APPENDIX

A. Calculation of region width $a$: Average hop distance

The rectangles in the EC algorithm merely represent virtual subnetworks that deliver information in sequence from one end of the network to the other. As a parameter of the analysis as well as the algorithm definition, the region width, $a$, determines the number of hops on multihop data delivery paths: The narrower the regions are, the more hops a packet needs to traverse. Hence, ideally $a$ should be wide enough to reduce the multihop energy consumption caused by multiple transmissions. However, transmissions over longer distances use a higher transmission power leading to higher consumption amounts. Hence, the high transmission power needed over longer distances tradeoffs with the large number of transmissions required for shorter distances.

For the scenario in Figure 1, we pick our objective as the minimization of end-to-end energy consumption in delivery of a single packet over a multihop path of distance $d$.

In Figure 1, each hop $i$ has a distance of $d_i$ and a deviation angle of $\theta_i$. A cascade of these transmissions establish a multihop path of distance $d$. Representing hop $i$ as a vector $\overrightarrow{d_i} = |d_i|e^{j\theta_i}$ in polar coordinates (with the $0^\circ$ angular direction set to be from source $S$ towards destination $D$), we have $\overrightarrow{d} = \sum_{i=1}^{K} |\overrightarrow{d_i}|e^{j\theta_i}$. Hence, $\sum_{i=1}^{K} |\overrightarrow{d_i}| \sin \theta_i = 0$ and $\sum_{i=1}^{K} |\overrightarrow{d_i}| \cos \theta_i = |\overrightarrow{d}|$. Therefore, each hop of this multihop progression is dependent on the others since the total distance and angular deviation should be $d$ and $0$, respectively.

This is a complicated mathematical problem related with spatial geometry and deals with additional topics such as node connectivity, coverage, and multihop propagation [1], [2].

Since our major focus is energy equalization and conservation but not a detailed analysis of multihop propagation patterns, we can simplify the analysis and treat each distance to be equal to an expected single-hop distance, $\overline{d}$. For a transmission range of $R$ and uniform node density, this average distance to a neighbor node is $\overline{d} \approx \int_{0}^{R} \frac{2x^2}{\pi R^2} dx = \frac{2R}{3}$, where $x$ is a dummy variable. Similarly, the deviation in each
step leads to an average progress from \( S \) to \( D \) equal to \( \overline{dE[\cos \theta_i]} \approx \frac{2R}{3} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos \theta_i \, d\theta_i \approx \frac{4R}{3\pi} \), where \( E[.] \) is the expectation operator. Hence, we can approximate the number of hops to be equal to \( m = \frac{3d\pi}{4R} \).

Using the energy model in Equation (1) (see the paper), the consumption amount in a multipath route of \( m = \frac{3d\pi}{4R} \) hops with respect to different transmission ranges, \( R = R_{\text{min}} \), which minimizes energy consumption, is then obtained as in Figure 2. The result in this figure suggests that, considering only the multi-hop data delivery energy amounts, energy consumption is minimized for a CH transmission range of around \( R_{\text{min}} = 87m \). The question is now to find the rectangle width \( a \) that provides an average CH-to-CH distance of \( \overline{d} = R_{\text{min}} = 87m \) between two neighbor rectangular regions. To determine this, we compute the average distance between two neighbor CHs in two neighboring regions, when node density is uniform.

![Fig. 2. Finding the CH transmission range, \( R \).](image)

![Fig. 3. Distance between two randomly selected nodes in two neighbor regions, \( R_1 \) and \( R_2 \).](image)

For a rectangle width \( a \), and network width \( W \), two CH nodes have random coordinates of \( CH_1(x_1, y_1) \), \( CH_2(x_2, y_2) \) as shown in Figure 3, with coordinates having distributions of \( p(x_1) = p(x_2) = \frac{1}{a}, p(y_1) = \)
\( p(y_2) = \frac{1}{W} \). The square of this distance, \( d^2 \) is then also a random variable, which gives:

\[
d^2 = (x_1 + x_2)^2 + (y_1 - y_2)^2,
\]

\[
E[d^2] = \frac{1}{W^2a^2} \int_0^W \int_0^W \int_0^a \int_0^a d^2 \, dx_1 \, dx_2 \, dy_1 \, dy_2 = \frac{7a^2 + W^2}{6}.
\]

(1)

If we set \( \sqrt{E[d^2]} \approx E[d] = R_{\text{min}} \), this means that the average distance of a hop is \( R_{\text{min}} = 87m \), suggesting a minimum end-to-end energy consumption in data delivery. With this, we can calculate the region width \( a \) as:

\[
a = \sqrt{\frac{6R_{\text{min}} - W^2}{7}} \approx 71m.
\]

(2)

REFERENCES
