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Joint Rate and Power Adaptation for MC-CDMA over Tempo-Spectral Domain

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***Abstract*— This paper studies the area of link adaptation for multicarrier code division multiple access (MC-CDMA) radio interface and adapts the rate and transmission power jointly over temporal as well as spectral domains. On one side, frequency domain spreading lends a high degree of frequency diversity to MC-CDMA. On the other, it has been identified as the main hurdle when it comes to spectral domain rate allocation in MC-CDMA. This paper presents band based MC-CDMA (BB-MC-CDMA) architecture as an efficient scheme to practically realize the spectral domain rate allocation in MC-CDMA, without undergoing the drawbacks of other published techniques for this purpose. BB-MC-CDMA asks for the elimination of frequency interleaving, usually considered to be an essential ingredient of MC-CDMA. It also suggests an intelligent reduction in the spreading factor in a channel aware fashion. Furthermore, based on BB-MC-CDMA architecture, joint rate and power adaptation over time and frequency (JRPATF) has been shown to outperform the frequency domain power allocation only.**

Keywords; Multi-carrier CDMA, MC-CDMA; Transmit Power Allocation; Dynamic Rate Adaptation; Adaptive Modulation; Link Adaptation; Bandwidth Allocation. Adaptive Frequency Hoping

I. INTRODUCTION

Recently, MC-CDMA (Multi-Carrier Code Division Multiple Access) has attracted significant research interest as potential air-interface for 4G cellular systems [1]. MC-CDMA is a diversity based transmission scheme, which simultaneously harnesses the advantages of OFDM (Orthogonal Frequency Division Multiplexing) and CDMA (Code Division Multiple Access). Hence, it enables the materialization of a completely ISI (Inter Symbol Interference) free system with added features of frequency diversity and robustness against narrow band interference.

The diversity based transmission schemes generally, require no CSI at the transmitting end. However, they are required to be designed with a fixed link margin, which results in inefficient utilization of the channel and eventually becomes a bottle neck to realize a high performance physical layer, as envisioned for 4G systems. Whereas, if CSI is known at transmitting end, link adaptation has been shown [2,3] to improve the spectral efficiency or alternatively minimize the system transmit power by efficiently utilizing the variability of fading channels. The link adaptation schemes adaptively changes the transmission parameters (such as transmission power, symbol rate, constellation size, coding rate and/or coding scheme and any combination of them) in response to time-varying link conditions.

The area of link adaptation has been exhaustively studied for multicarrier systems like OFDMA [3,4]. Comprehensive link adaptation solutions have been found for these systems where rate and/or power can be adapted over temporal and/or spectral domains. Although, some valuable contributions have also been made toward MC-CDMA, they are either limited to power adaptation over spectral domain [5-7] or rate adaptation over time [8]. However, MC-CDMA still lacks a comprehensive link adaptation policy where both rate and power can be adapted jointly over time as well as frequency, which has been found to be exceptionally useful for OFDM systems [3]. This is mainly because the spectral domain rate adaptation in MC-CDMA is a tricky business compared with other multicarrier systems.

In MC-CDMA every information symbol is spread over a set of sub-carriers and frequency interleaving is employed to absorb the available frequency diversity to its full extent. This prohibits the rate adaptation on per sub-carrier basis, as carried out in OFDMA. To solve this problem, an Equivalent Carrier Concept (ECC) has been floated by [9,10] with slightly different definitions of ECC.

In [9], *equivalent carrier* consists of a band of contiguous sub-carriers bearing the chips from same information symbol, whereas in [10], frequency interleaving is used and *equivalent carrier* is made up of non-contiguous sub-carriers. Then authors have proposed OFDM based bit loading and power allocation algorithm on *equivalent carrier*. However, in [9], a loss in adaptation gain is observed with increase in channel delay spread, whereas, [10] asks for an essential use of Orthogonality Restoring Combining (ORC) which is notorious for its noise enhancing characteristics. *Therefore, the problem of joint rate and power adaptation over time and frequency in MC-CDMA is still an open issue to be discussed further.*

A Band Based MC-CDMA (BB-MC-CDMA) scheme has been recently proposed in [6] as an efficient technique for frequency domain power allocation (FDPA) purpose. In this paper, we identify and exploit the inherent immunity of BB-MC-CDMA architecture against the problems faced by [9,10] and extends it to Joint Rate and Power Allocation over Time and Frequency (JRPATF) by relaxing some of the constraints assumed in [6]. JRPATF exhibits a further gain of around 2 dB over FDPA. Rest of the paper is organized as follows; section II describes the BB-MC-CDMA briefly. Section III introduces the system model, formulates the problem mathematically and finally solves it. Simulations are conducted in Section IV whereas conclusions are drawn in section 0. NOTE: bold face **CAPITAL** and **lowercase** represent matrices and vectors respectively, whereas scalars are represented by regular faced letters.

II. BB-MC-CDMA

BB-MC-CDMA (Figure 1) has been indirectly proposed as an efficient method of frequency domain power allocation in MC-CDMA. It mitigates the severe loss in multiuser orthogonality experienced by Water-filling like Carrier-Based spectral

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domain Power Allocation (CBPA) schemes [5,7,11]. In order to preserve the multiuser orthogonality, BB-MC-CDMA advocates the following:

1. Elimination of frequency interleaving, as it is a diversity enhancement feature and not desirable as long as CSI is known at transmitter. Therefore, BB-MC-CDMA spreads the signal over a band of *N* contiguous sub-carriers.
2. Intelligent adjustments to the spreading factor according to the delay spread of radio channel, such that $\Delta f \times N \ll B_c$, where Δf and B_c represent the inter-carrier spacing and channel coherence bandwidth respectively.

Therefore, BB-MC-CDMA divides the available bandwidth $(\Delta f \times N_u)$ into *K* bands, each consisting of *N* contiguous sub-carriers; such that $\Delta f \times N \ll B_c$, N_u being the total number of used carriers. In this way, highly correlated fading is experienced over all carriers in a band. Hence each of these bands can be shared by up to *N* users in MC-CDMA fashion without undergoing any multiuser orthogonality loss usually caused by frequency selective fading in conventional MC-CDMA systems.

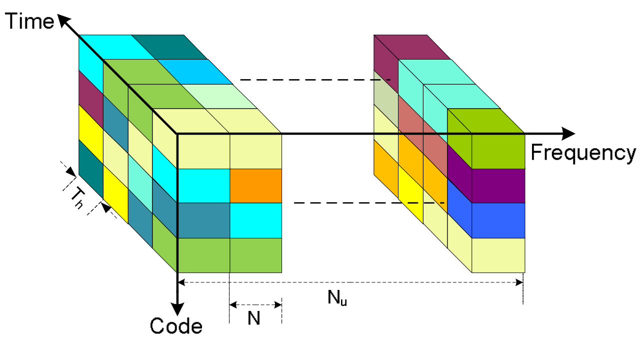

Figure 1 BB-MC-CDMA with $K = N_u/N$, slot size = T_h

III. Joint Rate and Power Allocation over Time-Frequency

Unlike ECC [9], the adaptive (in channel aware fashion) adjustments to the spreading factor of BB-MC-CDMA will overcome the loss in adaptation gain which ECC is suffering from. In addition to this, with the elimination of MAI, ORC is no longer an essential requirement of BB-MC-CDMA, as imposed by ECC [10]. Even MRC (which otherwise is an optimal combining scheme for single user MC-CDMA [12] only) can be safely employed without any degradation in multi-user orthogonality.

Thus, BB-MC-CDMA architecture is inherently immune to the problems faced by [9,10] for spectral domain rate adaptation in MC-CDMA, as explained above. Therefore, it can be adopted as a way forward to practically realize JRPATF in MC-CDMA by relaxing the constraints assumed in [6], as described in the following sections.

A. System Model

MC-CDMA downlink is considered with simultaneously *J* active users (Figure 2). Perfect and error free CSI from all users is assumed to be known at base station, either through an error free feedback channel (FDD mode) or through reciprocity of channel (TDD mode). Since BB-MC-CDMA is an MAI free system, therefore, the channel gain over each band can serve as CSI for a known noise density N_o. Moreover, due to a highly correlated fading over each band, the channel gain over a band can be obtained by linearly averaging the channel gain over subcarriers in the band as given by (1)

$$CSI = H_{jk}^2 = \left[\frac{1}{N}\sum_{m=0}^{N-1}\left|H_{j,k,m}(i)\right|\right]^2 \tag{1}$$

Where $H_{j,k,m}$ is complex channel gain for j^{th} user on m^{th} sub-carrier in k^{th} band.

On the basis of CSI, the link adaptation algorithm at the base station jointly computes the sub-carrier allocation, the bit-loading and power allocation matrices represented by $\boldsymbol{S}_{[JxK]}$, $\boldsymbol{Q}_{[JxK]}$ and $\boldsymbol{\Lambda}_{[JxK]}$ respectively. Based on $\boldsymbol{Q}$ and $\boldsymbol{S}$ the adaptive modulator loads the appropriate number of information bits on selected bands for respective users. The power amplifier then allocates the necessary power ($\boldsymbol{\Lambda}$) to all users on the corresponding bands such that promised QoS (BER in our case) can be realised.

B. Problem Formulation

Let the matrices $\boldsymbol{\Lambda(i)} = \{\lambda_{jk}(i)|\lambda_{jk}(i) \in [0,\theta], \forall\, j,k\}$ and $\boldsymbol{S(i)} = \{s_{jk}(i)|s_{jk}(i) \in [0\,,1], \forall\, j,k\}$ such that

$$s_{jk}(i) = \begin{cases} 1, & \lambda_{jk}(i) \neq 0 \\ 0, & oterwsie \end{cases} \tag{2}$$

represents the band power and band selection indices respectively over i^{th} OFDM frame whereas the suffixes *j, k* represent k^{th} band of j^{th} user. The modulation index over each of these bands is represented by $\boldsymbol{Q(i)} = \{q_{jk}(i)|q_{jk}(i) \in [0,2,4,\ldots\log_2 M],\ \forall\, j,k\}$ with θ and M being the maximum power and modulation order allowed on a single band respectively.

Unlike [6], the multiuser data rates and transmission power are allowed to vary both in temporal as well as spectral domains in this paper. Therefore, minimization of total transmit power for a fixed BER requirement becomes a natural choice for cost function which is dual to maximization of system throughput for a given constant power budget.

Mathematically, if the vector $\{R_1(\mathrm{i}), \mathrm{R}_2(\mathrm{i}) \ldots . R_J(\mathrm{i})\}$ represents the individual users data rate requirements such that $R(i) = \sum_{j=1}^{J} R_j(\mathrm{i})$ is the total system throughput at any instant of time *"i"*, then the cost function can be written as

$$\lambda(i) = min\sum\nolimits_{j=0}^{J-1}\sum\nolimits_{k=0}^{K-1}\lambda_{jk}\,(i) \tag{3}$$

subject to the following constraints

1. Individual rate requirement

$$R_j(i) = \sum\nolimits_{k=0}^{\mathrm{K}-1} q_{jk}(\mathrm{i}),\quad \forall \mathrm{j}$$
$$where\ q_{jk} \in \{0,2,4,\ldots\,\mathrm{q} = \log_2 M\} \tag{4}$$

2. Multiple access constraint: No more than N users can share the same band, as N is the spreading factor

$$s_k(i) = \sum\nolimits_{j=0}^{J-1} s_{jk}(i) \leq N \quad \forall i,k \tag{5}$$

3. Fixed BER requirement: Let the function $f(P_e,\, N_o, q_{jk})$ give the signal power on a band with unit channel gain, single sided noise spectral density (N_o) and bit-loading co-efficient q_{jk} required for a BER (P_e). Then for a band with

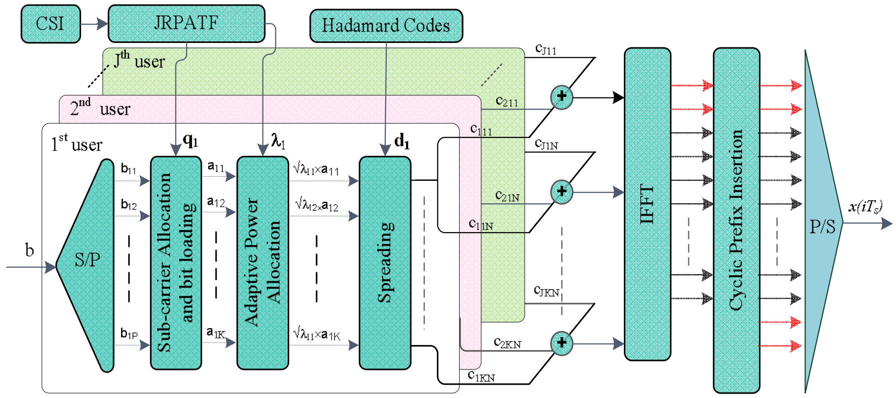

Figure 2 Block Diagram for JRPATF in MC-CDMA Downlink

non unity channel gain H_{jk}^2 and fixed values of P_e and N_o ,the required transmitted power can be written as:

$$\lambda_{jk} = \frac{f(q_{jk})}{H_{jk}^2} \tag{6}$$

$$\Rightarrow \lambda = \min_{q_{jk}\in\{0,2,4,\ldots q\}} \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} \frac{f(q_{jk})}{H_{jk}^2} \tag{7}$$

However, because of the discrete nature of variables involved like q_{jk} and s_{jk} , the optimization problem is discrete and combinatorial in nature. Therefore, to make it more tractable following relaxations are assumed

1. q_{jk} is relaxed to assume any real value from closed interval [0, q] i.e. a continuous constellation size.
2. Similarly, s_{jk} can take any real value from closed interval [0, 1]*.

Though, these assumptions are not very practical, they provide a convenient abstraction where the problem could readily be treated as continuous optimization problem which would be analytically more tractable. Furthermore, with the assumption of above relaxations, optimization will be performed over larger set hence, the solution obtained will give the lower bound to λ. After applying these relaxations, our problem transforms to:

$$\lambda_{lb} = \min_{\substack{q_{jk}\in[0,q] \\ s_{jk}\in[0,1]}} \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} \frac{s_{jk} \times f(q_{jk})}{H_{jk}^2} \tag{8}$$

subject to (5) and

$$R_j = \sum_{k=0}^{K-1} s_{jk} q_{jk}, \quad \forall j \tag{9}$$

C. Solution

Similar problem has been solved by many for OFDMA/OFDM [4,13] where sub-carriers are exclusively assigned to only one user, which is not the case here, as any number of users can share a band as long condition (5) is satisfied. Our problem can be converted to a Convex Optimization [14] problem through a simple change of variables $r_{jk} = s_{jk} q_{jk}$, which will render a handy framework of Lagrange Multipliers to find the solution. After performing the change of variables, our problem becomes:

$$\lambda_{lb} = \min_{\substack{r_{jk}\in[0,qs] \\ s_{jk}\in[0,1]}} \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} \frac{s_{jk}}{H_{jk}^2} f\left(\frac{r_{jk}}{s_{jk}}\right) \tag{10}$$

Subject to following constraints.

$$R_j = \sum_{k=0}^{K-1} r_{jk}, \quad \forall j \tag{11}$$

$$s_k = \sum_{j=0}^{J-1} s_{jk} \le N \quad \forall k \tag{12}$$

where the Lagrangian can be written as follows:

$$\begin{aligned} L = {} & \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} \frac{s_{jk}}{H_{jk}^2} f\left(\frac{r_{jk}}{s_{jk}}\right) \\ & + \sum_{j=0}^{J-1} \alpha_j \left(R_j - \sum_{k=0}^{K-1} r_{jk}\right) \\ & + \sum_{k=0}^{K-1} \beta_k \left(N - \sum_{j=0}^{J-1} s_{jk}\right) \end{aligned} \tag{13}$$

α and β are the Lagrange Multipliers. This problem has been solved by [4] for OFDMA i.e. for N=1, and the optimum solution (r_{jk}^*, s_{jk}^*) obtained is given below:

$$r_{jk}^* = s_{jk}^* f'^{-1}(\alpha_j^* H_{jk}^2) \tag{14}$$

$$\text{and} \qquad \alpha_j^* = \begin{cases} \dfrac{f'(0)}{H_{jk}^2}, & f'^{-1}(\alpha_j H_{jk}^2) < 0; \\ \alpha_j, & 0 \le f'^{-1}(\alpha_j H_{jk}^2) \le q; \\ \dfrac{f'(q)}{H_{jk}^2}, & f'^{-1}(\alpha_j H_{jk}^2) > q; \end{cases} \tag{15}$$

$$\text{and} \qquad s_{jk}^* = \begin{cases} 0, & \beta_k < \eta_{jk}(\alpha_j^*) \\ 1, & \beta_k > \eta_{jk}(\alpha_j^*) \end{cases} \text{where} \tag{16}$$

$$\eta_{jk}(\alpha) = \frac{1}{H_{jk}^2}\left[f\{f'^{-1}(\alpha_j H_{jk}^2)\} - \alpha_j H_{jk}^2 f'^{-1}(\alpha_j H_{jk}^2)\right] \tag{17}$$

However, to fit the solution within scope of our problem and practical limitations, we introduce some changes and withdraw the relaxations (assumed above) as follows: To avoid the hybrid multiple access, we propose to allocate k^{th} band to only N or fewer users with least values of η_{jk}. Therefore, (16) changes to the following:

* *Fractional value 'ξ' of s_{jk} would necessarily mean a fractional occupancy of the k^{th} band by j^{th} user. i.e for $\xi \times T_h$ time interval. This means we are relaxing our multiple access scheme from strictly CDMA to a hybrid of CDMA and TDMA. Where any number of users can be accommodated over each band (some in TDMA fashion and others in CDMA fashion) such that $s_k(i) = \sum_{j=0}^{J-1} s_{jk}(i) \le N \quad \forall i, k$.*

$$\mathbf{\Omega}_{[\mathrm{N}\times\mathrm{K}]} = \arg \underbrace{least}_{N\ out\ of\ J} \ \eta_{jk}(\alpha_j^*) \quad \forall\, j, \tag{18}$$

$$s_{jk}^* = \begin{cases} 1, & \mathrm{j} \in \mathbf{\Omega} \\ 0, & j \notin \mathbf{\Omega} \end{cases} \quad \forall\ \mathrm{k} \tag{19}$$

Consequently, for any fixed vector α_j, the constraint (11) might not fulfill. To meet any such situation we have developed an iterative search method (Figure 3) where value of α_j is increased iteratively for each user until constraint (11) is satisfied. This algorithm converges, since η_{jk} is a decreasing function of α_j . Furthermore, to avoid the continuous constellation size, we use the iterative search method for band allocation purposes only; the bit loading is performed using single user Greedy bit-loading algorithm (Figure 4) over the allocated bands.

IV. SIMULATIONS AND RESULTS

As mentioned earlier, we have simulated the downlink only. However, the logic behind BB-MC-CDMA concept remains equally valid for uplink as well. Simulation parameters are adopted from MATRICE† and shown in TABLE I. It is worth mentioning that the spreading factor remains fixed at N=32 in case of no-adaptive MC-CDMA as specified by MATRICE, however, it changes to N=4 in accordance with the coherence bandwidth of radio channel used (BranE‡) as proposed by BB-MC-CDMA concept. Only six prominent paths from BranE power delay profile are simulated for convenience.

TABLE I. SIMULATION PARAMETERS

No	Parameter	Symbol	Value
1.	RF carrier frequency	F_c	5 GHz
2.	Badnwidth	Bo	50MHz
3.	Useful symbol duration	T_s	17.75 µs
4.	FFT size	N_{FFT}	1024
5.	Guard interval	T_g	3.75 µs
6.	Baseband modulation	q	QAM
7.	Parallel data streams	R_j	46 bps
8.	Spreading factor	N	4 and 32
9.	CSI updation period	T_h	20 Symbols
10.	Increment Step size	Δ	.5
11.	Initial Threshold	α_{th}	1
12.	Spreading codes	Hadamard Walsh	
13.	Combining technique	MRC	
14.	BranE Power Profile	[0, 138.9, 312.5, 711.8, 1284.7, 1753.5] ns	
15.	BranE Delay Profile	[-4.90, -0.28, 0.10, -5.42, -13.45, -20.92] dB	

Furthermore, adaptive Quadratur Amplitude Modulation (QAM) is used with $q_{jk} \in \{0,1,2,3,4\}$. The closed form expression for the energy required to transmit q bits to j^{th} user over k^{th} band $f(P_e, N_o, q_{jk})$ for QAM can be shown to take the following form after an easy manipulations of algebraic equations as explained in [4].

$$f(P_e, N_o, q_{jk}) = \frac{N_0}{3}\left[erf^{-1}\left(\frac{P_e}{4}\right)\right]^2 (2^{q_{jk}} - 1) \tag{20}$$

In order to illustrate the functioning of JRPATF algorithm, a snapshot of band and rate allocation has been shown (Figure 5) at an instant *"i"* for each of the eight active users. It can be seen that none of the bands is shared by more than four users as spreading factor is assumed to be four. Furthermore, since the users 1- 4 are assigned orthogonal code and same codes are reused for user 5-8, therefore, band allocation is exclusive for user pairs (1,5), (2,6) ,(3,7) and so on.

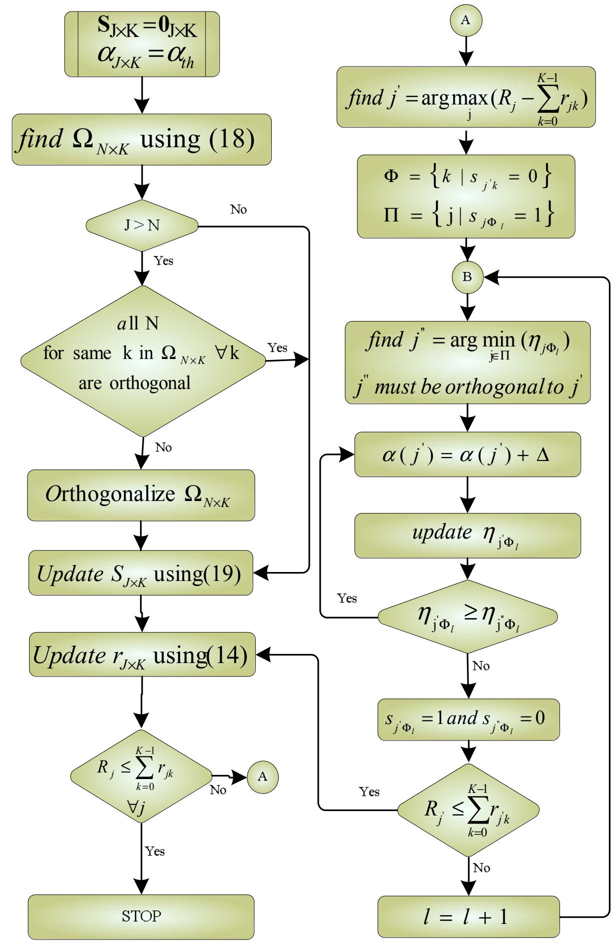

Figure 3 Flow Chart for JRPATF

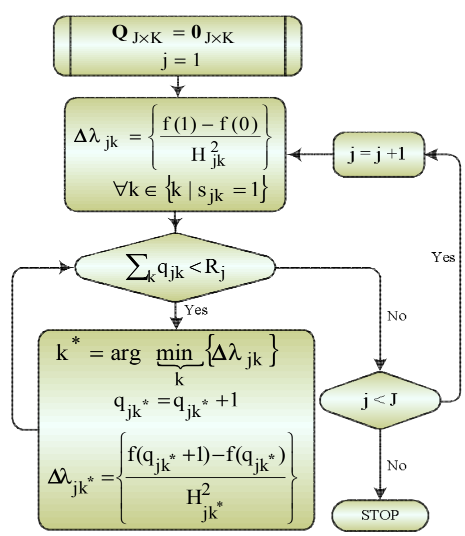

Figure 4 Single User Greedy Bit-Allocation

The BER performance of different adaptive and non-adaptive schemes is compared against system load and E_b/N_o in Figure 6 and Figure 7 respectively. In Figure 6, it can be seen that JRPATF clearly outperforms the non-adaptive as well adaptive schemes like CBPA [7]

† (MC-CDMA Transmission Techniques for Integrated Broadband Cellular Systems) a European-IST project. http://www.ist-matrice.org/

‡ ETSI, Broadband Radio Access Networks (BRAN), HIPERLAN/2

and FDPA [6] where only frequency domain power allocation has been performed. The same trend can be observed in Figure 7 where it outperforms FDPA through ~2dB in the higher SNR region. However, a rather interesting phenomenon is observed in Figure 7, where both FDPA and JRPATF are shown to outperform even the single user performance in AWGN channel.

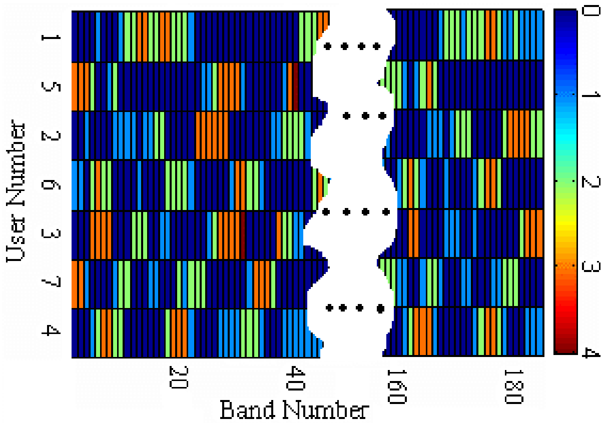

Figure 5 A Snapshot of Band Allocation and Bit-Loading. Colors represent bit-load on a band.

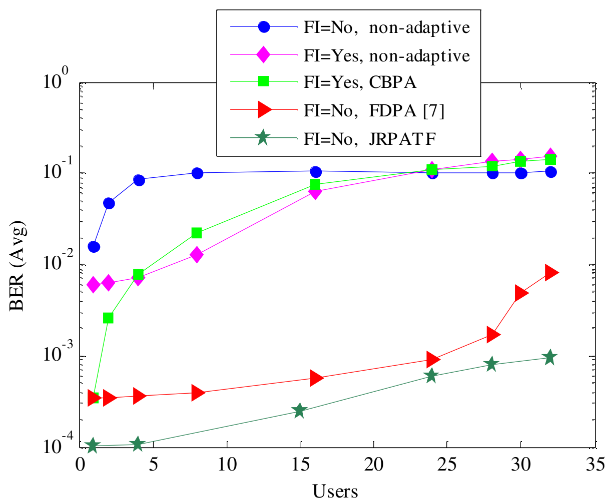

Figure 6 BER performance of adaptive and non-adaptive MC-CDMA against system load (E_b/N_o=9dB, F_d = 33.36Hz)

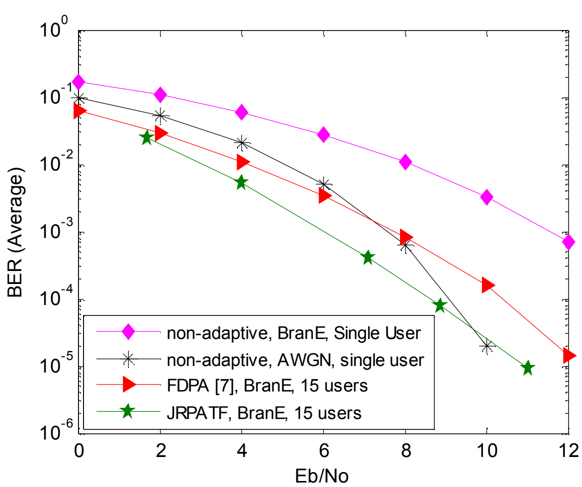

Figure 7 BER performance of adaptive and non-adaptive MC-CDMA against E_b/N_o. (F_d = 33.36Hz)

However, this is quite understandable and can be attributed to the link adaptivity enjoyed by the FDPA and JRPATF where only the bands with suitable channel gain are selected for transmission. This increases the received SNR in case of FDPA and JRPATF.

V. CONCLUSION

This paper presented BB-MC-CDMA architecture as a way forward to practically implement the Joint Rate and Power Allocation in MC-CDMA over Time-Frequency domain. It asks for channel aware dynamic adjustments in spreading factor N, such that band of sub-carriers bearing chips from same information symbol lies within coherence bandwidth (B_c) of the channel. In this way it can overcome the loss in adaptation gain with increase in channel delay spread as observed in [9], and also allows for the use of MRC eliminating the necessary use of ORC required by Equivalent Carrier Concept floated in [10]. Furthermore, JRPATF exhibits a further gain of ~2dB over FDPA where only spectral domain power allocation was performed.

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