Semiconductor microstructure in a squeezed vacuum: Electron-hole plasma luminescence

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We consider a semiconductor quantum-well placed in a waveguide microcavity and interacting with the broadband squeezed vacuum radiation, which fills one mode of the waveguide with a large average occupation. The waveguide modifies the optical density of states so that the quantum well interacts mostly with the squeezed vacuum. The vacuum is squeezed around the externally controlled central frequency \( \omega_0 \), which is tuned above the electron-hole gap \( E_g \), and induces fluctuations in the interband polarization of the quantum well. The power spectrum of scattered light exhibits a peak around \( \omega_0 \), which is moreover non-Lorentzian and is a result of both the squeezing and the particle-hole continuum. The squeezing spectrum is qualitatively different from the atomic case. We discuss the possibility of observing the above phenomena in the presence of additional nonradiative (e-e, phonon) dephasing.

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I. INTRODUCTION

The modification of the spontaneous emission of an atom placed in a cavity has been predicted and observed a long time ago.\(^1,2\) Recently, cavity effects have been observed also in quantum cavity dots and quantum wells which were placed in a microcavity made of distributed Bragg mirrors (DBRs). Similarly, effects of nonclassical radiation, such as squeezed vacuum states produced in the process of parametric down conversion, were considered in the context of interaction with atoms.\(^3\) An important theoretical prediction was made by Gardiner,\(^4\) that an atom coupled to a squeezed reservoir will exhibit two linewidths in its resonance fluorescence spectrum. To observe this effect, it is necessary that most of the electromagnetic modes which are resonant with the atomic transition are occupied by squeezed vacuum with a large average photon occupation.

In this paper, we consider the coupling of a two-band system of a semiconductor quantum well to a squeezed reservoir of photons occupying the modes of an ideal optical waveguide (without leakage). This presents a generalization of the atomic case in several ways: (1) There is a continuum of electron-hole excitations in the band; (2) each particle-hole excitation is detuned differently with respect to the squeezing energy \( \omega_0 \) (which is externally controlled); and (3) inevitable additional nonradiative relaxation and dephasing. The solid-state environment involves more types of interactions which have to be considered together, but it offers a possibility to observe these quantum optical effects, as the quality of cavities improves. Specifically, we consider the luminescence of the scattered squeezed state, in the regime where the nonradiative dephasing \( \gamma_1 \) is of the order or smaller than the radiative transition rate \( \Gamma \), and the relaxation \( \gamma_2 \) of electrons is small compared to \( \Gamma \). In this regime, where the radiation may be assumed to be a reservoir, we will argue that the effect of Coulomb interaction is mainly to give rise to dephasing. We calculate the optical spectra of the unshifted luminescence, i.e., the scattered radiation with the same frequency as the incoming radiation and compare it qualitatively to the atomic case. The power spectrum (Fig. 1) has a non-Lorentzian peak at the frequency \( \omega_0 \), a consequence of the strong energy dependence of the correlation times of the fluctuating polarization of different e-h pairs. The squeezing spectrum exhibits reduced fluctuations in one quadrature (Fig. 2), the minimum of which is proportional to \( \sqrt{\Gamma + \gamma_2 - |\Lambda|} \), where \( \Lambda \) is a measure of the squeezing correlations of the field.

II. MODEL

We will first present our model system and the results for the scattered radiation, and later discuss its possible realization. The free part of the Hamiltonian \( H = H_0 + H_I + H_c \) is given by (\( \hbar = 1 \))

\[
H_0 = \sum_{k} \omega_k \hat{b}_k^\dagger \hat{b}_k + \sum_{k} \epsilon_k \hat{c}_k^\dagger \hat{c}_k + \sum_{k} \epsilon_k \hat{t}_k^\dagger \hat{t}_k.
\] (1)

The operators \( \hat{c}_k \) and \( \hat{t}_k \) denote annihilation operators of the free electrons in the conduction and valence bands of the

![FIG. 1. (Color online) Unshifted luminescence (\( L \)) for squeezing \( \Lambda=0.9 \Gamma \), bandwidth \( B=50 \Gamma \), and phonon dephasing \( \gamma_2=0 \) (dashed curve), \( \gamma_2=3 \Gamma \) (dotted curve), and \( \gamma_2=\Gamma \) (solid curve). Inset: particle-hole line shapes for different detuning \( \delta_k=0,0.9 \Gamma \) and \( 3 \Gamma \) (from left to right). The curves are normalized to the total power.](Image)
quantum well, with in-plane momentum \( k \) (omitting spin). The operators \( b_{\lambda q} \) denote the photon annihilation operators of the waveguide mode \( \lambda \) with wave number \( q \). For simplicity we confine ourselves to the case of normal incidence, i.e., \( q \) is orthogonal to the electronic in-plane momentum \( k \). The interaction of the electrons and the photons in the dipole approximation is given by\(^7\)

\[
H_i = \sum_{\lambda k q} \frac{\epsilon(\omega_{\lambda q})}{\sqrt{\omega_{\lambda q}}} u^\dagger_{\lambda q} b_{\lambda q} (d_{\lambda q}^+ u^\dagger_k + d_{\lambda q}^- u^-_k) + \text{h.c.},
\]

where \( \epsilon(\omega_{\lambda q}) = \sqrt{\omega_{\lambda q}}/V, \) \( u^\dagger_k \) is an overlap integral of the electronic envelope and waveguide mode functions, \( d_{\lambda q} \) is dipole matrix element [we suppress its weak dependence on \( k \) (Ref. 9)] and \( V \) is the waveguide volume. Finally, the Coulomb interaction is given by\(^8\)

\[
H_c = \frac{1}{2} \sum_{\lambda k k' q q'} U_{\lambda q} (2 e^{i\pi q y} d_{\lambda q}^+ d_{\lambda' q'} + c_{\lambda q'}^+ c_{\lambda' q} + \text{h.c.})
+ \frac{1}{2} \sum_{\lambda k q} \frac{\epsilon(\omega_{\lambda q})}{\sqrt{\omega_{\lambda q}}} u_k^\dagger u_{\lambda q} d_{\lambda q}^+ d_{\lambda q}^-, \quad \text{(3)}
\]

where \( U_{\lambda q} \) is the bare Coulomb interaction.

We assume that the radiation acts as a reservoir with correlations of a two mode broadband squeezed vacuum\(^4\)

\[
\langle b_{\lambda q} b_{\lambda' q'}^\dagger \rangle = \mathcal{N}(\omega_{\lambda q}) \delta_{\lambda \lambda'} \delta(\omega_{\lambda q} - \omega_{\lambda q'}),
\]

\[
\langle b_{\lambda q} b_{\lambda' q'} \rangle = \mathcal{M}(\omega_{\lambda q}) \delta_{\lambda \lambda'} \delta(2\omega_0 - \omega_{\lambda q} - \omega_{\lambda q'}),
\]

\[
\langle b_{\lambda q} \rangle = 0,
\]

where \( \mathcal{N}(\omega_{\lambda q}) \) is the average photon occupation of the mode \( \lambda \) and wave number \( q \), and \( \mathcal{M}(\omega_{\lambda q}) \) is the squeezing parameter.\(^4\) We assume that \( \mathcal{N}, \mathcal{M} \gg 1 \) within the bandwidth \( \omega_0 \pm B/2 \) of one squeezed mode (denoted by \( \lambda = s \)) and they vanish for other empty modes (denoted by \( \lambda = e \)).

The energy of the central frequency \( \omega_0 \) is tuned higher than the electron-hole gap \( E_g \), and the bandwidth of the squeezed radiation \( B \) is assumed to be much larger than the radiative bandwidth \( \Gamma \), but much smaller than the conduction bandwidth. The radiation induces interband transitions between the valence and conduction bands. In principle, when the photon occupation is large, it is possible to find the system in a regime where the dephasing rates due to nonradiative and radiative scattering are comparable, and nonradiative relaxation is small. In this regime the system, initially an unexcited full valence band, reaches a stationary state in which there is a large occupation of the conduction band in the energy stripe where the photon occupation is large.

In order to understand qualitatively the effect of the squeezed reservoir on the electron-hole pair, it is instructive to begin with the equation of motion for \( \langle P_k(t) \rangle = \langle \epsilon_k^\dagger(t) \epsilon_k(t) \rangle \), the interband polarization, without taking into account explicitly the \( H_c \) part of the Hamiltonian. Using the Markov approximation\(^8\) we find in the rotating frame of frequency \( \Delta \epsilon_k = \epsilon_k^e - \epsilon_k^s \),

\[
\frac{d\langle P_k(t) \rangle}{dt} = - \left( \Gamma + \gamma_2 \right) \langle P_k(t) \rangle + e^{2i\delta_k/t} \Lambda(P_k(t)),
\]

where \( \delta_k = \omega_0 - \Delta \epsilon_k \) is the detuning frequency, \( \gamma_2 \) is the dephasing rate due to electron-electron and electron-phonon scattering. The radiative decay coefficients \( \Gamma \) and \( \Lambda \) are given for the squeezed mode \( \lambda = s \) by

\[
\Gamma(\Delta \epsilon_k) = \rho(\Delta \epsilon_k) |A_s(\Delta \epsilon_k)|^2 \left[ \Lambda(\Delta \epsilon_k) + \frac{i}{2} \right],
\]

\[
\Lambda(\Delta \epsilon_k) = \rho(\Delta \epsilon_k) |A_s(\Delta \epsilon_k)|^2 |A_s(2\omega_0 - \Delta \epsilon_k)|^2,
\]

where \( \rho \) is the optical density of states, and \( A_s(\Delta \epsilon_k) \) is the electron-photon coupling strength. It is assumed that there is only a small number of empty resonant modes, thus neglecting their influence in the dynamics of \( \langle P_k \rangle \).

The special features of Eq. (5) are (1) the coupling of \( P_k \) to \( P_{k'} \) due to the squeezing correlations [Eq. (4)] in the radiation, and Eq. (2) the absence of a coupling to the population inversion \( \Delta n_k = n_k^e - n_k^s \), due to the zero average of the field, Eq. (4). The solution of Eq. (5) exhibits two complex frequencies

\[
\Omega_{1,2}(k) = i(\Gamma + \gamma_2) + \delta_k \mp i/|\Lambda|^2 - \delta_k^2.
\]

For \( \delta_k < |\Lambda| \), the frequencies \( \Omega_{1,2}(k) \) have two different imaginary parts, which can be interpreted as overdamping of the polarization, whereas for \( \delta_k > |\Lambda| \) the \( \Omega_{1,2}(k) \) have two different real parts, reminiscent of underdamping.

The incoming squeezed vacuum radiation cannot induce average interband polarization. However, it induces fluctuations \( \langle P_k(t) + \tau) P_{k'}(t') \rangle \) and \( \langle P_{k'}(t + \tau) P_{k'}(t) \rangle \) of the polarization, which in turn give rise to scattered radiation in all the resonant waveguide modes. These correlators can be measured by power and squeezing spectra. First we calculate the power spectrum of the scattered radiation into an initially empty mode of the waveguide (\( \lambda = e \)). In the long-time limit it is given by
where $N_{e,q}$ is the occupation of the photon mode $\lambda=e$ with wave number $q$.

Let us begin by considering the Coulomb interaction. The effective mean field Hamiltonian which usually leads to coherent excitonic correlations in the $e-h$ plasma is given by

$$H_{eff} = -\sum_{k,q\neq 0} U_q(n^c_{k,q}n^a_{k,q}c^\dagger_k + n^a_{k,q}n^c_{k,q}v^\dagger_k + p^a_{k-q}v^c_k + p^c_{k}v^a_k),$$

(9)

where $n^c_{k,u} = n^c_{k,u}^a = n^a_{k,u}$ are band occupations and $p_k = \langle P_k \rangle$. For the dynamics of $\langle P_k(t+\tau)P_{k'}(t) \rangle$ with respect to $\tau$, under the condition $p_k=0$ this interaction can only induce energy level shifts. Going beyond the mean field approximation it is simplest to consider the equation of motion for the correlations $C_{kk'} = \langle P_k(t)P_{k'}(t) \rangle$.

This equation contains contributions of the form $(1-n^c_{k}-n^c_{k'})\sum_k U_q U^*_{-k} C_{kk'}$, which in principle give rise to an excitonic effect. \cite{11}

In the presence of the photon reservoir which we consider here, we assume the band occupations $n^c_{k,u}$ to be close to $1/2$ (see below), leading to a reduction of the excitonic effect through the phase filling prefactor $(1-n^c_{k}-n^c_{k'})$. As a result of the above considerations we will assume from now on that the effect of Coulomb interaction is twofold. First it gives rise to energy shifts which we assume constant over the range $B$ and include in $\epsilon^{(uc)}_k$. Second, it induces dephasing which we assume is included in phenomenological constant $\gamma_2$.

Let us turn next to the interaction of the electronic band with the squeezed reservoir. The equation of motion for $\langle P_k(t+\tau)P_{k'}(t) \rangle$ with respect to $\tau$ can be derived from the explicit equation of motion for the operator $P_k(t)$ (see the Appendix), which is a form similar to Eq. (5).

The solution for $\tau>0$ is given by

$$\langle P_k(t+\tau)P_{k'}(t) \rangle = G(\tau)\langle P_k(t)P_{k'}(t) \rangle + H(\tau)e^{i\delta}(\langle P_k(t)P_{k'}(t) \rangle),$$

(10)

where

$$G(\tau) = \frac{1}{2} \left[ e^{i\Omega_1(\tau)}(1-i\frac{\delta_k}{\Delta_k}) + e^{i\Omega_2(\tau)}(1+i\frac{\delta_k}{\Delta_k}) \right],$$

$$H(\tau) = \frac{\Lambda}{2\Delta_k} \left[ e^{i\Omega_1(\tau)} - e^{i\Omega_2(\tau)} \right],$$

(11)

where $\Delta_k = \sqrt{|\Lambda|^2 - \delta_k^2}$. For $\tau<0$ the solution is given by taking $G(-\tau)$ and $H(-\tau)$. The dependence of the last term on $\tau$ in Eq. (10) is due to the nonstationary nature of the squeezed radiation. \cite{4}

The off diagonal $k\neq k'$ elements of equal time correlations $\langle P_k(t)P_{k'}(t) \rangle$ and $\langle P_k(t)P_{k'}(t) \rangle$ which serve as initial conditions in Eq. (10) vanish in equilibrium. However here the system is coupled to two reservoirs: the nonradiative thermal bath and the squeezed reservoir. It can be shown\footnote{That there are nonzero steady state off-diagonal correlations of two kinds: normal $\langle P_k(t)P_{k'}(t) \rangle$ and anomalous $\langle P_k(t)P_{k'}(t) \rangle$. They are smaller than the diagonal correlations $\langle P_k(t)P_k(t) \rangle$ by a factor proportional to $(\gamma_1/\Gamma)$.} that as a result there are nonzero steady state off-diagonal correlations of two kinds: normal $\langle P_k(t)P_{k'}(t) \rangle$ and anomalous $\langle P_k(t)P_{k'}(t) \rangle$. These smaller than the diagonal correlations $\langle P_k(t)P_k(t) \rangle$ by a factor proportional to $(\gamma_1/\Gamma)$.\footnote{Qualitatively this can be estimated by referring to the equation $\partial P_k = d\sum_{\lambda}e^{i\omega_\lambda}n_\lambda b_\lambda \Delta n_k$. The off-diagonal correlations are driven by the squeezed reservoir since there exists a nonzero steady state difference $\Delta n_k$. This is given by the rate equation}

$$\partial \Delta n_k(t) = - (\Gamma + \gamma_1) \Delta n_k(t) + \gamma_1,$$

(12)

which leads to the above estimate.

### III. RESULTS

We shall treat the luminescence in the limit of $\gamma_1/\Gamma \rightarrow 0$ so that only the diagonal correlations $\langle P_k(t)P_{k}(t) \rangle$ contribute to Eq. (8). Moreover, since $\langle P_k(t)P_k(t) \rangle = 0$, and $\langle P_0(t)P_{k}(t) \rangle = \langle n_0^c \rangle (1-n_0^c)$, Eq. (10) shows that $\langle P_0(t)P_{k}(t) \rangle$ are stationary in this limit.

Substituting Eq. (10) with $k=k'$ and Eq. (11) in the integral (8) we obtain

$$\mathcal{L}(\omega_{qz}) = 2|A_\lambda(\omega_{qz})|^2 \Re \int dk \langle n_0^c \rangle (1-n_0^c) \times$$

$$\left[ \frac{i}{\Delta_{\epsilon_1} - \omega_{qz} + \Omega_1(k)} \left( 1 - i\frac{\delta_k}{\Delta_k} \right) \right] + \left[ \frac{i}{\Delta_{\epsilon_2} - \omega_{qz} + \Omega_2(k)} \left( 1 + i\frac{\delta_k}{\Delta_k} \right) \right],$$

(13)

where we assume $\lambda=e$ from now on. The line shape of an individual particle-hole transition [the integrand of Eq. (13)] has two distinct limits (see inset of Fig. 1): when $\delta_k \rightarrow 0$, it consists of two superimposed regular Lorentzian line shapes of different widths $\Gamma + \gamma_2 \pm |\Lambda|$. When $\delta_k \gg \Lambda$ the line shape is a single Lorentzian of width $\Gamma + \gamma_2$, which is just the radiation width acquired without squeezing. In between those limits the line shapes are asymmetric with short tails lying on the side of the central frequency $\omega_0$. The superposition of such line shapes produces a non-Lorentzian peak at $\omega_0$ in the unsqueezed luminescence spectrum $\mathcal{L}(\omega_{qz})$. Fig. 1. For simplicity $\langle n_0^c \rangle$, $\mathcal{N}$.($\Delta_{\epsilon_1}$), $A_\lambda(\omega_{qz})$, and $\mathcal{M}(\Delta_{\epsilon_2})$ were taken constant in the energy range $\omega_0 \pm B/2$ (these will be assumed also below for the squeezing spectrum).

The width of the peak is of the order of $\Gamma + \gamma_2$, and the maximum is (in the $B \gg \Gamma$ limit)

$$\mathcal{L}_{max} = \mathcal{L}_0 \frac{\Gamma + \gamma_2}{\sqrt{(\Gamma + \gamma_2)^2 - |\Lambda|^2}},$$

(14)

where $\mathcal{L}_0 = {4\pi^2|A_\lambda(\omega_0)|^2}\rho_{el}(n_0^c)(1-n_0^c)$ with $\rho_{el}$ the electronic density of states.

We now turn to the squeezing spectrum which is defined\footnote{In terms of the fluctuations of the field quadrature $X_\theta = \frac{1}{2}[E^{+}(t)e^{i\theta}+E^{-}(t)e^{-i\theta}]$} in terms of the fluctuations of the field quadrature $X_\theta = \frac{1}{2}[E^{+}(t)e^{i\theta}+E^{-}(t)e^{-i\theta}]$. 075333-3
where $\Delta \omega_{q} = \omega_{q} - \omega_{0}$ and $\theta$ determines the choice of 
quadrature. For the limit $\gamma_{1} \ll \Gamma$ we again neglect the 
off-diagonal correlations $\langle P_{k}(t)P_{k'}(t)\rangle (k \neq k')$ and 
obtain in the long-time limit

$$S_{\theta}(\omega_{q}) = \frac{1}{2} \psi^{2} \text{Re} \int d^{2} k (n_{k}^{e}) (1 - n_{k}^{e}) \times \left[ \frac{1 + |\Lambda| \cos(2 \theta + \alpha)}{\Delta k} \right]^{-1} \left| \frac{\Delta \omega_{q} + \Delta_{k} - (\Gamma + \gamma_{2})}{\Delta \omega_{q} - \Delta_{k} - (\Gamma + \gamma_{2})} \right|,$$

where $\alpha$ is the phase of $\Lambda$, Eq. (6), and $\psi$ is a geometrical 
factor. Figure 2 shows the squeezing spectra for the in-phase 
$(2 \theta + \alpha = 0)$ and out-of-phase $(2 \theta + \alpha = \pi)$ quadratures, 
normalized to the total emitted power. The minimum of the 
out-of-phase quadrature is proportional to

$$S_{\theta}(\omega_{q}) = \frac{1}{2} \psi^{2} \text{Re} \int d^{2} k (n_{k}^{e}) (1 - n_{k}^{e}) \times \left[ \frac{1 + |\Lambda| \cos(2 \theta + \alpha)}{\Delta k} \right]^{-1} \left| \frac{\Delta \omega_{q} + \Delta_{k} - (\Gamma + \gamma_{2})}{\Delta \omega_{q} - \Delta_{k} - (\Gamma + \gamma_{2})} \right|,$$

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APPENDIX: THE POLARIZATION FLUCTUATION EQUATION

Here we derive the effective equations of motion which lead to the solution (10) for the fluctuations. We employ a consistent truncation of the hierarchy of equations of motion at the level of three- and four-body correlations. We further assume the previous conditions $\Gamma \gg \gamma_i$. The equation of motion for $\langle P^i_k(t)P_k(t') \rangle$ with respect to one time argument is derived from the Hamiltonian $H = H_0 + H_I$

$$\dot{\langle P^i_k(t)P_k(t') \rangle} = i \Delta \epsilon_i \langle P^i_k(t)P_k(t') \rangle$$

$$+ i \sum_q A_q^\dagger \langle b_q^\dagger(t) \Delta n_q(t) P_k(t') \rangle.$$ (A1)

The derivative of $\langle b_q^\dagger(t) \Delta n_q(t) P_k(t') \rangle$ is given by the two terms

$$\langle b_q^\dagger(t) \Delta n_q(t) P_k(t') \rangle = i \omega_q \langle b_q^\dagger(t) \Delta n_q(t) P_k(t') \rangle$$

$$+ i \sum_k \langle P^i_k(t) \Delta n_q(t) P_k(t') \rangle,$$ (A2)

$$\langle b_q^\dagger(t) \Delta n_q(t) P_k(t') \rangle = 2i \sum_q A_q^\dagger \langle b_q^\dagger(t) b_q^\dagger(t) P^i_k(t) P_k(t') \rangle$$

$$- 2i \sum_q A_q^\dagger \langle b_q^\dagger(t) b_q^\dagger(t) P_k P_k(t') \rangle.$$ (A3)

We now assume that $\langle P^i_k(t) \Delta n_q(t) P_k(t') \rangle \approx \langle P^i_k(t) P_k(t') \rangle \times \langle \Delta n_q(t) \rangle$ is very small, since in the steady state $\langle \Delta n_q(t) \rangle \ll 1$ (note that the correlation $\langle P^i_k(t) \Delta n_q(t) P_k(t') \rangle$ cannot decouple to contributions with an average polarization $\langle P^i_k \rangle$).

Next we assume that at the second order in the system-reservoir interaction $A_q$, the correlations can be decoupled by approximating for the total density matrix $\rho_{tot} = \rho_{sys} \otimes \rho_{res}$. As a result we get for the correlations in the right-hand side of Eq. (A3)

$$\langle b_q^\dagger(t) b_q^\dagger(t) P^i_k(t) P_k(t') \rangle \approx \langle b_q^\dagger(t) b_q^\dagger(t) \rangle \langle P^i_k(t) P_k(t') \rangle,$$

$$\langle b_q^\dagger(t) b_q^\dagger(t) P_k(t) P_k(t') \rangle \approx \langle b_q^\dagger(t) b_q^\dagger(t) \rangle \langle P_k(t) P_k(t') \rangle.$$ Substituting all the contributions in Eq. (A3) back into Eq. (A1) we have for $\langle P^i_k(t) P_k(t') \rangle$

$$\dot{\langle P^i_k(t) P_k(t') \rangle} = i \Delta \epsilon_i \langle P^i_k(t) P_k(t') \rangle - 2 \sum_{qq'} A_q^\dagger A_q e^{i\omega_q t}$$

$$\times \int dt' e^{-i\omega_q t'} \langle b_q^\dagger(t') b_q^\dagger(t) \rangle \langle P^i_k(t) P_k(t') \rangle$$

$$+ 2 \sum_{qq'} A_q^\dagger A_q e^{i\omega_q t} \int dt' e^{-i\omega_q t'} \langle b_q^\dagger(t') b_q^\dagger(t) \rangle$$

$$\times \langle P_k(t') P_k(t') \rangle.$$ (A4)

Similarly, the equation for the correlation $\langle P_k(t) P_k(t') \rangle$ is given by

$$\dot{\langle P_k(t) P_k(t') \rangle} = - i \Delta \epsilon_i \langle P_k(t) P_k(t') \rangle - 2 \sum_{qq'} A_q^\dagger A_q e^{i\omega_q t}$$

$$\times \int dt' e^{i\omega_q t'} \langle b_q^\dagger(t') b_q^\dagger(t) \rangle \langle P^i_k(t) P_k(t') \rangle$$

$$+ 2 \sum_{qq'} A_q^\dagger A_q e^{i\omega_q t} \int dt' e^{i\omega_q t'} \langle b_q^\dagger(t') b_q^\dagger(t) \rangle$$

$$\times \langle P_k(t') P_k(t') \rangle.$$ (A5)

We now employ the correlations (4) and the Markov approximation to obtain

$$\dot{\langle P^i_k(t) P_k(t') \rangle} = (i \Delta \epsilon_i - \Gamma(\Delta \epsilon_i)) \langle P^i_k(t) P_k(t') \rangle$$

$$+ \Lambda(\Delta \epsilon_i) e^{2i\delta_i(t)} \langle P_k(t) P_k(t') \rangle$$

$$\dot{\langle P_k(t) P_k(t') \rangle} = (i \Delta \epsilon_i - \Gamma(\Delta \epsilon_i)) \langle P_k(t) P_k(t') \rangle$$

$$+ \Lambda(\Delta \epsilon_i) e^{-2i\delta_i(t)} \langle P^i_k(t) P_k(t') \rangle.$$ (A6)
Modification of spontaneous emission in a waveguide was considered, for example, in Ref. 1 and in a microwaveguide in the context of semiconductors in Ref. 24.


V. F. Gantmakher and Y. B. Levinson, Carrier Scattering in Metals and Semiconductors (Elsevier Science, New York, 1987), Chap. 4.


