AN INVESTIGATION INTO MECHANICAL FAILURE
OF COMPOSITE PROPELLANTS.

by

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A summary of a thesis to be presented for partial fulfillment of the
requirement for a Collaborative PhD at the University of Surrey.
A typical composite propellant consists of a matrix of either carboxyl or hydroxyl terminated polybutadiene loaded with between 80 to 90 percentage by weight of particulate solid. Its mechanical properties are non-linear and the failure processes complex.

The initiation and spacial aspects of the failure have been investigated under uniaxial conditions. Poisson's ratio is treated as a strain dependent parameter and related to the work required to fracture the propellant.

The cantilever beam technique has been adapted to measure this work of fracture. The results show that the magnitude of this work is dependent on the type of propellant tested and the velocity of the crack front, but can be considered independent of the sample size and geometry. A geometrical argument based on the stress analysis of the sample is used to show that the crack velocity varies systematically with crack length and load. This leads to fracture criteria based on a critical strain and yield stress, and to the conclusion that the majority of the irreversible work is dissipated near to the crack tip. The amount of irreversible work is estimated and deducted from the measured work of fracture to give the "surface energy" of the composite.

Selected fracture surfaces have been studied using the scanning electron microscope and the photographs have been interpreted to give details of the fracture processes. The observed surface damage is evidence for the large amount of irreversible work required to propagate a crack in these types of materials.

The measured work of fracture is used to assess the resistance to failure of various different types of composite propellant. An understanding of the mechanism of crack propagation can lead to safer operation of the rocket motor.
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A thesis submitted to the
Faculty of Mathematical and Physical Sciences
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SUMMARY

A typical composite propellant consists of a matrix of either carboxyl or hydroxyl-terminated polybutadiene loaded with between 80 to 90 percentage by weight of particulate solid. Its mechanical properties are non-linear and the failure processes complex.

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The measured work of fracture is used to assess the resistance to failure of various different types of composite propellant. An understanding of the mechanism of crack propagation can lead to safer operation of rocket motors.
Dedicated to

my Parents
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1. INTRODUCTION

1.1 Introductory Remarks

The following introductory sections are a review of some of the past work published on polymers, composite materials and propellant properties. A comprehensive review is impossible in the space available. Therefore the reader who requires more detailed information is referred to more authoritative text books for polymer properties 1, 2, 3, 4, 5, conference proceedings 6 and the CPIA publication 7 for propellant properties, together with the many references given by them. What follows are in the author's opinion the more important parts of the past work having regard to the investigation described in this thesis.

The understanding of the structural integrity of rocket motors has made considerable advances in the past few years. However, although the important mechanics of the problem have been established and methods worked out to handle them, as yet no technique has been adopted to categorise composite propellants nor to make quantitative measurements which would allow these methods to be used with complete certainty.

The aim of this work was, therefore, to investigate composite propellant failure and to establish a method to obtain data which would allow the comparison of different types of propellants. The best propellant for the particular use envisaged could then be selected from the possible alternatives. The method could also be used in a propellant chemistry development programme to make "better" composite propellants and hence more reliable rocket motors.

1.2 Composite propellants and the rocket motor

Solid propellants are unique materials in that they must simultaneously be the source of propulsive energy and one of the integral structural components of rocket motors. A typical rocket motor is shown
in Figure (1). A motor consists of a case, which must be strong enough to withstand the internal pressure during combustion, with a constricting throat and nozzle assembly at one end. Most of the space inside the case is filled with the solid propellant charge which has a central hole called the conduit. The cross-sectional shape of the conduit is usually complex, which can be seen in Figure (1) as a six-pointed star. The combustion of the propellant is initiated by an igniter which is usually situated at the opposite end of the conduit from the nozzle.

To produce the hot gas required for propulsion the propellant is a heterogeneous mixture of both organic fuel and inorganic oxidizer. The usual oxidizer, ammonium perchlorate, is present to the extent of 80 - 90 per cent by weight. To achieve such a high solids loading it is normal to have a multimodal particle distribution in the size range 10 to 750 μm. The smaller crystals tend to be roughly spherical, whereas the larger crystals which are present in smaller proportions are more irregular in shape. Small quantities of other solids may be added as ballistic modifiers and larger quantities of Aluminium, up to 15 per cent by weight, may also be used as a replacement for some of the ammonium perchlorate. Traces of bonding agent and other additives are essential constituents of a typical propellant.

The organic fuel is based on a polybutadiene pre-polymer of the form

$$R - \overset{n}{\sum} CH_2 = CH - CH = CH - CH_2 - R$$

where R can be either the carboxyl radical, -COOH, or the hydroxyl radical, -OH. Other types of radicals have been used for propellants but are not relevant to this work. The pre-polymer which is a viscous liquid is mixed with the solid material, cast into the rocket motor case and then cured at 60°C to give a cross-linked rubber matrix, containing
the dispersed solid filler, of the desired shape. The name of the type
of propellant produced is derived from the terminating radical. Hence,
Carboxyl-terminated polybutadiene (CTPB) and Hydroxyl-terminated
polybutadiene (HTPB) are two types of propellant which have been investiga-
ted in this work.

Solid propellant is formulated to burn smoothly and to evolve hot
gases in a predictable manner. The motors are designed to provide a
specific performance to meet the requirements of a planned application,
and the performance is given in terms of motor thrust as a function of
burn time. The thrust of a motor is directly proportional to the chamber
pressure

\[ T = P_c A_t C_f \quad (1) \]

where \( T \) is the thrust (Newtons)
\( P_c \) is the chamber pressure (N/m\(^2\))
\( A_t \) is the nozzle throat area (m\(^2\))

and \( C_f \) is the thrust coefficient, a dimensionless efficiency factor
which has a value of less than unity. Chamber pressure, \( P_c \), is propor-
tional to the exposed propellant burning surface area, \( A_b \), determined from
conduit shape and length,

\[ P_c = \left[ \frac{A_b}{A_t} \right] \frac{1}{1-n} \quad (2) \]

where the constant \( n \), is less than unity. The study of the internal
ballistics of a solid rocket motor is primarily one of the control and
variations of \( A_t \) and the burning rate.

The burning surface area is completely determined by charge design
while the burning rate is governed primarily by propellant ingredients
and the chamber pressure. Hence the principal ballistic design problem
is choice of a suitable propellant and the proper manipulation of the
charge geometry to achieve the desired thrust - versus - time performance.
However, if during use the structural integrity of the propellant charge should fail, then there would be an increase in burning area. This will cause an increase in chamber pressure resulting in faster burning, and the process can rapidly lead to catastrophic failure of the rocket motor. Therefore limitations must be placed on the use of rocket motors and the propellant investigated to ascertain its resistance to failure. Hence, structural integrity considerations impose constraints upon ballistic designs.

1.3 Properties of highly filled elastomers

Rigid particles in a rubbery or elastomeric material give a composite material whose properties depend on whether the filler interacts strongly or weakly with the polymeric binder. A general increase in modulus and sometimes also in strength is achieved by the use of finely divided solids. Other factors, such as uniformity of dispersion, particle size, shape and distribution of filler and volume fraction also have important influences on the magnitude of these changes.

If the filler is only weakly bonded to the binder the properties can be treated by classical theory for spheres immersed in an elastic matrix.

A mathematical theory was worked out by Einstein for a low concentration of rigid spheres. With no interaction between the spheres

\[ \eta_r = 1 + 2.5V_f \quad \text{(3)} \]

where \( V_f \) is the volume fraction of the spheres. The relative viscosity \( \eta_r \) is the ratio of the viscosity of the suspension to the viscosity of the ideal fluid. Since the publication of this basic analysis numerous attempts have been made both theoretically and empirically to extend the formula to high concentrations of non-spherical filler particles, but with only limited success. For high concentrations of strongly bonded
filler particles the situation is even worse and no satisfactory
empirical or theoretical relationship has been established. 10

The response of such highly filled materials not only exhibit a
strong time and temperature dependence (see section 1.4) but also indicate
significant non-linear stress-strain behaviour. Of particular importance
are the large departures from linear viscoelastic behaviour at relatively
low strain levels 11 This behaviour is attributed to the separation of
the soft matrix from the hard filler particles and the formation of voids. 12
This phenomenon is called dewetting and was first observed in experiments
on filled vulcanised rubbers. 13

Experiments conducted on composites under an applied hydrostatic
pressure also give evidence for the suppression of voids. The results show
a more linear behaviour and an increase in the strength of the composite.
This is due to the pressure reinforcing the filler-binder bond and
suppressing the formation and growth of the voids. 14, 15

The extent of this dewetting and non-linearity also depends on the
state of strain and strain history. As a result, the linear theory of
viscoelasticity will be severely limited in its application to composites
with a loading density of rigid filler particles near to its maximum
value. Composite propellant (see section 1.1) is such a material.

There seems to be a critical strain for the initiation of dewetting
in some propellant systems. This is probably related to the strength of
the adhesive bond between binder and filler. 16 The heterogeneity of
composite propellant results in stress concentrations between the filler
particles. 17 Hence, when the propellant is deformed the stress
distribution is localized in these concentrations. The local stress
increases until it exceeds the binder-filler interface bond strength,
thus causing dewetting and voids form around the filler particles.
Cohesive failure would occur if the cohesive strength of the binder were exceeded, resulting in the formation of vacuoles (small voids) in the binder.

The similarity between cohesive and adhesive failure, as pointed out by Williams, is recognised by the fact that they both lead to the initiation of voids, the former in the binder and the latter at the interface. Since the binder is essentially a film around the solid inclusions, any cohesive failure in it leads directly to an adhesive separation at the interface, because the adhesive force in a normal propellant is weaker than the cohesive strength of the binder.

The strain-rate and temperature will affect both the binder-filler bond strength and binder mobility, hence any analytical representation of this behaviour will be complex. However experiments by Kruse indicate that the temperature dependence of this binder-filler bond is not very important, at least in the propellant composition studied.

The dewetting phenomenon tends to soften the composite material so that subsequent stretching produces a stress-strain curve which is displaced downwards. This behaviour is now known as the "Mullins' effect" due to the early work of Mullins who examined the phenomenon in detail. He postulated that a breakdown of particle-to-particle association and possibly particle-to-rubber association could account for the effect. Beuche proposed a molecular argument based on the assumption that the centres of filler particles are displaced in an affine manner during deformation of the rubber. The polymer network chains which are attached at both ends to filler particles break when the particles have separated enough to stretch the chains to near their full elongation. Whether the polymer chains pull loose from the particle surface or break along the backbone will depend upon the relative strengths of the bonds involved. A schematic drawing of the process is shown in Figure (2).
Dewetting always precedes rupture in propellants which are dilating by binder-oxidizer separation and may be considered to be the first step in the failure process. These dewetted regions are relatively weak and are the sites for the progressive failure of the propellant sample. Microscopic studies \(^2\) have shown that propellant rupture occurs in two steps. First, when deformed in tension, the binder separates from the oxidiser, which results in the formation of elliptical voids around the filler particles, Figure (3). Secondly, a tear is initiated under conditions of tensile strain in the binder near to the apex of a void and propagates perpendicularly to the direction of straining. The dewetting in propellants can occur in localized regions and hence the strain is not uniform throughout the test section. Dewetting is also observed in biaxial loading where a number of oxidizer particles effectively agglomerate, permitting dewetting around the periphery only. It is believed that dewetting will also occur in specimens loaded in triaxial tension, in which the dewetted agglomerates will be roughly spherical. In all cases the processes of dewetting are essentially the same i.e. separation followed by tearing.

1.4 Modelling Temperature dependence

Temperature dependence is a fundamental aspect of the properties of polymers. Classical viscoelastic theory has been successful in characterising its influence on the behaviour of polymers.\(^2\) The time and temperature dependencies are inter-related and a change in temperature scale is usually equivalent to a change in the time scale.\(^22\) This is known as time - temperature equivalence and enables data measured over a limited time scale at a series of temperatures to be combined to give a master curve which represents the behaviour of the material at a selected temperature but over a very wide time scale.
The method used is based on empirical relationships for the temperature shift factor $a_T$ and variables corrected for their temperature dependence, which are called reduced variables. The assumptions made have now been given a molecular interpretation by various molecular theories of polymer viscoelasticity.\textsuperscript{25, 26}

The best known relationship is that devised by Williams, Landel and Ferry (WLF)\textsuperscript{27}

$$\log a_T = \frac{C_1(T - T_0)}{C_2 + T - T_0} \quad \text{(4)}$$

Where $T$ is the test temperature and $C_1$ and $C_2$ are constants for the reference temperature $T_0$. If this reference temperature is such that $T_0 - T_g = 50^\circ C$, where $T_g$ is the glass transition temperature, then $C_1$ and $C_2$ are general constants with values of 8.86 and 101.6 respectively.

A more useful version of the equation is

$$\log a_T = -\frac{17.44(T - T_g)}{51.6 + T - T_g} \quad \text{(5)}$$

Values of $a_T$ can then be calculated if the glass transition temperature is known or can be measured.\textsuperscript{28} Otherwise it may be deduced from the experimental data.

The WLF method of reduction gives a good correlation for filled polymers,\textsuperscript{29} to such an extent that it is standard practice in the propellant industry.

Uniaxial data in the form of isothermal log/log plots of modulus or maximum stress (both normalised to a standard temperature) and of strain, all against strain-rate, may be shifted along the rate axis by an amount $\log a_T$. The magnitude of $a_T$ is calculated for the particular test temperature. In this way the shifted plots are superimposed to give a
continuous master-curve. This technique enables measurements of a tensile variable made over a range of strain-rates \( R \) and of temperature to be plotted on a single scale, that of log reduced strain-rate, \( \log R_a \), to give a single curve characteristic of the propellant tested.  

1.5 Theoretical failure criteria

Theoretical failure criteria can be classified as either microscopic or macroscopic. The macroscopic criteria treat the composite propellant as a continuum and general theories, such as thermo-dynamics, elasticity, viscoelasticity, etc., are used to describe the behaviour. For microscopic criteria the propellant is treated as a conglomeration of particles and microscopic theories involving such phenomena as binder-filler interaction, effect of particle size and shape, and interparticle friction, etc., must be combined to give overall criteria.  

The different approaches are all describing the same process, therefore, a microscopic failure theory must be extended to the macroscopic range before it can be considered a true failure criterion. 

1.5.1 Macroscopic failure theories

The classical macroscopic failure theories of maximum principal stress, maximum strain, maximum shear stress and maximum total energy have been applied to metallic materials with a fair degree of success. This is due to the fact that most metals yield at relatively low strains and yielding is generally classified as failure. However, the large strains associated with polymer failure give considerable deviations from the predictions of classical infinitesimal theory.

Finite elasticity theory has been applied to certain rubbers with reasonable results. The method involves generating a three-dimensional failure surface using the three principal stresses or
strains. These surfaces are defined so that any stress combination which occurs within the boundary will not cause rupture. However, this approach does not include the effects of time and temperature and a general case, therefore, would involve a failure surface in a five-dimensional space. Even so, a composite material is sensitive to history of loading, and the damage is not only cumulative, but the rate of accumulation also depends on the amount of previous damage, and such cases cannot be treated by the failure surface method.

Some of these problems have been overcome by constructing a uniaxial failure envelope which has been developed for filled elastomers by Smith. This envelope normally consists of a log-log plot of reduced failure stress (see section 1.4) versus the strain at break. Any calculated point (reduced stress and corresponding strain) which falls within this envelope represents a condition under which the propellant will not fail. This method is extensively used to characterise propellants in the laboratory but has limitations in its uses for multi-axial stress states and fatigue.

The time dependency has been modelled using a viscoelastic approach consisting of a series of springs and dashpots. The method is essentially linear and does not fully describe the nonlinear behaviour of composite propellants. An excellent review article of the problems involved and techniques used has been produced by Williams.

The energetic approach to macroscopic failure has been based on Griffith's hypothesis. The theory was formulated for brittle fracture and involved the energy balance between the elastic strain energy and the release of this energy in the form of the surface energy of a crack (see section 1.6). For rubbers the concept was
modified to a characteristic energy for tearing \(^{37}\), which is very similar to the Griffith's surface energy. It should be noted that in a solid material the surface energy is not the same as the surface tension.\(^{38}, 39\)

The strain energy can be released by means of three mechanisms, they are surface energy, kinetic energy and heat or dissipation energy. These are combined into what is referred to as the work of fracture, i.e., the energy in the form of work required to create unit area of crack regardless of the processes involved.\(^{40}\) The Griffith's approach has been extended by these considerations to failure in viscoelastic materials,\(^{41}\) (see section 1.5.2).

If the experimental difficulties involved in measuring the thermodynamic quantities, in the appropriate form for the energy equation, could be overcome this approach would become a very useful failure criterion.

1.5.2 Microscopic failure theories

The "weakest link" theory has been used with extreme-value statistics to formulate a failure theory.\(^{42}\) The theory considers the ultimate strength of a volume of material as the strength of its weakest unit volume. The material is treated as an elastic continuum containing a random distribution of flaws.\(^{43}\) While the theory has been successfully used to describe brittle-type failures\(^{44}, 45\), it is not directly applicable to the ductile-type failures associated with composite propellants.

As the filler particle size and distribution has an effect on the mechanical properties of propellants\(^{46}\), a granular mechanics approach has been used similar to that in the study of soil behaviour. The propellant was treated as various sized spherical particles
interconnected by an elastic binder. The filler particles were assumed to behave in an elastic manner and have a unique coefficient of friction. By the use of a statistical analysis the behaviour of the composite under compressive and shear stress loads was successfully modelled. The fitting parameters have to be determined from experimental data. However the theory does not include time effects and breaks down when applied to tensile loads.

The effect of cumulative damage has been described by the hypothesis used by Miner for the fatigue of metals. Failure occurs at the $M$th load cycle when

$$\sum_{i=1}^{M} \frac{n_i}{N_i} = 1 \quad (6)$$

where $n_i$ is the number of cycles at the $i$th stress level, and $N_i$ is the number of cycles at the $i$th stress level required to cause failure.

This linear expression was modified for non-linear behaviour and applied to propellant failure in the form

$$\sum_{i=1}^{M} \left(\frac{n_i}{N_i}\right)^{x_i} = 1 \quad (7)$$

where the empirical coefficient $x_i$ is stress dependent and is less than one for large stresses and greater than one for small stresses.

1.5.3 Other failure theories

Empirical criteria have been used in several laboratories. A typical example is that a failure will occur sometime in a rocket motor if the strain exceeds one half of the average strain at break measured by uniaxial test.
A theory that uses the thermal activation concept to drive the failure process has been proposed for unfilled rubbers below their glass transition temperature. However, a molecular reaction rate model as first proposed by Tobolsky and Eyring has been applied to the dilatational failure of filled elastomers. In the notation of reference (53), if the number of bonds per unit cross-sectional area of thread is \( N \), then the rate of breaking of the bonds under an applied stress \( S \), assuming that repair was possible, was calculated as

\[
\frac{1}{N} \frac{dN}{dt} = \frac{kT}{h} \exp \left( \frac{-\Delta F}{RT} \right) 2 \sinh \left( \frac{S \lambda}{2NT} \right)
\]

In this relationship \( \Delta F \) is the free energy of activation, \( \lambda \) is the average distance projected in the direction of stress between equilibrium positions in the displacement process, \( T \) is the absolute temperature, and \( k \), \( h \) and \( R \) are Boltzmann's constant, Planck's constant, and the gas law constant, respectively. When the thread breaks, \( N \) equals zero, since the number of remaining bonds is zero.

Experimental results indicate that the argument of the hyperbolic sine is independent of temperature. This allows the approximate integration of the differential form of the equation from the unstressed condition to the breakage of the bonds, and if the time failure is denoted as \( t_f \) then

\[
\ln \left( \frac{t_f ST}{T} \right) = A + B - CS
\]

where \( A \) and \( B \) are constants. The term \( C = \frac{\lambda}{2 N_0 kT} \) where \( N_0 \) is the number of unbroken bonds per unit area in the unstressed material, and is found to be a material constant. The value of the constants can be found from three measurements of \( t_f \) at independent conditions of temperature and stress level.
When applied to dilatational failure, the number of unbroken bonds will reach zero at the point at which the void initiates and begins to propagate along the particle boundary. Further assumptions are required if the equations are to be used to determine the actual failure of a test sample.

The reaction rate model of failure has also been investigated by electron paramagnetic resonance (EPR) and infrared (IR) spectroscopic techniques in an attempt to investigate the bond rupture phenomena.

Farris has suggested from the results of his extensive investigation of propellant dilatation that the maximum allowable strain in use should be less than that at which the propellant starts to dewet. This observation is based on results from a gas dilatometer and have been interpreted to give an expression for the following stress strain function for a highly filled elastomer such as propellant. 

\[ S = E \varepsilon \exp \left( -\frac{B \Delta V}{\varepsilon} \right) \] (10)

where \( S \) is the stress, \( \varepsilon \) the strain, \( E \) the modulus and \( \Delta V/V_0 \) the dilatation. The constant \( B \) does not appear to depend strongly upon rate or temperature, which indicates that the vacuole growth process modifies the material behaviour in the same way over a broad range of test conditions. In the simple model the modulus is assumed to remain constant, not a realistic assumption, but the resulting effects are negligible compared to those caused by the dewetting processes.

Taking the vacuole initiation at the interface as being stress controlled and the void growth to be a function of the viscoelastic properties of the binder, the equation can be modified for constant strain \( R \), to give
\[
\frac{\Delta V}{V} = k \int_0^t R \exp\left(\frac{S(t) - S_0}{S^*}\right) dt \quad (11)
\]

where \(S(t)\) is the stress at time \(t\), \(S_0\) the mean stress and \(S^*\) the standard deviation of the stress distribution derived from the vacuole distribution which appears to be Gaussian. The magnitude of the exponential exponent is calculated from the experimental data.

For a constant load the equation becomes

\[
\frac{\Delta V}{V} = kt e(t) \exp \left(\frac{S - S_0}{S^*}\right)^2 \quad (12)
\]

and as the stress is constant

\[
\frac{\Delta V}{V} = A_0 e(t) \quad (13)
\]

The dilation has an approximately linear dependency on strain and time.

1.6 Fracture Mechanics Approach

1.6.1 Classical fracture theories

Fracture mechanics has developed rapidly in the past few years but had its origin in the work of Griffith's on the failure of a thin sheet of brittle material.\(^{35}\) He proposed an energy balance concept that a crack would increase in size provided that

\[
\frac{\partial u}{\partial c} > \frac{\partial S}{\partial c}
\]

where \(u\) is the elastic strain energy, \(S\) the surface energy and \(c\) is the length parameter of the crack. This leads to an expression which related the stress to initiate failure, \(\sigma_{cr}\), to the size of the inherent characteristic flaws which are present in most materials.\(^{58}\)

\[
\sigma_{cr} = k \sqrt{\frac{E \lambda_{cr}}{\alpha}} \quad (14)
\]
where \( K \) is a constant for the geometry and crack shape under consideration, \( E \) the material modulus, \( a \) the size of the flaw and \( \gamma_G \) the energy required to create a unit of new surface area called the surface energy.

The idea was extended by Orowan to ductile materials, then

\[
\sigma_{\gamma} = K' \sqrt{\frac{E(\gamma_G + P)}{a}}
\]

where \( P \) is the irreversible (plastic) work done by the particular dissipation process involved in the propagation of a crack of length \( 2a \).

It is now common practice to combine the irreversible work term with the Griffith's surface energy and refer to a term called work of fracture, \( \gamma_F \).

The stress distribution around cracks in infinite plates and other geometries derived from classical elastic theory was applied by Irwin to give a mathematical basis to the Griffith's idea and introduced the term \( G \) called the strain energy release rate. This term is related to the stress field intensity factor \( K \), by

\[ K^2 = \frac{E \gamma}{(1-\mu^2)} \quad \text{for plane stress, i.e. thin sheets} \]

and

\[ K^2 = \frac{E \gamma}{(1-\mu^2)} \quad \text{for plane strain, i.e. thick sheets} \]

where \( \mu \) is Poisson's ratio. The parameter \( K \) represents a combination of the effects of crack dimensions and the normal stress field influencing the crack behaviour and can be evaluated for a large number of different loading systems and sample geometries. These relationships are generalisations of the Griffith's infinite sheet concept.
Crack propagation occurs when the applied force gives a critical value $G_c$, called fracture toughness, and this value is related to the surface energy by

$$G_c = 2\gamma$$ ---- (16)

The analysis gave

$$G = \frac{F^2}{2w} \frac{dC}{da}$$ ---- (17)

where $F$ is the applied load, $w$ the crack width and $C$ the compliance of the system used to load the crack. This relationship can be used to measure the magnitudes of these parameters (see section 4).

In a composite the material controlling the extension of a crack is contained in a small "active" region close to the crack-tip. Large amounts of irreversible work will be dissipated in the process of fracture. Wells suggested in 1961 that even when considerable irreversible work occurred, it was possible to use the amount by which the crack opened as a measure of the work done in extending the crack. This was called Crack Opening Displacement (COD) and could be related to the $G$ and $K$ concept of fracture. He showed using the analysis of Dugdale for a crack in an infinite plate, that the crack opening displacement, $\delta$ was

$$\text{COD} = \delta = \frac{\pi \sigma^2 a}{E \sigma_y}$$ ---- (18)

where $\sigma_y$ is the uniaxial yield stress of the material and $\sigma$ is the tensile stress remote from the crack. But for the same conditions $G$, (regarded as an energy release rate) is defined as

$$G = \frac{\pi \sigma^2 a}{E}$$ ---- (19)

so that we have

$$G = \delta \sigma_y = \frac{K^2}{E}$$ ---- (20)
providing a direct link with the concepts of the previous theory. This relationship has been applied to polymers and will be referred to later (Section 4).

A review paper of these ideas with their limitations and shortcomings has been published. Even so, classical fracture theories have become design criteria for combating brittle fracture of metals and plastics in the presence of cracks or flaws.

The Griffith-type energy approach has been developed for rubbers by Rivlin and Thomas who introduced the tearing energy, T, defined as

\[ T = \frac{\partial U_c}{\partial A} \quad (21) \]

where \( U_c \) is the total energy in a test piece having a crack surface of 2A. For a crack of length \( a \) in the edge of a sheet of uniform thickness in simple extension then

\[ U = k \alpha^2 tw \quad (22) \]

where \( k \) is a strain-dependent term, \( t \), the sheet thickness and \( W \) the strain-energy density in the rubber far removed from the crack. It therefore follows that

\[ T = 2kW a \quad (23) \]

and this applies to materials which are highly elastic and which have non-linear stress-strain relationships. At small strains \( k \) approaches the classical-elasticity value of \( T \) and the relationship becomes identical with the strain-energy release rate of linear fracture mechanics. It is therefore possible to derive \( T \) theoretically in terms of the sample dimensions and applied forces or to measure it experimentally in the laboratory.

A generalised theory of fracture mechanics has been proposed which introduces a loss function \( \Phi \). The loss function as defined
varies with the external constraint $\sigma_o$, the temperature $\Theta$ and the rate of crack propagation $R$ and any other factor affecting the loss characteristics of the material

$$\gamma_F = \gamma_C \Phi(\sigma_o, \Theta, R)$$

The loss function reduces to unity if the material is everywhere perfectly elastic. The theory is still under development and has yet to be compared with sensible experimental results.

1.6.2 Crack propagation and growth

Classical fracture mechanics as described in the last section has been developed for materials which are in essence rate-insensitive where crack propagation is concerned. In these materials the crack is considered to be unstable if it propagates at a high velocity (usually the order of magnitude of the shear wave speed) as in brittle fracture; otherwise it is termed stable and will not propagate.

The Griffith's energy approach models the point of instability. For plastic materials the Griffith's energy is modified, as previously described, and the crack growth is modelled using a time independent dissipation mechanism and the point of instability becomes less well defined. However for a viscoelastic material the dissipation mechanism not only depends on the history of the state of stress or strain but is strongly time and temperature dependent and a point of instability becomes very difficult to define.

For polymers, which are not strongly viscoelastic, the fracture mechanical parameters have been experimentally determined for various crack-tips speeds. It is found that $K_c$, the critical stress field intensity factor, depends on crack-tip speed for crack growth in air, in various organic solvents, and under fatigue-crack conditions. In all cases the relationship is not well defined and has not been modelled with complete certainty.

*The crack may increase very slowly but is still referred to as a stable crack.*
When a viscoelastic material containing a crack is stressed, some time will be required before the material at the crack tip is strained sufficiently to allow crack propagation. This time span is known as the initiation time for crack propagation.\(^\text{80}\) The rate of subsequent crack propagation may vary over many orders of magnitude. Temperature also has an effect on the material parameters controlling the processes and on the critical crack length.\(^\text{81}\)

The rate of energy dissipation has been estimated for a thermorheologically simple linear viscoelastic material\(^\text{82}\) and compared with experimental data.\(^\text{83}\) The time-dependent processes have been modelled for the viscoelastic joint between elastic materials but have not been tested experimentally.\(^\text{84}\)

However, all these theories replace the failing material near the crack-tip by a very idealised model. A more realistic approach has been proposed\(^\text{85}\) in which although the bulk material is assumed to be linearly viscoelastic, the nature of the failure zone is quite arbitrary and, therefore, could include material which is highly non-linear, rate dependent, and even discontinuous. The viscoelastic nature of the bulk material is modelled by a generalised power law. It was also assumed that the instantaneous crack-tip velocity depends only on the instantaneous stress intensity factor and is independent of the history of both this factor and the stresses. This may not be correct for fatigue cracking.

An equation for fracture initiation time has been derived\(^\text{85}\) and found to be very similar to the elasticity relation for critical stress, except that a secant compliance appears in place of an elastic constant. The crack-tip velocity in the opening mode of failure depends on the effective stress intensity factor and has been
mathematically modelled but measurements are required to specify the parameters. The theory has been applied to both unfilled and filled polymeric materials with reasonable success and the work is being extended to account for geometric and material non-linearities in the bulk material and for other modes of crack growth.
FIG. 2 NETWORK REACTION BETWEEN FILLER PARTICLES ON STRETCHING
FIG. 3 DEWETTING AND TEARING AROUND A PARTICLE EMBEDDED IN A POLYMER.
2. UNIAXIAL TESTS

2.1 Introduction

The uniaxial test was used initially because it was simple to perform and monitor. The strains were applied at various constant rates and then held at a particular constant value and the propellant behaviour measured. A constant strain test was chosen because it applies similar stresses to those encountered by propellants in the rocket motor during storage. The propellant's response during the straining phase of the tests was also measured.

Three cross-head speeds were used for the tests so that a nominal strain of 0.25 was applied in 125 seconds for Series A tests, 600 seconds for Series B tests and 60 seconds for Series C tests.

2.2 The experimental technique

The constant strain capability of the CTPB propellant I was tested in the uniaxial dumb-bell configuration shown in Figure (4)a.

2.2.1 Apparatus

The straining device and the propellant samples used are illustrated in Figure (5). Strains were applied manually by slowly rotating the screw and moving the attached jaws along the side guide-rods. The magnitude of the strains applied to the sample were measured by comparison of the surface grid marks with standard prepared gauges which are also shown in Figure (5).

The experiments were monitored photographically, a flash-light source being used to reduce surface heating. Good photographic contrast on the propellant surface was obtained using yellow light together with blue gauge marks. A semi-silvered mirror was incorporated into the optical system to allow a digital time display to be superimposed on the photographic record of the propellant surface. The complete experimental rig can be seen in Figure (6).
At a later stage of the investigation a bench model, Instron 1026 tensile tester became available and was used to replace the manual straining rig. The Instron 1026 comprises of an electronic load-weighing system of strain-gauged load cells, a controlled driving system and an accurate pen and chart recorder. The load cells have a full scale load range from 0-50 gms to 0-50 kgms. The driving system provides cross-head speeds in the range 0.5 mm/min to 500 mm/min and the chart recorder may be driven at paper speeds of 50 to 1000 mm/min or at a chosen ratio of the cross-head speed.

The film records of the experiments were analysed on a film reader, the screen of which is a Minmac trace reader. The Minmac manufactured by D-Mac of Glasgow has a digitising area of 480 mm x 480 mm. The X and Y co-ordinates of any point in this area can be obtained using a viewing sight. The position of the sight is determined by tensioned wire driven linear potentiometers. The analogue signals proportional to the co-ordinates are transmitted by flexible cables to an electronic unit and displayed on a digital voltmeter. Scaling and zeroing facilities enable the operator to select the required origin and scale. The electronic unit drives an electric typewriter for hard copy records and a punch for tape output compatible with the computer used for the analysis.

The manufacturer's specification is given as a true resolution of 1 part in 1350 at maximum scale.

2.2.2 Sample preparation and method of testing

The CTPB propellant I, the composition of which is shown in Appendix A (84% by weight of solid), was mixed and cast into standard dumb-bell moulds. The moulds were cured for 7 days at 60°C. Samples were cut from the moulds, sealed into polythene
bags and stored over molecular sieve at 20°C. The samples are stored dry because the absorption of water vapour by the propellant changes the physical properties. \(^8^7\) This absorption is very slow and a period of many weeks is required to give any measurable change.

Each sample to be tested was identified by means of a number and a letter, the number indicating the position in the mould and the letter the particular mould from which the sample was cut. Samples to be tested were chosen at random.

The gauge marks were stamped onto the surface of the propellant in blue ink. The use of the grid technique for strain measurement has been reviewed \(^8^8\) and compared to the Moire strain measurement method \(^8^9\), but for this particular work it was considered that a simple grid was sufficient. Also the heterogeneous nature of the propellant surface could have made a Moire type interference pattern difficult to interpret. \(^9^0\) However, it is acknowledged that a Moire technique would have given a more comprehensive strain distribution.

The strain was applied at a constant rate and photographs were taken at pre-determined intervals during the straining. After the selected strain had been achieved photographs were taken at intervals upto and including the time of break. A photograph of the gap between the failure surfaces was taken 100 seconds after the sample failed, and 100 seconds later a photograph of the rejoined, but not compressed, sample was taken. A typical sequence of events is shown in Figure (7). A constant nominal strain of 0.25 was applied, assuming a nominal gauge length of 30 mm, in 125 seconds for Series A tests, in 600 seconds for Series B tests and 60 seconds for Series C tests. Thirty, fourteen
and twelve samples respectively were tested at the different strain rates. The experiments were carried out at a temperature of $20 \pm 1^\circ C$ and at ambient humidity. No precautions were taken to keep the samples dry as the tests did not last more than 1,200 seconds, in which time no measurable change in bulk properties could have taken place.

2.3 Results of Series A and B tests

Series A and Series B samples were tested on the manual rig with a 10 mm spaced lateral grid as shown in Figure (5). A preliminary analysis of the data was conducted to test for random position of break and reproducibility of propellant properties.

2.3.1 Random position of break

Samples from one particular mould were selected as part of the Series A tests. The time to break and the position of the break were noted. It can be seen from Figure (8) that the position of break is randomly distributed about the centre position. It is therefore concluded that no systematic flaw or "weak spot" was introduced into the propellant during the preparation of the samples. There was also no evidence of mould to mould variation in properties.

The distance from a particular end to the centre of the break surface was measured. The average of these measurements was 32 mm, with a standard deviation of 8. This would position the average break sufficiently near the centre, ie 35 mm, to warrant the assumption of a random distribution of break position.

2.3.2 Effective gauge length

As stated in section 2.1.2 a nominal gauge length of 30 mm was assumed for the tests. The photographically measured true propellant strain did not equate with the applied nominal strain...
because of the flow of the propellant from the grips. The average applied true strain was measured as 0.24 with a standard deviation of 0.005.

The measured gap between the propellant pieces after failure was used as a measure of actual displacement, and hence used to determine the nominal strain (no permanent extension of the samples being observed after these particular experiments). The effective gauge length of each propellant sample was then calculated. The average value was 47.5 mm with a standard deviation of 3.0. This result agrees very well with the value of 50 mm given by previous work with the same test piece geometry, considering the error in the indirect measurement of displacement caused by non-planar failure surfaces.

2.3.3 Time to break distribution

The variation of time to break for both Series A and Series B tests are shown in Figure (9) and it can be seen that the time values vary over two orders of magnitude. The recorded time to break was the lapse time from the beginning of the test to the time at which the sample finally failed. Therefore the distribution for the slower-strain rate tests, i.e. Series B tests, is displaced to longer times. The peak of the Series A distribution curve occurs at about 250 seconds whereas the corresponding peak for the Series B tests occurs at about 700 seconds. The shapes of the curves are similar to those of a Gaussian or normal distribution. This wide variation in time to break cannot be explained by differences in the amount of strain applied, or by variations in the rate of applying the strain. The samples tested have an exponential survival distribution, as can be seen from the linear plots in Figure (10). The data extrapolates to a survival ratio of unity at zero time.
This indicates, with respect to the mode of failure being studied, that the samples were essentially flow-free when manufactured in agreement with the conclusions from section 2.3.1.

The linear plots also implies that the mechanism controlling the rate of the failure process can be described by a probability type of relationship. This type of failure depends on the rate of straining as indicated by the separate plots of Figure (10). The slope for the Series A samples is $1.4 \times 10^{-3}$ sec$^{-1}$ and that for the Series B samples $3.3 \times 10^{-3}$ sec$^{-1}$. A factor of 5 increase in the rate of straining changes the slope by a factor of nearly 2.5. This is not unexpected as it is well known that the properties of filled and unfilled elastomers depend on strain rate.\(^9\) The temperature also has an effect on the ultimate properties of elastomers.\(^9\) However, the temperature effects have not been investigated in this particular work.

No attempt was made to derive any empirical relationship from this limited information. Although the indication that an exponential influence exists has been used in section 2.4.1 and can be expected from the various theories discussed in section 1.5. The non-linear response of such a highly loaded system also makes the theoretical approach to the description of the failure processes a very difficult if not impossible task.\(^9\)

2.4 Discussion of results from Series A and B tests

It has been suggested that the strain-rate dependency of the ultimate properties of filled elastomers are due to changes in the filler matrix interaction rather than truly viscoelastic phenomena.\(^9\) This interaction is known as dewetting and results in the formation of voids. The tendency of the polymer to pull loose from the surface of the filler particles when the sample is strained and the subsequent
formation of voids around these particles leads to an overall density decrease. The process may be characterised by bulk volume changes during straining. A convenient way of investigating this volume change is to use Poisson's ratio.

2.4.1 Variation of Poisson's ratio with uniaxial strain

Poisson's ratio can be defined as

\[ \mu = \frac{\text{Transverse fractional contraction} (\Delta w)}{\text{Axial fractional extension} (\varepsilon)} \]

The fractional contraction is the change per unit width

\[ |\Delta w| = \frac{w_1 - w_0}{w_0} \]

\(w_0\) being the original unstrained sample width. The fractional extension is the change per unit length

\[ |\varepsilon| = \frac{\ell_1 - \ell_0}{\ell_0} \]

\(\ell_0\) being the original unstrained length of the reference grid.

From the photographic records it was possible to measure the increase in length of the longitudinal grid marked on the surface, \((\ell_1 - \ell_0)\), and at the same time to measure the change in width, \((w_1 - w_0)\), of the sample. Assuming that the cross-sectional area changes isotropically, the ratio of the two fractional changes, regarding a fractional contraction as having a negative value, would give a measure of Poisson's ratio.

The accuracy when measuring small changes must be considered. In this case the changes were measured on approximately a 10 mm square. The measurements were carried out on a film reader of \(\times 10\) magnification to an accuracy of better than 0.2 mm, ie a change of 1 in 500 could be detected. However the shadow at the edge of the sample and the finite width, \((1/5\ mm)\), of the gauge marks reduced this to a reproducible measurement of change to
about 1 in 250. The initial increments of strain measurement could therefore be subject to considerable error.

The variation of Poisson's ratio with strain is shown in Figure (11) for samples from the Series A tests. The considerable scatter at low strains can be seen. Also shown is the plot for a sample straining at constant volume which indicates that the propellant is dilating, ie dewetting, as expected. The magnitude of this effect is considered in section 2.4.2. Samples from the Series B tests behaved in a similar manner.

Noting the exponential nature of the results, as discussed previously, the relationship between Poisson's ratio and strain was taken to be of the form:

\[ \mu = c \exp (-me) \]  

which is shown in Figure (12). The values of the intercept, c, and the exponent, m, were computed for each sample using a "least - square - fit" technique. The variation of the values obtained can be seen in Figure (13) for the constant c, and Figure (14) for the exponent m. The amount of scatter is considered typical for tests on materials like composite propellants.

The average values obtained are listed below:

Series A tests (30 samples)  
exponent \( m = 2.9 \) with a standard deviation of 0.4  
constant \( c = 0.56 \) with a standard deviation of 0.06  

Series B tests (14 samples)  
exponent \( m = 3.1 \) with a standard deviation of 0.2  
constant \( c = 0.59 \) with a standard deviation of 0.02  

The differences between the exponent values were shown by the Student \( t \) test to be of no significance and a value of \( m = 3.0 \) was taken as a good approximation for both series of tests. This
is in agreement with Farris's work (section 1.5.3), in which his exponent, $B$ (in equation 10), all be it, in a slightly different equation, was considered independent of strain-rate.

The values of the constant $c$ are significantly different, the Student "t" test gives a probability that the distributions are the same of less than 0.5%, and therefore the constant depends on strain-rate. This constant also gives the value of Poisson's ratio for zero strain and the calculated values are more than 0.50, which is considered doubtful for real materials. However similar results have been obtained with a gas dilatometer, and it was suggested that propellant when initially strained does in fact undergo a volume decrease due to a rearrangement and closer packing of filler particles. This type of behaviour may explain the small knee observed at the beginning of the stress-strain curve for some uniaxial tests. Laboratory density measurements were unsuccessful in detecting this density increase after a small amount of tensile strain.

In this reported work the relationship was calculated using the measured true strain, $\varepsilon$, and it is suspected that there is a critical strain for the initiation of dewetting. This is analogous to the yield strain in a Bingham type material. Therefore, the relationship should have been written in terms of an effective strain, $\varepsilon_{\text{eff}}$, where

$$\varepsilon_{\text{eff}} = \varepsilon - \varepsilon_{\text{crit}} \quad \text{(26)}$$

$\varepsilon_{\text{crit}}$ being the critical strain for the onset of voiding. The direct measurement of $\varepsilon_{\text{crit}}$ is very difficult because of the large errors inherent at low strains, but can be interpolated from the results by calculating the value of strain which
represents a Poisson's ratio of 0.50. Values of $\varepsilon_{\text{crit}}$ are shown in Table 1 and it can be seen that the value depends on strain rate, but has an approximate value of 0.05.

It has therefore been shown that for the propellant tested the Poisson's ratio depends on strain, in the form:

$$\mu = 0.5 \exp(-3.0\varepsilon_{\text{eff}})$$  \hspace{1cm} (27)

where the effective strain, $\varepsilon_{\text{eff}}$, depends on strain rate but has the approximate value of

$$\varepsilon_{\text{eff}} = \varepsilon - 0.05$$

It should be mentioned that equation 27 breaks down at effective strains of above about 0.40, the actual limit value depending on the particular propellant tested. If the propellant survives to this high strain then it behaves as a filled foam with a constant value of Poisson's ratio.

2.4.2 Calculation of dilatation on straining

The volume change can be calculated if we consider a unit cube. After straining the length of the sides become, assuming that the material is isotropic, $(1 + \delta \ell)$, $(1 + \delta w)$ and $(1 + \delta w)$. Hence the dilatation D, i.e. the change in volume per unit volume on straining is

$$D = \frac{\Delta V}{V} = (1 + \varepsilon)(1 - \mu\varepsilon)^2 - 1$$ \hspace{1cm} (28)

where $\varepsilon = \delta \ell$ and $\mu = \frac{\delta w}{\delta \ell}$,

which expands to

$$\mu^2 \varepsilon^2 - 2 \mu \varepsilon + \left[1 - \frac{D + 1}{1 + \varepsilon}\right] = 0$$ \hspace{1cm} (29)

For small strains this reduces to

$$2\mu = \frac{\varepsilon - D}{\varepsilon(1 + \varepsilon)}$$ \hspace{1cm} (30)

and if there is no volume change

$$D = 0, \text{ and}$$

$$2\mu = \frac{1}{1 + \varepsilon}$$ \hspace{1cm} (31)

which gives for vanishingly small strains

$$\mu = 0.5$$
<table>
<thead>
<tr>
<th>SAMPLE NO</th>
<th>( \varepsilon \text{ crit} )</th>
<th>SAMPLE NO</th>
<th>( \varepsilon \text{ crit} )</th>
<th>SAMPLE NO</th>
<th>( \varepsilon \text{ crit} )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>16</td>
<td>0.066</td>
<td>1</td>
<td>0.067</td>
</tr>
<tr>
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<td>17</td>
<td>0.055</td>
<td>2</td>
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<tr>
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<tr>
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<td>4</td>
<td>0.065</td>
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<td>5</td>
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<tr>
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<td>0.069</td>
</tr>
<tr>
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<td>0.080</td>
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<tr>
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<tr>
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<td>10</td>
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<td>11</td>
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<tr>
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<td>0.049</td>
</tr>
<tr>
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<tr>
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<tr>
<td>15</td>
<td>0.061</td>
<td>30</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

**Average** \( \varepsilon \text{ crit} = 0.04 \)  
**Standard Deviation** = 0.02

**Values of strain for Poisson's ratio 0.50**
However, for finite strains with no volume change, i.e. $D = 0$, $\mu$ decreases with increasing tensile strain as shown by the dashed line in Figure (11). For a tensile strain of one per cent the value of Poisson's ratio is

$$\mu = 0.495$$

Substituting the known relationship for Poisson’s ratio, i.e. equation 27, into equation 28 gives

$$D = (1 + \varepsilon) \left(1 - \frac{1}{2} \varepsilon \exp - 3.0 \varepsilon_{\text{eff}}\right)^2 - 1 \quad (32)$$

and hence the dilatation can be calculated from known values of applied tensile strain.

Measured values of $\varepsilon$ and calculated values of $\mu$ were computed in equation 28, using the computer program shown in Appendix B, to give the plots of dilatation shown in Figure (15). The computed values of predicted dilatation, using equation 32, are also shown in Figure (15) for the cases where $\varepsilon_{\text{eff}}$ is equal to the applied true strain ($\varepsilon$) and where $\varepsilon_{\text{eff}} = \varepsilon - 0.05$. The agreement is good and shows that the majority of computed values occur within the two envelopes. Also plotted on this graph are some results from Parris's reported work using a gas dilatometer shifted along the strain axis to superimpose at the origin. The values are similar but the shape is different at higher strains because the data was for an American type of propellant with a higher strain capability than the tested propellant. The samples used in the gas dilatometer also have a constraint at the tabs and so the recorded average dilatation would be less than that calculated from my tests particularly at high strains.

The scatter on the computed values is shown in Figure (16) as the distribution plot of the dilatation at the 0.20 true strain level. The two envelope values are shown to contain the peak values of the distribution. No correlation was found between the
time to break and the distribution. It was suspected that the samples with the shorter time to break would correspond to those with the largest dilatation at the 0.20 strain level. However the data did not support this hypothesis. It was concluded that the dilatation was not uniformly spread throughout the samples.

2.5 Results from Series C tests

One of the disadvantages of the gas dilatometer is that it measures the average dilatation of the sample under test. Whereas the grid method and analysis technique described in the previous sections can be used to measure the surface strain distribution. The non-uniformity of the surface strain field could be seen from the variation of width during the tests of Series A and B samples. Therefore Series C samples were tested at a strain-rate of 0.25 mm/mm/min on the Instron tensile tester to measure this distribution.

2.5.1 Test of surface grid

A grid with rectangular elements of nominally 5 mm by 2 mm was stamped onto a sheet of white card. This card was then placed in the same position as a sample would occupy in the test machine and a series of photographs taken. These photographs were then analysed on the film reader at the maximum magnification of about X 20, the results were computed and compared to the grid spacing measured, using a vernier travelling microscope. The variation of a given pair of lines was measured for both methods to an accuracy of ± 0.01 mm.

The dimensions of sixty elements were measured with the following results;

<table>
<thead>
<tr>
<th>Travelling microscope measurements</th>
<th>Average</th>
<th>Maximum</th>
<th>Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 mm sides</td>
<td>1.97 mm</td>
<td>+ 0.10</td>
<td>- 0.09</td>
</tr>
<tr>
<td>5 mm sides</td>
<td>5.00 mm</td>
<td>+ 0.16</td>
<td>- 0.06</td>
</tr>
</tbody>
</table>
The variation in spacing of the grid was at least $\pm 0.1 \text{ mm}$ for both the 2 and 5 mm sides of the elements.

These results were then compared to those obtained from the analysis of two separate photographs.

Analysis of photographs:

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>Maximum</th>
<th>Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 mm sides (1)</td>
<td>2.03</td>
<td>$+0.17$</td>
<td>$-0.14$</td>
</tr>
<tr>
<td>(2)</td>
<td>2.03</td>
<td>$+0.14$</td>
<td>$-0.13$</td>
</tr>
<tr>
<td>5 mm sides (1)</td>
<td>5.10</td>
<td>$+0.10$</td>
<td>$-0.08$</td>
</tr>
<tr>
<td>(2)</td>
<td>5.11</td>
<td>$+0.15$</td>
<td>$-0.07$</td>
</tr>
</tbody>
</table>

The similarity of the results confirmed the acceptability of the photographic technique and the difference in magnitude of the average values was accounted for by about a $+2\%$ error in the scaling of the digitiser. This type of systematic error did not affect the calculated strain values.

The grid was arranged on the surface of the propellant sample so that the 5 mm side of the elements were at right angles to the edge. The small lateral strains were therefore measured with maximum accuracy. The longitudinal strains were measured from changes of the 2 mm sides and because of the measuring error only strains larger than 0.05 were considered.

The samples also had a 5 mm by 2 mm grid ruled onto one side surface. The photographic recording rig was modified to include a 45 degree mirror so that the side grid was recorded on the same negative as the front surface grid. Some typical photographs can be seen in Figure (17). The quality of this grid was not so good as the main front surface grid. However, this side grid was not used in the analysis described later but was used to determine if the sample was deforming isotropically.
2.5.2 Test for transverse isotropic behaviour

Both the side grid and the front grid were analysed for a limited number of Series C samples. The Poison's ratio was then computed from the average change of 10 grid elements i.e. for approximately a 10 mm square. Figure (18) shows the results and how they vary with time for both grids. It can be seen that Poison's ratio decreases with increasing time i.e. with strain, in the same way for both the front and side surfaces during the initial loading. The value then remains practically constant during the constant strain part of the test, only decreasing slightly just before failure.

No significant difference could be detected between the behaviour of the side and front surface of the sample. Therefore the results were interpreted as evidence that the propellant sample behaved with transverse isotropic behaviour during the uniaxial tests.

2.5.3 Time to break

The variation of the time to break for the samples of Series C tests can be seen from the distribution plot shown in Figure (19). The times again vary over two orders of magnitude and have a similar shape of distribution to those shown in Figure (9). The peak of the curve occurs at about 300 seconds.

The similarity of shape and magnitude of the distribution plots for the three tests leads to an interesting conclusion. For the same amount of applied strain, in this case 0.25, the "life-time" distribution does not depend directly on the rate at which the strain was applied. There must be a secondary influence as the "life-time" of a sample would vary if tested with extreme values of strain-rate. However, for these reported tests the peak in the distribution curve occurred between about 100 and 200 seconds after the constant strain
level had been reached. The total time for all the distribution
curves was constant at about 1,000 seconds. It was therefore concluded
that the rate at which the failure processes evolve must depend on
factors which are virtually independent of initial strain-rate. This
is discussed in Section 2.7.2 where the stored energy is used to compare
the results.

2.5.4 Surface strain distribution

The strain distribution was computed from grid measurements using
the program shown in Appendix B. The longitudinal surface strain
distribution for a typical sample can be seen in Figure (20) for
applied average nominal strains of 0.10, 0.20 and 0.25. The corres-
ponding plots for the lateral surface strain are shown in Figure (21).
The site of the final failure could be determined very early in the
test as can be seen from the peak in the distribution, (a) in
Figure (20), of the bottom elements at grid position 5. Later in the
test a crack appeared as the local strain differentially increased
until failure. This type of behaviour was observed for all the samples
tested. Although for a few samples more than one initial "failure"
site was detected and only later in the test did one of them pre-
dominate and lead to failure. The lateral strain also showed the
higher local strain level in grid position 5, but as can be expected
in this case the peak level occurred in the top element. The magnitude
of the computed longitudinal strains were artificially high across the
crack. This was because the strains were calculated from the surface
displacements which included the crack width.

The position of the first observed crack was marked during the
test. This area of the failure surface was then studied under the
scanning electron microscope. Reasons for the initiation of failure
at this site are discussed in Section 3.3.
2.6 Discussion of Results from Series C tests

The results from one sample of the series C tests were analysed in the same way as all the samples from Series A and B tests. The object of the analysis was to determine if the large variations observed for the A and B tests were the result of similar variations over the surface of one sample. Due note must be taken of the scatter due to experimental error in the Series C test results.

2.6.1 Variations of Poisson's ratio with uniaxial strain

The Poisson's ratio variation with strain followed the same relationship as the previous tests, i.e. a good fit was obtained with:

$$\mu = c \exp \left( - \frac{\varepsilon\epsilon}{m} \right) \quad (25)$$

The values of the intercept, c, and the exponent, m, were computed for each element on the surface of the sample using a "least-square fit" technique with the error in y direction.

The average values obtained are listed below:

- exponent $m = 3.1$ with a standard deviation of 0.5
- constant $c = 0.57$ with a standard deviation of 0.10

These values are very similar to the previous results but have larger standard deviations. The exponent value is close to the average value of $m = 3.0$ taken as a good approximation for the previous results. Whereas the average value of $c$ is slightly higher than that expected probably due to the larger scatter of this data.

The critical strain for the onset of dewetting, i.e. the value of strain to give a Poisson's ratio of 0.5 was calculated for each element. The average value was determined as:

$$\varepsilon_{\text{crit}} = 0.05$$

with a standard deviation of 0.03. Again this value agrees very well with the previous values given in Table (1).
It was therefore concluded that the equations:

$$\mu = 0.5 \exp(-3.0 \, e_{\text{eff}})$$

where $e_{\text{eff}} = e - 0.05$, describes the propellant behaviour for all the tests carried out under uniaxial conditions.

2.6.2 The variation of Poisson's ratio with time

It has been shown, see Figure (18), that under constant nominal strain the average Poisson's ratio does not change with time. However from the detailed analysis of the surface grids it has also been shown that the strain varies considerably over the surface of the sample. This variation changes as the failure process evolves. Therefore the distribution of Poisson's ratio for each element would also change with time. The variation of Poisson's ratio with time is shown in Figure (22) for a row of 5 elements around the final position of break. The data has considerable scatter but this indicates that the region of propellant close to the failure site, i.e. near the crack, relaxes whereas the element containing the crack undergoes a monotonically decreasing Poisson's ratio up to the failure time.

It was hoped that detailed computer analysis of these changes would give useful information for the failure process. However no sensible correlation could be obtained because of the experimental or inherent scatter of the results. It was therefore decided to investigate the scatter of data at a particular strain level during the loading.

2.6.3 The distribution of propellant dilatation

It has been shown in Section 2.4.2 that the propellant dilatation can be calculated from strain measurements. This was performed for each element on the surface of the Series C sample. The results showed considerable scatter particularly for elements near to and
containing the crack. However the trend is similar to the previous data and is shown in the form of a distribution curve of dilatation at the 0.20 strain level in Figure (23). This distribution has the same peak value as before, see Figure (16), but has a larger range of values partially due to the experimental error. The isolated results at high strain levels are due to the elements concerned containing a crack. However the distributions are similar and therefore the variation in properties measured from a batch of tests are reflected in the actual variation over the surface of one member of that batch. The spatial aspect of the failure processes are therefore important and must be considered.

2.7 Conclusions

In this section of work it has been shown that the time to failure of composite propellant has an inherent range of values resulting in an appreciable scatter of results. A similar variation was observed in the surface strain distribution of a tested propellant sample and the resulting calculated propellant dilatation. An attempted correlation of the influence of time on these factors was not successful. A statistical analysis to describe these phenomena in detail would have required considerable experimental effort which was not considered to be the best approach to the basic problem. This problem being in essence the selection of the material parameter best suited to describe the failure processes and then to investigate its significance and variation. To this end, a qualitative model of the failure processes was considered and the empirical relationship between Poisson's ratio and the true strain reviewed and compared with other equations of the same form.
2.7.1 Qualitative model of propellant failure

It has been shown that during straining the Poisson's ratio changes similarly for all the samples irrespective of time to break. Hence it is concluded that the void initiation and growth, which is assumed to affect Poisson's ratio, depends on strain, and that the time to break depends on other less well defined factors. These factors could include the following:

1. Viscoelastic stress relaxation in the binder.
2. Stress concentrations between filler particles.
3. Interactions between voids of dewetted particles.
4. Rupture of binder filaments as the failure plane propagates.

These factors have been included in a suggested qualitative model of failure, in that: when a strain is applied to a sample of propellant:

(a) Voids form in the propellant as the local stress increases. The stress distribution is localised by stress concentrations between filler particles and increases until it exceeds the binder-filler interface bond strength thus causing dewetting and voids around the filler particles. Cohesive failure could also occur if the cohesive strength of the binder is exceeded resulting in the formation of voids in the binder.

(b) Secondly two processes occur within the sample, further dewetting of particular solid surfaces leading to void growth and viscoelastic relaxation of the binder which tends to reduce and redistribute the local stresses. These processes lead to the growth of certain voids at the expense of others and tend to concentrate and link the voids eventually leading to either of the following:
C₁) A particular region of the propellant is stressed "to the limit" due to the linking and growth of voids and a failure surface propagates rupturing binder filaments as the surfaces separate.

or

C₂) The viscoelastic nature of the binder sufficiently relaxes the stresses so that the propellant can accommodate the applied strain in a semi-stable condition.

The model can be used to suggest a reason for the large variation in the time to break. The variation could be due to the uncertainty inherent in the mechanism involved in stage b of the model. The variation in time to break is due to the heterogeneous nature of the propellant in which the particle size of the solid loading covers a range of sizes and the distribution of particles within the matrix is not entirely uniform. The shape of the oxidiser particles are also very irregular (see Section 3.1).

Various combinations of these facts would give cases of rapid build-up of local stress and very rapid propagation of failure, hence the time to break would be short. On the other hand slow build-up of local stresses with slow propagation of failure gives a long time to break. The failure processes in both cases are similar with the magnitude of the build-up controlling the eventual time to break.

The initiation and growth of failure mentioned above, are the dominant factors until the time when the crack in the sample begins to grow in length. Then, the rapid decrease in cross-sectional area of the sample results in an increase in stress which leads to a very rapid propagation of failure and the sample breaks.
A great deal more work is required before a full quantitative explanation of the failure process can be given for the complete temperature and time range of interest. However, from the results described a working hypothesis was made that the driving mechanism for the failure process was the "energy input" into the sample. The nonlinear viscoelastic effects are assumed to be accounted for by variations in the amount of this energy available to the failure process. As a first approach the elastic case is considered and then compared to the results from independent tests.

2.7.2 Modelling the spatial aspects of failure

It is assumed that at each failure site in the propellant a small spherical void is produced and grows then a measure of the created surface area can be made from volume changes. The values of \( \frac{\Delta A}{A} \) can be calculated from the volume changes as given by equation 32. The variation of this created surface area with stored elastic energy is shown in Figure (24) and can be represented as

\[
\frac{\Delta A}{A} \propto \exp \left( -\frac{B}{J} \right) \quad \cdots \quad (34)
\]

where \( J \) is the stored energy and \( B \) is a constant. The stored elastic energy \( J \) was calculated from \( \frac{1}{2} Es^2 \) using 3.9 MPa as the elastic tensile modulus (see Appendix C). The linear plot shown in Figure (24) has a slope \( B' \) of magnitude \( 1.16 \times 10^5 \) J/m\(^3\).

It may be useful to consider the significance of this slope. Its value is about a factor of ten larger than the stored elastic energy at a strain of 0.10, i.e. \( 2 \times 10^{4} \) J/m\(^3\). If we assume that the distribution of this stored energy is Boltzmannian, then the number of sites, \( N_i \), with a given energy, \( J_i \), is given by

\[
\frac{N_i}{N_0} = C \exp \left( -\frac{J_i}{J_0} \right)
\]
where $J_0$ is the total energy and $N_0$ the total number of possible sites.

If the increase in surface area due to dewetting is proportional to the amount of stored energy, then the number of sites where dewetting is taking place, is:

$$
\int_{J_0}^{\infty} \frac{N_i}{N_0} dJ_i = C \int_{J_0}^{\infty} \exp - \frac{J_i}{J_0} dJ_i
$$

$$
\frac{N}{N_0} = D \exp - \frac{J^*}{J_0}
$$

where $J^*$ is the critical energy for the onset of dewetting and $N$ the number of sites where dewetting is taking place. By comparing the above equation with equation 34 the value of $J^*$ is taken to equal that of $B$ which has a magnitude of $1.16 \times 10^5$ J/m$^3$. Hence, the critical stored energy for the onset of dewetting is about 100 kJ/m$^3$. To obtain a value for the surface energy from this critical energy requires the number of unit surface areas produced per unit volume. To calculate this ratio a measure of void size, shape and distribution is necessary. However, an estimation of this figure can be made from the surface strain distribution results from section 2.6 which suggest that a 2 mm element contains at least one possible failure site. A value of 500 is therefore taken as the number of units of surface area produced per unit volume. An estimation of the surface energy is therefore

$$
\gamma = \frac{1.16 + 10^5}{500} J/m^2
$$

$$
\gamma = 232 \text{ Joules/m}^2
$$

This is a reasonable value for the surface energy of an elastomeric type of material$^{99}$. 

A value of surface energy has thus been derived from uniaxial tests. A number of assumptions were made including elastic behaviour and the amount of surface area produced per unit volume. However, propellant is a viscoelastic material and does not have a constant value of surface energy as it would vary with time and crack speed. Therefore, independent experimental measurements of the surface energy were made and the values compared (see section 4).
FIG. 4 TENSILE TEST PIECES

(a) TEST PIECE THICKNESS 10 mm
NOMINAL GAUGE LENGTH 30 mm

(b) TEST PIECE THICKNESS 10 mm
GAUGE LENGTH L
OF 50, 60, 70 AND 80 mm
FIG 6
EXPERIMENTAL RIG
FIG 7  TYPICAL SEQUENCE OF EVENTS
SAMPLES 5 AND 12 WERE USED IN OTHER TESTS AND NOT INCLUDED IN PHOTOGRAPH

FIG. 8  POSITION OF BREAK SHOWN WITH RESPECT TO POSITION IN MOULD
SERIES A SAMPLES

MOST PROBABLE VALUE
200-300 SEC

SERIES B SAMPLES

MOST PROBABLE VALUE
650-750 SEC

FIG. 9 VARIATION OF TIME TO BREAK
FIG. 11 THE VARIATION OF POISSON'S RATIO WITH STRAIN - SERIES A TESTS

- $\mu = C \exp (-\alpha \epsilon)$
- --- FOR SAMPLE WITH NO VOLUME CHANGE
\[ \mu = C \exp(-me) \]
INTERCEPT \( C = 0.59 \)
SLOPE \( m = 2.9 \)

FIG.12 EMPIRICAL REPRESENTATION OF POISSON'S RATIO - SERIES A TESTS
RESULTS FROM 30 SAMPLES TESTED AT 25 PER CENT CONSTANT STRAIN APPLIED IN 125 ± 5 SECONDS
RESULTS FROM 14 SAMPLES TESTED AT
25 PER CENT CONSTANT STRAIN APPLIED
IN 600 ± 10 SECONDS
FIG. 15  DILATATION OF COMPOSITE PROPELLANT
FIG. 17 PHOTOGRAPHS OF SURFACE GRIDS

CONSTANT STRAIN

0.25 STRAIN

ZERO STRAIN
FIG. 18 VARIATION OF POISSON'S RATIO WITH TIME
FIG. 20  LONGITUDINAL SURFACE STRAIN DISTRIBUTION
SERIES C TEST
FIG. 21  LATERAL SURFACE STRAIN DISTRIBUTION
SERIES C TEST
FIG. 23 DILATATION DISTRIBUTION FOR A SERIES C SAMPLE
STEREOSCAN STUDIES OF FAILURE SURFACES

The principle of the scanning electron microscope and its application to studying fracture surfaces have been reported elsewhere. The specialised use required to observe composite propellant surfaces has also been reported.

The inorganic constituents of composite propellant, i.e. Ammonium perchlorate and Aluminium, and representative failure surfaces were observed under magnifications ranging from 15 to 3,500 times. A few samples were viewed at low magnification without a conductive metallic coating, whereas the majority of the samples were coated to eliminate charging effects.

3.1 Study of propellant constituents

The inorganic filler in propellant is by far the largest part of the composite. Its particle shape and size distribution is therefore of predominance importance. Specimens of the actual materials used in Propellant I were obtained and studied under both optical and scanning electron microscopes. In Figure (25) stereoscan photographs are shown of typical particles. The top two photographs show unmilled and milled, i.e. fine, Ammonium Perchlorate (AP). The unmilled AP consists of basically cubic crystals with irregular surfaces. The milling process just breaks-up these crystals into very irregular shapes and a large distribution of sizes. The heterogeneous nature of the propellant results from the irregular shape and size of these crystals.

The optical study under polarised light showed that the large AP crystals contained inclusions i.e. holes. These flaws result from the manufacturing process which was by crystallization from concentrated solution. The flaws produced weakened crystals which became apparent when the failure surfaces were investigated.

The bottom photographs in Figure (25) show on the left micro-AP as uniform and spherical in shape, and on the right Aluminium (Al) pieces of irregular shape. These constituents only make up about 10% of the total
filler. The micro-AP is added to fill the interstitial holes between the larger particles which is necessary to obtain high solid loading. The present tendency in propellant manufacturing is to replace the unmilled AP by micro-AP which results in a propellant with a greater strain capability.

3.2 General features of the failure surface

The surface produced by cutting a propellant sample is shown at the top left of Figure (26). The surface is essentially flat with only a few cleaved crystals visible on the relatively smooth surface. It can be compared with a typical failure surface shown at the top right of Figure (26). The failure surface is far more irregular and contains numerous broken embedded crystals. This is a feature common to all the propellant failure surfaces studied.

The bottom two photographs of Figure (26) show close-ups of typical large broken crystals. The crystals are not only fractured but are also cracked into small pieces. This structural damage was probably caused during the mixing of the manufacturing process or due to internal stresses caused during the cool down from the cure temperature. The numerous inclusions must also have weakened the crystal structure. The large crystals then broke into two or more pieces during the propagation or initiation of the propellant failure. A large amount of surface damage is visible on all the photographs with small fragments of crystals scattered over the surface. A propellant sample was broken over a piece of white card and a considerable amount of loose crystal fragments was collected.

No difference in surface structure was observed between the areas of slow crack growth, i.e. near the point of failure initiation, and the regions of fast crack propagation.

3.3 Study of initiation region

The area of the sample where the failure crack initiated was marked with ink during the tensile test. Small pieces of both surfaces surrounding
this initiation site were cut and mounted side-by-side on the stereoscan sample holder. Two typical areas of failure initiation are shown in Figure (27). A matching pair of large flat crystals is shown in the dark area at the bottom of the upper left photograph. More conclusive evidence can be seen in the photograph at the top right. The join of the two parts runs from side to side between the two crystals. A close-up of the two halves of the crystal is shown in the photograph at the bottom left of Figure (27). The two inclusions, one containing a particle, and surface markings can be matched to show that the crystal broke into two parts close to the site of the initiation of failure. A detailed view of the lower half of the crystal pair is shown in the bottom right photograph of Figure (27). The small particles attached to the surface could be dust from the molecular sieve used to keep the failure surfaces dry, and possibly the product of some surface reaction.

It is suggested that the reason for the initiation of fracture may be due to a weakened large crystal breaking into two halves. This either causes the tearing of voids formed around the crystal or accelerates the growth and initiation of further voids in close proximity. The propagating failure plane would also break weakened crystals still not completely dewetted, resulting in the predominance of broken crystals on the failure surface.

The replacement of these "weak" large crystals by smaller more regular crystals suppresses failure initiation and growth and may result in propellant with higher strain capability.
**FIG 25** PROPELLANT CONSTITUENTS
FIG 26 FAILURE SURFACES
FAILURE SITE

CRYSTAL PAIR

CRYSTAL PAIR

INCLUSION IN CRYSTAL

FIG. 27 INITIAL FAILURE AREA
4.1 The concept of failure energy and methods of measurement.

From the viewpoint of continuum mechanics, the energy concept of fracture, adhesive and cohesive failures are similar. The essential difference involves the interpretation of the energy required to create a new surface area. For viscoelastic materials, the energy dissipated by relaxation and other flow processes at the crack-tip increases the energy required to create a new surface area. The total energy needed to create unit area of new surface is termed the work of fracture. For a brittle material with no dissipation of energy, the classical Griffith's approach (see section 1.6.1), results in a unique value for this work which is called the surface energy. The counterpart of this in viscoelastic materials can be called the intrinsic failure energy. The work of fracture is therefore the sum of the intrinsic failure energy and the energy dissipated viscoelastically.

A model adhesive joint between a cross-linked amorphous rubber and a rigid polymeric substrate has been investigated. The intrinsic failure energy and the dissipated energy were separated and their magnitude measured. For these reported tests, the dissipated energy was orders of magnitude greater than the intrinsic adhesive failure energy. Any cohesive failure in the bond material would contribute energy to the measured work of fracture.

Adhesive interlayer fracture and cohesive fracture have also been studied by pressure loading a blister of material until it increases in size. A review of this blister test and of the general problems of adhesive and cohesive fracture has been made by Williams et al.

Other test methods have been used to investigate fracture of polymers and glass fibre composites. The single edge notched sample in tension has
been used to measure the failure energy of polystyrene\textsuperscript{110} and polymethylmethacrylate (PMMA)\textsuperscript{111}, together with the notched bending beam specimen, to study fracture of a glass fibre composite.\textsuperscript{112} The double-cantilever beam technique has been used to measure the fracture energy of some epoxy resin materials\textsuperscript{113} and oriented polyvinyl chloride.\textsuperscript{114} All these materials have values of fracture energy which depend on the rate of testing ie the crack-tip speed. The magnitude of the viscoelastic dissipation at the crack-tip depends on the rate of propagation of the crack. Attempts have also been made to correlate the fracture processes with measurements of fracture toughness and impact strength.\textsuperscript{70} The double-cantilever beam test has also been used extensively in fracture studies of metals and weld-melts.\textsuperscript{115}

The double-cantilever beam specimen has been found to be useful in characterising the fracture behaviour of isotropic, homogeneous, brittle and semi-brittle polymers and metals for both stable and unstable crack propagation.\textsuperscript{116} However the technique has not been used for weak ductile materials because of excessive yielding or failure of the specimen before crack propagation. In order to make it possible to measure the work of fracture on materials such as composite propellant, the specimen was reinforced with wooden tabs. The resulting test-piece was similar to the sandwich double-cantilever beam sample used to study fracture in rubber-modified acrylics\textsuperscript{117} and is illustrated in Figure (28).

4.2 The fracture test specimen

The rectangular test specimen was cut from sheets of propellant and wooden tabs were bonded onto the long side with an epoxy adhesive. The geometry of the specimen and typical dimensions are shown in Figure (28). A 1mm side groove of width about 1.5mm was cut along the centre of both faces to control the direction of crack growth. A notch was cut at the
loading end to give a controlled start to the crack propagation. The sample was loaded via pins which were free to rotate in holes through the wooden tabs.

The behaviour of the standard double-cantilever beam sample, i.e. with no reinforcement, has been determined and the stored elastic energy computed for different crack lengths, a. The compliance can be calculated from the ratio of displacement of the ends of the sample against the applied load.119

The strain energy release rate, \( G \), is then

\[
G = \frac{F^2}{2w} \frac{dC}{da} \quad (17)
\]

where \( F \) is the applied load, \( w \) the crack width and \( C \) the compliance of the system. However, before this equation can be used the variation of compliance with crack length i.e. \( \frac{dC}{da} \), must be known.

If the sample behaved as a classical cantilever beam there would be a cubic relationship between the slope of the force-deflection chart and crack length.118 However the log-log plot for the modified fracture test specimen is not linear as can be seen from Figure (29). The slope of the force-deflection curve was determined for three different specimens. The variation is considerable, particularly at short crack lengths due to slight differences in sample geometry and material properties. It would have been possible to fit empirically a curve to the data.119 Even the standard cantilever beam specimen deviates from classical cantilever beam theory. For the reported work on PMMA118 the slope of the force-deflection curve against crack length gave a linear log-log plot with an exponent 2.67. To use this approach for the reinforced fracture test specimen it would have been necessary to derive the compliance relationship for each sample. Therefore it was decided to use the area under the force-deflection curve as a measure of the work done.
If $dA$ is the work done to extend the crack by a length $da$, then

$$\gamma = \frac{1}{2w} \frac{dA}{da} \quad (35)$$

where a unit extension of crack produces $2w$ units of new surface area.

The method used to calculate $dA$ is described in section (4.4).

4.3 Stress analysis of specimen geometry

The theoretical stress and strain distribution over the surface of the fracture test specimen was investigated using an elastic finite element computer program. Part of the element map is shown in Figure (30) and it consists of 594 simple triangular elements with 383 nodal points. The crack-line area was investigated in detail for several crack lengths, for this reason the element mesh was concentrated along the crack-line.

The computer program was used in the plane stress mode because the length to thickness ratio of the typical sample was greater than 1:15 and plane stress conditions were assumed. The element map and boundary conditions were tested under uniaxial tension conditions, i.e., the wooden tab was displaced 1.0 mm in the Y direction under the action of a constant applied load. The calculated resulting boundary forces and moment of the forces were found to balance and the resulting stresses and strains equated to those applied to the model. The program and element map were then considered to be working correctly.

The nodal points along the central crack line were released to introduce cracks of $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ the length of the sample. The loading end of the wooden tab was displaced in the Y direction the same distance as was measured during a test of a typical sample with the corresponding crack length. The other nodal points along the boundary were displaced a linearly decreasing distance to simulate the rotation of the wooden tab. The applied load was also measured from a typical test for each crack length. Therefore the boundary conditions of the computed model had been matched to those of a typical sample under test.

The computer print-out was interpreted to give the analysis results discussed below.
4.3.1 Surface stress and strain distribution

The surface displacements in the Y direction are shown in Figure (31) for the case of the \( \frac{1}{2} \) crack. The plot shows the distortion of five lines initially parallel to the wooden tabs in the X direction, at 2mm and 3mm spacing. The distorted lines consist of two straight sections joined by a curved section over the crack-tip. The amount of distortion decreases as the Y distance from the crack-tip increases. The upper boundary bonded to the wooden tab does not distort. The spacing of the lines in the section of the sample adjacent to region A in Figure (31) is the same as the original grid. Most of the surface strain occurs in front of the crack-tip. The linear part of the crack boundary can be seen to be rotating about the hinge point.

The displacements in the X direction are minimal compared to those in the Y direction. Along the centre line there is less than 1mm of movement at the hinge end and less than 0.5mm near the crack-tip. The other surface displacements decrease, with distance from this centre line to a zero value at the upper boundary.

For this elastic analysis the stress is proportional to the strain so that the stress and strain distributions are the same. The stress in the Y direction calculated at points along the centre line is shown in Figure (32) for three crack lengths. It can be seen that the distribution is approximately linear with a small compressive region at the hinge end and a narrow peak at the crack-tip. This peak represents a stress concentration at the crack-tip which decreases with distance from the centre line and crack-tip. The stress rapidly falls to zero in the cracked part of the sample.
A good overall approximation of this distribution up to the crack-tip can be taken as a straight line as shown in Figure (32). The only serious deviation from this line is within 10 mm of the crack-tip. From this data the stress at the crack-tip (given by the intersections of the straight line and the crack position) is 0.45, 0.50 and 0.34 MPa for the $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ crack respectively.

The data used in the computer analysis is shown in Table 2.

<table>
<thead>
<tr>
<th>TABLE (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>COMPUTED STRESS ANALYSIS DATA</strong></td>
</tr>
<tr>
<td>EFFECTIVE MODULUS (MPa)</td>
</tr>
<tr>
<td>-------------------------</td>
</tr>
<tr>
<td>$\frac{1}{4}$ CRACK</td>
</tr>
<tr>
<td>$\frac{1}{2}$ CRACK</td>
</tr>
<tr>
<td>$\frac{3}{4}$ CRACK</td>
</tr>
<tr>
<td>POISSON'S RATIO</td>
</tr>
<tr>
<td>PLANE STRESS CONDITIONS ASSUMED</td>
</tr>
</tbody>
</table>

As the program assumed elastic behaviour the magnitudes of the stress could be scaled by using different values of moduli. The value of modulus that gives the same computed end load as measured experimentally is called the effective modulus, $E_{eff}$. The method of using different values of modulus for each program run is a standard way of modelling a viscoelastic material with an elastic technique. The effective modulus decreasing with increasing crack length is considered to be due to the strain-rate dependence of the modulus. The strain-rate at the tip of a $\frac{1}{4}$ crack is higher than that at the tip of a $\frac{1}{2}$ crack. Hence the effective modulus for a $\frac{1}{4}$ crack should be higher than that for a $\frac{1}{2}$ crack (see Table 2).
The modulus measured from the slope of the stress-strain curve of a typical tensile test using the sample of geometry shown in Figure (4b) decreases with increasing strain as shown in Figure (33). From this data the nominal strain corresponding to the values of the effective modulus does not agree with the computed values. This is not surprising in a material which is strain-rate dependent. However the computed analysis is considered to be a reasonable approximation away from the crack-tip where rate effects are less apparent.

4.3.2 Computed crack shapes

It was shown in the previous section that the linear part of the crack boundary behaved as if it were rotating about the hinge point. The reinforcing tabs on the side of the sample therefore constrain the sample causing it to open like a hinge. This explains why the classical cantilever approach where the arms act like a bending beam, does not apply in this case. However the presence of the wooden tabs should not affect the shape of the crack-tip provided that the distance from the crack to the tabs is long compared with typical crack-tip dimensions.

The crack-tip shape for the three crack lengths is shown in Figure (34). The boundary from 15mm outwards is linear and is the part which rotates about the hinge point. It can be seen that the crack-tips are similar in shape, the slight differences are probably due to the modelling technique. The shape resembles a typical parabolic crack in an elastic sheet.

The effect of Poisson's ratio on the crack-tip shape was also investigated for the $\frac{1}{2}$ crack case. The resulting shapes for Poisson's ratios of 0.40 and 0.30 are compared to the previously used value of 0.49 in Figure (35). It can be seen that little difference in shape
results from this range of Poisson's ratios. A 7.5 percent reduction in the value of the effective modulus for the 0.40 case and a 15 percent reduction for the 0.30 case were required to match the end loading respectively.

The computed crack shapes have been compared to the experimental crack shapes in section 4.5.2.

4.4 The experimental technique

The Instron tensile tester (see section 2.2.1), was used for these fracture tests. The jaws were modified to accommodate pins which allowed the tabs of the fracture test specimen to rotate freely during the test. The rectangular test specimen was cut from 10mm thick sheets of propellant and wooden tabs were bonded onto the long sides with an epoxy adhesive (twin-pack Araldite). The typical specimen was 144mm long and 26mm wide. A 1mm/side groove of width about 1.5mm was cut along the centre of both faces to control the direction of crack growth. A notch was cut at the loading end to give a controlled start to the crack propagation. The geometry of the specimen is illustrated in Figure (28).

The crack growth was recorded photographically and the operation of the camera also produced a marker on the load-displacement chart. Analysis of this chart allowed the applied load and end displacement to be measured at the various crack lengths photographed. The crack length was measured from the film on the film reader and digitiser described previously (see section 2.2.1). A piece of standard mm graph paper was photographed close to the sample surface and the film used to scale the digitiser. Photographs were taken at intervals of about 5mm of crack extension. Three typical photographs of various stages of the test are shown in Figure (36).
The cross head was reversed during the test and returned to the start position. At the origin the cross head was switched to continue the test and reopen the crack. The cycle was sometimes repeated several times during the test. This exercise established an origin on the load-displacement chart and also gave an estimation of the viscoelastic nature of the propellant. A trace of a typical fracture test load-displacement chart is shown in Figure (37).

The work done extending the crack was calculated from the area contained between lines drawn from adjacent crack length markers to the origin. The area OAB marked on Figure (37) represents work done that is only about three percent smaller than the actual work done extending the crack from A to B. The area method of calculating the work done is therefore acceptable in this case. Cycles at other crack lengths gave similar results with a maximum difference of about five percent.

The areas between each pair of crack length marks was calculated from the co-ordinates of the crack length markers and the origin. The corresponding crack extension was measured from the film record and the work of fracture computed using the program shown in Appendix D. The average crack width, w, was measured from each fractured sample with a vernier travelling microscope.

4.5 Fracture test results

Tests were carried out at cross head speeds of 0.5, 5.0, 50 and 100 mm/min which gave a range of strain-rates that included those used for the uniaxial tests on propellant I. The samples were tested at a temperature of 20 ± 1°C and at ambient humidity.

Propellant I was tested together with two other C,T,P,B type propellants, referred to as II and III, all having the same, ie 84%, solids loading by weight. Two H,T,P,B propellants, referred to as IV and V, with
87% solid loading by weight were also investigated for their fracture
behaviour. The compositions of the five propellants tested are given in
Appendix A.

4.5.1 Variation of the work of fracture

The effect of varying the depth of the side groove was investigated
by testing a sample with 3mm deep side grooves and comparing the results
with standard 1mm deep grooved samples. Figure (38) shows that the
calculated work of fracture is independent of crack width. It can also
be seen that the work of fracture decreases with increased crack length.
Some of the scatter resulted from non-planar crack-tip growth and
measuring errors. The co-ordinates on the chart were measured to
within 0.1mm and the crack-tip position to within 0.02mm. It was
estimated that the technique used gave the work of fracture to an
accuracy of 10 percent.

The dependence of the work of fracture on the volume of the samples
was investigated using samples of various widths. The results for
three sample sizes are shown in Figure (39). The largest sample of
width 155mm, ie larger than the 140mm length, twisted at the beginning
of the test and this portion of the plot is dotted in Figure (39).
There was a volume effect with this large specimen which was probably
due to the drastic change in the aspect ratio of the sample and the
corresponding change in stress distribution. The other samples showed
no significant volume dependence of the work of fracture. Tests with
a series of samples with the same aspect ratio confirmed this result.

It can be inferred from these measurements that the majority of
the irreversible work, ie dewetting, occurred close to the line of the
crack-tip as predicted from the stress analysis of section 4.3 and was
not distributed throughout the bulk of the sample. This contrasts
with the uniaxial tests in which the work was distributed throughout the volume of the sample.

The crack started slowly from the notch-root, but rapidly increased in length and had its greatest velocity after about 10mm of crack growth. This initial part of the test was not included in the analysis. The crack-tip velocity then decreased as the crack length increased. The work of fracture has been shown to decrease with crack length. Hence the work of fracture depends directly on crack-tip velocity. The magnitude of the crack-tip velocity, and consequently the work of fracture depends on the rate of testing, ie the cross head speed. The effect of the cross head speed on the magnitude of the work of fracture is shown in Figure (40).

4.5.2 Experimental crack shapes

Selected enlarged images of the crack-tip were traced and the profile of the crack-tip measured. The accuracy of the measurements of the crack surface displacement was ± 0.1mm. A print of the trace of a fracture sample surface is shown in Figure (41). The crack-tip surface was irregular and not always symmetrical about the centre line. Consequently the average value at 1mm spacing is shown in Figure (42) for several propellants. For comparison the predicted computer crack-tip shape is also plotted.

The propellant crack-tip shapes are sharper than that produced by computed elastic analysis even for the case of Poisson's ratio equal to 0.30. This discrepancy is considered to be due to the local increase in modulus because of the high rates of strain at the crack-tip. It was not possible to modify the computer technique to include this phenomenon. However an estimation of the effect on the crack shape of increasing the value of the local modulus has been made\(^6\) and the agreement was then within the experimental error.
It was noticed during the analysis of the crack-shapes that the crack-tip profile for a particular propellant did not vary significantly along the sample length. This fact has been used in the modelling of the fracture behaviour described in section 4.6.

4.5.3 Stereoscan investigation of failure surfaces

Samples cut from along the surface of the fractured samples were investigated under the Stereoscan as described in Section 3. The photographs showed broken crystals and similar features to the uniaxial samples investigated in section 3.2. No difference was detected on the fracture surface for cracks of various speeds. The notch served to initiate the crack which then propagated in the same manner as with the uniaxial samples.

The variation in crystal size distribution observed on the surface of the various propellants corresponded to the multimodal distribution of Ammonium perchlorate in the original mix (see Appendix A). The difference between the carboxyl and hydroxyl propellants was not easily identified except by the fact that the more highly loaded hydroxyl propellant had a slightly more irregular surface. The surface of propellant V did not show any large crystals and there was less evidence of crystal failure during the fracture process.

4.6 Analysis of fracture results

An analysis of the fracture process was made making various assumptions which were inferred from the previous conclusions. They were:

1. The sample pivots about the opposite end of the centre-line to the loading point -H in Figure (41).
2. A critical strain exists at which the propellant fails.
3. The crack-tip shape does not vary as the crack propagates along the sample.
4. A critical stress exists near to the crack-tip.
5. The stress varies linearly in front of the crack-tip.
These assumptions are approximations of the actual behaviour but if a microscopic rather than a macroscopic view is taken then they are considered to be reasonable. It will be shown from the analysis that the fracture sample acts similarly to a series of uniaxial samples.

4.6.1 Analysis of crack-tip velocity

An analysis of the fracture sample was carried out and a method of calculating $b_{\text{crit}}$ developed. In the triangle $\Delta HBC$ in Figure (41), let the distance $BD$ be referred to as $b_{\text{crit}}$.

\[
\begin{align*}
\text{With a crack length of } a, \text{ then from similar triangles} \\
\frac{(\ell - a)}{b_{\text{crit}}} &= \frac{y}{y} \\
\therefore \quad a &= - b_{\text{crit}} \frac{q + q}{y} \\
\text{Differentiate w.r.t. time} \\
\dot{a} &= \frac{b_{\text{crit}} q}{y^2} \\
\text{where } y \text{ is the cross head speed.} \\
\text{Substituting for } y^2, \text{ this becomes} \\
\dot{a} &= \frac{y}{lb_{\text{crit}}} (\ell - a)^2 \quad \text{(36)}
\end{align*}
\]

61
The crack tip velocity depends on the square of the uncracked length of the sample. The crack-tip velocity decreases as the crack length increases. From equation 36 it can be seen that the crack velocity tends to zero as the crack length approaches the length of the sample. Therefore the crack will only go right across the sample when the tabs are nearly vertical. This fact has been confirmed experimentally. From equation 36 a value of $b_{\text{crit}}$ can be calculated as the other terms are known.

4.6.2 Analysis of applied load

Using the previously mentioned assumptions a method of determining $T_{\text{crit}}$ has been developed. For a linear stress distribution in front of the crack-tip and a crack width, $w$, then,

\[ F = T_{\text{crit}} \left( \frac{Q - a - x}{Q - a} \right) \]

The moment of $Fdx$ about $H$ is

\[ \text{moment} = wF (Q - a - x) \, dx \]

Therefore the total moment along $HB'$ becomes

\[ \text{Total moment} = \int_0^{Q-a} wF(Q - a - x) \, dx \]

\[ \left( Q - a \right) \]

[Diagram not shown]
Substituting for $F$ gives

\[
\text{Total moment} = \int_0^\infty \frac{w_{\text{crit}}}{(L - a)} \left( \frac{L - a - x)^2}{L - a} \right) dx
\]

Hence,

\[
\text{Total moment} = \frac{1}{3} w_{\text{crit}} (L - a)^2
\]

Assuming that the moments on the boundaries balance, then

\[
L^2 = \frac{1}{3} w_{\text{crit}} (L - a)^2
\]

or

\[
L = \frac{1}{3} \frac{w_{\text{crit}}}{L} (L - a)^2 \quad \text{(37)}
\]

The applied load also depends on the square of the uncracked length of the sample. The load also decreases as the crack length increases.

A value of $T_{\text{crit}}$ can be obtained from equation 37 as the other terms are known.

4.7 Discussion of Results

The assumptions of the previous section can be tested by plotting the crack-tip velocity and applied load against the uncracked length of the sample on a log scale and determining the slope. In both cases the slope should have a value of 2.0.

4.7.1 Crack-tip velocity and critical strain

The log-log plots of crack-tip velocity against $(L - a)$ for several propellants are shown in Figure (43). It can be seen that the data is approximately linear and has a slope close to the predicted value of 2.0. All the samples tested were analysed with a 'least square fit' method using the computer program shown in Appendix D. The average values of the slope exponents are shown for the five propellants tested in Table 3. The grand average is 1.9 with a
standard deviation of 0.3. This is close to the predicted value taking into account the considerable scatter of the results.

**TABLE 3**

**EXPONENT VALUES OF FRACTURE TEST**

<table>
<thead>
<tr>
<th>PROPELLANT</th>
<th>TYPE</th>
<th>CROSS HEAD (mm/min)</th>
<th>CROSS HEAD (mm/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a vs (L - a)</td>
<td>L vs (L - a)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>5.0</td>
<td>50</td>
</tr>
<tr>
<td>I</td>
<td>2.25</td>
<td>1.79</td>
<td>1.72</td>
</tr>
<tr>
<td>II</td>
<td>1.80</td>
<td>1.50</td>
<td>1.54</td>
</tr>
<tr>
<td>III</td>
<td>1.89</td>
<td>2.05</td>
<td>1.82</td>
</tr>
<tr>
<td>IV</td>
<td>2.34</td>
<td>1.86</td>
<td>2.13</td>
</tr>
<tr>
<td>V</td>
<td>1.65</td>
<td>1.90</td>
<td>1.87</td>
</tr>
<tr>
<td>AVERAGE</td>
<td>1.99</td>
<td>1.82</td>
<td>1.82</td>
</tr>
<tr>
<td>GRAND</td>
<td>1.9</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>AVERAGE</td>
<td>1.9</td>
<td>0.3</td>
<td></td>
</tr>
</tbody>
</table>

Equation 36 was then used to calculate a value for the term $b_{crit}$

$$\dot{a} = \frac{y}{L_{crit}} \frac{(L - a)^2}{(L - a)^2} \quad (36)$$

The plot of crack-tip velocity against $(L - a)^2$ is shown in Figure (44).

The slope of this plot is equal to:

$$\text{Slope} = \frac{y}{L_{crit}}$$

hence, as the cross head speed $y$ and the sample length, $L$, is known the value of $b_{crit}$ can be calculated. The value of $b_{crit}$ was computed,
for all the samples tested, from a "least square fit" to the plot of equation 36.

Selected series of photographs from a few tests were also analysed and the actual $b_{\text{crit}}$ value measured as the distance $BD$ in Figure (41). Good agreement was found between the average values obtained from the two methods as shown in Figure (44).

The average values obtained are shown in Table 4 for the five propellants tested.

<table>
<thead>
<tr>
<th>PROPELLANT TYPE</th>
<th>CROSS HEAD SPEED (mm/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td>I</td>
<td>4.9 ± 0.3</td>
</tr>
<tr>
<td>II</td>
<td>7.0 ± 0.3</td>
</tr>
<tr>
<td>III</td>
<td>8.1 ± 0.5</td>
</tr>
<tr>
<td>IV</td>
<td>5.7 ± 0.3</td>
</tr>
<tr>
<td>V</td>
<td>7.9 ± 0.3</td>
</tr>
</tbody>
</table>

The error shown above is the standard deviation.

The mean values of the average values are considered to be independent of cross head speed and hence crack-tip velocity. These mean values are listed below:

<table>
<thead>
<tr>
<th>Propellant</th>
<th>$b_{\text{crit}}$ (mm)</th>
<th>standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>6.5</td>
<td>1.5</td>
</tr>
<tr>
<td>II</td>
<td>8.6</td>
<td>1.7</td>
</tr>
<tr>
<td>III</td>
<td>8.3</td>
<td>1.5</td>
</tr>
<tr>
<td>IV</td>
<td>5.7</td>
<td>1.0</td>
</tr>
<tr>
<td>V</td>
<td>7.7</td>
<td>1.3</td>
</tr>
</tbody>
</table>
The standard deviations are high not because of the experimental error but due to sample to sample variation, which is considerable for a material like composite propellant. The experimental error for one sample gave a typical standard deviation of about one fifth of the value quoted above.

The value of $b_{\text{crit}}$ is not the strain at the crack-tip but is a measure of the maximum local deformation in the propellant at the crack-tip just before the propellant fails and the crack extends.

4.7.2 The applied load and critical stress

The log-log plots of applied load against $(l - a)$ for several propellants are shown in Figure (45). The data is approximately linear and has a slope close to the predicted value of 2.0. The value of the slope for each sample tested was computed. These values are also given in Table 3. The grand average is 1.9 with a standard deviation of 0.2. This is close to the predicted value taking into account the considerable scatter of the results.

The values of $T_{\text{crit}}$ were calculated from plots of equation 37.

$$L = \frac{1}{3\epsilon} w T_{\text{crit}} (l - a)^2 \quad (37)$$

The plot of applied load against $(l - a)^2$ is shown in Figure (46). The plot does not go through the origin, the zero shift being the weight of the sample. Values of $T_{\text{crit}}$ were calculated from the shape of similar plots for each of the samples tested. The average computed values of $T_{\text{crit}}$ are shown in Table 5.

The magnitudes of $T_{\text{crit}}$ are very similar to the uniaxial tensile maximum stress as can be seen from the case given in Appendix C.
The error shown above is the standard deviation. The values of $T_{\text{crit}}$ depend on strain rate in the same way as does the uniaxial tensile maximum stress. It cannot therefore be considered as a constant parameter of the material.

### 4.7.3 Calculation of the intrinsic failure energy

It can be seen from Figure (41) that the lines $AH$ and $CH$, do not correspond with the crack edge. The computed stress analysis predicted that the edge of the crack would hinge about the point $H$. However the propellant samples did not behave exactly in this way. It has been assumed from this difference that the volume of propellant between these lines and the crack surface contained the majority of the failure damage. The width of this failure zone, called $b_0$, was measured from the films. The measured values for each propellant did not depend significantly on the testing speed. The average values are shown in Table 6.

The measurement of these values was difficult not only because of the subjective drawing of the lines $AH$ and $CH$, but because the crack edge was irregular. The accuracy of the measurements was estimated to
be about 0.2mm. It can be seen from Table 6 that the values for the propellants are similar with an average value of about 5mm.

The value of the intrinsic failure energy, taken as the work of fracture at zero rate of crack growth, can be estimated by considering the work done on an element in this damage zone. An element of length $b_0$ is stretched to $b_{\text{crit}}$ before it breaks. Therefore the work done on the element is

$$\text{Work done} = \frac{1}{2} T_{\text{crit}} (b_{\text{crit}} - b_0)$$

If all this energy is converted into new surface area then

$$2\gamma = \frac{1}{2} T_{\text{crit}} (b_{\text{crit}} - b_0)$$

or

$$\gamma = \frac{1}{4} T_{\text{crit}} (b_{\text{crit}} - b_0) \quad \text{(38)}$$

This equation is very similar to the COD$_c$ concept discussed in section 1.6.1. In this section an equation for the surface energy was given as

$$2\gamma = G = \delta \sigma_y$$ \quad \text{(20)}

where $\delta$ is the crack opening displacement and $\sigma_y$ is the uniaxial yield stress. The actual physical significance of $\delta$ is not fully understood and no method exists to measure it directly in a material like composite propellant. However $\sigma_y$ relates to the maximum uniaxial stress which the material can sustain before failure and if this is taken to be equal to the maximum tensile stress then it is easily measured. Therefore a value of $\delta$ can be estimated if the surface energy, $\gamma$, is known. For propellant I, $\gamma$ has been
calculated to be 242 J/m² (see below) and the maximum tensile stress, measured independently and shown in Appendix C, is 0.50 MPa. The value of $\delta$ obtained from equation 20 is about 1mm (actual value 0.97mm). From the measured crack profiles, shown in Figure (42), a COD of 1mm would occur at a distance of less than 0.5mm from the crack-tip. The calculated value of $\delta$ is also close to the measured value of $(b_{\text{crit}} - b_0)$ for Propellant I, as given in Table 6. The $(b_{\text{crit}} - b_0)$ distance represents a deformation in the propellant at the crack-tip and not an opening of the crack surfaces.

The values of the intrinsic failure energy have been calculated for each propellant tested using equation 38. The $T_{\text{crit}}$ value corresponding to the slowest crack-tip speed was used together with the tabulated values of $(b_{\text{crit}} - b_0)$. The resulting values are shown in Table 6.

**TABLE 6**

**CALCULATED VALUES OF INTRINSIC FAILURE ENERGY**

<table>
<thead>
<tr>
<th>PROPellant Type</th>
<th>$b_0$ (mm)</th>
<th>$b_{\text{crit}} - b_0$ (mm)</th>
<th>CALCULATED $\gamma_{\text{INT}}$ (J/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>4.4 ± 0.9</td>
<td>2.1</td>
<td>242</td>
</tr>
<tr>
<td>II</td>
<td>5.4 ± 1.1</td>
<td>3.2</td>
<td>264</td>
</tr>
<tr>
<td>III</td>
<td>6.2 ± 1.1</td>
<td>2.1</td>
<td>258</td>
</tr>
<tr>
<td>IV</td>
<td>4.0 ± 1.0</td>
<td>1.7</td>
<td>293</td>
</tr>
<tr>
<td>V</td>
<td>5.4 ± 0.9</td>
<td>2.3</td>
<td>323</td>
</tr>
</tbody>
</table>

The error shown above is the standard deviation.
The value of the intrinsic failure energy for Propellant I, 242 J/m², is close to the value of 232 J/m² calculated from uniaxial data in section 2.7.2. The agreement is excellent. Propellants II and III have approximately the same value of intrinsic failure energy as can be expected from their compositions. (see Appendix A). Also the values relate to the propellants' potential performance as structural materials. From standard mechanical properties and subjective judgement Propellant I is the least acceptable material and Propellant V is the best propellant that the present "state-of-the-art" can produce. Optimisation of the intrinsic failure energy could therefore be used as an aid in a propellant chemistry development programme to make "better" composite propellants and hence more reliable rocket motors.

4.8 Work of fracture variation with crack-tip velocity

The intrinsic failure energy relates to zero, or very slow crack-tip growth when no work is done by dispersive forces. However these conditions do not apply in practice, when a crack propagates the magnitude of the work of fracture depends on the amount of work done at the crack-tip by the failure processes. This work depends on the crack-tip velocity.

Plots of the measured work of fracture against crack-tip velocity are shown in Figures (47) and (48) for the carboxyl and hydroxyl propellants tested. The log-log plots are linear and so the work of fracture depends on the crack-tip velocity raised to a fractional power. At the very slow crack-tip velocity end of the scale, the line approaches that of the value of the intrinsic failure energy calculated in the previous section.

The power-law type relationship applies to at least three decades of crack-tip velocity ie from 0.01 to 10mm per second, and the work of fracture depends on about the 1/5th power of the crack-tip velocity. The actual values for the measured slopes are:
Propellant I  0.23  
Propellants II and III  0.11  
Propellant IV  0.19  
Propellant V  0.17  
Average value  0.18  

It can be seen that there is no significant difference between the carboxyl and hydroxyl propellants. The magnitude of the exponent is considered to depend on the failure process which dissipates the energy and this would be similar for all the composite propellants tested. At higher velocities the curve would increase at a higher rate than that of the extrapolated relationship due to kinetic energy considerations and possible temperature rises at the crack-tip. 121

The calculated values of the intrinsic failure energy and the relationship for the work of fracture can be combined with existing failure analysis techniques 122 to give more reliable predictions of rocket motor failures.
FIG. 28 THE FRACTURE TEST SPECIMEN

FOR TYPICAL SAMPLE

\[ l = 144 \text{ mm} \]
\[ b = 26 \text{ mm} \]
\[ w = 8 \text{ mm} \]
FIG. 29 COMPLIANCE OF FRACTURE SPECIMEN

SLOPE OF FORCE-DEFLECTION CHART

CLASSICAL BEAM THEORY - SLOPE 3.0

CRACK LENGTH (L mm)

COMPLIANCE OF FRACTURE SPECIMEN

200

100
FIG. 31 SURFACE DISPLACEMENT MAP FOR $\frac{1}{2}$ CRACK
FIG. 32  Y-AXIS STRESS DISTRIBUTION ALONG CENTRE LINE
ONSET OF DEWETTING

SAMPLED FAILED AT 0.22 STRAIN

FIG. 33 TENSILE MODULUS CHANGE WITH STRAIN
FIG. 34 COMPUTER CRACK SHAPES
FIG. 35  EFFECT OF POISSON’S RATIO ON CRACK SHAPE

$E_{eff} = 1.57$ MPa

$\mu = 0.30$

$\mu = 0.40$

$\mu = 0.49$
ZERO

HALF CRACK

THREE-QUARTER CRACK

FIG 36 TYPICAL SEQUENCE OF FRACTURE TESTS
**Fig. 37 Typical Fracture Test Record Chart**

- **Crack Length Markers**
- **1/2 Crack**
- **Area Approximates Work Done Extending Crack**
- **3/4 Crack**

**Propellant I**
Cross Head 0.5 mm/Min
FIG. 38 EFFECT OF CRACK WIDTH

PROPELLANT I
CROSS HEAD 5 mm/MIN

WORK OF FRACTURE $\gamma$ (J/m$^2$)

- CRACK WIDTH 3 mm
- CRACK WIDTH 7 mm

CRACK LENGTH (mm)
PROPELLANT V
CROSS HEAD 5 mm/min

FIG 4.1 TRACING OF SAMPLE SURFACE
FIG. 42  COMPARISON OF CRACK-TIP SHAPES

- COMPUTER CRACK $\mu = 0.3$
- PROPELLANT IV 0.47 CRACK
- PROPELLANT V 0.53 CRACK
- PROPELLANT I 0.49 CRACK
- PROPELLANT III 0.48 CRACK
FIG. 43 CRACK-TIP VELOCITY AGAINST CRACK LENGTH
FIG. 45 LOAD AGAINST CRACK LENGTH
PROPELLANT IV
CROSS HEAD 50 mm/MIN

SLOPE $1.65 \times 10^{-3}$
$T_{\text{CRIT}} = 1.01 \text{ MPa}$
COMPUTER VALUE
$T_{\text{CRIT}} = 0.95 \text{ MPa}$

FIG. 46 SAMPLE LOAD RELATIONSHIP
FIG. 47  WORK OF FRACTURE  C.T.P.B.  PROPELLANT

- PROPELLANT II
- PROPELLANT III

INTRINSIC VALUE 262 J/m²
SLOPE 0.11

INTRINSIC VALUE 242 J/m²
SLOPE 0.23

CRACK-TIP VELOCITY \( \dot{\alpha} \) (mm/SEC)
FIG. 48 WORK OF FRACTURE H.T.P.B. PROPELLANT
The most important results which have been presented in this thesis are summarised as follows:

The work of fracture required to propagate a crack in composite propellant depends on the crack-tip velocity. For the carboxyl and hydroxyl propellants tested the work of fracture depended on about the $1/5^{th}$ power of the crack-tip velocity. This relationship applied to crack-tip velocities from 0.01 to 10 mm per second with typical work of fracture values of 300 to 500 J/m$^2$ and 1300 to 1800 J/m$^2$ respectively depending on the particular propellant tested.

The existence of an intrinsic failure energy was established for composite propellants and a value deduced for a particular carboxyl propellant from both uniaxial tests and direct measurements. Its magnitude was about 240 J/m$^2$, which was significantly smaller than the corresponding work of fracture measured at the lowest crack-tip velocity. The method which used data from uniaxial tests required an estimation of the number of possible failure sites per unit length of sample. A value was established from surface strain distribution measurements and taken as one per 2 mm. length of sample. These surface strain distribution measurements were also used to derive a relationship for the dependence of Poisson's ratio on strain of the form:

$$\mu = 0.5 \exp (-3.0e_{\text{eff}})$$

where $e_{\text{eff}}$ was the effective local strain. The amount of propellant dilation was also calculated and compared with reported direct measurements using a gas dilatometer.

The crack-tip shapes were also measured and the fracture results analysed to show that the Well's crack opening displacement hypothesis applies to composite propellant. A critical strain, $b_{\text{crit}}$, related to the local deformation at the crack-tip, and a critical stress, $T_{\text{crit}}$, which was similar to the maximum tensile stress were calculated from the fracture test results.
A cantilever beam sample modified with reinforcing wooden tabs was used for the fracture tests. An elastic analysis of the sample geometry was computed to determine the surface strain and stress distributions. An approximation of these distributions was then used in the analysis of the fracture results.

The failure surfaces have been investigated using a scanning electron microscope. The surfaces were found to be very irregular on a microscopic scale and covered with numerous crystal fragments. No differences were detected between the surfaces produced by slow or fast cracks or between the surfaces of carboxyl (C.T.P.B.) and hydroxyl (H.T.P.B.) propellants. The considerable amount of surface damage was evidence of the large amount of irreversible work done during the fracture process.

The following suggestions are made for possible further work on the mechanical failure of composite propellant:

1. The effect of temperature on the properties of the propellant measured by the uniaxial and fracture tests should be investigated. The data analysis techniques described in this thesis could then be extended to include temperature variations. It is suggested that temperature effects would not have a dramatic affect on the analysis and they could be included by using the reduced variables method as described in section 1.4. A temperature reduced crack-tip velocity would have the effect of extending the derived relationship so that it applied to a larger range of crack-tip velocities.

2. Higher loading speeds resulting in larger crack-tip velocities should also be investigated. The present crack length recording system would require modification to follow the increased crack speed as 10 mm per second is the upper limit of the present manual technique. A conducting surface grid could be used as a crack length recording method. The effect of the kinetic energy of the crack surfaces and possible temperature rises at the crack-tip could then be determined.
3. A series of specially produced propellants with different amounts of solid loadings, cross-linking density and various additives could also be investigated and the work of fracture measured. The variation of the intrinsic failure energy and the work of fracture with these factors could then help to identify the controlling aspects of the failure process. Composite propellant could then be produced with the highest possible optimum failure strength.
ACKNOWLEDGEMENTS

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Mr. K. Bills and Dr. R. Farris, Solid Rocket Motor division, Aerojet General, Sacramento, Calif.

and the many others, I met on my recent visits to the U.S.A.

Finally I wish to acknowledge the help of my wife in the preparation of this work and her patience and encouragement over the past six years.
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# COMPOSITION OF PROPELLANTS

## C.T.P.B. PROPELLANTS:

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<tr>
<td>coarse</td>
<td>31.6</td>
</tr>
<tr>
<td>Trona regular</td>
<td>47.4</td>
</tr>
<tr>
<td>Aluminium powder</td>
<td></td>
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<tr>
<td>So 4000 ± 200 cm$^{-1}$</td>
<td>5.0</td>
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<tr>
<td>Carboxyl binder</td>
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Propellant II and III:

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<td>23.7</td>
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<tr>
<td>Trona regular</td>
<td>47.4</td>
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<td>Trona So 2000 ± 100 cm$^{-1}$</td>
<td>7.9</td>
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<td>Aluminium powder</td>
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</tr>
<tr>
<td>So 3300 ± 100 cm$^{-1}$</td>
<td>5.0</td>
</tr>
<tr>
<td>Carboxyl binder</td>
<td>16.0</td>
</tr>
</tbody>
</table>

These propellants thus had a 84.0% content of solid loading by weight.

Propellants II and III used different lots of binder prepolymer.

## H.T.P.B. PROPELLANTS

<table>
<thead>
<tr>
<th>Propellant IV</th>
<th>Wt %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium perchlorate</td>
<td></td>
</tr>
<tr>
<td>coarse</td>
<td>21.6</td>
</tr>
<tr>
<td>Trona regular</td>
<td>43.2</td>
</tr>
<tr>
<td>Trona So 2000 ± 100 cm$^{-1}$</td>
<td>7.2</td>
</tr>
<tr>
<td>Aluminium powder</td>
<td></td>
</tr>
<tr>
<td>So 3500 ± 100 cm$^{-1}$</td>
<td>15.0</td>
</tr>
<tr>
<td>Hydroxyl binder</td>
<td>13.0</td>
</tr>
</tbody>
</table>
Propellant V

<table>
<thead>
<tr>
<th>Ingredient</th>
<th>Wt %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ammonium perchlorate</td>
<td></td>
</tr>
<tr>
<td>Trona regular</td>
<td>49.0</td>
</tr>
<tr>
<td>Trona So 2000 ± 100 cm⁻¹</td>
<td>13.0</td>
</tr>
<tr>
<td>Trona So 8000 ± 200 cm⁻¹</td>
<td>10.0</td>
</tr>
<tr>
<td>Aluminium powder</td>
<td></td>
</tr>
<tr>
<td>So 3500 ± 100 cm⁻¹</td>
<td>15.0</td>
</tr>
<tr>
<td>Hydroxyl binder</td>
<td>13.0</td>
</tr>
</tbody>
</table>

These propellants thus had a 87% content of solid loading by weight.
Program to calculate the strain distribution and volume change Mk 9

Jim Buswell Dec 1971

begin real e, u, D, A, Volch, eff, Svolch, eo, w, sumr1, sumr2, suml, sumw;
integer q, p, r, norows, nostrip, s, i, j, k, l;
integer array name [1: 100], comment [1: 500], bits[1: 50];
lineprinter;
sameline;
begin read q, norows, nostrip, A, eo;
r := norows + 1;
p := nostrip + 1;
s := 1;
read inst ring (name, s);
begin comment CBS ; arrays stored in backing store;
array: grid[1: q+1, 1: r+1, 1: p+1, 1: 2], refl[1: r+1, 1: p+1], refw[1: r+1, 1: p+1],
length[1: r+1, 1: p+1], stin[1: r+1, 1: p+1], width[1: r+1, 1: p+1],
stin[1: r+1, 1: p+1], ratio[1: r+1, 1: p+1], totlen[1: r+1, 1: p+1],
totwid[1: r+1, 1: p+1];
switch ss := out, stop;
boolean procedure key(n);
value n;
integer n;
begin elliott(7, 0, 0, 0, 3, 3, n); key := n ≠ 0
end;

bits[3] := 1;
for i := 1 step 1 until q do
begin if i > 2 then begin 1 := bits [i];
read inst ring (comment, 1);
bits[1 + 1] := 1;
end;
for j := 1 step 1 until r do
for k := 1 step 1 until 3 do
read grid [i, 1, j, k];
end;

comment A The origin of grid is the bottom left corner of the lowest strip across the sample,
The cells are numbered with row first then strip number..............
eg Bottom left cell is 1,1.
Next bottom cell is 2,1.

Dimension of the cells are calculated from the difference of the squares hence allows for any distortion of the grid when strained.
NB All readings are taken straight up edges of rows left one first.
The scale of length (y axis) is set at half true scale.
To end program after first part is grid dimensions KEY (3) ;
begin top of form ;
print $ell_2s_30? Strain distribution and volume change Mk 9 ? ;
print $ell_2s_30? D( e+1 )(1-eu)(1-ou) -1 ? ,
$ell_2s_30? Volch= e(1 + \exp -Ao) (1 + e)(0.25e^\exp -Ao) A, aligned(1,3),
$ell_55? w = lateral strain ?, \ell_55? u = Poissons Ratio ?,
$ell_55? Volch = Volume change ? ,
$ell_55? Effective Strain = e - eo ?,
$ell_55? Evolch = Effective volume change ?,
$ell_55? ;
s := 1 ;
print outstring (name, s ) ;
print $ell_3s_40? Undeformed grid dimensions in mm ? , $ell_333? ;
sumr := sumrw := 0 ;
for l:= 1 step 1 until norows do
for j:= 1 step 1 until nostrips do
begin refl[l, j] := 0.5*((( grid[l, l, j+1, 3] - grid[l, l, j, 2])\times 2
+ (grid[l, l+1, j, 1] - grid[l, l, j, 1])\times 2)\times 0.5
+(( grid[l, l+1, j+1, 2] - grid[l, l+1, j, 2])\times 2
+ ( grid[l, l+1, j+1, 1] - grid[l, l+1, j, 1])\times 2)\times 0.5) ;
sumr := sumr + refl[l, j] ;
refw[l, j] := 0.5*((( grid[l, l+1, j, 2] - grid[l, l, j, 2])\times 2
+ (grid[l, l+1, j, 1] - grid[l, l, j, 1])\times 2)\times 0.5
+(( grid[l, l+1, j+1, 2] - grid[l, l+1, j, 2])\times 2
+ ( grid[l, l+1, j+1, 1] - grid[l, l+1, j, 1])\times 2)\times 0.5) ;
sumrw := sumrw + refw[l, j] ;
print $ell_1s_30? initial length cell ?, digits(3), l,f,?,digits(3), j ,
\ell = ?, aligned (3,4) , refl[l, j]*0.02 ,
$ell_55? initial width cell ?,digits(2), l,r,?,digits(2) , j ,
\ell = ?, aligned (3,4) ,refw[l, j]*0.01 ,\ell_55? ;
end ;

print $ell_1s_30? total length of undeformed sample = ?,
aligned(3,4) , 0.02*sumr/norows ,
$ell_1s_30? average width of undeformed sample = ?,
aligned (4,4) , ( 0.01*sumrw ) / nostrip , \ell_55? ;
end ;

for i:= 2 step 1 until q do
begin top of form ;
s := 1 ;
print $ell_112?? , outstring ( name, s ) ;
print $ell_13?? Deformed grid of photo number ?, digits(3), i ,
\ell = ?, Dimensions in mm ?, \ell_13?? ;
for l:= 1 step 1 until norows do
for j:= 1 step 1 until nostrip do
begin length[l, j] := 0.5*((( grid[l, l, j+1, 3] - grid[l, l, j, 2])\times 2
+ (grid[l, l+1, j+1, 1] - grid[l, l, j, 1])\times 2)\times 0.5
+(( grid[l, l+1, j+1, 2] - grid[l, l+1, j, 2])\times 2
+ ( grid[l, l+1, j+1, 1] - grid[l, l+1, j, 1])\times 2)\times 0.5) ;
width [l, j] := 0.5*((( grid[l, l+1, j, 2] - grid[l, l, j, 2])\times 2
+ (grid[l, l+1, j, 1] - grid[l, l, j, 1])\times 2)\times 0.5
+(( grid[l, l+1, j+1, 2] - grid[l, l+1, j, 2])\times 2
+ ( grid[l, l+1, j+1, 1] - grid[l, l+1, j, 1])\times 2)\times 0.5) ;
print $ell_1s_10? Deformed length cell ?, digits(3), l,e,?,digits(2), j ,
\ell = ?, aligned (3,4), length[l, j]*0.02 ,
$ell_1s_10? Deformed width cell ?,digits(2), l,e,?,digits(2), j ,
\ell = ?, aligned(3,4), width[l, j]*0.01,\ell_17? ;
end ;
for $l:=1$ step $1$ until $r$ do
begin
totlen[$l$, $j$] := grid[$i$, $1$, $p$, $2$] - grid[$i$, $1$, $1$, $2$];
print $\text{££l320? total length of each grid row = ?}$,
aligned(3,4), totlen[$l$, $j$]$=0.02$, $\text{££l320?}$;
end;

for $j:=1$ step $1$ until $p$ do
begin
totwid[$l$, $j$] := grid[$i$, $r$, $j$, $1$] - grid[$i$, $1$, $j$, $1$];
print $\text{££l320? total width of each grid strip = ?}$,
aligned(3,4), totwid[$l$, $j$]$=0.01$, $\text{££l320?}$;
end;

if key(2) then goto stop;

for $i:=2$ step $1$ until $q$ do
begin
top of form;
$z:=1$;
print $\text{££l320?}$;
print outstring (name, s);
print $\text{££l320? Photo number ?, digits(2), i}$,
$\text{££s10? Minimum strain 0.005 ?, ££l320?}$;
$1:=\text{bits[i]}$;
print $\text{££s10?}$;
print outstring (comment, 1);
print $\text{££l320?}$;
sumv := sumw := 0;
for $l:=1$ step $1$ until norows do
for $j:=1$ step $1$ until nostrip do
begin
length[$l$, $j$] := $0.5*(((\text{grid}[i,1,j+1,2] - \text{grid}[i,1,j,2])1^2$
$+ (\text{grid}[i,1,j+1,1] - \text{grid}[i,1,j,1]1^2)5.5$
$+ (\text{grid}[i,1+j,1,2] - \text{grid}[i,1+j,1,1]1^2)5.5$
$+ (\text{grid}[i,1+j,1,1] - \text{grid}[i,1+j,1,1])3^2)5.5$);
sumv := sumv + length[$l$, $j$];
width[$l$, $j$] := $0.5*(((\text{grid}[i,1+1,j,2] - \text{grid}[i,1,j,2])1^2$
$+ (\text{grid}[i,1+1,j,1] - \text{grid}[i,1+j,1]1^2)5.5$
$+ (\text{grid}[i,1+j,1] - \text{grid}[i,1+j,1,1])3^2)5.5$);
sumw := sumw + width[$l$, $j$];
if (length[$l$, $j$] / refil[$l$, $j$] $\leq 1.005$ then
begin
print $\text{££l320? cell ?, digits(2), L, 2, ?, digits(2), j}$,
$\text{££s10? Small strain value for e ?, ££l320?}$;
goto out;
end;

stnil[$l$, $j$] := (length[$l$, $j$] / refil[$l$, $j$]) - 1;
* := stnil[$l$, $j$];
if (width[$l$, $j$] / refw[$l$, $j$] $\geq 0.995$ then
begin
print $\text{££l320? cell ?, digits(2), L, 2, ?, digits(2), j}$,
$\text{££s10? Small strain value for w ?, ££l320?}$;
goto out;
end;
TEXT BOUND INTO

THE SPINE
\[ \text{stnw}[l, j] := \left( \frac{\text{width}[l, j]}{\text{refw}[l, j]} - 1 \right); \]
\[ \text{ratio}[l, j] := -\frac{\text{stnw}[l, j]}{\text{stln}[l, j]} ; \]
\[ \text{Evolch} := \text{eff} := 0; \]
\[ w := \text{stnw}[l, j]; \]
\[ u := \text{ratio}[l, j]; \]
\[ D := \frac{((e+1)(1-e^u)(1-e^u)) - 1}{}; \]
\[ \text{Volch} := (e^((1+\exp(-A*e))\times((1+e)\times(e^\exp(-A*e)\times0.25 - 1)))); \]
\[ \text{if } e > 0 \text{ then } \text{eff} := e - \infty; \]
\[ \text{Evolch} := (\text{eff}^\times(1+\exp(-A\cdot\text{eff}))\times((1+\text{eff})\times(\text{eff}^\exp(-A\cdot\text{eff})\times0.25 - 1))); \]
\[ \text{print } \text{Cell }, \text{digits}(1), 1, \text{digits}(2), j, \text{digits}(2) = \text{?}, \text{aligned}(1, 4), \text{eff} = \text{?}, \text{aligned}(1, 4), \text{of} \]
\[ \text{eff} = \text{?}, w = \text{?}, w, \text{volch} = \text{?}, u = \text{?}, \text{eff} = \text{?}, \text{aligned}(6), D = \text{?}, \text{Volch} = \text{?}, \text{scaled}(6), \text{Volch}, \text{eff} = \text{?}, \text{Evolch} = \text{?}, \text{scaled}(6); \]
\[ \text{out } \text{end}; \]

\[ \text{print } \text{eff} = \text{?}, \text{total length of sample } = \text{?}, \text{aligned}(4, 4), \text{of} \]
\[ 0.02 \times \text{suml}/\text{norows}, \text{eff} = \text{?}, \text{average longitudinal strain } = \text{?}, \text{of} \]
\[ \text{aligned}(1, 4), (\text{suml} / \text{suml}) - 1, \text{of} \]
\[ \text{eff} = \text{?}, \text{average width of sample } = \text{?}, \text{of} \]
\[ \text{aligned}(4, 4), (0.01 \times \text{sumw}) / \text{nostrip}, \text{of} \]
\[ \text{eff} = \text{?}, \text{average lateral strain } = \text{?}, \text{aligned}(1, 4), \text{of} \]
\[ (\text{sumw} / \text{sumw}) - 1, \text{volch} = \text{?}, \text{average poisons ratio } = \text{?}, \text{of} \]
\[ \text{aligned}(1, 4), -((\text{sumw} / \text{sumw}) - 1)/((\text{suml} / \text{suml}) - 1); \]
\[ \text{stop } \text{end}; \]

\textbf{Comment :} Order for data tape is
\[ q - \text{number of photographs analysed} \]
\[ \text{norows} - \text{number of rows up sample (maximum no. 99), used to calculate p} \]
\[ \text{nostrip} - \text{number of strips across sample (maximum no 99), used to calculate p} \]
\[ A - \text{value of exp. exponent} \]
\[ e0 - \text{value of zero strain term} \]
\[ \text{instring(name,s)} - \text{only type identification at beginning of tape} \]
\[ \text{instring(comment,1)} - \text{after first photo print the time etc at the} \]
\[ \text{beginning of each data string} \]
\[ \text{coordinates } - x \text{ value and y value as given in comment A; } \]
\[ \text{end } \]
## APPENDIX C

**PROPELLANT I - STRAIN RATE 0.25 MIN⁻¹ - TEMP 20°C**

<table>
<thead>
<tr>
<th>MAXIMUM STRESS (MPa)</th>
<th>MAXIMUM STRAIN</th>
<th>SMALL-STRAIN TENSILE MODULUS (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.536</td>
<td>0.242</td>
<td>4.17</td>
</tr>
<tr>
<td>0.571</td>
<td>0.272</td>
<td>3.82</td>
</tr>
<tr>
<td>0.529</td>
<td>0.253</td>
<td>4.46</td>
</tr>
<tr>
<td>0.570</td>
<td>0.268</td>
<td>4.56</td>
</tr>
<tr>
<td>0.550</td>
<td>0.243</td>
<td>4.25</td>
</tr>
<tr>
<td>0.452</td>
<td>0.238</td>
<td>3.82</td>
</tr>
<tr>
<td>0.503</td>
<td>0.254</td>
<td>4.21</td>
</tr>
<tr>
<td>0.537</td>
<td>0.260</td>
<td>4.11</td>
</tr>
<tr>
<td>0.547</td>
<td>0.267</td>
<td>4.38</td>
</tr>
<tr>
<td>0.439</td>
<td>0.297</td>
<td>3.82</td>
</tr>
<tr>
<td>0.438</td>
<td>0.294</td>
<td>3.82</td>
</tr>
<tr>
<td>0.478</td>
<td>0.300</td>
<td>3.01</td>
</tr>
<tr>
<td>0.433</td>
<td>0.262</td>
<td>3.50</td>
</tr>
<tr>
<td>0.494</td>
<td>0.319</td>
<td>2.60</td>
</tr>
<tr>
<td>0.455</td>
<td>0.290</td>
<td>4.03</td>
</tr>
<tr>
<td><strong>0.50</strong></td>
<td><strong>0.27</strong></td>
<td><strong>3.9</strong></td>
</tr>
</tbody>
</table>

**STANDARD TENSILE PROPERTIES**

- **STANDARD DEVIATION 0.06**
- **STANDARD DEVIATION 0.03**
- **STANDARD DEVIATION 0.4**
APPENDIX D

Program to calculate Work of Fracture

K & LSF Model
Jim Buswell

OCTOBER 1972 UNCLASSIFIED

begin
real C1,C2,+.XIS,1.CS,devi.max;
integer FSL,r,j,s,k,nosamp,psn,ULPOLY;
integer array name[1:500];
  linewriter;
  sameline;
  top of form;
  ULPOLY:= ? ;

comment Least squares fit for Crack velocity against 1- c
Need to add LENGTH to standard DATA TAPE
Least squares fit for Load against 1- c ;

read nosamp ;
for k := 1 step 1 until nosamp do
begin
  top of form ;
  s:= 1 ;
  read string(name,s) ;
  read n,FSL,XIS,CS,t,t ;

  integer array data [ 1:n+1:1:3 ] ;
  library least squares fit ;
  C1:= ( FSL*XIS*9.8062 ) / ( CS*5000000 ) ;
  C2:= ?*-0.001 ;
  comment C1 units are Nm ( joules ),
C2 units are meters
  crack length given from data tape ;
  s:= 1 ;
  print C1234? Piaxia3 Fracture Results ? ,
  C1421? Test number ? ; outstring ( name,s ) ,
  C1441? Number of .points = ? , digits(3), n ,
  C1212? Full scale load (FSL) = ? .digits(3) , FSL , $ Kr ? ,
  C1212? Cross head speed (XIS) = ? .aligned(3.2),XIS , $ mm/min ? ,
  C1212? Chart speed (CS) = ? .aligned(3.3), CS , $ mm/min ? ,
  C1212? Crack width (t) = ? .aligned(2.2), t , $ mm ? ,
  C1212? Sample length (l) = ? ,aligned(3.3), l , $ mm13?? ;
for i:= 1 step 1 until n do
for j:= 1 step 1 until 3 do
begin
  read data [ i,j ] ;
  print digits(4) , data [ i,j ] ;
end ;

print C15?? ;
for i:= 1 step 1 until n-1 do
begin
  A[i] := abs( 0.5 *(( data[i+1,1]*data[i,2] )-( data[i,1]*data[i+1,2] ))) ;
  W[i] := A[i]*C1 ;
  G[i] := W[i]*1000/(C2*( data[i+1,3]- data[i,3] )) ;
  crkvel[i] := 5*(data[i+1,3]-data[i,3])*CS/((data[i+1,2]-data[i,2])*600 ) ;
  x[i] := (l-t data[i,3]+ data[i+1,3])*0.05 ;
  logx[i] := ln(x[i]) ;
  logy[i] := ln(crkvel[i]) ;
  L[i] := ( data[ i, 1 ] + data[i+1,1 ] )*0.0005*FSL ;
  logL[i] := ln(L[i]) ;
crack length given from data tape average of two readings

\[ \text{crkvel} \ldots \text{is crack front velocity and units are mm per second} \]

\[ \begin{align*}
\text{print } & \text{££s30? Total Crack length } \text{?, ££s33? = ?}, \text{aligned } (3.1), \\
\text{(data[1,3]+data[i+1,3])*0.05} & \text{, £ mm } ?, \text{££f17?}, \\
\text{££s3u? Area on chart } \text{?, ££s38? = ?}, \text{aligned}(3.3), \text{ A[i]* U,U1,} \\
& \text{£ small squares } ? \text{, ££s5??}, \\
\text{£ Segment } ? \text{, digits(2) , i } \text{, ££s11??,} \\
\text{£ Work done extending crack }?, \text{aligned}(3.1), \\
\text{(data[i+1,3] - data[i,3])*0.1,} \\
\text{£ mm } ?, \text{££s14d = ?}, \text{aligned } (3.3), \text{ W[i] , £ Joules ? , ££12s29?} \\
\text{£ Calculated Work of fracture (griffiths gamma ) }? \text{,££s37? = ?}, \\
\text{aligned } (4.2), \text{ G[i] , £ Joules / sq metro } ? \text{, ££ 12s30 ??,} \\
\text{£ incremental crack front velocity } \text{££s13? = ?}, \\
\text{aligned } (3.3), \text{ crkvel[i], £ mm per second ££s3u??,} \\
\text{£ sample length remaining } \text{£ s29? = ?}, \text{aligned}(3.2), \\
x[i] , \text{ £ mm£13??};
\end{align*} \]

\[ \begin{align*}
\text{end ;}
\end{align*} \]

\[ \begin{align*}
\text{least squares fit (n=1,0.1,logx,logy,poly,max,poen,resids,devi );} \\
\text{top of form ;}
\end{align*} \]

\[ \begin{align*}
\text{s := 1 ;} \\
\text{print ££12s2u??, outstring( name, s ),} \\
\text{££15s2u? Results crack velocity against sample length ? ,} \\
\text{££13s2u? Slope exponent (2.00) = ?,aligned(2,2),} \\
\text{poly[1], ££12s2u? bcrit} \\
\text{= ?, aligned (2.2),} \\
\text{XIS/60*1*exp(poly[0])}, \text{£ mm } ?, \\
\text{££12s2u? maximum distance from line = ?}, \\
\text{aligned(2.2), max, ££s10? at position ££s5??, digits(2), poen,} \\
\text{££12s2u? standard deviation of fit = ?}, \\
\text{aligned (2.2), devi ;}
\end{align*} \]

\[ \begin{align*}
\text{least squares fit (n=1,0.1,logx,logy,poly,max,poen,resids,devi );} \\
\text{print ££15s20? Results load against sample length ? ,} \\
\text{££13s20? slope exponent (2.00) = ?,aligned(2,2),poly[1],} \\
\text{££12s20? Critical tension at crack tip = ?,aligned(3.3),} \\
\text{exp(poly[0]))*1*3.0 , £ kg per mm } ?, \\
\text{££12s20? maximum distance from line = ?}, \text{aligned(2,2), max,} \\
\text{££s10? at position ££s5??, digits(2), poen,} \\
\text{££12s20? standard deviation of fit = ?}, \text{aligned(2,2),devi ;}
\end{align*} \]

\[ \begin{align*}
\text{print ££15s50? finished £ 15?? ;}
\end{align*} \]