \[ I(12) - 0.007191*I(152) = 0 \]

**THICKENER 1**

**MASS BALANCES-MIXING**

WMTHICK: \[ WM(154) - W(3) - W(154) = 0 \]
SMTHICK: \[ SM(154) - S(154) = 0 \]
IMTHICK: \[ IM(154) - I(154) = 0 \]

**MASS BALANCES-SEPARATION**

WMTH1OUT: \[ WM(154) - W(165) - W(155) - W(7) = 0 \]
SMTH1OUT: \[ SM(154) - S(155) - S(7) = 0 \]
IMTH1OUT: \[ IM(154) - I(7) = 0 \]

**WATER INPUT**

WATT1: \[ W(3) - 0.012266*W(154) = 0 \]

**VAPOUR**

VAPTH1: \[ W(165) - 0.008282*WM(154) = 0 \]

**UNDERFLOW**

WUTFH1: \[ W(7) - 2.276*I(7) = 0 \]
SUFTH1: \[ S(7) - 0.724*W(7) = 0 \]

**THICKENER 2A**

**CONSTANT SPLIT OF STREAMS 200 & 205**

WTHK2A: \[ W(24) - W(200) - W(205) = 0 \]
STHK2A: \[ S(24) - S(200) - S(205) = 0 \]
W200: \[ W(200) - 0.9*W(24) = 0 \]
S200: \[ S(200) - 0.9*S(24) = 0 \]

**MASS BALANCES-MIXING**

WMTH2A: \[ WM(7) - W(200) - W(51) - W(7) - W(324) = 0 \]
SMTH2A: \[ SM(7) - S(200) - S(7) - S(324) = 0 \]
IMTH2A: \[ IM(7) - I(7) = 0 \]

**MASS BALANCES-SEPARATION**

WMTH2K2A: \[ WM(7) - W(55) - W(30) - W(134) = 0 \]
SMTH2K2A: \[ SM(7) - S(55) - S(30) = 0 \]
IMTH2K2A: \[ IM(7) - I(55) = 0 \]

**WATER INPUT**

W51: \[ W(51) - 0.026346*W(7) = 0 \]

**FIX THE WATER INPUT TO #7**

VAP134: \[ W(134) - 0.001196*WM(7) = 0 \]

**FIX U/F**

W55: \[ W(55) - 3*I(55) = 0 \]
S55: \[ S(55) - 0.545455*I(55) = 0 \]
W324: \[ W(324) - W(87) = 0 \]
S324: \[ S(324) - S(87) = 0 \]

**THICKENER 2B**

**MASS BALANCES - INPUTS**

WMTH2B: \[ WM(55) - W(191) - W(205) - W(54) - W(55) = 0 \]
SMTH2B: \[ SM(55) - S(191) - S(205) - S(55) = 0 \]
IMTH2B: \[ IM(55) - I(55) = 0 \]

**MASS BALANCES - OUTPUTS**

WMTH2K2B: \[ WM(55) - W(11) - W(130) - W(87) = 0 \]
SMTH2K2B: \[ SM(55) - S(11) - S(87) = 0 \]
IMTH2K2B: \[ IM(55) - I(11) = 0 \]

**WATER INPUT FIXED TO WATER IN #55**

W54: \[ W(54) - 0.019983*W(55) = 0 \]

**VAPOUR LOSS FIXED TO THE TOTAL WATER IN INPUTS**
Appendix 6

W130: W(130) - 0.002511*WM(55) = 0

*******************************************************************************U/F (FIXED COMPOSITION)*******************************************************************************

W11: W(11) - 3.245370*I(11) = 0
S11: S(11) - 0.384259*I(11) = 0

*******************************************************************************THICKENER 4*******************************************************************************

*******************************************************************************MASS BALANCES-INPUTS*******************************************************************************

WMTH4: WM(11) - W(124) - W(60) - W(20) - W(21) = 0
SMTH4: SM(11) - S(124) - S(60) - S(11) - S(21) = 0
IMTH4: IM(11) - I(124) - I(11) = 0

*******************************************************************************MASS BALANCES-OUTPUTS*******************************************************************************

WMTH4K: WM(11) - W(136) - W(17) - W(28) = 0
SMTH4K: SM(11) - S(17) - S(28) = 0
UXTH4K: U(17) = 0

*******************************************************************************WATER INPUT FIXED TO WATER IN	*******************************************************************************

W20: W(20) - 0.054365*W(11) = 0

*******************************************************************************VAPOUR LOSS FIXED TO THE TOTAL WATER IN INPUTS*******************************************************************************

W136: W(136) - 0.001189*WM(11) = 0

*******************************************************************************SUMP INPUT (FIXED COMPOSITION)*******************************************************************************

W60: S(60) - 0.024*W(60) = 0
W601: W(60) - 0.010612*W(11) = 0

*******************************************************************************U/F (FIXED COMPOSITION)*******************************************************************************

W17: W(17) - 3.10696*I(17) = 0
S17: S(17) - 0.23913*I(17) = 0

*******************************************************************************GANGUE*******************************************************************************

*******************************************************************************O/F - FIXED RATIO OF INERTS TO WATER*******************************************************************************

I124: I(124) - 0.018640*W(124) = 0

*******************************************************************************O/F SPLITTING-THK#1*******************************************************************************

*******************************************************************************ALL O/F FROM THK#1 GOES TO 5MOL PLANT, #16=0*******************************************************************************

W155: W(155) - W(44) - W(45) = 0
S155: S(155) - S(44) - S(45) = 0

*******************************************************************************O/F SPLITTING-THK2A*******************************************************************************

*******************************************************************************CRYSTALLIZER-MIXING*******************************************************************************

WM5MXT: WM(44) - W(44) - W(59) = 0
SM5MXT: SM(44) - S(44) = 0

*******************************************************************************WATER INPUT FIXED TO THE WATER IN #44*******************************************************************************

W59: W(59) - 0.040844*W(44) = 0

*******************************************************************************SEPARATION*******************************************************************************

WM5MX: WM(44) - W(43) - W(58) - W(61) = 0
SM5MX: S(44) - S(58) - S(61) = 0

A6-17
**OF AND U/F (FIXED COMPOSITION)**

\( W58: \quad W(58) - 4.291005 \cdot S(58) = 0 \)
\( W61: \quad W(61) - 2.773585 \cdot S(61) = 0 \)

**VAPOUR - FIXED TO THE TOTAL WATER INPUT**

\( W43: \quad W(43) - 0.054153 \cdot W(44) = 0 \)

**CENTRIFUGES**

\( WM5MCIN: \quad WM(61) - W(41) = 0 \)
\( WM5MCOUT: \quad WM(61) - W(40) - W(62) = 0 \)
\( S61: \quad S(61) - S(40) - S(62) = 0 \)

**FIX STEAM INPUT TO THE SODIUM BORATE IN INPUT**

\( W41: \quad W(41) - 0.092143 \cdot S(61) = 0 \)

**OF AND U/F (FIXED COMPOSITION)**

\( W62: \quad W(62) - 5.134969 \cdot S(62) = 0 \)
\( WAT40: \quad W(40) - 0.483680 \cdot S(40) = 0 \)

**CAKE LOSS**

\( W40: \quad W(40) - W(52) - W(91) = 0 \)
\( S40: \quad S(40) - S(52) - S(91) = 0 \)

**FIX COMPOSITION OF THE CAKE**

\( W91: \quad W(91) - 0.022255 \cdot W(40) = 0 \)
\( S91: \quad S(91) - 0.022255 \cdot S(40) = 0 \)

**SMOL M/L TANK**

\( W53: \quad W(53) - W(58) - W(62) = 0 \)
\( S53: \quad S(53) - S(58) - S(62) = 0 \)
\( W24: \quad W(24) - W(53) - W(102) - W(298) = 0 \)
\( S24: \quad S(24) - S(53) - S(102) = 0 \)

**FACTOR WATER INPUT TO THE TOTAL WATER IN INPUTS**

\( W298: \quad W(298) - 0.063171 \cdot W(53) = 0 \)

**OUTPUTS**

\( W70: \quad W(70) - 11.658228 \cdot S(70) = 0 \)
\( W73: \quad W(73) - 2.861004 \cdot S(73) = 0 \)

**VAPOUR FIXED TO THE TOTAL WATER INPUT**

\( W42: \quad W(42) - 0.078229 \cdot W(190) = 0 \)

**CENTRIFUGES**

\( WM10MCIN: \quad WM(73) - W(73) - W(38) = 0 \)
\( WM10MCOUT: \quad WM(73) - W(71) - W(113) = 0 \)
\( S73: \quad S(73) - S(113) - S(71) = 0 \)

**FIX STEAM INPUT TO THE SODIUM BORATE IN INPUT**

\( W38: \quad W(38) - 0.087221 \cdot S(73) = 0 \)

**OF AND U/F - FIXED COMPOSITION**

\( W71: \quad W(71) - 8.174312 \cdot S(71) = 0 \)
\( W113: \quad W(113) - 1.100840 \cdot S(113) = 0 \)

**WASH**

\( W98: \quad W(98) - 0.238274 \cdot W(113) = 0 \)
\( S98: \quad S(98) - 0.042752 \cdot W(98) = 0 \)

**MASS BALANCES**

\( WMWSBIN: \quad WM(113) - W(113) - W(98) = 0 \)
\( SMWSBIN: \quad SM(113) - S(113) - S(98) = 0 \)

---

A6-18
Appendix 6

WMWSHOUT: WM(113) - W(227) - W(230) = 0
SMWSHOUT: SM(113) - S(227) - S(230) = 0

***************FIX THE COMPOSITION OF WASH OUTPUT******************

W230: W(230) - 19.408163*S(230) = 0
W227: W(227) - 1.100840*S(227) = 0

***************MIXING*************************

W229: W(229) - W(230) - W(71) = 0
S229: S(229) - S(230) - S(71) = 0
W126: W(126) - W(70) - W(229) = 0
S126: S(126) - S(70) - S(229) = 0

***************SEPARATION*************************

W137: W(137) - 1.100840*S(137) = 0
W227SPL: W(227) - W(138) - W(137) = 0
S227: S(227) - S(138) - S(137) = 0

***************CONVERTER*************************

WMCONIN: WM(138) - W(138) - W(234) - W(45) - W(84) - W(148) = 0
SMCONIN: SM(138) - S(138) - S(234) - S(45) = 0

WMCONOUT: WM(138) - W(102) - W(120) = 0
SMCONOUT: SM(138) - S(102) - S(120) = 0

***************FIX WATER TO THE TOTAL WATER INPUT******************

W148: W(148) - 0.01126*W(234) - 0.01126*W(45) - 0.01126*W(138) = 0

***************FIX STEAM TO THE SODIUM BORATE IN INPUTS************

W84: W(84) - 0.076587*SM(138) = 0

***************FIX COMPOSITION OF OUTPUTS**********************

W120: W(120) - 0.481481*S(120) = 0
W102: W(102) - 4.347594*S(102) = 0

***************CAKE LOSS (FIXED COMPOSITION)************************

WC120: W(120) - W(92) - W(93) = 0
S120: S(120) - S(92) - S(93) = 0
W92: W(92) - 0.481481*S(92) = 0
S93: S(93) - 0.04535*S(120) = 0

***************10MOL COLD XTALS AREA***************************

***************CRYSTALLISERS**************************

***************MASS BALANCES- INPUTS**************************

WMCLDIN: WM(126) - W(126) - W(146) - W(106) = 0
SMCLDIN: SM(126) - S(126) - S(106) = 0

***************MASS BALANCES- OUTPUTS**************************

WMCLDOUT: WM(126) - W(141) - W(109) - W(29) = 0
SMCLDOUT: SM(126) - S(109) - S(29) = 0

***************FIX WATER TO THE TOTAL WATER INPUT******************

W146: W(146) - 0.021734*W(126) = 0

***************FIX VAPOUR TO THE TOTAL WATER INPUT**************

W141: W(141) - 0.012059*W(126) = 0

***************U/F AND O/F (FIXED COMPOSITION)*********************

W109: W(109) - 16.543859*S(109) = 0
W29: W(29) - 3.926108*S(29) = 0

***************CENTRIFUGES*******************************

WMCLCIN: WM(29) - W(29) - W(39) = 0
WMCLCOUT: WM(29) - W(112) - W(106) = 0
S29: S(29) - S(112) - S(106) = 0

***************FIX STEAM INPUT TO THE SODIUM BORATE INPUT********

W39: W(39) - 0.042604*S(29) = 0

***************U/F AND O/F (FIXED COMPOSITION)*********************

W112: W(112) - 1.173913*S(112) = 0
W106: W(106) - 6.246377*S(106) = 0

***************10MOL ML TANK******************************

***************MIXING**************************

WMML10IN: WM(109) - W(109) - W(299) = 0

A6-19
Appendix 6

WMML10OUT: \( WM(109) - W(118) = 0 \)
S109: \( S(109) - S(118) = 0 \)

***************FIX WATER INPUT TO THE TOTAL WATER INPUT***************

W299: \( W(299) - 0.046964 \cdot W(109) = 0 \)

***************STREAM SPLITTING***************

W118: \( W(118) - W(117) - W(21) = 0 \)
S118: \( S(118) - S(117) - S(21) = 0 \)
W117: \( W(117) - 17.518519 \cdot S(117) = 0 \)
WG117: \( W(117) - 0.263359 \cdot W(118) = 0 \)
W112OUT: \( W(112) - W(234) - W(193) = 0 \)
S112: \( S(112) - S(234) - S(193) = 0 \)
W234: \( W(234) - 1.173913 \cdot S(234) = 0 \)
W243: \( S(234) - 0.120475 \cdot S(45) = 0 \)

***************STREAM SPLITTING***************

W161: \( W(161) - 0.483680 \cdot S(161) = 0 \)
W167: \( W(167) - 0.481481 \cdot S(167) = 0 \)

***************MIXING***************

W169: \( W(169) - W(160) - W(167) = 0 \)
S169: \( S(169) - S(160) - S(167) = 0 \)
W179: \( W(179) - W(168) - W(161) = 0 \)
S179: \( S(179) - S(168) - S(161) = 0 \)

***************FLUID BED DRYER***************

SFBD169: \( S(169) - S(49) = 0 \)
WFBD169: \( W(169) - W(57) - W(49) = 0 \)
W5MFBD: \( W(49) - 0.417793 \cdot S(49) = 0 \)

***************ROTARY DRYER***************

SROT179: \( S(179) - S(94) = 0 \)
WROT179: \( W(179) - W(94) - W(95) = 0 \)
W5MROT: \( W(94) - 0.417793 \cdot S(94) = 0 \)

***************WYSMONT DRYERS***************

***************MIXING***************

W239: \( W(239) - W(137) - W(193) = 0 \)
S239: \( S(239) - S(137) - S(193) = 0 \)
SWYS239: \( S(239) - S(215) = 0 \)
WWYS239: \( W(239) - W(215) - W(78) = 0 \)
W215: \( W(215) - 0.894632 \cdot S(215) = 0 \)

***************PRODUCT LOSS***************

WWYS215: \( W(215) - W(77) - W(245) = 0 \)
SWYS217: \( S(215) - S(77) - S(245) = 0 \)
W77: \( W(245) - 0.146067 \cdot W(77) = 0 \)
S77: \( S(245) - 0.146067 \cdot S(77) = 0 \)

***************TOTAL FLOWS IN PP***************

TOTFLW(n=1..C): \( PP(n) - W(n) - S(n) - I(n) = 0 \)

***************LINKING 5MOL AND AB AND 5MOL SHIPPING***************

LINKAB: \( S(94) + W(94) - FMAB - FMSHIP = 0 \)

***************BAP***************

***************REACTOR AREA- MASS BALANCES***************

BA106: \( BA(106) - BA(1) - BA(5) = 0 \)
IF106: \( IF(106) - IF(1) = 0 \)
SS106: \( SS - SSL(2) - SSL(5) = 0 \)
SS106TOT: \( SS - SSL(106) - SSS(106) = 0 \)
Appendix 6

**COMPOSITION**

WF1: \[ WF(1) - 0.567543*AB = 0 \]
IF1: \[ IF(1) - 0.737667*AB = 0 \]
SULPAC: \[ SA - 0.507372*AB = 0 \]
SSEQ: \[ SSL(2) - 1.391*SA - WF(2) = 0 \]
WF2: \[ WF(2) - 0.04*SA = 0 \]
SS106: \[ SSS(106) - 0.167921*BA(106) = 0 \]
BA1: \[ BA(1) - 1.228628*AB = 0 \]
WATR: \[ WR - 0.447316*AB = 0 \]

**PRIMARY CLASSIFIER**

BAPC: \[ BA(106) - BA(7) - BA(8) = 0 \]
WRPC: \[ WF(106) - WF(7) - WF(8) = 0 \]
IFPC: \[ IF(106) - IF(7) - IF(8) = 0 \]
SSLPC: \[ SSL(106) - SSL(7) - SSL(8) = 0 \]
SSSPC: \[ SSS(106) - SSS(8) = 0 \]

**SECONDARY CLASSIFIER**

BAML: \[ BA(1) - BA(8) - BA(9) = 0 \]
WFMI: \[ WFM(1) - WF(8) - WF(9) - WF(36) = 0 \]
SSLM1: \[ SSLM(1) - SSL(8) - SSL(9) - SSS(8) = 0 \]
IFM1: \[ IFM(1) - IF(8) = 0 \]
BAM1OUT: \[ BAM(1) - BA(11) - BA(10) = 0 \]
WFM1OUT: \[ WFM(1) - WF(11) - WF(10) = 0 \]
SSM1OUT: \[ SSLM(1) - SSL(11) - SSL(10) = 0 \]
IFM1OUT: \[ IFM(1) - IF(11) - IF(10) = 0 \]

**FIX WATER INPUT TO THE WATER IN #8**

WF36: \[ WF(36) - 1.62796*WF(8) = 0 \]

**HTMB**

**MASS BALANCES**

BA5: \[ BA(5) - BA(215) = 0 \]
WF5: \[ WF(5) - WF(215) = 0 \]
SSL5: \[ SSL(5) - SSL(215) = 0 \]

**PRIMARThICKENER**

BA7: \[ BA(7) - BA(13) - BA(12) = 0 \]
WF7: \[ WF(7) - WF(13) - WF(12) = 0 \]
SSL7: \[ SSL(7) - SSL(13) - SSL(12) = 0 \]
IF7: \[ IF(7) - IF(13) - IF(12) = 0 \]

IF12: \[ IF(12) - 0.001871*WF(12) = 0 \]
SSL12: \[ SSL(12) - 0.508330*WF(12) = 0 \]
BA12: \[ BA(12) - 0.360711*WF(12) = 0 \]
IF13: \[ IF(13) - 0.329815*WF(13) = 0 \]
SSL13: \[ SSL(13) - 0.508355*WF(13) = 0 \]
BA13: \[ BA(13) - 0.360708*WF(13) = 0 \]
Appendix 6

************************SECONDARY TFHCYENF-R****************************

***************************MASS BALANCES*****************************

BAMST: BAM(4) - BA(14) - BA(13) - BA(10) = 0
WFMST: WFM(4) - WF(14) - WF(113) - WF(13) - WF(10) = 0
SSLMST: SSLM(4) - SSL(14) - SSL(13) - SSL(10) = 0
IFMST: IFM(4) - IF(13) - IF(10) = 0

BAMSTOUT: BAM(4) - BA(215) - BA(17) = 0
WFMSTOUT: WFM(4) - WF(215) - WF(17) = 0
SSLMSTOUT: SSLM(4) - SSL(215) - SSL(17) = 0
IFMSTOUT: IFM(4) - IF(17) = 0

WF215: VvT(215) - 0.615023*WFM(4) = 0
EFC17: IF(17) - 0.12544*VvT(17) = 0
SSL17: SSL(17) - 0.44736*WF(17) = 0
BAC17: BA(17) - 0.12244*WF(17) = 0

BA215: BA(215) - 0.122449*VvT(215) = 0
SSL215: SSL(215) - 0.447409*VvT(215) = 0

WATER INPUT TO THE WATER IN #10***************

WF113: WF(113) - 0.13182*WF(10) = 0

***************************CONSTANT SPLIT RATIO**************************

WATER INPUT TO THE WATER IN #257*******************

WF85: WF(85) - WF(257) - WF(110) = 0

**************************FILTER AID******************************

IF86: IF(86) - 0.5*BA(257) - 0.5*WF(257) - 0.5*SSL(257) = 0

IF6: IF(86) - 0.5*BA(257) - 0.5*WF(257) - 0.5*SSL(257) = 0

WF10: WF(110) - 0.75188*WF(257) = 0

**************************PRESSURE LEAF FILTERS********************

BA15: BA(15) - BA(12) - BA(85) = 0
WF15: WF(15) - WF(12) - WF(85) = 0
SSL15: SSL(15) - SSL(12) - SSL(85) = 0
IF15: IF(15) - IF(12) - IF(85) = 0

**************************COMPOSITION*****************************

IF18: IF(18) - 1.85874*WF(18) = 0
BA18: BA(18) - 0.35502*WF(18) = 0
SSL18: SSL(18) - 0.50186*WF(18) = 0
Appendix 6

Cryalliser Balances

BA89: BA(89) - BA(65) - BA(64) - BAS(65) - BAS(64) = 0
WF89: WF(89) - WF(65) - WF(64) - WF(63) = 0
SSL89: SSL(89) - SSL(65) - SSL(64) = 0

WF63: WF(63) - 0.02899*WF(89) = 0
WF65: WF(65) - 0.116135*WF(89) = 0
BAS65: BAS(65) - 0.13331*WF(65) = 0
BA65: BAS(65) - 0.51381*BA(65) = 0
SSL65: SSL(65) - 0.29168*SSL(65) = 0
BAS64: BAS(64) - 0.058635*WF(64) = 0

BAM66: BA(66) - BA(64) - BA(65) = 0
BASM66: BAS(66) - BAS(64) - BAS(65) = 0
WF66: WF(66) - WF(65) - WF(64) = 0
SSL66: SSL(66) - SSL(64) - SSL(65) = 0

BA66: BA(66) - BA(40) - BA(101) - BAS(40) - BAS(101) = 0
WF66: WF(66) - WF(40) - WF(101) - WF(59) = 0
SSL66: SSL(66) - SSL(40) - SSL(101) = 0

WF59: WF(59) - 0.0754*WF(66) = 0
WF101: WF(101) - 0.79*WF(66) = 0
WF40: WF(40) - 11.9563*BA(40) = 0
BAS40: BAS(40) - 0.01272*WF(40) = 0
SSL40: SSL(40) - 0.49372*WF(40) = 0
BAS101: BAS(101) - 0.31398*WF(101) = 0
BA101: BAS(101) - 3.75566*BA(101) = 0

Filtered Strong Liquor Tanks - Mix

BA62: BA(62) - BA(49) - BAS(49) = 0
WF62: WF(62) - WF(49) - WF(21) = 0
SSL62: SSL(62) - SSL(49) = 0

WF21: WF(21) - 0.493752*WF(49) = 0

Mass Balances

BA89OUT: BA(89) - BA(62) - BA(19) = 0
WF89OUT: WF(89) - WF(62) - WF(19) = 0
SSL89OUT: SSL(89) - SSL(62) - SSL(19) = 0

IST Stage Vacuum Filter

BA69: BA(69) - BA(73) - BA(71) = 0
Bas69: BAS(69) - BAS(73) - BAS(71) = 0
WF69: WF(69) - WF(73) - WF(71) = 0
SSL69: SSL(69) - SSL(73) - SSL(71) = 0

Stream Split

BA73: BA(73) - 0.408097*BA(69) = 0
BAS73: BAS(73) - 0.408097*BA(69) = 0
WF73: WF(73) - 0.408097*WF(69) = 0
SSL73: SSL(73) - 0.408097*SSL(69) = 0

LF-2 Mass Balances

BAM2: BAM(2) - BA(101) - BA(73) = 0
BASM2: BASM(2) - BAS(101) - BAS(73) = 0
WFM2: WFM(2) - WF(101) - WF(73) = 0
SSLM2: SSLM(2) - SSL(101) - SSL(73) = 0
BAM2OUT: BAM(2) - BA(24) - BA(25) - BA(50) = 0
BAMS2OUT: BASM(2) - BAS(24) - BAS(25) - BAS(50) = 0

A6-23
Appendix 6

WFM2OUT:  WFM(2) - WF(24) - WF(25) - WF(50) = 0
SSLM2OUT:  SSLM(2) - SSL(24) - SSL(25) - SSL(50) = 0

**************************************************************************COMPOSITION**************************************************************************

WF50:  WF(50) - 0.081794*WF(101) = 0
BA50:  BA(50) - 0.071343*WF(50) = 0
BAS50:  BAS(50) - 0.0117710*WF(50) = 0
SSL50:  SSL(50) - 0.114894*WF(50) = 0
BA25:  BA(25) - 0.07200*WF(25) = 0
BAS25:  BAS(25) - 8.46948*VvT(25) = 0
SSL25:  SSL(25) - 0.49372*WF(25) = 0
BA24:  BA(24) - 0.09364*WF(24) = 0
SSL24:  SSL(24) - 0.01272*VvT(24) = 0

**************************************************************************MIXING IN THE IST STAGE REPULP TANK**************************************************************************

BA27:  BA(27) - BA(25) - BA(26) - BA(70) - BA(116) = 0
BAS27:  BAS(27) - BAS(25) - BAS(70) - BAS(116) = 0
WF27:  WF(27) - WF(25) - WF(26) - WF(70) - WF(116) = 0
SSL27:  SSL(27) - SSL(25) - SSL(26) - SSL(70) - SSL(116) = 0

**************************************************************************II STAGE VACUUM FILTER**************************************************************************

BA119:  BA(119) - BA(98) - BA(72) - BA(26) = 0
WF119:  WF(119) - WF(98) - WF(72) - WF(26) = 0
SSL119:  SSL(119) - SSL(98) - SSL(72) - SSL(26) = 0

**************************************************************************CONSTANT SPLIT OF #119 (USED FOR WASHING)**************************************************************************

BA72:  BA(72) - 0.096688*BA(119) = 0
WF72:  WF(72) - 0.096686*WF(119) = 0
SSL72:  SSL(72) - 0.096686*SSL(119) = 0
BA98:  BA(98) - 0.735707*BA(119) = 0
WF98:  WF(98) - 0.735707*WF(119) = 0
SSL98:  SSL(98) - 0.735707*SSL(119) = 0
BA26:  BA(26) - 0.1673114*BA(119) = 0

**************************************************************************SF-3**************************************************************************

WFM3:  WFM(3) - WF(72) - WF(27) = 0
SSLM3:  SSLM(3) - SSL(72) - SSL(27) = 0
BASM3:  BASM(3) - BAS(27) = 0
BAM3OOUT:  BAM(3) - BA(29) - BA(30) - BA(116) = 0
BASM3OOUT:  BASM(3) - BAS(29) - BAS(30) - BAS(116) = 0
WFM3OOUT:  WFM(3) - WF(29) - WF(30) - WF(116) = 0
SSLM3OOUT:  SSLM(3) - SSL(29) - SSL(30) - SSL(116) = 0

**************************************************************************COMPOSITION**************************************************************************

WFC116:  WF(116) - 0.036242*WF(27) = 0
BA116:  BA(116) - 0.06795*WF(116) = 0
BAS116:  BAS(116) - 0.01977*WF(116) = 0
SSL116:  SSL(116) - 0.01079*WF(116) = 0
BAS30:  BAS(30) - 5.151364*WF(30) = 0
SSL30:  SSL(30) - 0.009717*WF(30) = 0
BA29:  BA(29) - 0.06892*WF(29) = 0
BAS29:  BAS(29) - 0.02005*WF(29) = 0

**************************************************************************SPLIT #29**************************************************************************

BAS29OOUT:  BAS(29) - BA(69) - BA(70) = 0
WFC29OOUT:  WF(29) - WF(69) - WF(70) = 0
SSL29OOUT:  SSL(29) - SSL(69) - SSL(70) = 0

**************************************************************************ASSUME CONSTANT COMPOSITION**************************************************************************

BAC69:  BA(69) - 0.108058*BA(29) = 0
BASC69:  BAS(69) - 0.108058*BA(29) = 0
SSL69:  SSL(69) - 0.108058*SSL(29) = 0

A6-24
Appendix 6

WFC69: WF(69) - 0.108058*WF(29) = 0

BA96: BA(96) - BA(30) - BA(97) - BA(98) = 0
BAS96: BAS(96) - BAS(30) - BAS(97) = 0
WF96: WF(96) - WF(30) - WF(97) - WF(112) - WF(98) = 0
SSL96: SSL(96) - SSL(30) - SSL(97) - SSL(98) = 0

WF112: WF(112) - 0.205377*WF(98) - 0.205377*WF(30) - 0.205377*WF(97) = 0

WATER INPUT TO THE WATER INPUTS

WF103: WF(103) - 0.13942*WF(24) = 0

MASS BALANCE IN

BA4101JT: BA(41) - BA(80) - BAS(80) = 0
WF41OUT: WF(41) - WF(80) = 0
SSL41OUT: SSL(41) - SSL(80) = 0

BA125: BA(125) - BA(50) - BA(71) = 0
BAS125: BAS(125) - BAS(50) - BAS(71) = 0
WF125: WF(125) - WF(50) - WF(71) = 0
SSL125: SSL(125) - SSL(50) - SSL(71) = 0

SPLIT STREAM #124

BA125OUT: BA(125) - BA(124) - BA(49) = 0
BAS125OUT: BAS(125) - BAS(124) - BAS(49) = 0
WF125OUT: WF(125) - WF(124) - WF(49) = 0
SSL125OUT: SSL(125) - SSL(124) - SSL(49) = 0

CONSTANT SPLIT RATIO

BA124: BA(124) - 0.44714*BA(125) = 0
BAS124: BAS(124) - 0.44714*BA(125) = 0
WF124: WF(124) - 0.44714*WF(125) = 0
SSL124: SSL(124) - 0.44714*SSL(125) = 0

SETTLERS

BA96SET: BA(96) - BA(120) - BA(119) = 0
BAS96SET: BAS(96) - BAS(120) - BAS(119) = 0
WF96SET: WF(96) - WF(120) - WF(119) = 0
SSL96SET: SSL(96) - SSL(120) - SSL(119) = 0

COMPOSITION

BA120C: BA(120) - 0.06744*WF(120) = 0
BAS120C: BAS(120) - 0.06744*WF(120) = 0
SSL120C: SSL(120) - 0.06744*WF(120) = 0
BA119C: BA(119) - 0.06744*WF(119) = 0

BIRD CENTRIFUGES

BA120: BA(120) - BA(97) - BA(56) = 0
BAS120: BAS(120) - BAS(97) - BAS(56) = 0
WF120: WF(120) - WF(97) - WF(56) = 0
SSL120: SSL(120) - SSL(97) - SSL(56) = 0

CONSTANT COMPOSITION OF THE CAKE

BA56: BA(56) - 0.067396*WF(56) = 0
BAS56: BAS(56) - 0.067396*WF(56) = 0
SSL56: SSL(56) - 0.067396*WF(56) = 0

CONSTANT COMPOSITION OF THE OF

BAS97: BAS(97) - 0.067396*WF(97) = 0

DRYER

BA56DRY: BAS(34) - BAS(33) = 0
WF56DRY: WF(56) - WF(31) = 0
SSL56DRY: SSL(56) - SSS(34) - SSS(33) = 0
BA56C: BASC56 - BAS(56) - BA(56) = 0
BAS33: BAS(33) - 0.052635*BAS(34) = 0
SSS33: SSS(33) - 0.052635*SSS(34) = 0
TOTBAP(n=1): BAP(n) - BA(n) - BAS(n) - WF(n) - SSL(n) - SSS(n) - IF(n) = 0

**LINKING BA AND ABA AND BA SHIPPING**

FIVMO: FMAB - ABFM - WMOL = 0
WFMO: WMOL - 0.417793*ABFM = 0
FFBIN: FMFB - FMAB = 0

**FURNACE FEED BIN**

CHILR: ABCR - ABFM = 0
HAMBIN: ABHMB - ABCR = 0
HAMML: ABHM - ABHMB = 0
SCRBIN: ABSFB - ABHM = 0
SCRENI: ABSFB - ABPR - ABREJ = 0
SCRENI: ABREJ - 0.011439*ABSFB = 0

**ABA PLANT**

FEDBIN1: BAAB - ABABA - WABA = 0
WATABA: WABA - 0.775862*ABABA = 0
FEDBIN2: BAFB - BAAB = 0

**FURNACE FEED BIN**

ABAFF1: BAFFB - BAFB = 0

**SCREEN FEED**

ABACBLR: ABACR - ABABA = 0
ABAVBIN: ABAHM - ABACR = 0

**SEPARATOR SURGE**

ABASURG: ABAHM - ABAPR - ABALOS = 0
ABALS: ABALOS - 0.086346*ABAHM = 0

**TEST BINS**

ABATBIN1: ABAPROD - ABAPR = 0

**SHIPPING**

FMSC1: FMSHIP - FMSC - FMREJ = 0
FMSC2: FMREJ - 0.007676*FMSHIP = 0
FMSC3: FMSC - FMS - FMP = 0

**10MOL SCREENING**

TMSC1: S(77) + W(77) - TMP - TMREJ - TMST - TMBULK = 0
TMSC2: TMREJ - 0.016038*S(77) - 0.016038*W(77) = 0
TMSTOR: \[ TMST - 0.00693*S(77) - 0.00693*W(77) = 0 \]

FMSIL: \[ FMBULK - PENTS - FMS = 0 \]

PNT1: \[ S(49) + W(49) - PENTS - PENTREJ = 0 \]

PNT2: \[ PENTREJ - 0.01604*S(49) - 0.01604*W(49) = 0 \]

PPPACK1: \[ PPP - FMP - TMP = 0 \]

PPPACK2: \[ FMP - FM2550 - FM50100 - FMPPBAG = 0 \]

PPPACK3: \[ TMP - TM2550 - TM50100 - TMPPBAG = 0 \]

PPSHIP1: \[ FMSHIP1 - FMP - PENTS - FMBULK = 0 \]

PPSHIP2: \[ TMSHIP - TMP - TMBULK = 0 \]

PPSII-IPI: \[ FMSHIP - FMP - PENTS - FMIBULK = 0 \]

PPSII-IIPI2: \[ TMSHIP - TNT - TNIBULK = 0 \]

ABSHIPI: \[ ABPR - ABP = 0 \]

ABSHIP2: \[ ABBSH - ABP = 0 \]

ABSHIP4: \[ ABP - AB2550 - AB50100 - ABPPBAG = 0 \]

ABSHIP5: \[ ABSHIP - ABP = 0 \]

ABASHIPI: \[ ABAPROD - ABAP = 0 \]

ABASHIPI2: \[ ABABSH - ABAP = 0 \]

ABASHIPI3: \[ ABAP - ABA2550 - ABA50100 - ABAPPBAG = 0 \]

ABASHIPI4: \[ ABASHIP - ABAP = 0 \]

BAPSC1: \[ BASH - BAREJ - BADYB = 0 \]

BAPSC2: \[ BAREJ - 0.04155*BASH = 0 \]

DAYB: \[ BADYB - BASEL - BABS = 0 \]

SILOS: \[ BASIL - BABP - BASTOR = 0 \]

BAPACK1: \[ BABSH - BABP = 0 \]

BAPACK2: \[ BABP - BA2550 - BA50100 - BAPPBAG = 0 \]

BASHIPI: \[ BASHIP - BABP - BABS - BASTOR = 0 \]

TOTSHIP: \[ TOTSH - FMBULK - PPP - TMBULK - BASHIP - ABSHIP - ABASHIP = 0 \]

GENERATE
Tables A6.1-A6.3 are given in Volume II (pp A6.28-A6.39)
A6.3 The Results of Marginal Allocation of Environmental Burdens

Fig. A6.1 Nuclear electricity allocated by the marginal approach

Fig. A6.2 Hydro-electricity allocated by the marginal approach

Fig. A6.3 Coal reserves allocated by the marginal approach
Fig. A6.4 Oil reserves allocated by the marginal approach

Fig. A6.5 Gas reserves allocated by the marginal approach

Fig. A6.6 Other nonrenewable resources allocated by the marginal approach
Fig. A6.7 Renewable resources allocated by the marginal approach

Fig. A6.8 CO emissions allocated by the marginal approach

Fig. A6.9 NOx emissions allocated by the marginal approach
Appendix 6

Fig. A6.10 VOC emissions allocated by the marginal approach

Fig. A6.11 Emissions of metals to air allocated by the marginal approach

Fig. A6.12 Dust emissions allocated by the marginal approach
Fig. A6.13 Halide emissions allocated by the marginal approach

Fig. A6.14 Waste water discharge allocated by the marginal approach

Fig. A6.15 Emissions of metals to water allocated by the marginal approach
Fig. A6.16 Total dissolved solids (TDS) allocated by the marginal approach

Fig. A6.17 Total suspended solids (TSS) allocated by the marginal approach

Fig. A6.18 Emissions of oil and greases allocated by the marginal approach
Fig. A6.19 Chemical oxygen demand (COD) allocated by the marginal approach

Fig. A6.20 Biological oxygen demand (BOD) allocated by the marginal approach

Fig. A6.21 Solid waste (including overburden) allocated by the marginal approach
A6.4 The Results of Marginal Allocation of Environmental Impacts

Fig. A6.22 Direct global warming potential (GWP) allocated by the marginal approach

Fig. A6.23 Indirect global warming potential (GWPI) allocated by the marginal approach

Fig. A6.24 Ozone depletion potential allocated by the marginal approach
Fig. A6.25 Acidification potential allocated by the marginal approach

Fig. A6.26 Nutrification potential allocated by the marginal approach

Fig. A6.27 Photochemical oxidants creation potential (POCP) allocated by the marginal approach
Fig. A6.28 Human toxicity allocated by the marginal approach

Fig. A6.29 Landfill volume (excluding overburden) allocated by the marginal approach
Appendix 7 is given in Volume II
VOLUME 2. CONTAINS CONFIDENTIAL INFORMATION, SEE PAGE ii