KAM THEORY, LINDSTEDT SERIES AND THE STABILITY OF THE UPSIDE-DOWN PENDULUM

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Abstract. We consider the planar pendulum with support point oscillating in the vertical direction; the upside-down position of the pendulum corresponds to an equilibrium point for the projection of the motion on the pendulum phase space. By using the Lindstedt series method recently developed in literature starting from the pioneering work by Eliasson, we show that such an equilibrium point is stable for a full measure subset of the stability region of the linearized system inside the two-dimensional space of parameters, by proving the persistence of invariant KAM tori for the two-dimensional Hamiltonian system describing the model.

1. Introduction

1.1. The state of the art. The upside-down pendulum with the support point oscillating with a frequency $\omega$ large enough has been extensively studied in literature as a simple model exhibiting a quite nontrivial behaviour; see [5] and the references quoted therein.

The stability of the upside-down position can be proven by the averaging method (see for instance [19], Ch. 9): the result is that if the support point oscillates fast enough then the upward equilibrium position becomes stable. However such a kind of analysis is not completely rigorous both because no explicit control on the corrections can be obtained and because it can lead to incorrect results, as already pointed out in [8] and [6]. In fact the averaging method approach can be followed also for studying the stability of the downward position, and the result one finds in doing so is that such a position is always stable – provided that $\omega$ is large enough to make it possible to apply the averaging, say $\omega > \omega_1$, for some $\omega_1$ –, a result obviously unacceptable as by varying $\omega$ above $\omega_1$ one can lose even the linear stability (as it follows from Mathieu’s equation theory). A rather complete review on the averaging method can be found in [2].

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